An Approach to Predicting Static Properties of Tree Stems

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ABSTRACT

Taper equations make it possible to solve many different problems concerning tree stem mensuration, since they can be used to generate a three-dimensional stem representation (as a body of revolution). Estimating the centre-of-gravity location along the stem length by taper equations proves a suitable approach for predicting such a biomechanical property, at least as regards tree stems with monopodial branching (e.g., stems of many coniferous species). A validation is performed on Monterey pine trees. Models to expand the same approach to predicting stem mass moments of inertia are developed.

Key Words: *Biomechanics, Step Taper Equation, centre of gravity of tree stem, forest equipment design, Monterey pine*

INTRODUCTION

In the last decade, many research works on forest tree mensuration have been developed to analytically describe stem form by taper equations. Taper equations make it possible to solve many different problems concerning stem mensuration, since they can be used to generate a three-dimensional stem representation (as a body of revolution).

An important, often undervalued applicability of taper equations is the estimation of static properties, whose knowledge is especially useful in engineering analysis for design of machines that handle and/or process tree stems, and for the biomechanical study of stem stability.

The present work was planned to develop a model to predict the centre-of-gravity location of tree stems. In order to make the model practical as regards the parameters to be input, the study was oriented to an approach which, being based on a simplified general taper equation, requires only easily available stem data.

A SIMPLIFIED STEM TAPER EQUATION

In forest mensuration, tree stems, chiefly those characterized by monopodial branching (e.g., trees of many coniferous species), can be approximated to bodies of revolution with straight longitudinal axis and circular transversal sections, generated by the following taper equation:

$$d = D\left(\frac{H-h}{H-1.3}\right)^{b} \qquad (1)$$

where:

- d = stem diameter at h height
- H = total stem height
- D = stem diameter at breast height (i.e., stem diameter at 1.3 m-height
- b = equation's parameter

This equation does not prove fully correct from a theoretical point of view, because tree stems are usually regarded as bodies of form sectionable into truncated paraboloids of revolution, each characterized by a different exponent. In practice, however, equations as above (1), with a constant exponent for the whose stem profile, have found a fairly good applicability in forest mensuration. Despite their simple structure, they can provide rather acceptable approximations in stem taper estimations, at least in relation to tree species with monopodial branching [2].

Corona and Ferrara [1] have shown that the applicative profit of using equation (1) is that the b exponent can be directly calculated from already existing total stem volume equations or tables (even if elaborated by graphical smoothing). In fact, if, besides D and H, the stem volume (V) is known, the value of b can be easily computed by the following iterative algorithm. By rearranging the stem volume equation obtainable by integration of the equation (1)

$$V = \left(\frac{\pi}{4}\right)D^2 H \cdot \left(\frac{1}{(2b+1)\left(\frac{H}{H-1.3}\right)}\right)^{2b}$$

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it can be seen that:

$$b = \left\{ \frac{\left(\dfrac{H}{H - 1.3} \right)^{2b}}{\dfrac{2V}{\left(\dfrac{\pi}{4} \right) D^2 H}} \right\}^{-0.5} \tag{2}$$

Thus:

1) b can be calculated by introducing an approximate value of the exponent b' (a 0.7 starting value is generally recommended) into equation (2);
2) if b coincides with b', the calculation is ended; otherwise, the value of b has to be reintroduced as b' into the equation (2) to estimate a new value of b and so forth, until the value of b' and b coincide (an approximation not exceeding the sixth decimal figure requires less than 5-7 iterations).

THE CENTRE-OF-GRAVITY LOCATION

A prediction model

Considering a tree stem as a body of revolution generated by equation (1) and assuming the wood density (μ) constant throughout the stem, it can be seen that the centre of gravity is located on the longitudinal axis (the h-axis in Figure 1), which is symmetric in respect to the elements of the body. The location of the centre of gravity on axis h (G_h) can be estimated as:

$$G_h = \frac{\int_0^H \mu \left(\frac{\pi}{4} \right) d^2 h \delta h}{\int_0^H \mu \left(\frac{\pi}{4} \right) d^2 \delta h} \tag{3}$$

Substituting the value of d with that of equation (1), the following equation is obtained:

$$G_h = \frac{\mu \left(\frac{\pi}{4} \right) D^2 \int_0^H \left(\frac{H - h}{H - 1.3} \right)^{2b} h \delta h}{\mu \left(\frac{\pi}{4} \right) D^2 \int_0^H \left(\frac{H - h}{H - 1.3} \right)^{2b} \delta h}$$

which brings to

$$G_h = \frac{H}{2b + 2} \tag{4}$$

With reference to the bodies of revolution adopted as dendrometrical prototypes, it can be noted that the location of the centre of gravity for stems geometrically similar to apollonic paraboloids (b = 0.5) is equal to H/3 and that it is equal to H/4 for stems geometrically similar to cones (b = 1). However, the whole form of the tree stem rarely has an exact representation by the apollonic paraboloid or by the cone, since, in reality, the most prevalent whole stem forms are intermediate between such solids [3].

