Geometry of a Four-Wheel-Steered Off-Road Vehicle

H. Sakai, T. Matsuo, S. Murayama,
H. Hoshino, and R. Nagashima
Utsunomiya University,
Utsunomiya, Japan

ABSTRACT

An off-road vehicle with a front- and rear-wheel steering capability (4WS) can move sideways, and can avoid obstacles easily, with the same steering phase of both front and rear wheels. With the different steering phase of front and rear wheels, its minimum turning radius is half of that of a vehicle of the same size with only front-wheel steering (2WS). Furthermore, it can make the inner radius difference zero regardless of the wheel base. However, the practical breadth of turning with a minimum radius of a 4WS vehicle is wider than that of an articulated-frame steered vehicle of the same size because the practical minimum inner turning radius of the 4WS is shorter than the inner radii of the inner wheels and the practical minimum outer turning radius is longer than the outer radii of the outer wheels, whereas the practical minimum outer and inner turning radii of the frame-steering whose body is narrower than the overall width equal the outer and inner turning radii of the wheels, respectively. The distance between obstacles in slalom running of a 4WS is 57-62% and 68-71% of that of the 2WS and the frame-steered vehicle of the same size, respectively. The 4WS we tested can efficiently pass through a 48-year-old plantation of hinoki which the same-sized 2WS can not penetrate.

**Key words:** *off-road vehicle, four-wheel steering, slalom running.*

INTRODUCTION

The manoeuverability of off-road vehicles in forest harvesting is of interest to forest engineers. An off-road vehicle with a front- and rear-wheel steering capability (4WS) can make unique movements by its combination of the directions of the front and rear wheels. In forests, many switchbacks in a small

area often result in soil disturbances and downward slips along the slope with the vehicle becoming entangled among the trees and their branches. However, with the same steering phase of both front and rear wheels, the 4WS can move sideways, and can escape easily from such obstacles. And with the different steering phase of front and rear wheels, it has a minimum turning radius that is half of that of a vehicle of the same size with only front-wheel steering (2WS). Furthermore, it can make the inner radius difference zero regardless of the wheel base.

We analyzed the geometry of a 4WS and tested its manoeuverability comparing it with the same-sized 2WS.

THE 4WS TESTED

We tested the small off-road vehicle for forestry use made by the Oikawa Motors Company Ltd (Figure 1). It has four-wheel driving mechanisms with a front- and rear-wheel steering. Its overall width, the front and rear treads, and the width of the tires are 1.42 m, 1.17 m, and 0.25 m, respectively. The width of the body is 1.20 m, which is narrower than the overall width. It can extend the length of the shaft hydraulically together with the length of the bed to a maximum of 0.50 m. The driver can operate the rear steering hydraulically independently of the front steering, even when the vehicle is at a stop. He decides the steering angles of the rear wheels considering the positions of obstacles and the course in a forest. This rear-wheel steering is available not only for moving ahead but also for reversing direction, making switchbacks, and turning the 4WS body perpendicularly to the interval between two trees when passing between them.
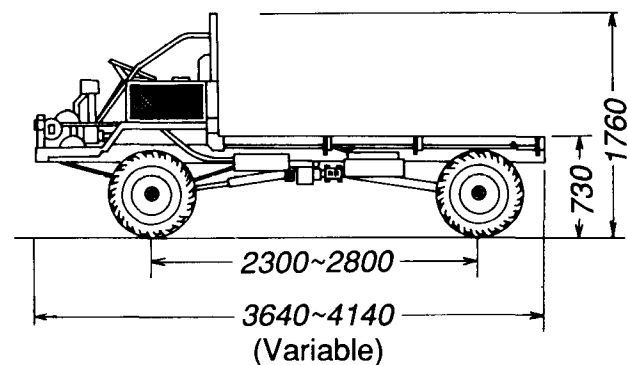
Figure 1. The off-road vehicle with front and rear steering apparatus (4WS).

Geometry of minimum turning radius

The turning radius of each wheel of the vehicle with only Ackermann-Jeantaud front steering is generally as follows [1] if it is turned slowly and centrifugal force can be ignored (Figure 2-a):

$$R_{fo} = L / \sin\theta_{fo}, \tag{1}$$

$$R_{fi} = L / \sin\theta_{fi},$$

$$b_f = R_{fo}\cos\theta_{fo} - R_{fi}\cos\theta_{fi}$$

$$= L(\cot\theta_{fo} - \cot\theta_{fi}),$$

$$R_{ro} = R_{fo}\cos\theta_{fo} + (b_r - b_f)/2$$

$$= L\cot\theta_{fo} + (b_r - b_f)/2, \text{ and}$$

$$R_{ri} = R_{fi}\cos\theta_{fi} - (b_r - b_f)/2$$

$$= L\cot\theta_{fo} - (b_r + b_f)/2,$$

where R is the turning radius, θ is the angle of steering, L is the wheel base, b_f is the front tread, b_r is the rear tread, and the suffixes fi, fo, ri, and ro indicate the inner-front wheel, outer-front wheel, inner-rear wheel, and outer-rear wheel, respectively. Considering the width of the tires, the outer radii of the outer wheels and the inner radii of the inner wheels are as follows:

