Economic Research-Ekonomska Istraživanja

ISSN: 1331-677X (Print) 1848-9664 (Online) Journal homepage: https://www.tandfonline.com/loi/rero20

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To cite this article: Joanna Stawska, Maciej Malaczewski & Agata Szymańska (2019) Combined monetary and fiscal policy: the Nash Equilibrium for the case of non-cooperative game, Economic Research-Ekonomska Istraživanja, 32:1, 3554-3569, DOI: 10.1080/1331677X.2019.1669063

To link to this article: https://doi.org/10.1080/1331677X.2019.1669063

Published online: 04 Oct 2019.

ECONOMIC RESEARCH-EKONOMSKA ISTRAŽIVANJA
2019, VOL. 32, NO. 1, 3554–3569
https://doi.org/10.1080/1331677X.2019.1669063

Combined monetary and fiscal policy: the Nash Equilibrium for the case of non-cooperative game

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ABSTRACT

The importance of the central bank and the government conducting their policies has increased recently, with more attention being given to the effectiveness of policy mix. The non-cooperative models of the monetary and fiscal game are frequently employed to study interactions between both authorities. The models assume that the authorities take into account each other's choices when making decisions. It is also important to remember when seeking equilibrium in the non-cooperative models that in the Nash Equilibrium (which is sought in this study) the parties try to come up with the best response to the opponent's decision. The aim of the paper is to present the Nash Equilibrium in a non-cooperative game between the government and the central bank using a non-cooperative model of a fiscal-monetary game (a policy-mix MODEL). This study demonstrates that in the Nash Equilibrium in the model, the budget deficit and interest rate of an EU member state depend on the exogenous data (external to the model), such as inflation target, base inflation and the Maastricht deficit limit. This study is enhanced by an analysis of the government and central bank's sensitivity to the deep parameters of economic variables.

ARTICLE HISTORY
Received 23 July 2018
Accepted 22 July 2019

KEYWORDS
Monetary policy; fiscal policy; game theory; Nash equilibrium; non-cooperative game

JEL CLASSIFICATION
C70; C72; E52; E62

1. Introduction

The interactions between the government's fiscal policy and the central bank's monetary policy influence the economy of a country. The combination of these policies is known in the economic literature as a 'policy-mix'. When studying the interactions between the central bank and the government's policies, one must bear in mind that the authorities' goals are different in many respects and sometimes even conflict with each other, which leads to coordination problems.

The monetary and fiscal policies are designed to stabilise economic activity and ensure sustainable development in the country by protecting price stability in the case of the monetary policy and supporting full employment or high economic

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growth in the case of the fiscal policy. These different goals explain why the government and the central bank have a problem coordinating their decisions, or why such coordination does not occur at all.

There have been numerous comprehensive studies examining the implications of potential conflicts between monetary and fiscal policies, as well as the benefits of coordinating them. Many of these studies are based on the game theory approach and models assuming a specific level of cooperation between the central bank and the government, which influence each other's behaviour by their decisions.

The paper aims to find the Nash Equilibrium with respect to a non-cooperative game between both these authorities. We propose using the non-cooperative game to examine the issue of coordination of fiscal and monetary policies in the EU member state. An original policy-mix model is constructed for a non-cooperative game taking account of specific constraints resulting from the behaviour of the two authorities as well as institutional limitations imposed by the Maastricht Treaty and the Stability and Growth Pact. This paper presents the reaction functions of the central bank and the government, which are different from those mentioned in the literature. Based on these, the Nash Equilibrium was calculated when designing the original equilibrium model for a non-cooperative monetary-fiscal game in the EU countries. Moreover, contrary to a generally applied approach based on discrete strategies, continuous strategies[1] were used in this equilibrium model, which additionally contributes strongly to the economic literature. The sensitivity of the government and the central bank to changes in the so-called deep parameters[2] of economic variables is also analysed. Parameters $\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{b}_1, \tilde{b}_2$ and $\tilde{b}_3$ - which are the non-linear combination of deep parameters – measure how strongly a given variable influences the optimal decision of the government or of the central bank.

It must be noted that the analysis of sensitivity conducted in this paper is an original study based on an original equilibrium model for a non-cooperative monetary-fiscal game in the EU countries. There is an apparent need to determine the level of the Nash Equilibrium for a non-cooperative fiscal-monetary game because few studies have used the mathematical models of policy mix thus far, especially with respect to the EU countries facing institutional restrictions, such as the Maastricht criteria, and due to the advancing economic integration of the EU. According to the authors, the Nash Equilibrium in the non-cooperative game model with institutional restrictions has not yet been thoroughly analysed.

