Rose-Hulman Undergraduate Mathematics Journal

Volume 7
Issue 2

Article 11

Optimizing a Volleyball Serve

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Hope College Mathematics REU Summer '06
Funded by NSF-REU Grant 0243834

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October 14, 2006

1 Introduction

An effective service in volleyball is crucial to a winning strategy. A good serve either will not be returned, resulting in the point, or it will be returned weakly, giving the serving team the advantage. One objective of an effective serve is to give the receivers as little time as possible to react. In this paper we construct a model of a served volleyball and use it to determine how to serve so that, after crossing the net, the ball hits the desired location in the minimal amount of time.

To form a model, the forces acting on the ball must be described mathematically. We consider the three most important forces in the order of their influence on the ball: first the force due to gravity, then air resistance, and finally the force from spin.

2 Dimensions, Parameters, and Notation

A standard volleyball court is a rectangle, 29 ft, 6 in. wide by 59 ft long. A net in the middle separates the court into two squares. The net in NCAA women's volleyball is 7 ft, 4 in. high. Extending from the end of each side of the court is an area, at least 6 ft wide, from which the ball is served. The server can stand in any part of this area to serve (see Figure 1).

A volleyball has a radius, r, of slightly over 4 inches and weighs about 0.60 lb [1]. The ball has shallow grooves along its outer shell which affect the way the air moves around the ball. When hit hard without spin, the ball tends to flutter in the air, moving erratically as it descends toward the ground. This movement, while important to consider when receiving a serve, is difficult to model and will not be considered in this paper.

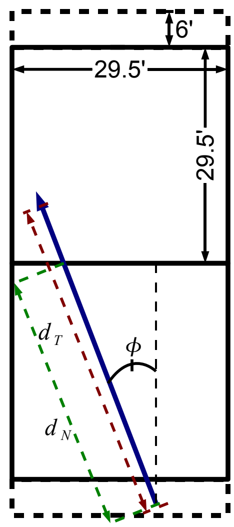

Figure 1: A diagram of the volleyball court. The ball can be served from anywhere in the dotted rectangle at the end of the court.

In the forthcoming models, x will denote the horizontal distance the ball travels and y will be the height of the ball. The initial conditions of the serve will be $x_0 = 0$ and $y_0 = h$, where h is the height from which the ball is served. The serve is struck with an initial speed, $||v_0||$, and an initial angle relative to the ground, θ. The angle ϕ represents the cross-court angle with respect to the sideline, so that a straight serve across the net has ϕ equal to 0. The total distance of flight is d_T and the distance to the net is d_N. The angular velocity of the ball is ω.

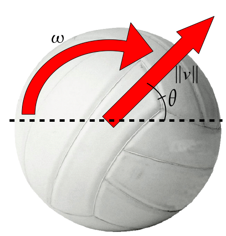

Figure 2: Parameters of a Spinning Volleyball

3 Gravity

3.1 Formulating the Model

The first model to be examined assumes that gravity is the only force acting on the ball. The magnitude of the force due to gravity is mg in the negative y-direction, where m is the mass of the ball and $g = 32.174 ft/sec^2$ is the acceleration due to gravity. Since there is no force in the x-direction, the force equations are:

$$F_x = 0$$

$$F_y = -mg.$$

With no horizontal force on it, the ball will move with constant velocity in the x-direction from the time it is served until the time it lands.

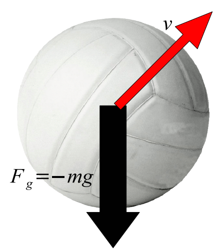

Figure 3: Gravity Force Diagram

3.2 Solving the Model

Since the ball moves with constant velocity in the x-direction, the time the ball is in the air is simply the distance, d, divided by the initial speed in the x-direction:

$$t = \frac{d}{||v_0||cos(\theta)}. \tag{1}$$

Equation 1 applies for all distances $0 \leq d \leq d_T$. Because the acceleration in the y-direction is constant, the following equation can be written for the y-position at time t:

$$y = h + ||v_0||sin(\theta)t - \frac{g}{2}t^2. \tag{2}$$

To find the time when the serve hits the floor, we set y equal to 0. Equation 2 thus becomes a quadratic equation, which can be solved for t:

$$t = \frac{-||v_0||sin(\theta) \pm \sqrt{||v_0||^2 sin^2(\theta) + 2hg}}{-g}. \tag{3}$$

By setting equations 1 and 3 equal to each other (with $d = d_T$), an equation for v_0 in terms of θ can be found:

$$||v_0|| = \sqrt{\frac{g(d_T)^2}{2d_T sin(\theta)cos(\theta) + 2hcos^2(\theta)}}. \tag{4}$$

Equation 4 reveals that for a given starting angle, there is at most one initial speed that will hit the target d_T feet away. If the ball is served any faster than $||v_0||$, it flies over the target. If the ball is served slower than $||v_0||$, it does not reach the target.

3.3 Applying the Model

The only force on the ball is downward, so the higher the ball is served, the longer it will stay in the air. Thus, the serve of minimum time will be the one that reaches the lowest maximum height, while still allowing the ball to clear the net.

