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Effects of celebrity endorsement on firms' competition: from industrial organisation perspective

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ABSTRACT

Product endorsement is employed by many firms and most studies have proceeded based on management perspectives and ignored the strategic effects (or indirect effects). However, as a non-price competition, product endorsement behaviour's strategic effects are valuable to be considered and convenient to be captured by industrial organisation perspective. So, this paper's purpose is to reveal the strategic effects of product endorsement by industrial organisation perspective. First, the results of this paper show that celebrity endorsement decreases the rival's benefits. Second, under a unique endorser, the lower efficiency (measured by marginal production cost) firm's celebrity endorsement improves price difference and dispersion, which are direct effects of product endorsement. The higher efficiency firm's endorsement promotes the producer surplus, consumer surplus and social welfare, which are called indirect effects. Finally, celebrity endorsement has trigger effects. If one firm launches celebrity endorsement, the rational reaction for its rivals is also to engage in a similar behaviour. This paper expands the celebrity endorsement issue from management to economics perspective.

1. Introduction

To improve their brand or demand, many firms select product endorsement according to consumer-based viewpoint. As MacCrachen (1989) issued, product endorsement is a meaning transfer from the producer to consumer with endorser by advertising. Product endorsement is extensively discussed in the management field (Baxter, Ilicic, & Kulczynski, 2015; Ding, Molchanov, & Stork, 2011; Hartmann & Apaolaza-Ibanez, 2012; Nicolau & Santa-María, 2013; Reed II, Forehand, Puntoni, & Warlop, 2012), while scant literature about product endorsement in economics exists (Garthwaite, 2014).

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The purpose of this paper is to offer an analysis about celebrity endorsement from industrial organisation theory (IOT) perspective for that some valuable conclusions such as the strategic effects of product endorsement cannot be obtained by management framework. What is the difference between management and economics? Management prefers to investigate micro-individual's behaviour and its conclusions depend on psychology heavily. This attribute constrains its efficiency of obtaining the common and robust conclusions about product endorsement from management or marketing perspective. Evidences show that the effects of celebrity endorsements vary across countries and cultures (Winterich, Gangwar, & Grewal, 2018). On the contrary, economics has the inherent strength of revealing common and robust economics behaviours. Furthermore, product endorsement advertising is a major non-price competition strategy, which also needs more studies from industrial organisation perspective. So, this paper proposes to survey the effects of celebrity endorsement on firms' competition based on the major hypothesis that product endorsement has both direct and indirect influences. As a pure theoretical study, all the conclusions are obtained by statics and comparative static analysis.

Extensive literature about product endorsement in management has been launched in recent years (Kima, Choeb, & Petrick, 2018; Rahman, 2018). Kim, Choe, & Petrick (2018) and Shih (2012) addressed the relationship between endorsement strategies and brand loyalty. Rahman (2018) investigated the positive effects of celebrity endorsement on brand awareness. Rahman (2018) issued that a firm can build brand awareness by celebrity endorsement. Erdogan (1999) reviewed the existing literature about celebrity endorsement. In Erdogan's reviewing paper, the history of celebrity endorsement is introduced and some related measures, such as match-up, performer Q rating are discussed. Miller & Laczniak (2011) discussed the effects of celebrity endorsement when a star steps out of bounds. Applied to three well-known situations, Miller and Laczniak (2011) examined the ethical implications of what initially were good choices for firms, their brands, and their consumers. Shih (2012) showed that manufacturer brands support high prices and boost those vivid impressions which are helpful in engendering consumer loyalty. Elberse & Verleun (2012) addressed the value of celebrity endorsement and find a positive pay-off to a firm's decision to sign an endorser, and that endorsements are associated with increasing sales in an absolute sense and also relative to competing brands. All those studies are based on management with the conclusion that celebrity endorsement promotes loyalty and purchase willingness and consumption promoting effects, which are the direct influence of endorsement.

There have been quite a few papers about celebrity in economics. For example, by connecting capital market to product market, Ding, Molchanov & Stork (2011) captured the influence of celebrity endorsement on stock and they found that endorsements bring technology industry firms with significant positive abnormal returns. Garthwaite (2014) examined the economic effects of endorsements and found that celebrity endorsements have demand spillover or business stealing in the publishing sector. Interestingly, Chung, Timothy, & Kannan (2013) and Popescu (2014) tried to reveal Economic Value of Celebrity Endorsements. Their studies illustrated that product endorsement in management implied economics theory, too. Barone & Jewell

(2014) declared endorsement creates advertising flexibility. Those studies involve the indirect effects of celebrity endorsement but they also are not pure economics perspective studies in the strict sense because they lack economic theories. Besides, more and more attentions are paid to B2B or industry sector (Agnihotria, Dingus, Huc, & Krush, 2016; Mehmet & Clarke, 2016; Pedeliento, Andreinia, Bergamaschia, & Salob, 2016).

Notice that nearly all the prior research ignored the behaviour of endorser because they took product endorser as an exogenous variable or only given celebrity endorsement to analyse its effects on the principal's profits. Different from the existing papers, especially those in management perspective, this study considers endorsers as profit-maximising economic agents and they make optimal decisions just like consumers and firms. In other words, the endorser's behaviour is also taken as a vertical endogenous variable. That means this study employs a three participants model: Endorser–Producer–Consumer. Endorser decides whether to accept the product endorsement invitation of the firms and which firm's product to endorse following the utility maximisation principle. Then, the firms decide their price and quantity based on the behaviour of the endorser. Finally, the consumer chooses the product to purchase based on his or her preference. Furthermore, this paper captures both the direct effects and indirect effect by connecting marketing theory to industrial organisation theory, which enriches the theory of celebrity endorsement and expands celebrity endorsement application.

Management is good at investigating specific value of firms' behaviour and economics has the inherent advantage of revealing general rules. More importantly, celebrity has both direct effects (e.g., purchase motivating & loyalty improving) and indirect effects (e.g., price arising, rivals' profit impacting & social welfare influence), while most of the prior studies based on management only focused on the direct impacts of endorsement. A possible reason for that is indirect effects is difficult to capture in marketing practice. Besides revealing the direct influence of celebrity endorsement, this paper aims to investigate its indirect effects by combining economics perspective with management. From the Match-Up Hypothesis, it learns that different product endorsement needs different celebrity choice. Generally, the more special the products, the unique (means the most famous) the celebrity is needed. That is why golf equipment industry prefers Tiger Woods and basketball industry likes Michael Jordan as the endorsement celebrity (Derdenger, 2018). But fashion product firms choose entertainment stars, while the pharmaceutical industry only takes a guy wearing white coats to endorse its products (Kennedy, Baxter, & Ilicic, 2019). The more unique the endorser, the more monopolistic the endorser is. Thus, this paper discusses both the monopolistic endorser and the competitive endorsers. Monopolistic endorser means there is a unique endorser, while competitive indicates two or more endorsers are available. First, this paper finds that celebrity endorsement lowers rival's benefits. Second, by measuring firm efficiency with marginal costs, this study finds that under a unique endorser, lower efficiency firms' celebrity endorsement improves price difference and price dispersion.[1] The higher-efficiency firm's endorsement promotes the producer surplus, consumer surplus and the social welfare. Finally, under competitive endorsers, firms compete both in outputs and in celebrity

endorsement. When one firm launches celebrity endorsement while the other does not, it has the similar conclusions to those under a unique endorser, which means the conclusions are robust.

This paper is organised as follows: Theory background is given in Section 2 and the model is outlined in Section 3. Some analysis and the main results about the unique endorser are presented in Section 4. The effects of endorsement are captured with the unique endorser. Competitive endorsers are addressed in Section 5. The effects of duopoly product endorsers are analysed and are compared with the unique endorser. Then some conclusions and discussions are outlined in the final section.

2. Theory background

Modern product endorsements can come with contracts worth substantial amounts of money. Many sport stars agree to participate in product endorsement campaigns with the understanding that the company will compensate them for the trouble; some stars donate the proceeds to charities they support, using product endorsement as a public relations campaign. For example, TCL Corporation in China, a famous company with TV and other family appliance products, signed Brazilian soccer star Ronaldo de Assis Moreira for endorsement with 10 million euros in 2007 (http://www.tcl.com/About/index.html). More interestingly, Celebrity Endorsement even was used in political field by Politicians (Sikorski, Knoll, & Matthes, 2018).