The structure of the paper is as follows. Section 2 presents the findings of a review of studies on the coordination of fiscal and monetary policies in the context of the game theory. In Section 3 our model of a non-cooperative game between the central bank and the government is introduced. The section also explains the model's assumptions and different variants of the Nash Equilibrium and provides a sensitivity analysis. The last section presents the conclusions.

2. The game theory framework for analysing policy mix coordination in the context of a non-cooperative game

Studies on policy mix have been conducted by researchers such as Clarida, Gali, and Gertler (2000), Buti (2003), Canzoneri, Cumby, and Diba (2006), Flanagan, Uyarra, and Laranja (2011), Badarau and Levieuge (2011), Saulo, Rego, and Divino (2013) and Cui (2016), all of whom found the coordination of monetary and fiscal policies to be beneficial. Special problems with coordinating policy mix are observed in the Euro area countries (Carlberg, 2012; Jacquet & Pisani-Ferry, 2007). Their fiscal policies influence national inflation, which has an effect on average inflation in the Euro area and consequently the ECB's decisions about common monetary policy. It should be noted, however, that the governments of the EU member states have limited freedom in designing their fiscal policies because of the limitations imposed by the Maastricht Treaty (Treaty on the Functioning of the European Union, 2007) and the Stability and Growth Pact (1997). There is also the question about the degree to which the ECB's common monetary policy influences each member of the Euro area and about the policy's implications for national fiscal policies. The implications are certainly not the same because of structural differences between countries and because the Euro-zone as a whole fails to meet the conditions of an optimum currency area.

According to Sargent and Wallace (1981) 'unpleasant monetarist arithmetic', in an expansionary fiscal policy environment with chronic budget deficits, the monetary authority cannot maintain control over inflation in the longer term regardless of its strategy. In other words, the central bank's monetary policy can be affected by a fiscal policy, which may lead to problems with stabilising prices (Sargent & Wallace, 1981). Stabilising an economy in the short term is problematic because the central bank and the government have different goals or preferences. To be optimally effective, both authorities should coordinate their actions and decisions (Bhattacharya & Kudoh, 2002; Buiter & Panigirtzoglou, 1999), but the central bank's pursuit of stable prices is disturbed in the long term by various factors hindering the coordination of fiscal and monetary policies (Bhattacharya, Guzman, & Smith, 1998; van Aarle, Bovenberg, & Raith, 1995).

In examining the monetary-fiscal interactions and their implications, models constructed in the game theory framework are useful (Bennett & Loayza, 2000; Libich & Stehlik, 2010). The game theory is increasingly used to study monetary and fiscal interactions. This article contributes to this area of research - with an additional focus on non-cooperative games. Considering that there may be a conflict of interest between the creators of the monetary and fiscal policies of a given country, to model these and similar potential conflicts between economic authorities, the game theory - a valuable analytical tool - should be used. Several macroeconomic policy applications can be found in the economic literature (Arora, 2012; Basar & Olsder, 1999; Neck & Behrens, 2003, 2009).

An analysis of these models shows that the coordination of fiscal and monetary policies supports the economy (Blake & Weale, 1998; Demertzis, Hughes, Hallet, & Viegi, 2002; Dixit & Lambertini, 2003) by reducing the risk of frictions, helps minimise the price stability costs and ensures greater stability of the financial system (Benassy, 2004; Chadha & Nolan, 2007; Sargent & Wallace, 1981). The models also provide insight into the mechanism of conflicts between the central bank and the government; an expansionary fiscal policy frequently leads to the tightening of the monetary policy (Bennet & Loayza, 2000), while an overly restrictive monetary policy may increase the cost of disinflation and the government's cost of fiscal policy, mitigating the impacts of disinflation (Wyplosz, 2002).

Game theory has been successfully employed by Pohjola (1986), Osborne and Rubinstein (1994), Camerer (2003), Osborne (2003), Canzoneri et al. (2006), Saulo et al. (2013) and Cui (2016) to study the interactions between monetary and fiscal policies. The assumption adopted by these and other authors is that a central bank seeks to keep inflation on target and prices stable, while the government's fiscal decisions are to ensure a high rate of economic growth or employment. In pursuing their goals, both authorities adjust their actions to their partner's choices (Dixit & Lambertini, 2000).

Each authority's preferences can be represented by an objective function that is optimised for selected constraints. In order to determine each authority's optimal behaviour, the so-called reaction function is constructed, which shows the likely response of one authority to a specific decision made by its partner. The reaction functions allow for identifying the level of equilibrium, called the Nash Equilibrium (Bennett & Loayza, 2000; Cechetti, 2000; Kishan & Opiela, 2000; Nash, 1950), where each authority's decision is its best answer to the opponent's choice (Gibbons, 1997).

The above game can be cooperative or non-cooperative. The cooperative game assumes that both authorities operate in the same macroeconomic circumstances and optimise their decisions by taking into account what their partner may consider important.