To solve for the angle that will barely get the ball over the net, y from equation 2 is set equal to the height of the net, h_N, plus the radius of the ball, $r_B = 0.35ft$. Substituting for t from equation 1 and $||v_0||$ from equation 4 and simplifying, we find the angle of the optimal serve to go distance d_T:

$$\theta = tan^{-1}\left(\frac{\frac{h_N + r_B - h}{d_N} + \frac{hd_N}{d_T^2}}{1 - \frac{d_N}{d_T}}\right). \tag{5}$$

This angle can be substituted into equation 4 to find $||v_0||$, which can then be substituted into equation 1 to find the time of the optimal serve.

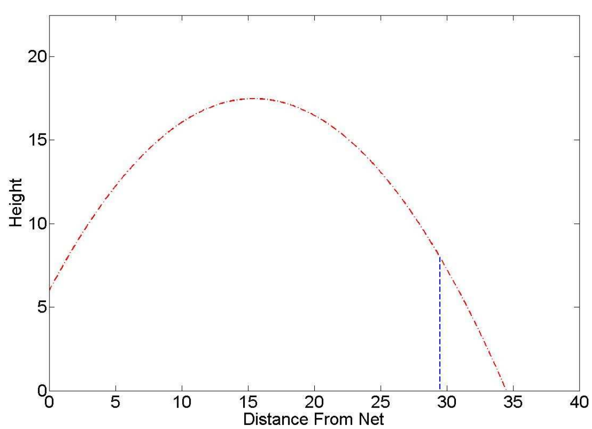

Figure 4: Example Serve Affected by Only Gravity

When the ball will just clear the net while still hitting the target, the optimal serve is hit. An example trajectory of such a serve is shown in figure 4. This serve was hit a total distance of 34.5 ft from a height of 6 ft. The optimal initial angle was 56.1 degrees and the corresponding initial velocity was $32.75 ft/sec$.

4 Gravity + Air Resistance

4.1 Formulating the Model

Drag, or air resistance, is a force that opposes the motion of the ball. The force always acts in a direction opposite to the direction of the velocity, and its magnitude is proportional to the square of the speed [2]:

$$||F_{drag}|| = K_d||v||^2. \tag{6}$$

Breaking the drag force into its x and y components, we get

$$F_x = -K_d||v||^2 cos(\theta) \tag{7}$$

$$F_y = -K_d||v||^2 sin(\theta) - mg. \tag{8}$$

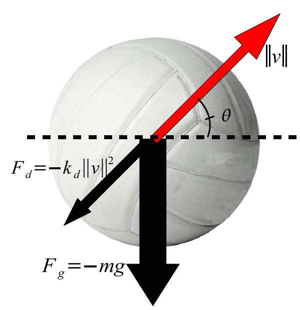

Figure 5: Drag Force Diagram

Writing $||v||cos(\theta)$ as v_x, $||v||sin(\theta)$ as v_y, and $||v||$ as $\sqrt{v_x^2 + v_y^2}$, the force equations become:

$$F_x = -K_d v_x \sqrt{v_x^2 + v_y^2} \tag{9}$$

$$F_y = -K_d v_y \sqrt{v_x^2 + v_y^2} - mg. \tag{10}$$

Using Newton's Second Law, $F = ma$, and the fact that acceleration is the derivative of velocity, equations 9 and 10 can be written as differential equations for the velocity of the volleyball:

$$v'_x = \frac{-K_d v_x}{m}\sqrt{v_x^2 + v_y^2} \tag{11}$$

$$v'_y = \frac{-K_d v_y}{m}\sqrt{v_x^2 + v_y^2} - g. \tag{12}$$

4.2 Solving the Model

To solve this model we must find the constant K_d experimentally. This can be accomplished by allowing the ball to free fall, so that $v_x = v'_x = 0$ for all t. Then equations 11 and 12 simplify to:

$$v'_x = 0 \tag{13}$$

$$v'_y = \frac{-K_d v_y^2}{m} - g. \tag{14}$$

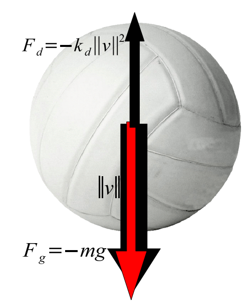

Figure 6: Drag Force Diagram for Free-Fall

Equation 14 can be solved with partial fraction decomposition and integration (see appendix). The resulting free-fall velocity is:

$$v_y = \sqrt{\frac{gm}{K_d}} tanh\left(t\sqrt{\frac{gK_d}{m}}\right). \tag{15}$$

Integrating this equation to find an equation for position is straightforward; substitute $u = 2 * cosh(t\sqrt{\frac{gK_d}{m}})$ in the integration. The derived equation for the y-position in free fall is:

$$y = \frac{m}{K_d}\left(ln(e^{2t\sqrt{\frac{gK_d}{m}}} + 1) - t\sqrt{\frac{gK_d}{m}}.\right) \tag{16}$$

Since the values of g and m are known, we need only to find the time, t, it takes the ball to drop distance y, and the value of K_d can be found.