$$R_{fo}' = R_{fo} + t_f, \tag{2}$$

$$R_{fi}' = R_{fi} - t_f,$$

$$R_{ro}' = R_{ro} + t_r, \text{ and}$$

$$R_{ri}' = R_{ri} - t_r,$$

where t_f is the half width of the front tires, and t_r is that of the rear tires.

R_{fo} with a maximum θ_{fo} is called the minimum turning radius of the vehicle. When the body exceeds the width over the track print in a minimum turn, the distance between the outer-front corner of the body and the center of the turn is called the practical minimum outer turning radius, R_o'', and the distance between the inner body and the center of the turn is the practical minimum inner turning radius, R_i''. If $b_f = b_r$ and $t_f = t_r$, $\tag{3}$

$$R_o'' = \{(R_{ro} + (b_w - b_r)/2)^2 + (L + L_f)^2\}^{1/2},$$

$$R_i'' = R_{ri}' \text{ (when } b_w \le b_r + 2t_r\text{), and}$$

$$R_i'' = R_{ri} - (b_w - b_r)/2 \text{ (when } b_w \ge b_r + 2t_r\text{),}$$

where b_w is the width of the body, and L_f is the front overhang.

The inner-rear wheel passes inside of the radius of the inner-front one. This inner radius difference, E, is

$$E = R_{fi} - R_{ri} \tag{4}$$

$$= L(1 / \sin\theta_{fi} - \cot\theta_{fi}) + (b_r - b_f)/2.$$

From Equations (1) and (4), θ_{fo}, θ_{fi}, R_{fi}, R_{ro}, R_{ri}, and E of a 2WS can be calculated one after another by R_{fo} and L.

On the other hand, the turning radius of each wheel of a 4WS vehicle with a different steering phase of front and rear wheels can be derived geometrically from Figure 2-b as follows when it circles a point:

$$R_{fo}\sin\theta_{fo} + R_{ro}\sin\theta_{ro} = L, \tag{5}$$

$$R_{fi}\sin\theta_{fi} + R_{ri}\sin\theta_{ri} = L,$$

$$R_{ro}\cos\theta_{ro} - R_{fo}\cos\theta_{fo} = (b_r - b_f)/2, \text{ and}$$

$$R_{fi}\cos\theta_{fi} - R_{ri}\cos\theta_{ri} = (b_r - b_f)/2.$$

Therefore,

$$R_{fo} = \{L\cos\theta_{ro} - \sin\theta_{ro} \times (b_r - b_f)/2\} \tag{6}$$

$$/ \sin(\theta_{fo} + \theta_{ro}),$$

$$R_{fi} = \{L\cos\theta_{ri} + \sin\theta_{ri} \times (b_r - b_f)/2\}$$

$$/ \sin(\theta_{fi} + \theta_{ri}),$$

$$R_{ro} = \{L\cos\theta_{fo} + \sin\theta_{fo} \times (b_r - b_f)/2\}$$

$$/ \sin(\theta_{fo} + \theta_{ro}), \text{ and}$$

$$R_{ri} = \{L\cos\theta_{fi} - \sin\theta_{fi} \times (b_r - b_f)/2\}$$
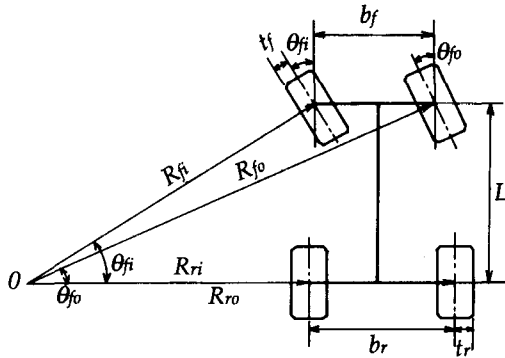
$$/ \sin(\theta_{fi} + \theta_{ri}).$$

From Figure 2-b, the relationship between θ_{fi} and θ_{fo} is

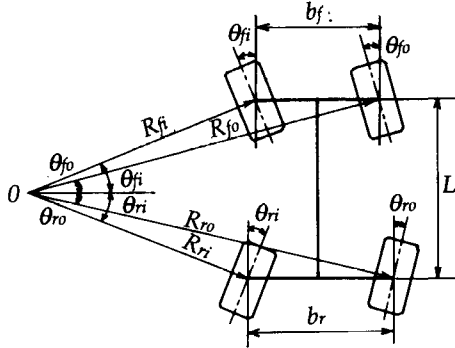$$R_{fo}\cos\theta_{fo} - R_{fi}\cos\theta_{fi} = b_f. \tag{7}$$

This R_{fo} is half of that of a vehicle with only front steering. And E can be reduced to zero regardless of the wheel base.