In the non-cooperative game model (Nash, 1951), the government - trying to maximise its objective function - is faced by restrictions such as budget constraints, the central bank's decisions and the macroeconomic conditions that cannot be controlled by either of the players. The central bank - seeking to optimise its decisions without compromising its main goal, which is keeping prices stable without deviating from the inflation target - has to deal with constraints such as the government's decisions, economic circumstances and specific impediments affecting the implementation of a monetary policy (e.g. a rule of monetary policy) (Chang & Liu, 2018; Davig & Leeper, 2011; Dixit & Lambertini, 2000, 2003; Gali & Monacelli, 2008; Leith & Wren-Lewis, 2006).

The non-cooperative models assume that the central bank and the government may choose not to cooperate for reasons such as different output and inflation targets, different weights attached to the targets, the adherence to different economic theories and different views on the fiscal and monetary policies' effect on the economy (Darnaut & Kutos, 2005; Hughes Hallett, Libich, & Stehlik, 2014). The non-cooperative models frequently produce suboptimal results characterised by higher deficits and higher real interest rates compared with the cooperative models (Bennet & Loayza, 2000; Blinder, 1982; Nordhaus, 1994). The non-cooperative game model by Dixit and Lambertini (2001) produced a similar result, with output and inflation respectively lower and higher than optimal.

In the Dixit and Lambertini (2000) model, the Nash Equilibrium for non-coordinated monetary and fiscal policies involves higher inflation and lower output compared with the best feasible allocation, which would not be optimal anyway because of disturbances caused by fiscal policy. This is because of the nature of the conflict between the central bank and the government, where the central bank pursues output and inflation levels lower than those set by the government. As a result, an inflationary fiscal policy is partially offset by an overly-contractionary monetary policy (Kuttner, 2002).

The Nordhaus (1994) model also assumes that each authority chooses a non-cooperative behaviour and designs its policy believing that the other authority will not change its policy. As a result, the budget deficit rises above a level that the authorities consider acceptable, because the government increases the deficit with the goal of reducing unemployment, whereas the central bank raises interest rates to fight inflation. It is for this same reason that interest rates are higher in the non-cooperative equilibrium (Nordhaus, 1994; Vieira, Machado, & Ribeiro, 2018).

In keeping with the Nordhaus (1994) approach, we also investigated a monetary-fiscal game to find the Nash Equilibrium. In the Nordhaus (1994) study, the implicit preferences of each player (the central bank and the government) serve as functions of policy variables. The central bank implements its monetary policy through interest rates, and the fiscal authority is responsible for the structural fiscal surplus ratio (government surplus at high employment divided by potential gross national product). The government's preference function consists of inflation, unemployment, the growth of potential output and the structural fiscal surplus ratio. For the central bank, the function includes inflation, unemployment and the growth of potential output. The unemployment rate measuring the utilisation of resources is a function of the government's and central bank's policies and of other predetermined and exogenous variables, such as capital stock, technology and foreign output. The rate can be replaced by the ratio of actual to potential output.

Dixit and Lambertini (2000) investigated the central bank authority and the government with the assumption that they pursued different goals. To this end, they constructed a model of strategic cooperation between both authorities using various loss functions, which differ in design from our proposal in this paper. Namely, to construct the loss function for the government, they used inflation (the difference between inflation and the government's inflation target), the level of output (the difference between output and the government's output target) and the parameters of severe losses related to taxes and/or expenditures. As far as the central bank's loss function is concerned, it consisted of inflation (the difference between inflation and the central bank's inflation target) and output (the difference between output and the central bank's output target).

3. The model – a non-cooperative game

In a non-cooperative game, the players make decisions independently of each other (see the section above). In our model, the government and the central bank are autonomous authorities pursuing individual goals. The government is intent on achieving the highest GDP growth possible, whereas the central bank makes efforts to keep inflation on target to ensure price stability. Each authority uses different policy tools to accomplish their goals: the budgetary surplus (or deficit) for the government and interest rates for the central bank. Because of their different goals, they exert different influence on the macroeconomic factors and indirectly influence each other's policies.

The main challenge in modelling a non-cooperative game is to optimise the objective function with respect to constraints. The optimisation procedure is explained step-by-step below.