To accomplish this, the volleyball was dropped from the 3rd floor of the Science Center Atrium at Hope College to the 1st floor. Ten trials were recorded with a camera at 60 frames per second. The ball was dropped a distance of 32.271 ft. Eight drops took exactly 1.485 seconds. The other two differed by just $\frac{1}{60}$ second. Inserting the height and time into equation 17, the proportionality constant K_d was found to be 0.00832. [1]

Once K_d was found, it was possible to write a Matlab program that approximated the trajectories of serves with various initial conditions. This program utilized the Runge-Kutta method of approximation to analyze equations 11 and 12.

4.3 Applying the Model

Armed with our newly obtained proportionality constant, K_d, we set out to test the validity of our drag model. We obtained access to a volleyball launcher used for practicing returning serves. This launcher shoots out a non-spinning serve at varied initial velocities and angles of inclination. By filming the flight of the volleyball, we were able to compare the volleyball launcher's serve to the model's predicted trajectory. Breaking the video into a stop-frame picture and overlaying the theoretical curve, the following image was obtained:

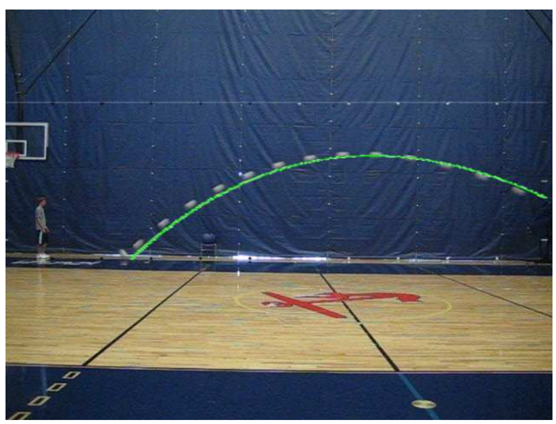

Figure 7: Stop-Frame Volleyball Flight

[1]The proportionality constant, K_d has the form:

$$K_d = \frac{C_d A \rho}{2} \tag{17}$$

where C_d is a dimensionless drag coefficient, ρ is the density of the air, typically $0.0732\frac{lbs.}{ft^3}$, and A is the cross-sectional area of the volleyball, $0.38079ft^2$ [2]. The dimensionless drag coefficient depends on the geometry of the volleyball. For most spherical objects, the drag coefficient is approximately 0.4 [3]. Our experimental value for C_d was 0.36.

Reassured of our model's accuracy, we can now compare the drag model to the theoretical gravity model. To understand the difference between the two models, first look back at figure 4. Recall that the ball was served with an initial angle of 56.1 degrees and an initial velocity of $32.75 ft/sec$ to obtain the optimal serve. If the ball is served in the drag model with these initial conditions, it does not come close to reaching the target of 34.5 ft (see figure 8). The ball must be served at a slightly lower angle, 53 degrees, and a slightly higher velocity, $36 ft/sec$ to overcome the drag force and reach the optimal serve.

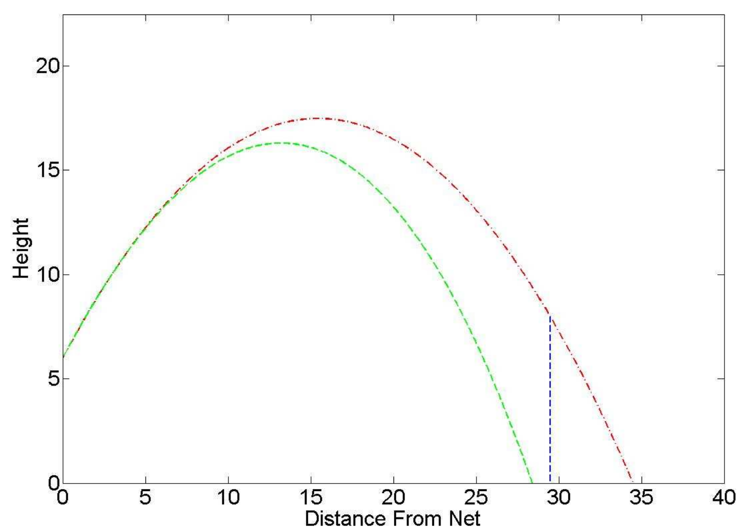

Figure 8: Gravity (Top Curve) and Drag (Bottom Curve) Models at the Same Initial Conditions

We can also vary the initial conditions to better understand how the height, total distance, and cross-court angle, ϕ affect the optimal time of the serve.

The height of the serve, as evidenced by figure 9, has little effect on the total time of the serve. Increasing the height of the serve by 2 ft has only a 0.015 second difference in the total time.

Total distance, on the other hand, has a large effect on the optimal time for a serve (see figure 10). Landing the ball close to the net requires the ball to be served with a high arc. Serving the ball toward the far end-line, however, allows the ball to reach a much lower maximum height. Because gravity is still the dominant force, this lower serve will have a correspondingly lower optimal time.