Celebrities have been employed for product endorsement since the late nineteenth century. Sherman (1985) gave an example from the early days of endorsement involving Queen Victoria in association with Cadbury's Cocoa. According to Erdogan's view (1999), the emergence of papers, cinema and internet all greatly extends the scope of endorsement as an advertising technique. Nicolau & Santa-María (2013) examined the relationship between two types of performances, one on the ground (of a tennis court) and the other on the floor (of the stock market). Reed II, Forehand, Puntoni, & Warlop (2012) developed the relationship between identity-based strategies and consumer behaviours. Zamudio (2016) employed a two-sided matching market model to capture the impacts of celebrity endorsement on consumers' attitudes and firm values.

Why is celebrity endorsement so popular and why can a person having nothing to do with the company promote the product or service? Several theories are trying to justify that and nearly all those theories hold the Product Match-Up Hypothesis (Erdogan, 1999). Product Match-Up Hypothesis maintains the celebrity and the endorsed brand should match-up each other and that means a matched celebrity can succeed and vice versa. First, the Source Credibility Theory (SCT) shows consumer's beliefs, opinions, attitudes and behaviour can be influenced by the information delivered from the credible source (Saldanha, Mulye, & Rahman, 2018; Wang & Scheinbaum, 2018). It learns from Desphands & Stayman (1994) that SCT is similar to social identity theory for that it issues that the interactions between celebrity endorser and consumers occurs because individuals trust people who have similar attitudes and value structures to them.

The Source Attractiveness Theory (SAT) contends consumers tend to form positive stereotypes about those individuals who are famous and successful and they are more likely to be famous and successful if they do as those successful individuals do (Bergkvist & Zhou, 2016; Saldanha, Mulye, & Rahman, 2018). In other words, attractive communicators are more helpful in changing beliefs and attractive endorsers create more purchase intentions (Kahle & Homer, 1985). Solomon (1996) showed that Performer Q (quotient) Ratings is a core concept of SAT, which reflects the celebrity's popularity among those who recognise the celebrity.

The Meaning Transfer Theory (MTT) is another famous theory about celebrity endorsement. MTT contains three stages in meaning transfer, formation of celebrity image, meaning transfers from celebrity to product and meaning transfer from product to consumers (Knoll, Matthes, Münch, & Ostermann, 2018; McCracken, 1988; Scheidt, Gelhard, Strotzer, & Henseler, 2018). Benefits of celebrity endorsement are marked, but the costs and risks of celebrity endorsement are also significant because the endorsement fees are usually high and any stages of the meaning transfer can turn out to be failed. News from Fortune show several celebrity endorsements that enraged consumers, Such as Michael Jordan and Nike, Nicole Kidman and United Arab Emirates' Etihad Airways, Actor Alec Baldwin and Capital One Bank, and so on.[2]

All the theories above based on marketing perspective were built upon psychological foundation. They are convenient to be used to explain the impact of celebrity endorsement on purchase motivating. But this impact is difficult to be measured and observed. Those celebrity endorsement theories mentioned above have fundamental differences: SCT assumes that consumers can be influenced by people who have common features with themselves; SAT implies that people are more like to be influenced by famous and successful one; while MTT shows that Meaning can be transferred between different individuals and objects. Besides, Match-Up Hypothesis is the strongest for MTT. Those theories are too microcosmic to reach common and robust conclusions. Furthermore, they are powerless on revealing the effects of celebrity on equilibrium price, rivals' strategies and social welfare, which are called indirect effects.

Nonprice competition is a major theory of Industrial organisation theory (Hatfield, Plott, & Tanaka, 2016; 2012; Spence, 1977), and (celebrity endorsement) advertising is a significant nonprice competition (Elliott & Lockard, 1996). Besides, Tirole (1988)'s industrial organisation theory (IOT) showed that firm's behaviours have strategic effects which influence the behavior of the rivals. On the other hand, many studies from IOT capture advertising behaviour, but few of them pay attention to celebrity endorsement (Andre, 2016; Garthwaite, 2014). A possible reason is that celebrity endorsement is a special type of advertising, while IOT just concerns with advertising. Advertising is a meso-economics concept, while celebrity endorsement is a micro-economics one. The issues that economics concerns are more macroscopic than management. The reason for a firm to engage in celebrity endorsement is that it can transfer meaning of the product from producer to consumer by celebrity endorsement advertising. So, this study combines MTT of marketing theory with nonprice competition theory of IOT to make a full frame of celebrity endorsement and purpose to obtain some general conclusions.

3. The model

Cournot quantity competition is a general model in industrial organisation (See Bian et al., 2018; Chen et al., 2019; Sacco & Schmutzler, 2011; Tao, Wang, & Yang, 2018). The model about celebrity endorsement is established here and the two firms are assumed to abide Cournot quantity competition, which means they make quantity decision by considering the action of the rival, simultaneously. In other words, this paper employs a duopoly model to capture the celebrity competition based on industrial organisation theory. There are many firms in nearly any industries, but most industries are similar to oligopoly structure in reality because no products between two firms are prefect substitutes. On the other hand, our conclusions can expand to multi-competition market easily. So, duopoly competition model makes sense. Denote the two firms by $i \in \{1, 2\}$. The firms produce substitutable products and simultaneously select celebrity product endorsers. They compete both in quantity and in product endorsement. Different from other articles, this paper considers product endorsers as endogenous variables because they should make endorsing decisions endogenously and our analyses begin with utility maximising consumers.

Consumers. From the industrial organisation theory (Chen, He, & Paudel, 2018; Chen, Huang, Mishra, & Wang, 2018; Chen, Wen, Wang, & Nie, 2017; Sacco & Schmutzler, 2011; Vives, 2008; Wang, Chen, & Nie, 2017), it assumes the representational utility function of duopoly market is $U(P_i, p_j) = A(q_i + q_j) - \frac{1}{2}q_i^2 - \frac{1}{2}q_j^2 - \lambda q_i q_j$ without considering any other factor besides quantity. Relevant study only needs to modify this function based on the variable concerned. Source Attractiveness Theory (SAT) shows that celebrity endorsement highlights consumer surplus and equilibrium prices, but none of those studies model the effects of celebrity endorsement on utility and prices besides equilibrium quantity.[3] Following SAT and given the price vector of the two producers $p = (p_1, p_2)$, the quantity vector $q = (q_1, q_2)$ and the celebrity's popularity (which is attained by the technique of performer Q rating in Erdogan (1999) or Shimp (1997) $x = (x_1, x_2)$, the utility function of the representative consumer is given as follows[4]

$$U(p_1, p_2, q_1, q_2, x_1, x_2) = A(1 + x_1)q_1 + A(1 + x_2)q_2 - \frac{1}{2}q_1^2 - \frac{1}{2}q_2^2 - p_1 q_1 - p_2 q_2 - \lambda q_1 q_2, \quad (1)$$

where $\lambda \in [0, 1]$ presents the product substitutability and the constant $A > 0$ means market capacity without product endorsement. $\lambda = 0$ means that goods are independent and $\lambda = 1$ manifests perfect substitutes (Chen, Chen, & Mishra, 2019; Chen, Wan, & Wang, 2015; Liu et al., 2012; Nie, 2012; Nie, Wang, Chen, & Chen, 2018; Nie & Chen, 2012; Nie & Wang, 2018; Raykov & Calantone, 2014). According to (1), the celebrity product endorsement improves the corresponding market size efficiently, which affects the firm's other strategies, including price, outputs and so on. Furthermore, this paper assumes that the product message and celebrity information are perfect match-up.[5] Therefore, the match-up degree is not focused in this work.

For $i, j \in \{1, 2\}$ and $i \neq j$, the corresponding inverse demand function is induced by (1), which is presented as

$$p_i = A(1 + x_i) - q_i - \lambda q_j. \quad (2)$$

From (2), q_i increases with x_i while decreases with x_j. This means that product endorsement can improve the corresponding demand and reduce the rival's demand.

The product endorsement has expansionary effects on the demand and crowding-out effects on rivals. Similarly, product endorsement promotes the price of the corresponding products while reduces the rival's price.