Let us consider a model of an economy in which fiscal policy and monetary policy are carried out by the government and the central bank, respectively. Because both authorities shape their policies based on their partner's decisions, their behaviours can be analysed in the game-theory framework. Therefore, the government seeks to maximise its objective function[3]:

$$F_F(\cdot) = g_y^2 - f_1(d) = g_y^2 - \alpha_0 (d - d_M)^2 \rightarrow max \tag{1}$$

subject to following constraint[4]:

$$g_y = f_2(\pi, d, r) = \alpha_1 \cdot d + \alpha_2 \cdot r + \alpha_3 \cdot \pi \tag{2}$$

where g_y is a rate of growth of GDP per capita, r is the interest rate, d is the budget deficit, d_M is the Maastricht deficit limit, π is the level of inflation, $\alpha_0, \alpha_1, \alpha_3 > 0$ and $\alpha_2 < 0$ are constant parameters[5]. From Equation (1), it follows that the government would like to keep the rate of economic growth as high as possible, but the level of the instrument it can employ – the budget deficit – is limited in use by the Maastricht criteria. Equation (2) implies that the economic growth rate depends not only on the level of budget deficit but also on the level of the interest rate – the central bank policy instrument – and on the level of inflation. A higher level of r affects growth rate negatively and a higher level of inflation rate is connected with a higher level of growth rate. Therefore, the decisions made by the government must also take into consideration the macroeconomic situation (level of inflation) and the decision of the central bank (level of interest rate).

At the same time, the central bank seeks to keep inflation as close to the target as possible, minimising the difference between actual inflation and the given target rate[6]:

$$F_M(\cdot) = (\pi - \pi^t)^2 \rightarrow min \tag{3}$$

subject to:

$$\pi = f_4(g_y, d, r) = \pi_0 + \beta_1 \cdot r + \beta_2 \cdot g_y + \beta_3 \cdot d \tag{4}$$

where $\pi_0 > 0$ is a base inflation, π^t is the inflation target, β_2 and $\beta_3 > 0$ and $\beta_1 < 0$. The central bank is in a similar situation as the government; the level of inflation depends not only on the level of interest rate but also on the economic growth rate and level of budget deficit.

The above functions allow the non-cooperative model of a fiscal-monetary game to be solved and, consequently, the Nash Equilibrium for the game.

3.1. Nash Equilibrium

To determine the reaction functions describing each player's response to its partner's decision and to find the Nash Equilibrium presenting the resulting state of the economy, let us first solve the interdependent Equations (2) and (4). As a result, the following equations are obtained:

$$g_y = \frac{(\alpha_1 + \alpha_3\beta_3)}{1 - \alpha_3\beta_2} \cdot d + \frac{(\alpha_2 + \alpha_3\beta_1)}{1 - \alpha_3\beta_2} \cdot r + \frac{\alpha_3}{1 - \alpha_3\beta_2} \cdot \pi_0 \tag{5}$$

and

$$\pi = \frac{1}{1 - \alpha_3\beta_2}\pi_0 + \frac{(\beta_1 + \beta_2\alpha_2)}{1 - \alpha_3\beta_2} \cdot r + \frac{(\beta_3 + \beta_2\alpha_1)}{1 - \alpha_3\beta_2} \cdot d \tag{6}$$

Solving the equations produces explicit macroeconomic restrictions. Obviously, $1-\alpha_3\beta_2>0$ implies that $\alpha_3\beta_2<1$.

The reaction functions for the government and the central bank are obtained by maximizing the one-variable function[7] (1) with respect to (5) and minimising the function (3) with respect to (6). The government's reaction function has the following form[8]:

$$\tilde{d} = \frac{(\alpha_2 + \alpha_3\beta_1)}{\left(\frac{\alpha_0(1-\alpha_3\beta_2)^2}{\alpha_1+\alpha_3\beta_3} - (\alpha_1 + \alpha_3\beta_3)\right)} \cdot r + \frac{\alpha_3}{\left(\frac{\alpha_0(1-\alpha_3\beta_2)^2}{\alpha_1+\alpha_3\beta_3} - (\alpha_1 + \alpha_3\beta_3)\right)} \cdot \pi_0 + \frac{\frac{\alpha_0(1-\alpha_3\beta_2)^2}{\alpha_1+\alpha_3\beta_3}}{\left(\frac{\alpha_0(1-\alpha_3\beta_2)^2}{\alpha_1+\alpha_3\beta_3} - (\alpha_1 + \alpha_3\beta_3)\right)} d_M \tag{7}$$

Analogously, the reaction function for the central bank is written as:

$$\tilde{r} = \frac{(1-\alpha_3\beta_2) \cdot \pi^t - \pi_0 - \beta_2\alpha_1 \cdot d}{\beta_1 + \beta_2\alpha_2} = \frac{(1-\alpha_3\beta_2)}{\beta_1 + \beta_2\alpha_2} \cdot \pi^t - \frac{1}{\beta_1 + \beta_2\alpha_2}\pi_0 - \frac{\beta_2\alpha_1}{\beta_1 + \beta_2\alpha_2} \cdot d \tag{8}$$