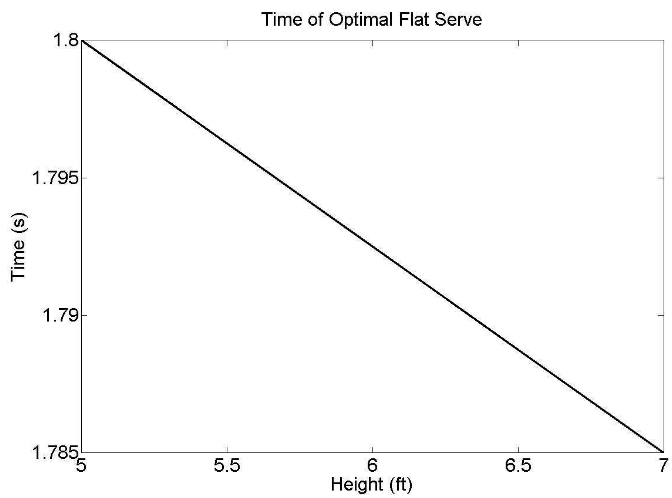

Figure 9: Height of Serve vs. Optimal Time

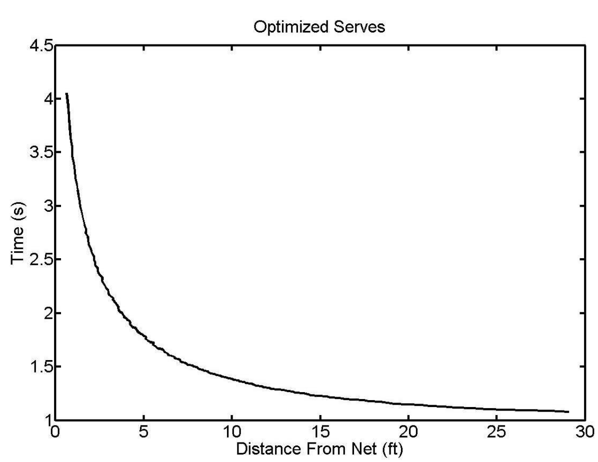

Figure 10: Distance of Serve vs. Optimal Time

The angle cross-court, ϕ, affects the serve in a different way. The optimal time actually remains constant as ϕ increases. That is, hitting the ball five feet past the net straight ahead takes the same time as hitting the ball five feet past the net on the other side of the court. The increased distance that comes with increasing ϕ is offset by the increased velocity that can be applied to the ball. Velocity increases linearly with ϕ (see figure 11). So a cross-court serve may not cut down on the optimal time, but it will allow for a greater serve velocity and perhaps catch the other team by surprise.

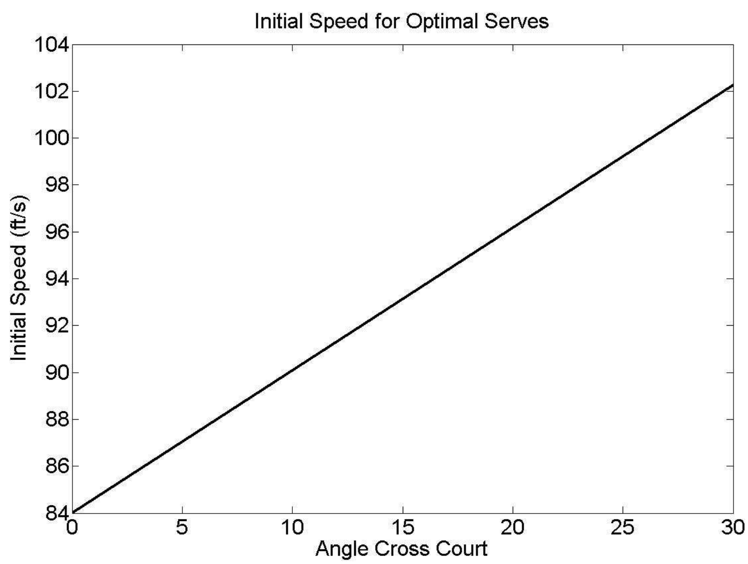

Figure 11: Increasing Velocity with Increased ϕ

5 Gravity + Drag + Spin

5.1 Formulating the Model

Last, we take into account the force due to spin. Spin produces a force that acts perpendicular to the velocity of the ball. To see this, imagine the case where the ball is served horizontally with top-spin, i.e. the top of the ball is spinning in the same direction as the ball's translational motion. On the microscopic level, the ball is moving through individual air molecules as it spins. On the top of the ball, the air molecules are being pulled forward by the spinning ball. However, the air is also being pushed backward as the ball, as a whole, moves. Thus, air molecules accumulate at the top of the ball. On the bottom of the ball, this does not occur. The spin of the ball compensates for the movement of the ball as a whole, pushing the air molecules backward off the ball so there is no accumulation. This difference in accumulation of air molecules creates a higher pressure on top of the ball. So a ball, moving through the air with top-spin, will experience a downward force due to the spin, which makes the ball drop to the ground faster than it would with no spin. Servers typically use top-spin, instead of back-spin, for this reason.