Product Endorser. Here, behaviour of the product endorser is addressed. The value of the product endorsement is pe, which is independent of the quantity of products. Moreover, denote $u(x)$ to be the opportunity cost of the endorser with the celebrity's popularity x. $u(x)$ is convex and $\frac{du(x)}{dx} > 0$. Furthermore, it is assumed that $u(x) = \frac{1}{2}x^2$. Only when the following constraints are met, the product endorser appears[6]

$$pe - u(x) \geq 0. \tag{3}$$

This is also called the participation constraint.

Producers. The profits of firms are presented as follows. For $i \in \{1, 2\}$,

$$\pi_i = p_i q_i - c_i q_i - pe_i, \tag{4}$$

where pe_i is the cost incurred by the product endorsement independent of the quantity of products or it is a type of transfer. c_i is the marginal cost independent of the product endorsement and q_i represents the outputs of firm $i \in \{1, 2\}$. This paper addresses asymmetrical efficiency situation in which the two firms have different marginal costs which measure the efficiencies of firms. Without loss of generality, this paper stipulates $c_1 < c_2$ throughout. Or, the first (high efficiency) firm owns the cost advantage compared with the second (low efficiency) firm.

Firm $i \in \{1, 2\}$ maximises its profits by pe_i, qr_i and q_i. According to (2) and (3), the profit function is concave both in pe_i and q_i. A unique solution for the above system exists. Moreover, the above functions are all consistent with the assumption about quality commitment in Reitzes (1992).

The above model is analysed in two cases. One is the monopoly product endorser, called monopolistic endorser-competitive principals-model, in which one firm launches the product endorsement while the other does not. In this case, the unique product endorser plays the monopolisation position. The other is the duopoly product endorsers, known as competitive endorsers-competitive principals-model, in which two firms may simultaneously launch the product endorsing.[7]

Here the benchmark model without product endorsement or $pe_1 = pe_2 = 0$ is introduced. The equilibrium outputs, equilibrium price, equilibrium profits and social welfare based on (2) and (4) are

$$
q_1^{b,*} = \frac{2(A - c_1) - \lambda(A - c_2)}{4 - \lambda^2}, \quad q_2^{b,*} = \frac{2(A - c_2) - \lambda(A - c_1)}{4 - \lambda^2}
$$
$$
p_1^{b,*} = A - \frac{(2 - \lambda^2)(A - c_1) + \lambda(A - c_2)}{4 - \lambda^2}, \quad p_2^{b,*} = A - \frac{(2 - \lambda^2)(A - c_2) + \lambda(A - c_1)}{4 - \lambda^2}
$$
$$
\pi_1^b = \left[\frac{2(A - c_1) - \lambda(A - c_2)}{4 - \lambda^2} \right]^2, \quad \pi_2^b = \left[\frac{2(A - c_2) - \lambda(A - c_1)}{4 - \lambda^2} \right]^2 \tag{5}
$$
$$
SW^0 = \frac{3}{2}(q_1^{b,*})^2 + \frac{3}{2}(q_2^{b,*})^2 - \lambda q_1^{b,*} q_2^{b,*}
$$
$$
= \frac{[3[2(A - c_1) - \lambda(A - c_2)]^2 + 3[2(A - c_2) - \lambda(A - c_1)]^2}{2(4 - \lambda^2)^2} .
$$
$$
\frac{-2\lambda[2(A - c_1) - \lambda(A - c_2)][2(A - c_2) - \lambda(A - c_1)]]}{2(4 - \lambda^2)^2} .
$$

The equilibrium of the benchmark model is employed to compare with other conclusions under product endorsement.

4. Monopoly product endorser

For the unique product endorser, the herein model is analysed. In this case, it assumes that $u(x) = \frac{x^2}{2} = c_0$. For the unique product endorser with the celebrity popularity x, this product endorser plays the monopoly position. The model will be discussed based on two different cases. One is that the first firm launches product endorsement and the other is that the second firm employs product endorsement. In these two cases, the product endorser plays the leading position.

4.1. High-efficient firm having product endorsement

The first firm launches while the second firm does not make product endorsement. In this case, $pe_2 = 0$ and $pe_1 \geq 0$. This endorser meets the participation constraint as the following problem

$$pe_1 \geq c_0. \tag{6}$$

The profit functions of the two producers are restated as follows.

$$\begin{aligned} \underset{q_1}{Max}\, \pi_1 &= \left[A(1+x) - q_1 - \lambda q_2\right]q_1 - c_1 q_1 - pe_1 \\ S.t.\ \pi_1 &\geq \pi_1^b, \end{aligned} \tag{7}$$

$$\underset{q_2}{Max}\, \pi_2 = (A - q_2 - \lambda q_1)q_2 - c_2 q_2. \tag{8}$$

The lower-level problem (7)-(8) is concave. Therefore, a unique solution for the lower-level problem exists, which is determined by the first order optimal conditions. The first optimal conditions of the lower-level problem imply

$$\frac{\partial \pi_1}{\partial q_1} = A(1+x) - 2q_1 - \lambda q_2 - c_1 = 0, \tag{9}$$

$$\frac{\partial \pi_2}{\partial q_2} = A - 2q_2 - \lambda q_1 - c_2 = 0. \tag{10}$$

Denote the solution to be $(q_1^{m1,*}, q_2^{m1,*})$. (6)-(10) imply that

$$q_1^{m1,*} = \frac{2(A + x - c_1) - \lambda(A - c_2)}{4 - \lambda^2}, \tag{11}$$

$$q_2^{m1,*} = \frac{2(A - c_2) - \lambda(A + x - c_1)}{4 - \lambda^2}. \tag{12}$$

The quantity of products and the price of product endorsement all decrease with the corresponding cost while increase with the rival's cost. The profits are determined

by the expressions $\pi_1^{m1} = [\frac{2(A+x-c_1)-\lambda(A-c_2)}{4-\lambda^2}]^2 - pe_1$ and $\pi_2^{m1} = [\frac{2(A-c_2)-\lambda(A+x-c_1)}{4-\lambda^2}]^2$. Moreover, for the endorser, the endorsement transfer is $pe_1^{m1,*}$ satisfying the relationship $\pi_1^{m1} - \pi_1^b = 0$. Or $pe_1^{m1,*} = 4(\frac{x}{4-\lambda^2})^2 + 4x[\frac{2(A-c_1)-\lambda(A-c_2)}{4-\lambda^2}]^2$. It points out that $pe_1^{m1,*} \geq \frac{x^2}{2}$. (If $pe_1^{m1,*} < \frac{x^2}{2}$, the unique endorser has no intention to act as an endorser.)

The profits of the two firms are $\pi_1^{m1} = [\frac{2(A-c_1)-\lambda(A-c_2)}{4-\lambda^2}]^2$ and $\pi_2^{m1} = [\frac{2(A-c_2)-\lambda(A-c_1)}{4-\lambda^2}]^2 + \lambda^2(\frac{x}{4-\lambda^2})^2 - \frac{2(A-c_2)-\lambda(A-c_1)}{(4-\lambda^2)^2} 2\lambda x$.

The social welfare (SW) is the consumer surplus (CS) plus the producer surplus as well as the endorser surplus. The above analysis implies that product endorsement hurts producer surplus but improves consumer surplus.

$$SW^{m1} = CS^{m1} + PS^{m1} + pe_1^{m1,*} - c_0 = \frac{3}{2}(q_1^{m1,*})^2 + \frac{3}{2}(q_2^{m1,*})^2 - \lambda q_1^{m1,*} q_2^{m1,*} - c_0$$
$$= \frac{1}{2(4-\lambda^2)^2} \{3[2(A+x-c_1)-\lambda(A-c_2)]^2 + 3[2(A-c_2)-\lambda(A+x-c_1)]^2 \quad (13)$$
$$- 2\lambda[2(A+x-c_1)-\lambda(A-c_2)][2(A-c_2)-\lambda(A+x-c_1)]\} - c_0.$$

The term $pe_1^{m1,*} - c_0$ represents the utility of the endorser. Moreover, $c_2 > c_1$ implies

$$q_1^{m1,*} - q_2^{m1,*} = \frac{(2+\lambda)(x+c_2-c_1)}{4-\lambda^2} > 0. \quad (14)$$

Furthermore, $q_1^{m1,*} - q_2^{m1,*} > q_1^{b,*} - q_2^{b,*} > 0$, $q_1^{m1,*} + q_2^{m1,*} > q_1^{b,*} + q_2^{b,*}$, $q_1^{m1,*} > q_1^{b,*}$ and $q_2^{m1,*} < q_2^{b,*}$. The first firm launching endorsement enlarges output difference and improves the total demand.