The Nash Equilibrium is determined by certain sizes of d and r that solve the set of Equations (7) and (8). In the Nash Equilibrium, the level of d is such that it is the best response of the government to the best response of the central bank for the given level of budget deficit. Similar can be written about the level of r; it follows from the fact that the Nash Equilibrium level of both decision variables is the intersection of both reaction functions, which describe the best responses of the given player to the given decision of the other player. The obtained levels of d and r in the Nash Equilibrium are given by the following formulas:

$$d^* = a_1^* \cdot \pi^t + a_2^* \cdot \pi_0 + a_3^* \cdot d_M \tag{9}$$

$$r^* = b_1^* \cdot \pi^t + b_2^* \cdot \pi_0 + b_3^* \cdot d_M \tag{10}$$

where:

$$a_1^* = \frac{(\alpha_2 + \alpha_3\beta_1)\cdot\left(\frac{(1-\alpha_3\beta_2)}{\beta_1+\beta_2\alpha_2}\right)}{\left(\left(\frac{\alpha_0(1-\alpha_3\beta_2)^2}{\alpha_1+\alpha_3\beta_3} - (\alpha_1+\alpha_3\beta_3)\right) + (\alpha_2+\alpha_3\beta_1)\left(\frac{\beta_2\alpha_1}{\beta_1+\beta_2\alpha_2}\right)\right)} > 0$$

$$a_2^* = \frac{\left(\alpha_3 - \frac{(\alpha_2+\alpha_3\beta_1)}{(\beta_1+\beta_2\alpha_2)}\right)}{\left(\left(\frac{\alpha_0(1-\alpha_3\beta_2)^2}{\alpha_1+\alpha_3\beta_3} - (\alpha_1+\alpha_3\beta_3)\right) + (\alpha_2+\alpha_3\beta_1)\left(\frac{\beta_2\alpha_1}{\beta_1+\beta_2\alpha_2}\right)\right)}$$

$$a_3^* = \frac{\left(\frac{\alpha_0(1-\alpha_3\beta_2)^2}{\alpha_1+\alpha_3\beta_3}\right)}{\left(\left(\frac{\alpha_0(1-\alpha_3\beta_2)^2}{\alpha_1+\alpha_3\beta_3} - (\alpha_1+\alpha_3\beta_3)\right) + (\alpha_2+\alpha_3\beta_1)\left(\frac{\beta_2\alpha_1}{\beta_1+\beta_2\alpha_2}\right)\right)} > 0$$

$$b_1^* = \left(\left(\frac{(1-\alpha_3\beta_2)}{\beta_1+\beta_2\alpha_2}\right) - \left(\frac{\beta_2\alpha_1}{\beta_1+\beta_2\alpha_2}\right)\cdot\frac{(\alpha_2+\alpha_3\beta_1)\cdot\left(\frac{(1-\alpha_3\beta_2)}{\beta_1+\beta_2\alpha_2}\right)}{\left(\left(\frac{\alpha_0(1-\alpha_3\beta_2)^2}{\alpha_1+\alpha_3\beta_3} - (\alpha_1+\alpha_3\beta_3)\right) + (\alpha_2+\alpha_3\beta_1)\left(\frac{\beta_2\alpha_1}{\beta_1+\beta_2\alpha_2}\right)\right)}\right)$$

$$b_2^* = -\left(\frac{1}{\beta_1+\beta_2\alpha_2} + \left(\frac{\beta_2\alpha_1}{\beta_1+\beta_2\alpha_2}\right)\frac{\left(\alpha_3 - \frac{(\alpha_2+\alpha_3\beta_1)}{(\beta_1+\beta_2\alpha_2)}\right)}{\left(\left(\frac{\alpha_0(1-\alpha_3\beta_2)^2}{\alpha_1+\alpha_3\beta_3} - (\alpha_1+\alpha_3\beta_3)\right) + (\alpha_2+\alpha_3\beta_1)\left(\frac{\beta_2\alpha_1}{\beta_1+\beta_2\alpha_2}\right)\right)}\right)$$

$$b_3^* = -\left(\frac{\beta_2\alpha_1}{\beta_1+\beta_2\alpha_2}\right)\frac{\left(\frac{\alpha_0(1-\alpha_3\beta_2)^2}{\alpha_1+\alpha_3\beta_3}\right)}{\left(\left(\frac{\alpha_0(1-\alpha_3\beta_2)^2}{\alpha_1+\alpha_3\beta_3} - (\alpha_1+\alpha_3\beta_3)\right) + (\alpha_2+\alpha_3\beta_1)\left(\frac{\beta_2\alpha_1}{\beta_1+\beta_2\alpha_2}\right)\right)} > 0$$

As can be seen, the Nash Equilibrium levels in Equations (9) and (10) depend on π_t, π_0 and d_M (the exogenous parameters). In other words, decisions of the government and the central bank are such that the levels of budget deficit and interest rate in the Nash Equilibrium can be expressed as functions of exogenous variables, such as the level of inflation target, base inflation and the Maastricht deficit limit.