If $b_f = b_r$, then symmetrically, $\qquad$ (8)
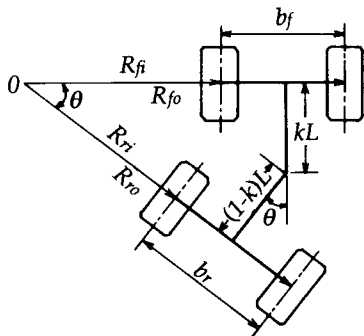
$$R_{fo} = R_{ro} = L / 2\sin\theta_{fo},$$

$$R_{fi} = R_{ri} = L / 2\sin\theta_{fi}, \text{ and}$$

$$b_f = b_r = (\cot\theta_{fo} - \cot\theta_{fi}) \times L / 2.$$

(a) Front-steered vehicle.

(b) Front- and rear- steered vehicle.

(c) Articulated-frame steered vehicle.

Figure 2. Minimum turning radii.

However, its practical minimum inner turning radius generally is

$$R_i'' = R_{fi}\cos\theta_{fi} + (b_f - b_w) / 2, \qquad (9)$$

which is shorter than R_{fi}' because the inner body with a very small turning radius exceeds the track print of the inner tires. The practical minimum outer turning radius is

$$R_o'' = \left\{ (R_{fo}\cos\theta_{fo} + (b_w - b_f) / 2)^2 \right. \qquad (10)$$

$$\left. + (L_o + L / 2)^2 \right\}^{1/2}$$

unless it is shorter than R_{fo}', where $L_o = \text{Max}(L_f, L_r)$, and L_r is the rear overhang. If $t_f = t_r$ and $L\le(b_f - b_w + 2t_f)\sin\theta_{fi} / (1 - \cos\theta_{fi})$, for instance, and if b_w is much narrower than b_f, L is very short, and θ_{fi} is small,

$$R_i'' = R_{fi}'. \qquad (11)$$

The turning radius of the articulated-frame steering is as follows if $b_f = b_r$ (Figure 2-c) [2]:

i) $k \ge 0.5$ $\qquad$ (12)

$$R_{ro}(= R_{max})$$

$$= L / \sin\theta \times \{k + (1 - k)\cos\theta\} + b_r / 2,$$

$$R_{ri} = R_{ro} - b_r,$$

$$R_{fi}(= R_{min}) = L / \sin\theta \times (k\cos\theta + 1 - k)$$

$$-b_f / 2, \text{ and}$$

$$R_{fo} = R_{fi} + b_f,$$

where k is the ratio of the center-pivot location, and θ is the angle of steering.

ii) $k \le 0.5$ $\qquad$ (13)

$$R_{fo}(= R_{max})$$

$$= L / \sin\theta \times (k\cos\theta + 1 - k) + b_f / 2,$$

$$R_{fi} = R_{fo} - b_f,$$

$$R_{ri}(= R_{min})$$

$$= L / \sin\theta \times \{k + (1 - k)\cos\theta\} - b_r / 2,$$

and

$$R_{ro} = R_{ri} + b_r.$$

iii) $k = 0.5$ (14)

$$R_{fo} = R_{ro}$$
$$= L(1 + \cos \theta) / 2 \sin \theta + b_f / 2, \text{ and}$$

$$R_{fo} - R_{fi} = R_{ro} - R_{ri} = b_f.$$

Therefore, $E = 0$. When the width of the body is narrower than the overall width ($b_w \leq b_r + 2t_r$), and when the distance between the outer-front corner of the body and the center of the turn is shorter than R_{fo}' as a vehicle we tested whose front overhang is not so long,

$$R_o'' = R_{fo}', \text{ and} \tag{15}$$

$$R_i'' = R_{ri}',$$

if $t_f = t_r$. When $b_w \geq b_r + 2t_r$, (16)

$$R_o'' = \left\{ (R_{fo} + (b_w - b_f) / 2)^2 + L_{f^2} \right\}^{1/2}, \text{ and}$$

$$R_i'' = R_{ri} - (b_w - b_r) / 2.$$

Measurement of turning radius

We measured the minimum turning radius R_{fo} of the vehicle on level land, with and without the 4WS mechanism, and calculated θ_{fo}, θ_{fi}, R_{fi}, R_{ri}, and R_{ro} from Equations (1) and (8) (Table 1). And the turning radius of an articulated-frame steered vehicle of the same size when $k = 0.5$ is assumed from Equation (14) (Table 1).