Further denote the prices to be

$$(p_1^{m1,*}, p_2^{m1,*}) = (A(1+x) - \frac{(2-\lambda^2)(A+x-c_1)+\lambda(A-c_2)}{4-\lambda^2},$$
$$A - \frac{(2-\lambda^2)(A-c_2)+\lambda(A+x-c_1)}{4-\lambda^2}). \quad (2)$$

Functions (2) indicates $p_1^{m1,*} > p_1^{b,*}$ and $p_2^{m1,*} < p_2^{b,*}$. Similar to Nie and Chen (2012), this paper addresses price dispersion. Price dispersion means the relative price difference. Denote the price dispersion to be $\eta^{m1} = \frac{|p_1^{m1,*} - p_2^{m1,*}|}{p_1^{m1,*} + p_2^{m1,*}}$. For the benchmark problem, it has $\eta^b = \frac{|p_1^{b,*} - p_2^{b,*}|}{p_1^{b,*} + p_2^{b,*}}$.

4.2. Low efficient firm launching product endorsement

The second firm launches while the first firm does not make product endorsement. In this case, $pe_2 > 0$ and $pe_1 = 0$. This endorser maximises the following problem

$$Max \ pe_2$$
$$S.t. \quad pe_2 \geq c_0. \quad (15)$$

The profit functions of the two producers are restated as follows.

$$\underset{q_1}{Max}\, \pi_1 = (A - q_1 - \lambda q_2)q_1 - c_1 q_1, \tag{16}$$

$$\underset{q_2}{Max}\, \pi_2 = \left[A(1+x) - q_2 - \lambda q_1\right]q_2 - c_2 q_2 - pe_2 \tag{17}$$
$$S.t.\ \pi_2 \geq \pi_2^b.$$

This constitutes a Stackelberg game. The lower-level problem is also concave. The first optimal conditions of the lower-level problem imply

$$\frac{\partial \pi_1}{\partial q_1} = A - 2q_1 - \lambda q_2 - c_1 = 0, \tag{18}$$

$$\frac{\partial \pi_2}{\partial q_2} = A(1+x) - 2q_2 - \lambda q_1 - c_2 = 0. \tag{19}$$

Denote the solution to be $(q_1^{m2,*}, q_2^{m2,*})$. (15)-(19) imply the following

$$q_1^{m2,*} = \frac{2(A - c_1) - \lambda(A + x - c_2)}{4 - \lambda^2}, \tag{20}$$

$$q_2^{m2,*} = \frac{2(A + x - c_2) - \lambda(A - c_1)}{4 - \lambda^2}. \tag{21}$$

The quantity of products and the price of product endorsement all decrease with the corresponding cost while increase with the rival's cost. The profits are correspondingly determined by the expression $\pi_1^{m2} = \left[\frac{2(A - c_1) - \lambda(A + x - c_2)}{4 - \lambda^2}\right]^2$ and $\pi_2^{m1} = \left[\frac{2(A + x - c_2) - \lambda(A - c_1)}{4 - \lambda^2}\right]^2 - pe_2$. Moreover, the endorsement transfer is $pe_2^{m2,*}$ satisfying the relationship $\pi_2^{m2,*} - \pi_2^{b,*} = 0$. Or $pe_2^{m2,*} = 4\left(\frac{x}{4 - \lambda^2}\right)^2 + 4x\left[\frac{2(A - c_2) - \lambda(A - c_1)}{4 - \lambda^2}\right]$. this paper also points out that $pe_2^{m2,*} \geq \frac{x^2}{2}$, while under $pe_1^{m1,*} \geq \frac{x^2}{2}$ and $pe_2^{m2,*} \geq \frac{x^2}{2}$, firm with low efficiency has no motivation to launch product endorsement.

The profits of the two firms are $\pi_2^{m2} = \left[\frac{2(A - c_2) - \lambda(A - c_1)}{4 - \lambda^2}\right]^2$ and

$$\pi_1^{m2} = \left[\frac{2(A - c_1) - \lambda(A - c_2)}{4 - \lambda^2}\right]^2 + \lambda^2 \left(\frac{x}{4 - \lambda^2}\right)^2 - \frac{2(A - c_2) - \lambda(A - c_1)}{(4 - \lambda^2)^2} 2\lambda x.$$

Again, the social welfare is the consumer surplus plus the producer surplus and the endorser's utility.

$$SW^{m2} = CS^{m2} + PS^{m2} + pe_2^{m2,*} - c_0 = \frac{3}{2}(q_1^{m2,*})^2 + \frac{3}{2}(q_2^{m2,*})^2 - \lambda q_1^{m2,*} q_2^{m2,*} - c_0$$
$$= \frac{1}{2(4 - \lambda^2)^2} \left\{ 3[2(A - c_1) - \lambda(A + x - c_2)]^2 + 3[2(A + x - c_2) - \lambda(A - c_1)]^2 \right. \tag{22}$$
$$\left. - 2\lambda[2(A - c_1) - \lambda(A + x - c_2)][2(A + x - c_2) - \lambda(A - c_1)] \right\} - c_0.$$

The term $pe_2^{m2,*} - c_0$ is the endorser's utility. It has $q_1^{m2,*} + q_2^{m2,*} = q_1^{m1,*} + q_2^{m1,*}$, $|q_1^{m2,*} - q_2^{m2,*}| < |q_1^{m1,*} - q_2^{m1,*}|$, $q_1^{m2,*} < q_1^{m1,*}$ and $q_2^{m2,*} > q_2^{m1,*}$. Moreover, the output difference is also denoted as follows:

$$q_1^{m2,*} - q_2^{m2,*} = \frac{(2+\lambda)(c_2 - c_1 - x)}{4 - \lambda^2}. \tag{23}$$

The sign of the above expression depends on parameters, including cost difference and the celebrity's popularity. Further denote the price to be $(p_1^{m2,*}, p_2^{m2,*})$. It has $p_1^{m2,*} < p_1^{b,*}$ and $p_2^{m2,*} > p_2^{b,*}$. Moreover, $q_1^{m2,*} + q_2^{m2,*} = q_1^{m1,*} + q_2^{m1,*}$ implies $p_1^{m2,*} + p_2^{m2,*} = p_1^{m1,*} + p_2^{m1,*}$. Price dispersion is $\eta^{m2} = \frac{|p_1^{m2,*} - p_2^{m2,*}|}{p_1^{m2,*} + p_2^{m2,*}}$. For the outputs and price, this paper has the following conclusion

Proposition 1. Under the unique endorser, price difference and price dispersion with the lower efficiency firm launching endorsement are all larger than those under the higher efficiency firm's launching endorsement. Moreover, $\eta^{m2} > \eta^b$.

Proof: See in Appendix. ■

Remarks: Firm's endorsement improves the price of the corresponding products. If the higher efficiency firm launches product endorsement, the price difference reduces. If the lower efficiency firm has product endorsement, the price difference increases. The endorsement has brand effects and this yields the above conclusions. The main conclusion of proposition 1 shows that production efficiency has important impact on product endorsement behaviour and high-efficiency firm has more motivation to engage in product endorsement innovation.

Considering the profit functions under the above two cases, this paper has the following conclusion.

Proposition 2. Under the unique endorser, product endorsement does not bring the firm any direct benefit but it decreases the profits of the rivals. This unique endorser would like to endorse for the high-efficiency firm. When the endorser has better celebrity, the firm without endorsement incurs a greater loss. Furthermore, profits satisfy the relationship $\pi_1^b = \pi_1^{m1} > \pi_1^{m2}$ and $\pi_2^b = \pi_2^{m2} > \pi_2^{m1}$.

Proof: See in Appendix. ■

Remarks: Under the unique endorser, endorsement increases the output of the endorsed firm. When the higher-efficiency firm launches endorsement, the extra profits are larger than that when the lower efficiency firm has endorsement. All the extra profits are taken by the endorser because he/she is the monopolistic endorsement offer. Therefore, the endorser would like to act as the endorser of the higher-efficiency firm. Proposition 1 implies that firm would prefer to launch endorsement advertising even if it brings the firm with no direct benefit because endorsement has strategy effects of reduce rival's profit. This conclusion cannot obtain by those studies based on management or marketing perspective, which partly explains why this study is valuable to be carry out.