The government's objective function and its limitations considered in this study comprise, in addition to output, inflation and interest rate, the level of budget deficit and the Maastricht deficit criterion. As far as the central bank's objective function and its limitations are concerned, inflation and the level of budget deficit are supplemented by the target inflation rate, interest rate and output. To the authors' best knowledge, using these variables to construct objective functions (and their limitations) for the monetary and fiscal authorities is an innovative approach in that it takes into consideration both economic mechanisms and the mandatory EU legislation (mainly laws capping budget deficits).

The results of these calculations are consistent with the findings of Engwerda, van Aarle, and Plasmans (2002), who studied the influence of a fiscal stabilisation policy on economic fluctuations in the EMU countries. The main assumption underlying their model was the ECB's adoption of a passive (non-strategic) monetary policy to control a common nominal interest rate. Among the limitations of the model, its focus on open-loop Nash Equilibria and Pareto solutions are noteworthy. External restrictions (e.g. arising under the Stability and Growth Pact) limit the fiscal activity of the government, making the stabilisation of output and prices in the Euro area less effective and leading to suboptimal macro-economic policy. Dixit and Lambertini (2000) stressed that if the policy preferences are fixed, then the outcomes can only be influenced by institutional constraints, changing the fiscal reaction function. Therefore, simple constraints used in the EMU have some desirable effects, but the design of optimal constraints of fiscal policy remains an interesting task for the researchers.

In addition, Foresti (2018) added that the degree of commitment to a rule and discretion, the level of coordination, the order of moves and others issues related to interaction between monetary and fiscal policies are all significant elements able to affect the policy mix outcome in the EMU. Lambertini and Rovelli (2004) did not consider whether or not the individual provisions of the Stability and Growth Pact are optimal, but they supported the view that discipline must be imposed on the fiscal policies designed by EMU member states. Otherwise, inflation may fluctuate too widely around its target rate, and interest rates may become too volatile. According to Raguseo and Sebo (2008), fiscal restrictions are not an effective tool against asymmetric shocks that can impair the performance of EMU economies. It is empirically known that fiscal restrictions that are not accompanied by full cooperation (a fully centralised budget) are more likely to exert intense pressure on the central bank.

Similarly, Hughes Hallett et al. (2014) indicated that if the government is too myopic, it will not be disciplined by the central bank. If in the currency union there is a moral hazard problem due to free-riding of small member countries, the disciplining channel is unlikely to be effective. Therefore, it is necessary in such cases to discipline fiscal policy in the long term by direct fiscal commitment arrangements (i.e. legislated and enforceable fiscal policy rules).

3.2. Reaction function analysis

In this section, we examine the sensitivity of the reaction function due to changes in the deep parameters of the modelled economy (α_1, α_2, α_3, β_1, β_2 and β_3). The reaction functions presented in the previous subsection (derived from Equations (7) and (8)) are written as:

$$\tilde{d} = \tilde{a}_1 \cdot r + \tilde{a}_2 \cdot \pi_0 + \tilde{a}_3 \cdot d_M \tag{11}$$

$$\tilde{r} = \tilde{b}_1 \cdot \pi^t + \tilde{b}_2 \cdot \pi_0 + \tilde{b}_3 \cdot d \tag{12}$$

where:

Table 1. Signs of the respective derivatives.

x	α_0	α_1	α_2	α_3	β_1	β_2	β_3
$\frac{\partial \tilde{a}_1}{\partial x}$	<0	>0	>0	?	>0	>0	>0
$\frac{\partial \tilde{a}_2}{\partial x}$	<0	<0	$=0$	>0	$=0$	>0	<0
$\frac{\partial \tilde{a}_3}{\partial x}$	>0	>0	$=0$	>0	$=0$	<0	>0
$\frac{\partial \tilde{b}_1}{\partial x}$	$=0$	$=0$	<0	>0	<0	>0	$=0$
$\frac{\partial \tilde{b}_2}{\partial x}$	$=0$	$=0$	>0	$=0$	>0	<0	$=0$
$\frac{\partial \tilde{b}_3}{\partial x}$	$=0$	>0	>0	$=0$	>0	>0	$=0$

Source: own calculations.