The usefulness of the simple equation (4) that allows one to estimate the centre-of-gravity location along the stem by simply knowing H, D and V is evident (the two latter values, together with H, are

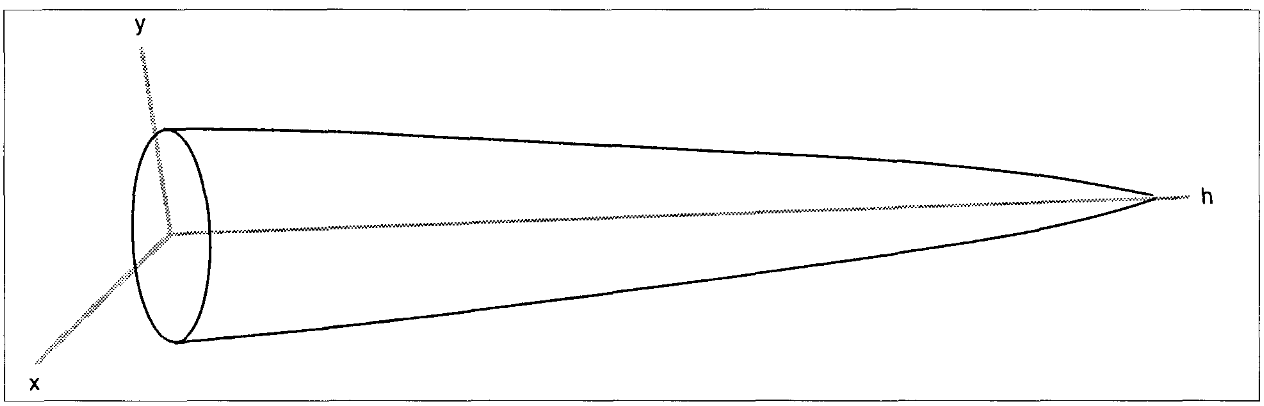

Figure 1. Tree stem seen as a body of revolution with straight longitudinal axis and circular transversal sections.

Table 1. Sampled stems' main characteristics.

tree	dbh	height	volume	centre-of-gravity location	
				observed	predicted
	(m)	(m)	(m^3)	(m)	(m)
1	0.22	14.80	0.261	4.46	4.28
2	0.12	8.60	0.050	2.54	2.50
3	0.15	12.20	0.123	4.00	4.13
4	0.36	10.88	1.022	6.45	6.42
5	0.24	18.20	0.399	5.53	5.38
6	0.20	17.00	0.287	5.62	5.41
7	0.29	17.76	0.554	5.23	5.30
8	0.18	15.10	0.171	4.66	4.43
9	0.26	18.26	0.452	5.44	5.24
10	0.25	20.63	0.470	6.24	6.16
11	0.20	18.55	0.265	5.81	5.62
12	0.26	20.36	0.515	6.50	6.20
13	0.18	17.63	0.220	5.46	5.19
14	0.14	13.50	0.102	4.15	4.30

Table 2. Test statistics of G_h estimation by equation (4) (B = bias; S = standard deviation of the residuals; PVE = percent of variation explained).

$$B = \frac{\Sigma(y_1 - \bar{y}_1)}{N} = 10.9 \text{ cm}$$

$$S = \left(\frac{\Sigma(y_1 - \bar{y}_1)^2 - \frac{\Sigma(y_1 - \bar{y}_1)^2}{N}}{N - 1} \right)^{0.5} = 14.6 \text{ cm}$$

$$PVE = 100\left(\frac{1 - \Sigma(y_1 - \bar{y}_1)^2}{\Sigma(y_1 - \bar{y}_1)^2} \right) = 97.1\%$$

used to compute b, as previously shown).

Validating the model

A validation to evaluate the soundness of the proposed model was carried out on 14 stems of *Pinus radiata D.Don*. The stems were sampled in a 20-year-old plantation (stocking: 540 stems/ha; standing volume: 240 m^3/ha) near Grosseto, in Central Italy (average rainfall: 673 mm/year; mean annual temperature: 15.0°C). The following parameters were measured on each sampled stem (Table 1): dbh and height; volume, by using the Heyer's method; weight and balance point, by horizontally suspending each stem.

The relative location of the centre-of-gravity location has a positive correlation with stem slenderness (Figure 2): dominant trees, characterized by low slenderness, are more coniform, so that present G_h values are lower than dominated trees, which, on the contrary, have more paraboloidical stems characterized by greater slenderness.