The magnitude of the force due to spin is proportional to the angular velocity and to the velocity of the volleyball. That is,

$$||F_s|| = K_s \omega ||v|| \tag{18}$$

where ω is angular velocity and K_s is the proportionality constant for spin [2].

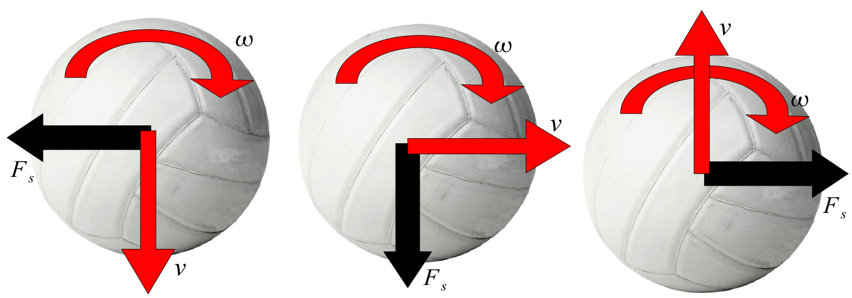

Figure 12: Direction of Spin Force for Various Velocity Directions

Since the force due to spin acts perpendicular to the velocity, when breaking the spin force into components, the force in the y-direction is dependent on the x-velocity and the force in the x-direction is dependent on the y-velocity. With top-spin, the force is in the negative y-direction when the ball is moving horizontally in the positive x-direction, and the force is in the positive x-direction when the ball is moving in the positive y-direction (see Figure 11). This leads to the following modification of equations 9 and 10:

$$F_x = -K_d v_x \sqrt{v_x^2 + v_y^2} + K_s \omega v_y \tag{19}$$

$$F_y = -mg - K_d v_y \sqrt{v_x^2 + v_y^2} - K_s \omega v_x. \tag{20}$$

Again using Newton's Second Law, we get:

$$v_x' = -\frac{K_d v_x}{m} \sqrt{v_x^2 + v_y^2} + \frac{K_s \omega v_y}{m} \tag{21}$$

$$v_y' = -g - \frac{K_d v_y}{m} \sqrt{v_x^2 + v_y^2} - \frac{K_s \omega v_x}{m}. \tag{22}$$

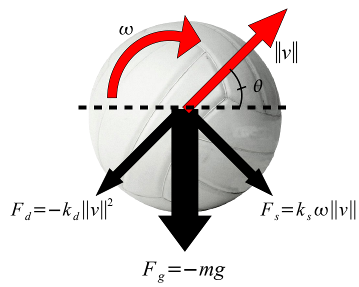

Figure 13: Spin Force Diagram

5.2 Solving the Model

As in the previous model, the spin constant, K_s can be found by dropping a ball in free-fall. By knowing the angular velocity of the ball and the amount of deflection from the non-spinning ball's landing place, the value of K_s can be determined.

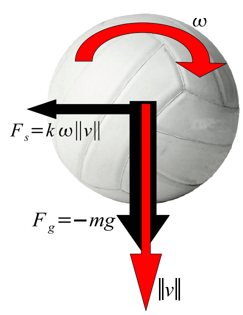

Figure 14: Spin Force Diagram in Free-Fall

In order to perform this free-fall experiment, a volleyball was rigged in the following manner. Two short, hollow cylinders were attached to opposite sides of the ball. Metal rods were inserted into holes in these cylinders so the ball and cylinders could rotate while holding the rods steady. A string was then wrapped around the ball. When the string was pulled free, the ball would spin rapidly while it was dropped from the third floor of the Science Atrium.

First, a non-spinning ball was dropped. Its time of fall and position of contact with the floor were recorded. Five of these drops were performed in order to obtain an average position against which to measure the deflection of the spinning drops. Next, a video camera was used to film the rotation of the ball just before each drop (So that the camera might more easily capture the angular velocity of the ball, a blue strip and a red strip of tape were applied to the volleyball). The position where the ball struck the ground was again recorded. The data for this experiment is shown in figure 15.