$$\tilde{a}_1 = \frac{(\alpha_2 + \alpha_3\beta_1)}{\left(\frac{\alpha_0(1-\alpha_3\beta_2)^2}{\alpha_1 + \alpha_3\beta_3} - (\alpha_1 + \alpha_3\beta_3)\right)} < 0$$

$$\tilde{a}_2 = \frac{\alpha_3}{\left(\frac{\alpha_0(1-\alpha_3\beta_2)^2}{\alpha_1 + \alpha_3\beta_3} - (\alpha_1 + \alpha_3\beta_3)\right)} > 0$$

$$\tilde{a}_3 = \frac{\frac{\alpha_0(1-\alpha_3\beta_2)^2}{\alpha_1 + \alpha_3\beta_3}}{\left(\frac{\alpha_0(1-\alpha_3\beta_2)^2}{\alpha_1 + \alpha_3\beta_3} - (\alpha_1 + \alpha_3\beta_3)\right)} > 0$$

$$\tilde{b}_1 = \frac{(1-\alpha_3\beta_2)}{\beta_1 + \beta_2\alpha_2} < 0$$

$$\tilde{b}_2 = -\frac{1}{\beta_1 + \beta_2\alpha_2} > 0$$

$$\tilde{b}_3 = -\frac{\beta_2\alpha_1}{\beta_1 + \beta_2\alpha_2} > 0$$

Reaction functions (11) and (12) have the following interpretation. For some given levels of interest rates r, base inflation π_0 and the Maastricht deficit limit d_M, the government sets the deficit at level $\tilde{d}$. For some given levels of target inflation π^t, base inflation π_0 and budget deficit d, the central bank sets interest rates at $\tilde{r}$. Parameters $\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{b}_1, \tilde{b}_2$ and $\tilde{b}_3$ measure how strongly a given variable influences the optimal decision of the government or of the central bank. For example, if the target inflation rate rises, then the central bank's optimal decision would be to cut interest rates (because of $\tilde{b}_1 < 0$).

Parameters $\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{b}_1, \tilde{b}_2$ and $\tilde{b}_3$ are the non-linear combinations of deep parameters α_0, α_1, α_2, α_3, β_1, β_2 and β_3, which describe the foundations of the analysed economy (e.g. the reaction of the inflation rate to changes in interest rates, the reaction of the economic growth rate to changes in the inflation rate, etc.). Table 1 shows the signs of the derivatives of the" tilde" parameters with respect to the deep parameters. A positive sign denoting an increase in a deep parameter (e.g. α_1) means that the value of the" tilde" parameter ($\tilde{a}_1$) grows too.

The sign of the respective derivative can be interpreted in the following way. Let us consider two identical economies that only differ in one of the deep parameters, for example α_1. In the economy, when the parameter α_1 has a higher value, the parameter $\tilde{a}_1$ (which in general is negative, $\tilde{a}_1 < 0$) has a lower absolute value, which means that it is closer to 0. This means that in an economy where the budget deficit has a stronger effect on the GDP growth rate (parameter α_1 is higher; see Equation (2)), the government reacts to an increasing interest rate by adopting policy leading to a higher deficit (see Equation (11)) than in the other economy. Next, when the interest rate has a stronger effect on the GDP growth rate (parameter α_2 is higher, see Equation (2)) the government reacts to increasing inflation by adopting policy leading to a higher deficit (see Equation 11) than in the other economy.

In the case of the parameter α_2 (interest rates), its positive impact on the optimal decision of the government (i.e. the level of deficit $(\partial\tilde{a}_1/\partial\alpha_2)$). Table 1 shows that the parameter α_2 expresses the positive impact of deficit on the optimal decision of the government (see Equation (11)) and the negative impact of current inflation on the optimal decision of the central bank (see Equation (12)). The impact of β_3, as well as α_0, is important only for optimal decision making by the government.

Of all 42 derivatives, only one $\left(\frac{\partial\tilde{a}_1}{\partial\alpha_3}\right)$ has an unidentifiable sign determined by values of unknown parameters. The other 41 derivatives have unambiguous signs (positive or negative, or derivative takes a value of 0). Table 1 presents them all.

It would be interesting to carry out a similar comparative static sensitivity analysis of the Nash Equilibrium levels, but the fact that they are determined through the rivalry between two opposing forces (the government and the central bank) makes it almost impossible (except in a few specific cases) to conclusively obtain the sign of similar derivatives in the general case. Given that a change in any of the deep parameters has a concurrent effect on the reaction functions and many of the" tilde" parameters, the final reaction of the Nash Equilibrium is difficult to deduce (to obtain analytically), but it can be calculated (obtained numerically) assuming certain values of all parameters.

Conclusions relevant to this study were also presented by Woroniecka-Leciejewicz (2015), according to whom changes in the central bank and government's priorities affect the optimal fiscal and monetary policy responses and consequently the Nash Equilibrium (equilibrium is a choice of policy mix). When the government wants to stimulate the rate of economic growth, the optimal budgetary response becomes more expansionary. The central bank's optimum monetary strategy becomes more expansionary when it redefines its priorities to accept higher inflation.