The measured location of the centre-of-gravity of each stem was compared with that predicted by equation (4), after having computed b by algorithm (2). In most cases, the residuals between actual and predicted values are positive, indicating that equation (4) tends to estimate G_h in a slightly lower location than the actual one. Actually, equation (1), by simplifying the stem profile, implicitly tends to overconsider the middle part and, on the other hand, to underconsider the bottom and the top parts of the stem (2). However, from a practical point of view, the prediction by equation (4) is satisfactory: the residuals between actual and predicted values are never greater than 5% (Table 1) and the percent of variation explained is high (Table 2).

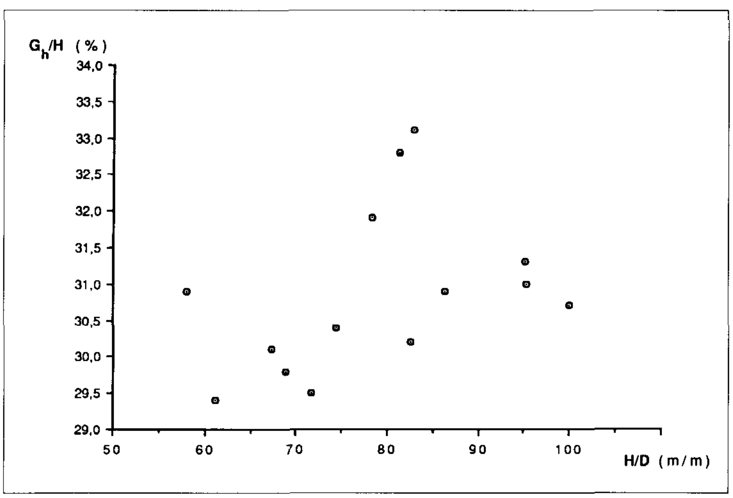

Figure 2. Relative centre-of-gravity location as a function of slenderness ratio.

Discussion

As already mentioned, the usefulness of equation (4) is that it allows estimation of the centre-of-gravity by easily measurable stem parameters (D, H and V). The parameters to be measured become only D and H, if a proper volume equation or table is available; for instance, with reference to the Monterey pine plantation in which model (4) validation was carried out, it can be stressed that the G_h values estimated by the b values computed using the local volume equation

$$V = 0.018550 + 0.346955 D^2 H$$
(std. err. est. = 0.029 m^3

prove quite satisfactory, being the bias equal to 12.1 cm and the percent of variation explained equal to 92.5%, in respect to the observed G_h values.

The model could be improved by taking into account wood density variations throughout the stem, but, even if such variations may be significant, a general model able to quantitatively explain them, so that it could be incorporated into (3), has not yet been stated [5].

PREDICTION MODELS OF STEM MASS MOMENTS OF INERTIA

The results obtained in predicting the centre-of-gravity location by equation (4) suggest an expansion of the proposed approach to predicting stem mass moments of inertia. The present study was not designed to experimentally investigate such a subject; however, it seems interesting to show the possible development of prediction models from a theoretical point of view.

Considering a tree stem as a body of revolution generated by equation (1) and assuming the wood density constant throughout the stem (μ), its mass moment of inertia (I_h) about the longitudinal axis (the h-axis in Figure 1) can be assessed as:

$$I_h = \int_0^H \mu \left(\frac{\pi}{32} \right) d^4 \, \delta h \tag{5}$$

Substituting the value of d with that of equation (1), the following equation is obtained:

$$I_h = \mu \left(\frac{\pi}{32} \right) D^4 \int_0^H \left(\frac{H-h}{H-1.3} \right)^{4b} \delta h$$

which brings to

$$I_h = \mu \left(\frac{\pi}{32} \right) D^4 \cdot (H-1.3)^{-4b} \left(\frac{H^{4b+1}}{4(b+1)} \right) \tag{6}$$

Hence, the mass moment of inertia about the longitudinal axis of a stem can be calculated by simply knowing H, D and V. For instance:

- considering stem no. 3 of Table 1 (D = 0.15 m; H = 12.20 m; V = 0.123 m^3), b is equal to 0.477, so that, with the stem mean wood density about 827 kg/m^3, I_h is equal to 0.214 kg·m^2;
- considering stem no. 4 of Table 1 (D = 0.36 m; H = 20.88 m; V = 1.022 m^3), b is equal to 0.627, so that, with the stem mean wood density about 814 kg/m^3, I_h is equal to 9.387 kg·m^2.