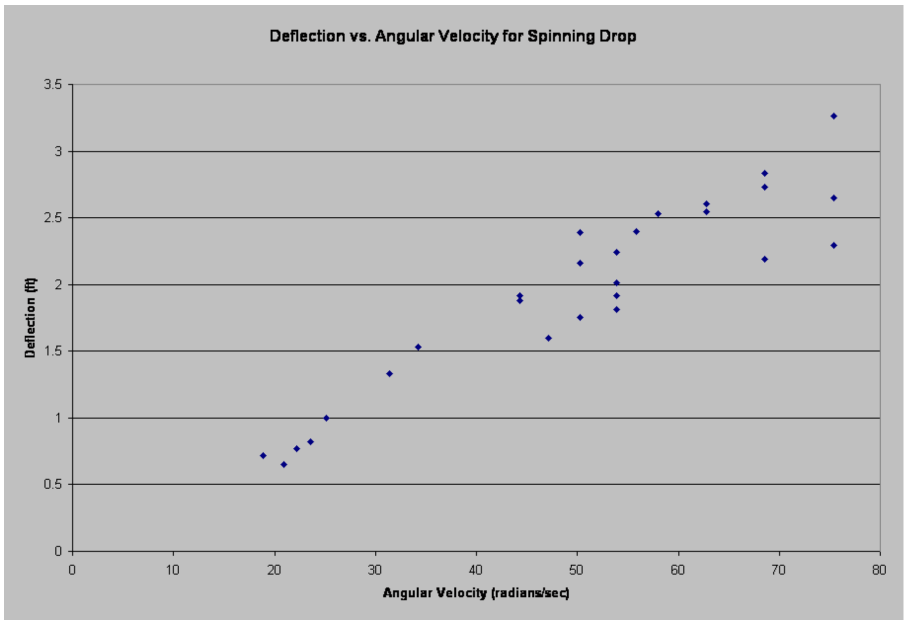

Figure 15: Spin Drop Experiment

In order to find the value of K_s from this data, several simplifying assumptions were made. First of all, since $v_x << v_y$ throughout the fall, we set $v_x = 0$ in equations 22 and 23, giving us:

$$v_x' = \frac{K_s}{m}\omega v_y \tag{23}$$

$$v_y' = -g - \frac{K_d}{m}v_y^2. \tag{24}$$

In equation 25, $\frac{K_d}{m}v_y^2 << g$, so we also make the approximation

$$v_y' = -g \tag{25}$$

Equation 26 can then be integrated to find $v_y = -gt$, and this can be substituted into equation 24 to obtain

$$v_x' = \frac{-K_s \omega g t}{m}. \tag{26}$$

Integrating twice, the x-position at a given time is:

$$x = \frac{-K_s \omega g}{6m} t^3. \quad (27)$$

Inserting the deflection in the x-direction, the angular velocity, and the time measurements into equation 28, a value of K_s can be found for each trial. The average of these values was 0.001452. [2]

5.3 Applying the Model

With the spin coefficient, K_s, now known, we can compare the three models. The Runge-Kutta method of numeric approximation will again be used, this time to analyze equations 21 and 22. Figure 16 shows a serve trajectory from each model. The initial conditions are the same for each: $||v_0|| = 45 ft/sec$ and $\theta = 45$ degrees. The spin model has 7 revolutions/sec. While it is clear the spin model lands in a shorter distance than the drag and gravity models, the figure does not show the time. The spin model hit in 1.66 seconds, compared to 1.80 seconds for drag and 1.97 seconds for gravity. Thus, the differences in the three models becomes apparent.

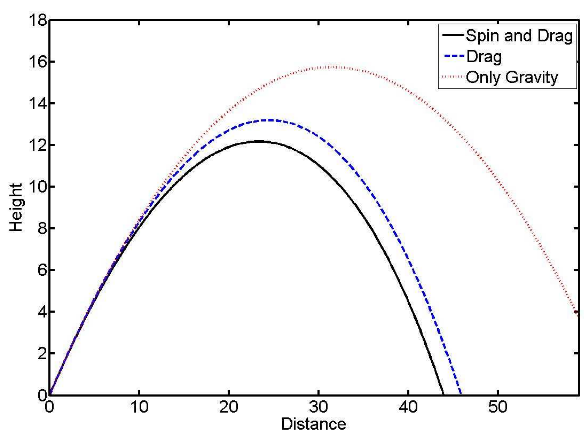

Figure 16: Comparing the Three Models

As in the drag model, the optimal serve time still decreases somewhat with increased serve height and quickly decreases with increased serve distance. Increasing ϕ

[2]This spin proportionality constant has been determined [2] to be:

$$K_s = C_s \frac{\rho}{2} A * r \quad (28)$$

where C_s is a dimensionless spin coefficient, ρ is the density of air, A is the cross-sectional area of the ball, and r is the radius of the ball. Typical spheres have a spin coefficient between 0.25 and 1.0 [4]. Solving for C_s in equation 27, the spin coefficient for our volleyball was found to be 0.30.

still has no effect on the overall time, but allows the ball to be served with much greater velocity.