Here this paper compares the social welfare under the above two cases. The following conclusion holds. $q_1^{m2,*} + q_2^{m2,*} = q_1^{m1,*} + q_2^{m1,*}$ and $|q_1^{m2,*} - q_2^{m2,*}| < |q_1^{m1,*} - q_2^{m1,*}|$

jointly imply the relationship $q_1^{m2,*} q_2^{m2,*} > q_1^{m1,*} q_2^{m1,*}$, $CS^{m2} < CS^{m1}$ and $PS^{m2} < PS^{m1}$. Therefore, the following conclusion is adhered.

Proposition 3. Under the unique endorser condition, the consumer surplus and the producer surplus of the first case are all larger than those of the second case. Moreover, $SW^{m2} < SW^b$ and $SW^{m2} < SW^{m1}$. With small c_0, celebrity endorsement improves social welfare. Otherwise, celebrity endorsement reduces social welfare.

Proof: See in appendix. ∎

Remarks: The higher-efficiency firm's endorsement yields larger price difference, larger consumer surplus and larger producer surplus. The first firm's endorsement improves the social welfare correspondingly. Celebrity endorsement improves consumer surplus, while reduces producer surplus. Taking the cost of celebrity endorsement, the above conclusions are achieved.

5. Duopoly product endorsements

Here it discusses the situation with two product endorsers.[8] Three cases are addressed: the first case is that the first (high efficiency) firm launches endorsement while the second (low efficiency) one does not; the second is the reverse meaning that the second firm has endorsement while the first firm does not; and the third case is that the two firms simultaneously choose their own product endorsement. It points out that the first case and the second case are different from section 3 because the endorsers are competitive in this section, so they do not have enough power to take all the extra profits resulting from endorsement. In this case, the maximum value of the endorser is equal to its opportunity costs or $pe_i = u(x_i) = \frac{1}{2} x_i^2$. Two endorsers are different in performer Q rating and value. High-efficient producer prefers the high-value endorser and the high Q rating endorser likes to endorse for high-efficient firm.

5.1. The first firm uniquely launching endorsement

In this case, $pe_2 = 0$ while $pe_1 > 0$. The profit functions are restated

$$\underset{q_1, qr_1}{Max}\, \pi_1 = \left[A(1 + x_1) - q_1 - \lambda q_2\right]q_1 - c_1 q_1 - u(x_1), \tag{24}$$

$$\underset{q_2}{Max}\, \pi_2 = (A - q_2 - \lambda q_1)q_2 - c_2 q_2. \tag{25}$$

π_1 is concave in q_1 and qr_1, and π_2 is concave in q_2. This implies the existence of a unique solution and this unique solution is determined by the first order optimal conditions. The first optimal conditions of the lower-level problem imply

$$\frac{\partial \pi_1}{\partial q_1} = A(1 + x_1) - 2q_1 - \lambda q_2 - c_1 = 0, \tag{26}$$

$$\frac{\partial \pi_1}{\partial qr_1} = Aq_1 - x_1 = 0, \tag{27}$$

$$\frac{\partial \pi_2}{\partial q_2} = A - 2q_2 - \lambda q_1 - c_2 = 0. \tag{28}$$

The equilibrium solution of (26)-(28) is

$$q_1^{c1,*} = \frac{2(A-c_1)-\lambda(A-c_2)}{4-2A^2-\lambda^2}, q_2^{c1,*} = \frac{(2-A^2)(A-c_2)-\lambda(A-c_1)}{4-2A^2-\lambda^2},$$

$$x_1^{c1,*} = A\frac{2(A-c_1)-\lambda(A-c_2)}{4-2A^2-\lambda^2}, p_1^{c1,*} = A - \frac{(2-2A^2-\lambda^2)(A-c_1)}{4-2A^2-\lambda^2} - \lambda\frac{(A-c_2)}{4-2A^2-\lambda^2},$$

$$p_2^{c1,*} = A - \frac{(2-A^2-\lambda^2)(A-c_2)}{4-2A^2-\lambda^2} - \lambda\frac{(A-c_1)}{4-2A^2-\lambda^2},$$

$$\pi_1^{c1} = \left(1-\frac{A^2}{2}\right)\left[\frac{2(A-c_1)-\lambda(A-c_2)}{4-2A^2-\lambda^2}\right]^2, \pi_2^{c1} = \left[\frac{(2-A^2)(A-c_2)-\lambda(A-c_1)}{4-2A^2-\lambda^2}\right]^2. \tag{29}$$

(29) implies $p_1^{c1,*} - p_2^{c1,*} = (2-A^2-\lambda^2-\lambda)\frac{(c_1-c_2)}{4-2A^2-\lambda^2} + \frac{A^2(A-c_1)}{4-2A^2-\lambda^2}$. The sign of price difference mainly depends on the cost difference and parameter A. Denote the price dispersion to be $\eta^{c1} = \frac{|p_1^{c1,*} - p_2^{c1,*}|}{p_1^{c1,*} + p_2^{c1,*}}$ and the corresponding social welfare to be $SW^{c1} = CS^{c1} + PS^{c1} + pe_1^{c1,*} - \frac{(qr_1^{c1,*})^2}{2} = \frac{3}{2}(q_1^{c1,*})^2 + \frac{3}{2}(q_2^{c1,*})^2 - \lambda q_1^{c1,*} q_2^{c1,*} - \frac{(x_1^{c1,*})^2}{2}$ and the term $pe_1^{c1,*} - \frac{(x_1^{c1,*})^2}{2}$ is the endorser's utility.

5.2. The second firm uniquely launching endorsement

In this case, $pe_2 > 0$ while $pe_1 = 0$. The profit functions are rewritten as

$$\underset{q_1}{Max}\,\pi_1 = (A-q_1-\lambda q_2)q_1 - c_1 q_1, \tag{30}$$

$$\underset{q_2, qr_2}{Max}\,\pi_2 = [A(1+x_2)-q_2-\lambda q_1]q_2 - c_2 q_2 - u(x_2). \tag{31}$$

The equilibrium solution of (30)-(31) is

$$q_1^{c2,*} = \frac{(2-A^2)(A-c_1)-\lambda(A-c_2)}{4-2A^2-\lambda^2}, q_2^{c2,*} = \frac{2(A-c_2)-\lambda(A-c_1)}{4-2A^2-\lambda^2},$$

$$x_2^{c2,*} = A\frac{2(A-c_2)-\lambda(A-c_1)}{4-2A^2-\lambda^2}, p_1^{c2,*} = A - \frac{(2-A^2-\lambda^2)(A-c_1)}{4-2A^2-\lambda^2} - \lambda\frac{(A-c_2)}{4-2A^2-\lambda^2},$$

$$p_2^{c2,*} = A - \frac{(2-\lambda^2-2A^2)(A-c_2)}{4-2A^2-\lambda^2} - \lambda\frac{(A-c_1)}{4-2A^2-\lambda^2},$$

$$\pi_1^{c2} = \left[\frac{(2-A^2)(A-c_1)-\lambda(A-c_2)}{4-2A^2-\lambda^2}\right]^2, \pi_2^{c2} = \left(1-\frac{A^2}{2}\right)\left[\frac{2(A-c_2)-\lambda(A-c_1)}{4-2A^2-\lambda^2}\right]^2. \tag{32}$$

From (32), it has $p_1^{c2,*} - p_2^{c2,*} = (2 - A^2 - \lambda^2 - \lambda)\frac{(c_1 - c_2)}{4 - 2A^2 - \lambda^2} - \frac{A^2(A - c_2)}{4 - 2A^2 - \lambda^2}$. By (32) and (29), this paper obtains $q_1^{c1,*} + q_2^{c1,*} > q_1^{c2,*} + q_2^{c2,*}$. Denote $\eta^{c2} = \frac{|p_1^{c2,*} - p_2^{c2,*}|}{p_1^{c2,*} + p_2^{c2,*}}$. Further denote the social $SW^{c2} = CS^{c2} + PS^{c2} + pe_2^{c2,*} - \frac{(x_2^{c2,*})^2}{2} = \frac{3}{2}(q_1^{c2,*})^2 + \frac{3}{2}(q_2^{c2,*})^2 - \lambda q_1^{c2,*} q_2^{c2,*}$ $- \frac{(x_2^{c2,*})^2}{2}$. The term $pe_2^{c2,*} - \frac{(qr_2^{c2,*})^2}{2}$ is the endorser's utility. Comparing (29) and (32), it has

Proposition 4. $x_1^{c1,*} > x_2^{c2,*}$, $x_1^{c1,*} > x_1^{c3,*}, x_2^{c2,*} > x_2^{c3,*}$ and $p_1^{c2,*} < p_1^{c1,*}$, $p_2^{c2,*} > p_2^{c1,*}$. Moreover, $\eta^{c2} > \eta^{c1}$.