Conclusions

This article considers the Nash Equilibrium for the case of a non-cooperative fiscal-monetary game between the governments and central banks in the EU member states. Both authorities are assumed to be independent in decision making and to optimally respond to each other's best decisions, thus keeping the system in a steady state. These assumptions were used to construct our original policy-mix model for a non-cooperative game. The model shows that in the Nash Equilibrium, the level of budget deficit and the interest rate - being the policy tools of the government and the central bank, respectively, - depend on exogenous factors such as the inflation target, base inflation and the Maastricht deficit limit. Therefore, the inflation target rate and the Maastricht deficit limit, which are set institutionally, are important determinants of the fiscal-monetary balance in the EU countries. This paper presents the reaction functions of the central bank and the government, which are different from those mentioned in the literature, and based on this, the Nash Equilibrium was calculated while designing the original equilibrium model for a non-cooperative monetary-fiscal game in the EU countries.

In this study, with the reaction functions for the government and the central bank, both authorities' sensitivity to changes in the deep parameters of economic variables were also analysed. The sensitivity analysis allows for examination of how strongly the parameters measuring variables' influence on the optimal decisions of the government and central bank change in response to variations in the deep parameters. It must be emphasised that the analysis of sensitivity conducted in this paper is an original study, as it was based on the original equilibrium model for a non-cooperative monetary-fiscal game in the EU countries.

Studies using mathematical policy-mix models to determine the Nash Equilibrium in a non-cooperative fiscal-monetary game are few, particularly for the case of countries such as the EU member states, which are subjected to institutional restrictions. This fact and the community's progressing economic integration increase the significance of this research gain. Its conclusions can also serve as a practical guideline for decision making by national economic authorities in the EU member states. Based on the analysis of the game between monetary and fiscal policy in EU countries, some suggestions have been made to improve the functioning of these policies in the countries discussed. This study demonstrates that in the Nash Equilibrium, budget deficit and interest rate depend on the inflation target (which is set by the monetary authorities), base inflation and the Maastricht deficit limit (influenced by the economic authorities of EU countries). It provides the main recommendation for policymakers to pay attention to these variables when constructing policy.

Notes

1. In our study we propose a model with continuous strategies (i.e., the game that allows players - the government and the central bank - to choose a specific strategy from a continuous set of alternatives). This approach is opposite to the discrete strategy, a very popular approach with a finite set of outcomes and inputs.
2. Deep parameters are thought to be invariant against policy change, and because of the stability of these parameters, economists use them to evaluate economic policy. The Lucas (1976) critique suggests that if we want to predict the effect of a policy experiment, we should model the deep parameters (relating to preferences, technology, and resource constraints) that are assumed to govern individual behavior, or so-called microfoundations.
3. Similar to Kuttner (2002) and Bennett and Loayza (2000).
4. As in Davig and Leeper (2011).
5. $\alpha_3 > 0$ for simplicity we assume that an inflation rate lower than the inflation target (which is under the control of the central bank) positively influences the GDP growth. The literature review informs that high inflation negatively affects the economy (e.g., Barro, 2013). On the other hand, low inflation can positively influence the behaviour of agents through the impact on their decision. Thus the low inflation can positively influence economic growth. This phenomenon is also investigated by e.g., Mallik and Chowdhury (2001) who found a positive relationship between inflation and economic growth. Thus

the non-linear relationship was found by e.g., Ghosh and Phillips (1998) – in their study based on 145 countries they found a positive relationship between inflation and economic growth when inflation is low and negative for high inflation. Due to the fact that inflation in the euro area is under the control of the ECB (the inflation target is set at the level no higher than 2%), we adopted a strong assumption of a linear and positive relationship between economic growth and inflation.

6. See Blinder (1982) and Bennett and Loayza (2000).
7. We skip the details of calculations in the text, but they are available upon request from the authors.
8. To satisfy sufficient condition:

$$\frac{\partial^2 F_F(d)}{\partial d^2} = 2 \cdot \frac{(\alpha_1+\alpha_3\beta_3)^2}{(1-\alpha_3\beta_2)^2} - 2\alpha_0$$

$$(\alpha_1+\alpha_3\beta_3) < \frac{\alpha_0(1-\alpha_3\beta_2)^2}{(\alpha_1+\alpha_3\beta_3)}$$

which implies

$$\frac{\alpha_0(1-\alpha_3\beta_2)^2}{(\alpha_1+\alpha_3\beta_3)} - (\alpha_1+\alpha_3\beta_3) > 0$$

Therefore we assume that parameters on the left-hand-side of this inequality are such that this condition is fulfilled.

Acknowledgements

This study was financed by funds from the National Science Centre for the period 2018–2021 for the research project entitled *The coordination of the monetary and fiscal policies in the studies of the monetary – fiscal interactions based on the game theory - the case of the European Union countries*, contract number: UMO-2017/26/D/HS4/00954.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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