The new parameter in the spin model, ω, also affects the optimal time and the velocity at which the ball can be served. The time greatly decreases with increased spin because the downward force of the spin propels the ball to the floor faster than without spin. This occurs in a linear relationship, as evidenced by figure 17. Increasing the spin from 0 to 12 revolutions/sec decreases the time of the optimal serve to the end-line by 0.15 seconds, a noticeable change.[3]

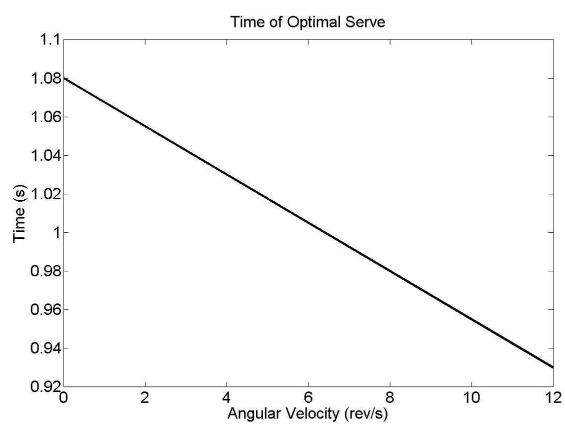

Figure 17: Spin vs. Time

6 Optimal Strategy

Combining the findings of our three models, the optimal strategy for minimizing the serve time can be found. Figures 10 and 17, in particular, will give a volleyball player valuable information from which to build a serving strategy. Increasing the height of the serve decreases the total time only slightly. Increasing the total distance has a large effect on the optimal time. Spinning the ball also has a measurable effect. So the serve with the minimum time in the air is struck when the ball is hit with as much top-spin as possible, toward the end-line. If the server wishes to serve cross-court, this does not affect the overall time, but may be used to serve the ball harder and perhaps catch the defense off-guard, leading to a ”shanked” pass.

[3]Increasing the spin also allows the server to hit the ball with a much greater initial velocity. Although this is not included in our consideration of an optimal serve, it will make the ball much more difficult for the opponent to return. Increasing the spin from 0 to 12 revolutions/sec allows for a 14% increase in initial velocity.

7 Appendix

7.1 Deriving the Free-Fall Velocity Equation for Drag

$$mv' = mg - K_d v^2$$

$$m\frac{dv}{dt} = mg - K_d v^2$$

$$\frac{dv}{mg - K_d v^2} = \frac{dt}{m}$$

$$\frac{dv}{(\sqrt{mg} - \sqrt{K_d}v)(\sqrt{mg} + \sqrt{K_d}v)} = \frac{dt}{m}$$

Partial Fraction decomposition

$$\left(\frac{A}{\sqrt{mg} - \sqrt{K_d}v} + \frac{B}{\sqrt{mg} + \sqrt{K_d}v}\right) dv = \frac{dv}{(\sqrt{mg} - \sqrt{K_d}v)(\sqrt{mg} + \sqrt{K_d}v)}$$

$$A(\sqrt{mg} + \sqrt{K_d}v) + B(\sqrt{mg} - \sqrt{K_d}v) = 1$$

$$(A + B)\sqrt{mg} = 1$$

$$(A - B)\sqrt{K_d}v = 0$$

$$\frac{1}{2\sqrt{mg}} = A = B$$

End Partial Fraction decomposition

$$\int \left(\frac{1}{2\sqrt{mg}}\right)\left(\frac{1}{\sqrt{mg} - \sqrt{K_d}v} + \frac{1}{\sqrt{mg} + \sqrt{K_d}v}\right) dv = \int \frac{dt}{m}$$

$$\left(\frac{1}{2\sqrt{mg}}\right)\left(ln(\sqrt{mg} + \sqrt{K_d}v) - ln(\sqrt{mg} - \sqrt{K_d}v)\right) = \frac{t}{m} + C$$

$$ln\left(\frac{\sqrt{mg} + \sqrt{K_d}v}{\sqrt{mg} - \sqrt{K_d}v}\right) = \frac{2t\sqrt{K_d g} + C}{\sqrt{m}}$$

$$\frac{\sqrt{mg} + \sqrt{K_d}v}{\sqrt{mg} - \sqrt{K_d}v} = e^{\left(2t\sqrt{\frac{K_d g}{m}} + C\right)}$$

$$\sqrt{mg} + \sqrt{K_d}v = e^{\left(2t\sqrt{\frac{K_d g}{m}} + C\right)}\sqrt{mg} - e^{\left(2t\sqrt{\frac{K_d g}{m}} + C\right)}\sqrt{K_d}v$$

$$\sqrt{K_d}v + e^{\left(2t\sqrt{\frac{K_d g}{m}} + C\right)}\sqrt{K_d}v = e^{\left(2t\sqrt{\frac{K_d g}{m}} + C\right)}\sqrt{mg} - \sqrt{mg}$$

$$\sqrt{K_d}v\left(e^{\left(2t\sqrt{\frac{K_dg}{m}}+C\right)}+1\right)=\sqrt{mg}\left(e^{\left(2t\sqrt{\frac{K_dg}{m}}+C\right)}-1\right)$$

$$v=\sqrt{\frac{mg}{K_d}}\left(\frac{e^{\left(2t\sqrt{\frac{K_dg}{m}}+C\right)}-1}{e^{\left(2t\sqrt{\frac{K_dg}{m}}+C\right)}+1}\right)$$