Proof. See in Appendix. ∎

Remarks: Product endorsement improves the brand and the price of the corresponding product, which is common conclusion. However, proposition 4 shows that high efficiency has more motivation to launch product endorsement again. Empirical evidence is that leading enterprise are more like to engage in endorsement advertising in reality. This yields the similar conclusions as monopoly condition of section 4, which illustrates the conclusions are robust.

5.3. Two firms making endorsement

In this case, the two firms simultaneously launch product endorsement. $pe_2 > 0$ and $pe_1 > 0$. The profits are

$$\underset{q_1, qr_1}{Max}\, \pi_1 = \left[A(1 + x_1) - q_1 - \lambda q_2\right]q_1 - c_1 q_1 - u(x_1), \tag{33}$$

$$\underset{q_2, qr_2}{Max}\, \pi_2 = \left[A(1 + x_2) - q_2 - \lambda q_1\right]q_2 - c_2 q_2 - u(x_2). \tag{34}$$

The corresponding equilibrium solution is

$$q_1^{c3,*} = \frac{(2 - A^2)(A - c_1) - \lambda(A - c_2)}{(2 - A^2)^2 - \lambda^2}, q_2^{c3,*} = \frac{(2 - A^2)(A - c_2) - \lambda(A - c_1)}{(2 - A^2)^2 - \lambda^2},$$

$$x_1^{c3,*} = A\frac{(2 - A^2)(A - c_1) - \lambda(A - c_2)}{(2 - A^2)^2 - \lambda^2}, x_2^{c3,*} = A\frac{(2 - A^2)(A - c_2) - \lambda(A - c_1)}{(2 - A^2)^2 - \lambda^2},$$

$$p_1^{c3,*} = A\left[1 + A\frac{(2 - A^2)(A - c_1)}{(2 - A^2)^2 - \lambda^2}\right] - \frac{(2 - A^2 - \lambda^2)(A - c_1)}{(2 - A^2)^2 - \lambda^2} - \lambda\frac{(A - c_2)}{(2 - A^2)^2 - \lambda^2},$$

$$p_2^{c3,*} = A\left[1 + A\frac{(2 - A^2)(A - c_2)}{(2 - A^2)^2 - \lambda^2}\right] - \frac{(2 - A^2 - \lambda^2)(A - c_2)}{(2 - A^2)^2 - \lambda^2} - \lambda\frac{(A - c_1)}{(2 - A^2)^2 - \lambda^2},$$

$$\pi_1^{c3} = \left(1 - \frac{A^2}{2}\right)\left[\frac{(2 - A^2)(A - c_1) - \lambda(A - c_2)}{(2 - A^2)^2 - \lambda^2}\right]^2, \pi_2^{c3} = \left(1 - \frac{A^2}{2}\right)\left[\frac{(2 - A^2)(A - c_2) - \lambda(A - c_1)}{(2 - A^2)^2 - \lambda^2}\right]^2.$$

$$\tag{35}$$

By (35), $x_1^{c3,*}>x_2^{c3,*}$. Similarly, the social welfare under this case is presented as follows: $SW^{c3} = \frac{3}{2}(q_1^{c3,*})^2 + \frac{3}{2}(q_2^{c3,*})^2 - \lambda q_1^{c3,*}q_2^{c3,*} - \frac{(x_2^{c3,*})^2+(x_1^{c3,*})^2}{2}$. Comparing the above three cases in this section, this paper gets the following conclusions

Proposition 5. $q_1^{c2,*}<q_1^{c3,*}<q_1^{c1,*}$ and $q_2^{c2,*}>q_2^{c3,*}>q_2^{c1,*}$. Moreover, $\pi_1^{c2}<\pi_1^{c3}<\pi_1^{c1}$ $\pi_1^{c1}>\pi_1^{b} = \pi_1^{m1}>\pi_1^{c2}$, $\pi_2^{c1}<\pi_2^{c3}<\pi_2^{c2}$ and $\pi_2^{c2}>\pi_2^{b} = \pi_2^{m1}>\pi_2^{c1}$.

Proof: See in appendix. ∎

Remarks: Under competitive endorsers, the two firms fiercely compete both in endorsement and in quantity. The output difference is reduced. Firms are impacted by rival's celebrity endorsement. When firm launches celebrity endorsement, the rival would correspondingly do celebrity endorsement to improve the profits. Compared with $\pi_1^{m1} = \pi_1^{b}$, the extra profits from celebrity endorsement under competitive endorsers are earned by the one-sided endorsement firm. However, the relationship between π_1^{b} and π_1^{c3} (or the relationship between π_2^{b} and π_2^{c3}) seems indeterminate. Proposition 5 illustrates that celebrity endorsement increases firms' quantity and profits and high-efficiency firm has higher motivation to launches product endorsement than the less efficiency one. More importantly, if one firm engages in product endorsement, the best reaction for the rival is to launch product endorsement, too. In other words, product endorsement has trigger effects, which is an important issue in IOT.

6. Conclusions and discussions

Celebrity endorsement is a common competition strategy especially for manufacturers such as perfumes and clothing producers. Many attentions about endorsement have been paid to the Business to Consumer (B2C) sector based on management, but more studies should be offered to the Business to Business (B2B) sector from industrial perspective because celebrity endorsement is far more than purchase motivating. Besides purchase motivating, celebrity endorsement has strategic effects to inhibit rival's quantity and profits and, has significant influence on social welfare. However, those effects are more convenient to be captured by economics perspective. So, by combining industrial organisation theory with marketing theory and using a triple consumer-endorser-producer model, this article captures both the direct and indirect effects of celebrity endorsement strategy. And this study addresses the product endorsement under duopoly both for the unique endorser and for duopoly endorsers. Under those two cases, output difference, price difference, price dispersion and social welfare are all compared. This paper finds that celebrity endorsement improves brand while hurts rivals' profits. More importantly, celebrity endorsement strategies under different conditions have different influence to social welfare.

The policy implications of the conclusions above are outlined as follows. For the celebrities, they should take the social welfare influence into account when they make their endorsement decision, but not only their own income maximisation. For the producer or the managers, they should notice the reactions of the rivals and the characteristic of the endorsers. For example, the best reaction for the less efficient

producer is to make endorsement investment as the higher efficient rival do, while it should not trigger endorsement competition if its efficient rival does not engage celebrity endorsement. Lastly, the government could motivate more efficient firms to choose endorsement strategy because it improves the social welfare. Firms can take the conclusions of this paper as a guideline to make endorsement competition after they realise the indirect influence of endorsement. Endorsers will learn from this article on when and whom they should endorse for by knowing more about the influences of their endorsement behaviour. And this article is also helpful for regulators to improve the social welfare based on celebrity endorsement.

The limitation of this study is that it takes celebrity behaviour as an endogenous variable, but little attention has been paid to celebrity's choice. For example, this paper does not consider celebrity's characteristic and preference, which may have significant effect on his or her endorsement choices. Besides, this paper does not employ monopolistic firm condition and only considers Cournot quantity competition, while other competition condition such as Stacklberg has considerable influence on product endorsement strategy. Some relationships are not obtained because they are ambiguous. These uncertainties come from cost difference, performer Q rating and other factors. Further studies can take the imperfect match-up hypothesis and endorsement uncertainties into consideration because some studies show characteristics changes or negative associations of celebrity endorsers influence celebrity endorsement and weak support for the match-up hypothesis. Besides, our further studies will endow endorsement failure a probability and consider the interaction between the producer and the endorser.