Adopting the same approach, the mass moment of inertia about an axis perpendicular to h-axis and through the butt of the stem (the x-axis in Figure 1) (I_x) (remember that $I_x = I_y$, being the stem seen as a body with circular transversal sections) can be assessed as:

$$I_x = \int_0^H \left[\left(\frac{\pi}{64} \right) \mu d^4 + \left(\frac{\pi}{4} \right) \mu d^2 h^2 \right] \delta h \tag{7}$$

Substituting the value of d with that of equation (1), the following equation is obtained:

$$I_x = \mu\pi \int_0^H \left[\left(\frac{D^4}{64} \right) \cdot \left(\frac{H-h}{H-1.3} \right)^{4b} + \left(\frac{D^2}{4} \right) \cdot \left(\frac{H-h}{H-1.3} \right)^{2b} \cdot h^2 \right] \delta h$$

which brings to

$$I_x = \mu \left(\frac{\pi}{4} \right) D^2 H \cdot \left(\frac{H}{H-1.3} \right)^{2b} \cdot \left\{ \left(\frac{D^2}{16[4(b+1)]} \right) \cdot \left(\frac{H}{H-1.3} \right)^{2b} + \left[\left(\frac{H^2}{2(b+1)} \right) \cdot (b+1) \cdot (2[b+3]) \right] \right\} \tag{8}$$

Hence, the mass moment of inertia about the x-axis can be calculated by simply knowing H, D and V. For instance:

- considering the stem no. 3 of Table 1 ($D = 0.15$m; $H = 12.20$ m; $V = 0.123$ m^3), b is equal to 0.477, so that, with the stem mean wood density about 827 kg/m^3, I_x is equal to 2590 kg·m^2;
- considering the stem no. 4 of Table 1 ($D = 0.36$ m; $H = 20.88$ m; $V = 1.022$ m^3), b is equal to 0.627, so that, with the stem mean wood density about 814 kg/m^3, I_x is equal to 52411 kg·m^2.

It also seems interesting to show the model of estimating the mass moment of inertia (I_{xh}) about an axis (xh) perpendicular to h-axis and through a stem section at height h from the butt. It is known that, using M for stem mass, the following relation exists between I_{xh} and I_x:

$$I_{xh} = I_x - MG_h^2 + M(G_h - h)^2$$

Hence:

$$I_{xh} = \mu\left(\frac{\pi}{4}\right)D^2 H \cdot \left(\frac{H}{H-1.3}\right)^{2b} \cdot \left[\left(\frac{D^2}{16[4(b+1)]}\right)\cdot\left(\frac{H}{H-1.3}\right)^{2b} + \left(\frac{H^2}{2(b+1)}\right)\cdot (b+1)\cdot[2(b+3)] + \left(\frac{h}{2(b+1)}\right)\left(\frac{h-H}{b+1}\right)\right] \quad (9)$$

For instance, considering the stem no. 4 of Table 1 (b = 0.627; μ = 814 kg/m^3), I_{xh} is equal to:

- 23014 kg·m^2, through a stem section at height $h = 4$ m;
- 18155 kg·m^2, through a stem section at height $h = G_h$;
- 129782 kg·m^2, through a stem section at height $h = 18$ m.

CONCLUSIONS

A simple model has been presented to predict the centre-of-gravity location of tree stems with monopodial branching. Models to evaluate stem mass moments of inertia have also been developed by the same approach as the centre-of-gravity location model.

Despite the assumptions in approximating the stem to a body of revolution generated from a unique taper curve for the whole profile, and taking no account of wood density variations throughout the stem, the validation of the centre-of-gravity location model has given satisfactory results. Actually, calculating the values of the centre-of-gravity location along the stem length by a taper equation proves a suitable method for predicting such stem biomechanical properties.

The proposed approach can be generalized: in particular, equations (3), (5) and (7) are usable also with more complex and sophisticated taper equations than those used in the present study. Moreover, they can be used to estimate the static properties of single parts of tree stems (i.e., logs or assortments) by varying integration limits.

The approach is not original. Forslund [3] examined the relationship between stem profile and G_h, elaborating a procedure to estimate numeric coefficients of a taper equation by measuring that parameter: the procedure was interesting from a theoretical point of view, but had poor practicability, since the mensuration of G_h is not currently feasible. A different perspective was examined by Burrows and Fridley [1], who analytically determined G_h, I_h and I_x, using a three-dimensional stem representation generated by a polynomial taper equation. Subsequent research proved that simple taper equations (even if with only one numeric coefficient, as the one used in the present study) are also able to generate stem representations to profitably estimate stem static properties [4].

Obviously, if a taper equation is not available for the trees of interest, taper data (stem diameters at various heights) are needed to estimate the considered biomechanical properties. The models presented in this paper are able to overcome this problem being based on easily available stem data (D, H, V). Their usefulness has to be regarded mainly in respect to the limited and often not available information about the forces to handle stems (or logs): they can contribute to the biomechanical study of stem responses to natural stresses (e.g., by wind or/and wet snow) or help to prevent costly failures or unnecessary overdesign of prototype forest equipment [4].

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