Plug in $t = 0$ and $v = 0$

$$0=\sqrt{\frac{mg}{K_d}}\left(\frac{e^C-1}{e^C+1}\right)$$

$$e^C - 1 = 0$$

$$C = 0$$

Plug in $C = 0$ into equation

$$v=\sqrt{\frac{mg}{K_d}}\left(\frac{e^{\left(2t\sqrt{\frac{K_dg}{m}}\right)}-1}{e^{\left(2t\sqrt{\frac{K_dg}{m}}\right)}+1}\right)$$

$$v=\sqrt{\frac{mg}{K_d}}\left(\frac{e^{\left(2t\sqrt{\frac{K_dg}{m}}\right)}-1}{e^{\left(2t\sqrt{\frac{K_dg}{m}}\right)}+1}\right)\left(\frac{e^{\left(-t\sqrt{\frac{K_dg}{m}}\right)}}{e^{\left(-t\sqrt{\frac{K_dg}{m}}\right)}}\right)$$

$$v=\sqrt{\frac{mg}{K_d}}\left(\frac{e^{\left(t\sqrt{\frac{K_dg}{m}}\right)}-e^{\left(-t\sqrt{\frac{K_dg}{m}}\right)}}{e^{\left(t\sqrt{\frac{K_dg}{m}}\right)}+e^{\left(-t\sqrt{\frac{K_dg}{m}}\right)}}\right)$$

$$v=\sqrt{\frac{mg}{K_d}}tanh\left(t\sqrt{\frac{K_dg}{m}}\right)$$

7.2 Matlab Program Exhibiting the Runge-Kutta Method of Numeric Approximation

```
%Program intro
disp('This program was written to calculate the fastest volleyball
serve'); disp('given several user-defined values. The program
assumes the weight'); disp('of the ball, the height of the net, the
size of the ball, and the'); disp('drag coefficient of the the ball.
Answer questions in feet');

%Input parameters
height = input('What height is the volleyball served from: \n'); phi
```

```
= input('What angle are you serving with respect to the sideline (in
degrees): \n'); dtonet2 = input('What is the shortest distance to
the net from where you are standing: \n'); dfromnet2 = input('How
far from the net would you like it to land: \n'); angularvelocity2 =
input('How much spin is put on the ball (in revolutions per second):
\n');

%Define assumed parameters
netheight = 7.6919; %in ft, includes ball size
global dragcoef; dragcoef = .35; global densityofair;
densityofair = .0729; %temp =70, altitude = 600 ft, in lb/ft^3
global areaball;
areaball = .3808; %in sqare ft
global massball;
massball = 0.59375; %in pounds
global gravity;
gravity = -32.174; %ft/s/s
global radius; %ft
radius = 0.3448;
global angularvelocity; %????
angularvelocity=angularvelocity2*2*pi; global spincoef; spincoef =
.298;

%Calculate true distance from and to net
phiradians = phi*pi/180; dtonet = dtonet2/cos(phiradians); dfromnet
= dfromnet2/cos(phiradians);

%Initial loop, moves large incriments
q=0; for theta=0:(pi/90):(pi/2)
    for speed=150:-5:0
        V0=[speed*cos(theta) speed*sin(theta)];
        [time,velocities] = ode45('velocityspinsolver',0:.005:5, V0);
        X = zeros (length(time),2);
        X(1,2)=height;
        for i = 1:(length(time)-1)
            X(i+1,1)=(X(i,1)+(time(i+1)-time(i))*velocities(i,1));
            X(i+1,2)=(X(i,2)+(time(i+1)-time(i))*velocities(i,2));
        end
        findnet=(X(:,1)<dtonet);
        if sum(findnet)~= length(time)
            if X(sum(findnet)+1,2)> netheight
                b=sum(X(:,2)>0);
                if (X(b,1)>dtonet) & (X(b,1)<(dtonet+dfromnet))
                    q=1;
                end
            end
```

```
        end
        if q==1
            break
        end
    end
    if q==1
        break
    end
end

%Fine tuned loop: This loop uses the results from the initial loop to
%obtain a result accurate to within a degree and a foot per second
q=0; for theta2=(theta-(pi/10)):(pi/180):(pi/2)
    for speed2=150:-1:(speed-20)
        V0=[speed2*cos(theta2) speed2*sin(theta2)];
        [time,velocities] = ode45('velocityspinsolver',0:.005:5, V0);
        X = zeros (length(time),2);
        X(1,2)=height;
        for i = 1:(length(time)-1)
            X(i+1,1)=(X(i,1)+(time(i+1)-time(i))*velocities(i,1));
            X(i+1,2)=(X(i,2)+(time(i+1)-time(i))*velocities(i,2));
        end
        findnet=(X(:,1)<dtonet);
        if sum(findnet)~= length(time)
            if X(sum(findnet)+1,2)> netheight
                b=sum(X(:,2)>0);
                if (X(b,1)>dtonet) & (X(b,1)<(dtonet+dfromnet))
                    q=1;
                end
            end
        end
        if q==1
            break
        end
    end
    if q==1
        break
    end
end

%Displays results
disp('Ball should be hit at a speed of ') disp(speed2) disp('and an
angle of ') disp(theta2*180/pi) disp(' degrees.') disp('A plot of
the trajectory of the ball is displayed') plot(X(:,1),X(:,2))
```

References

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