Notes

1. Firm's efficiency is generally measured by marginal costs in economics-oriented studies, especially in industrial organisation (Chen, Nie & Wang, 2015; Chen, Wen & Luo, 2016; Sacco & Schmutzler, 2011; Vives, 2008), and so as this paper.
2. FORTUNE: http://fortune.com/2015/04/19/celebrity-endorsements-gone-wrong/.
3. On one hand, the value of celebrity endorsement is that it promotes perchance by raising consumers' utility. On the other hand, celebrity endorsement is costly and all the expenditure should be reclaimed by higher prices and consumers increasing.
4. Performer Q Rating was first issued in Solomon (1996)' famous book named Consumer Behavior and then Shimp (1997) calculated celebrity's Q rating by questionnaire and Q equals to the percentage of sample who know the celebrity divided the percentage of the total sample rating the celebrity as 'one of the favorites', which measured the attractiveness of the celebrity.
5. Search costs of celebrity are ex-ante expenditure called sunk costs, which do not influence firm decision in economics perspective. And this study will ignore the fail endorsement cases, so we hold on the perfect match-up assumption.
6. Opportunity cost is a major concept in economics and which is different from accounting cost because you do not need to pay this cost but it influences individual's decision.
7. The difference between the two models is that in the former the endorser makes decision without rival, while in the latter both the endorsers and producers make choices under competitive condition.
8. Conclusions under multiple endorsers are almost the same as the duopoly model.

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Appendix

Proof of Proposition 1. It first considers the price difference.

$$p_1^{m1,*} - p_2^{m1,*} = Ax - \frac{(2-\lambda^2+\lambda)(x+c_2-c_1)}{4-\lambda^2},$$
$$p_2^{m2,*} - p_1^{m2,*} = Ax - \frac{(2-\lambda^2+\lambda)(x+c_1-c_2)}{4-\lambda^2}.$$

$c_2 > c_1$ implies $|p_1^{m1,*} - p_2^{m1,*}| < |p_2^{m2,*} - p_1^{m2,*}|$. $p_1^{m2,*} + p_2^{m2,*} = p_1^{m1,*} + p_2^{m1,*}$ implies the relationship $\eta^{m2} > \eta^{m1}$. Moreover, $p_1^{m2,*} < p_1^{b,*}$, $p_1^{m2,*} < p_1^{b,*}$ and $p_2^{m2,*} > p_2^{b,*}$ jointly imply $\eta^{m2} > \eta^{b}$. Conclusion is achieved and the proof is complete. ∎

Proof of Proposition 2. For the unique endorser, it has $\pi_1^{m1} = \pi_1^{b}$, $\pi_2^{m1} < \pi_2^{b}$, $\pi_2^{m2} = \pi_2^{b}$ and $\pi_1^{m2} < \pi_1^{b}$. It therefore has that product endorsement does not bring the firm any direct benefit but it decreases the profits of the rival's.

$c_2 > c_1$ indicates $ep^{m1,*} > ep^{m2,*}$. The endorser earns more from the higher-efficiency firm. Thus, this unique endorser would like to act as the endorsement of firm with the high efficiency.

$\frac{\partial \pi_2^{m1}}{\partial x} < 0$ and $\frac{\partial \pi_1^{m2}}{\partial x} < 0$ imply that when the endorser has better celebrity, the firm without endorsement undertakes a more loss. The relationship $\pi_1^{b} = \pi_1^{m1} > \pi_1^{m2}$ and $\pi_2^{b} = \pi_2^{m2} > \pi_2^{m1}$ seems obvious.

Conclusion is attained and the proof is complete. ∎

Proof of Proposition 3. $CS^{m2} < CS^{m1}$ and $PS^{m2} < PS^{m1}$ imply $SW^{m2} < SW^{m1}$. By comparing SW^{m1}, SW^{m2} and SW^{0},

$$SW^{m1}-SW^0 = \frac{1}{2(4-\lambda^2)^2}\{3[2(A+x-c_1)-\lambda(A-c_2)]^2 + 3[2(A-c_2)-\lambda(A+x-c_1)]^2$$
$$-2\lambda[2(A+x-c_1)-\lambda(A-c_2)][2(A-c_2)-\lambda(A+x-c_1)]\}-c_0-$$
$$\frac{3[2(A-c_1)-\lambda(A-c_2)]^2 + 3[2(A-c_2)-\lambda(A-c_1)]^2 -2\lambda[2(A-c_1)-\lambda(A-c_2)][2(A-c_2)-\lambda(A-c_1)]}{2(4-\lambda^2)^2}.$$

$$SW^{m2}-SW^0 = \frac{1}{2(4-\lambda^2)^2}\{3[2(A+x-c_2)-\lambda(A-c_1)]^2 + 3[2(A-c_1)-\lambda(A+x-c_2)]^2$$
$$-2\lambda[2(A+x-c_2)-\lambda(A-c_1)][2(A-c_1)-\lambda(A+x-c_2)]\}-c_0-$$
$$\frac{3[2(A-c_1)-\lambda(A-c_2)]^2 + 3[2(A-c_2)-\lambda(A-c_1)]^2 -2\lambda[2(A-c_1)-\lambda(A-c_2)][2(A-c_2)-\lambda(A-c_1)]}{2(4-\lambda^2)^2}.$$

It immediately knows that under small c_0, celebrity endorsement improves social welfare. Otherwise, celebrity endorsement reduces social welfare.

Conclusion is attained and the proof is complete. ∎

Proof of Proposition 4. $c_2>c_1$, (29) and (32) indicate $qr_2^{c2,*}<qr_1^{c1,*}$. $p_1^{c2,*}<p_1^{c1,*}$ and $p_2^{c2,*}>p_2^{c1,*}$ are derived from $A^2>0$. Here it proves $x_1^{c1,*}>x_1^{c3,*}, x_2^{c2,*}>x_2^{c3,*}$.

$$x_1^{c1,*}-x_1^{c3,*} = A\left[\frac{2(A-c_1)-\lambda(A-c_2)}{4-2A^2-\lambda^2} - \frac{(2-A^2)(A-c_1)-\lambda(A-c_2)}{(2-A^2)^2-\lambda^2}\right]$$
$$= [2(A-c_1)-\lambda(A-c_2)]\frac{-A^3(2-A^2)}{(4-2A^2-\lambda^2)[(2-A^2)^2-\lambda^2]} + A^3(A-c_1)\frac{1}{(2-A^2)^2-\lambda^2}$$
$$= A^2\{[2(A-c_1)-\lambda(A-c_2)]\frac{-(2-A^2)}{(4-2A^2-\lambda^2)[(2-A^2)^2-\lambda^2]} + (A-c_1)\frac{1}{(2-A^2)^2-\lambda^2}\}$$
$$= \frac{A^3}{(4-2A^2-\lambda^2)[(2-A^2)^2-\lambda^2]}[\lambda(A-c_2)(2-A^2)-(A-c_1)\lambda^2]>0.$$

Similarly, it has $x_2^{c2,*}>x_2^{c3,*}$. $x_1^{c1,*}>x_1^{c3,*}$ and $x_2^{c2,*}>x_2^{c3,*}$ are therefore achieved.
From (29) and (32), it has $|p_1^{c2,*}-p_2^{c2,*}|>|p_1^{c1,*}-p_2^{c1,*}|$. Moreover,

$$p_1^{c2,*} + p_2^{c2,*} = 2A - \frac{(2-A^2-\lambda^2-\lambda)(A-c_1)}{4-2A^2-\lambda^2} - \frac{(2-2A^2-\lambda^2-\lambda)(A-c_2)}{4-2A^2-\lambda^2}$$
$$= 2A - \frac{(2-A^2-\lambda^2-\lambda)[(A-c_1)+(A-c_2)]}{4-2A^2-\lambda^2} + \frac{A^2(A-c_2)}{4-2A^2-\lambda^2}$$
$$< 2A - \frac{(2-A^2-\lambda^2-\lambda)[(A-c_1)+(A-c_2)]}{4-2A^2-\lambda^2} + \frac{A^2(A-c_1)}{4-2A^2-\lambda^2} = p_1^{c1,*} + p_2^{c1,*}.$$

Therefore, $\eta^{c2}>\eta^{c1}$. Conclusion is attained and the proof is complete. ∎

Proof of Proposition 5. From (29, 32) and (35), it obtains $q_1^{c3,*}>q_1^{c2,*}$ and $q_1^{c3,*}<q_1^{c1,*}$. By (29) and (35), the following holds.

$$q_1^{c1,*} - q_1^{c3,*} = \frac{2(A-c_1)-\lambda(A-c_2)}{4-2A^2-\lambda^2} - \frac{(2-A^2)(A-c_1)-\lambda(A-c_2)}{(2-A^2)^2-\lambda^2}$$

$$= \left[\frac{\lambda(A-c_2)}{(2-A^2)^2-\lambda^2} - \frac{\lambda(A-c_2)}{2(2-A^2)-\lambda^2}\right] + (A-c_1)\left[\frac{2}{4-2A^2-\lambda^2} - \frac{(2-A^2)}{(2-A^2)^2-\lambda^2}\right]$$

$$= \left[\frac{\lambda(A-c_2)}{(2-A^2)^2-\lambda^2} - \frac{\lambda(A-c_2)}{2(2-A^2)-\lambda^2}\right] - (A-c_1)\frac{\lambda^2 A^2}{(4-2A^2-\lambda^2)\left[(2-A^2)^2-\lambda^2\right]}$$

$$= \lambda A^2 \frac{(2-A^2)(A-c_2)-\lambda(A-c_1)}{(4-2A^2-\lambda^2)\left[(2-A^2)^2-\lambda^2\right]} = \lambda A^2 \frac{q_2^{c3,*}}{(4-2A^2-\lambda^2)} > 0.$$

By the above analysis, it therefore has $q_1^{c2,*} < q_1^{c3,*} < q_1^{c1,*}$. Similarly, this paper immediately achieves $q_2^{c2,*} > q_2^{c3,*} > q_2^{c1,*}$.

Here it shows $\pi_1^{c2} < \pi_1^{c3} < \pi_1^{c1}$ and $\pi_2^{c1} < \pi_2^{c3} < \pi_2^{c2}$.

$$\pi_1^{c3} - \pi_1^{c2} = \left(1-\frac{A^2}{2}\right)\left[\frac{(2-A^2)(A-c_1)-\lambda(A-c_2)}{(2-A^2)^2-\lambda^2}\right]^2 - \left[\frac{(2-A^2)(A-c_1)-\lambda(A-c_2)}{4-2A^2-\lambda^2}\right]^2$$

$$= \left[(2-A^2)(A-c_1)-\lambda(A-c_2)\right]^2 \left\{\left(1-\frac{A^2}{2}\right)\left[\frac{1}{(2-A^2)^2-\lambda^2}\right]^2 - \left(\frac{1}{4-2A^2-\lambda^2}\right)^2\right\}$$

$$= \frac{\left[(2-A^2)(A-c_1)-\lambda(A-c_2)\right]^2}{2(4-2A^2-\lambda^2)^2\left[(2-A^2)^2-\lambda^2\right]^2}\left\{(2-A^2)(4-2A^2-\lambda^2)^2 - 2\left[(2-A^2)^2-\lambda^2\right]^2\right\}$$

$$= \frac{\left[(2-A^2)(A-c_1)-\lambda(A-c_2)\right]^2}{2(4-2A^2-\lambda^2)^2\left[(2-A^2)^2-\lambda^2\right]^2}\left\{2A^2\left[4-2A^2-\lambda^2+(2-A^2)^2-\lambda^2\right](2-A^2)-A^2(4-2A^2-\lambda^2)^2\right\}$$

$$= \frac{\left[(2-A^2)(A-c_1)-\lambda(A-c_2)\right]^2}{2(4-2A^2-\lambda^2)^2\left[(2-A^2)^2-\lambda^2\right]^2}\left\{2A^2\left[(2-A^2)^2-\lambda^2\right](2-A^2)+A^2\left[4-2A^2-\lambda^2\right]\lambda^2\right\} > 0.$$

$$\pi_1^{c1} - \pi_1^{c3} = \left(1-\frac{A^2}{2}\right)\left[\frac{2(A-c_1)-\lambda(A-c_2)}{4-2A^2-\lambda^2}\right]^2 - \left(1-\frac{A^2}{2}\right)\left[\frac{(2-A^2)(A-c_1)-\lambda(A-c_2)}{(2-A^2)^2-\lambda^2}\right]^2$$

$$= \frac{2-A^2}{2}\left[\frac{2(A-c_1)-\lambda(A-c_2)}{4-2A^2-\lambda^2} + \frac{(2-A^2)(A-c_1)-\lambda(A-c_2)}{(2-A^2)^2-\lambda^2}\right](q_1^{c1,*}-q_1^{c3,*}) > 0.$$

The above inequality holds because $q_1^{c1,*} - q_1^{c3,*} > 0$. It further proves that $\pi_1^b > \pi_1^{c2}$.

$$\pi_1^b - \pi_1^{c2} = \left[\frac{2(A-c_1)-\lambda(A-c_2)}{4-\lambda^2}\right]^2 - \left[\frac{(2-A^2)(A-c_1)-\lambda(A-c_2)}{4-2A^2-\lambda^2}\right]^2 =$$

$$\left[\frac{2(A-c_1)-\lambda(A-c_2)}{4-\lambda^2} + \frac{(2-A^2)(A-c_1)-\lambda(A-c_2)}{4-2A^2-\lambda^2}\right]\left[\frac{2(A-c_1)-\lambda(A-c_2)}{4-\lambda^2} - \frac{(2-A^2)(A-c_1)-\lambda(A-c_2)}{4-2A^2-\lambda^2}\right]$$

$$= \left[\frac{2(A-c_1)-\lambda(A-c_2)}{4-\lambda^2} + \frac{(2-A^2)(A-c_1)-\lambda(A-c_2)}{4-2A^2-\lambda^2}\right]A^2\left[\frac{(A-c_1)}{4-2A^2-\lambda^2} - \frac{4(A-c_1)-2\lambda(A-c_2)}{(4-\lambda^2)(4-2A^2-\lambda^2)}\right]$$

$$= \left[\frac{2(A-c_1)-\lambda(A-c_2)}{4-\lambda^2} + \frac{(2-A^2)(A-c_1)-\lambda(A-c_2)}{4-2A^2-\lambda^2}\right]A^2\lambda\frac{2(A-c_2)-(A-c_1)\lambda}{(4-\lambda^2)(4-2A^2-\lambda^2)}$$

$$= \left[\frac{2(A-c_1)-\lambda(A-c_2)}{4-\lambda^2} + \frac{(2-A^2)(A-c_1)-\lambda(A-c_2)}{4-2A^2-\lambda^2}\right]A^2\lambda\frac{q_1^{b,*}}{4-2A^2-\lambda^2} > 0.$$

It shows that $\pi_1^b < \pi_1^{c1}$.

$$\pi_1^b - \pi_1^{c1} = \left[\frac{2(A-c_1)-\lambda(A-c_2)}{4-\lambda^2}\right]^2 - \left(1-\frac{A^2}{2}\right)\left[\frac{2(A-c_1)-\lambda(A-c_2)}{4-2A^2-\lambda^2}\right]^2$$

$$= [2(A-c_1)-\lambda(A-c_2)]^2 \frac{(4-2A^2-\lambda^2)^2 - \left(1-\frac{A^2}{2}\right)(4-\lambda^2)^2}{(4-\lambda^2)^2(4-2A^2-\lambda^2)^2}$$

$$= [2(A-c_1)-\lambda(A-c_2)]^2 \frac{A^2}{2}\frac{-4(4-2A^2-\lambda^2+4-\lambda^2)+(4-\lambda^2)^2}{(4-\lambda^2)^2(4-2A^2-\lambda^2)^2} < 0.$$

The inequality holds because of $4-2A^2-\lambda^2+4-\lambda^2 > 4-\lambda^2$ and $4-\lambda^2 < 4$.this paper therefore has the relationship $\pi_1^{c2} < \pi_1^{c3} < \pi_1^{c1}$ and $\pi_1^{c1} > \pi_1^b > \pi_1^{c2}$. Similarly, it has the relationship $\pi_2^{c1} < \pi_2^{c3} < \pi_2^{c2}$ and $\pi_2^{c2} > \pi_2^b > \pi_2^{c1}$. Conclusions are attained and the proof is complete. ∎