It was recognized that the minimum turning radius of the 4WS was about half of that of the 2WS, and that it was smaller than that of the frame-steering. The ratio of the minimum turning radius and the wheel base [2], R_{fo}/L, is theoretically as follows when $b_f = b_r$: the 2WS, $1/\sin\theta_{fo}$; the 4WS, $1/2\sin\theta_{fo}$; the frame-steering when $k = 0.5$, $(1 + \cos\theta)/2\sin\theta + b_f/2L$. That of the 4WS was 0.99-1.10 (Table 1). It is about 50% and 75 % of that of the 2WS and the frame-steering, respectively.

The experimental values of the inner radius difference were similar to the theoretical values (Table 1). The 4WS and the frame-steering realized an inner radius difference of zero.

Breadth of turning

The breadth between the outer and the inner tires with a minimum turning radius B' can be given

as follows on level land without respect to switch-backs:

$$B' = R_o' - R_i', \tag{17}$$

where $R_o' = Max(R_{fo}', R_{ro}')$,

and $R_i' = Min(R_{fi}', R_{ri}')$.

The vehicle we tested was $b_f = b_r$ and $t_f = t_r$, then its theoretical B' values of the 2WS, 4WS, and the frame-steering when $k = 0.5$ are given by the following equations, respectively:

$$B' = L(1 / \sin \theta_{fo} - \cot \theta_{fo}) + b_r + 2t_r, \tag{18}$$

$$B' = L(1 / \sin \theta_{fo} - 1 / \sin \theta_{fi}) / 2 + 2t_r, \text{ and}$$

$$B' = b_r + 2t_r.$$

B' of the frame-steering equals the width of the vehicle. From Figure 2-b and as shown in Table 1, that of the 4WS is narrower than this width, and is the smallest. It was recognized that the real B' of the 4WS was narrower than this width.

The following practical breadth of a minimum turning B" also is an important factor to pass through a forest.

$$B'' = R_o'' - R_i''. \tag{19}$$

R_o'', R_i'', and B" of the vehicle are calculated from the real size (Table 1). B" of the 4WS is narrower with the smallest radius than that of the
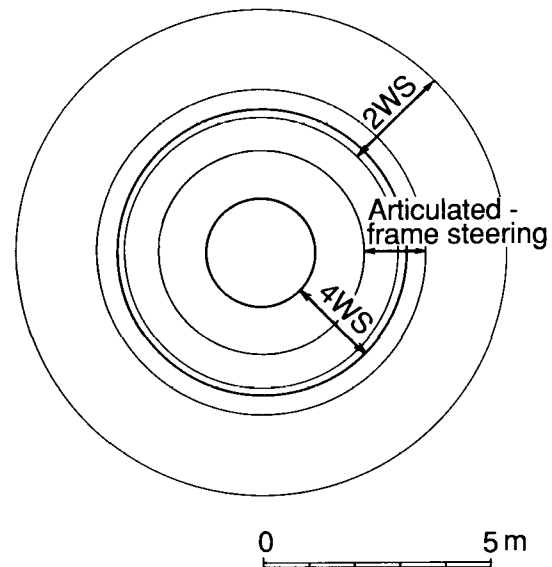
Figure 3. Practical breadth of a minimum turning, B", of the vehicle when L=2.435 m.

Wheel base	2.435 m			2.935 m		
Steering	2WS	4WS	Frame	2WS	4WS	Frame
θ_{fo}	29.7°	27.1°	-	29.7°	30.4°	-
θ_{fi}	38.1°	45.2°	45.1°	36.4°	47.8°	47.8°
		($=\theta_{ri}$)	($=\theta$)		($=\theta_{ri}$)	($=\theta$)
R_{fo}	4.92 m*	2.67 m*	3.52 m	5.93 m*	2.90 m*	3.90 m
R_{ro}	4.28 m	2.67 m	3.52 m	5.15 m	2.90 m	3.90 m
R_{fi}	3.95 m	1.72 m	2.34 m	4.95 m	1.98 m	2.73 m
R_{ri}	3.11 m	1.72 m	2.34 m	3.98 m	1.98 m	2.73 m
R_{fo}/L	2.02*	1.10*	1.45	2.02*	0.99*	1.33
Inner radius difference, E	0.85 m*	0 m*	-	0.90 m*	0 m*	-
(Theoretical values)	(0.84 m)	(0 m)	(0 m)	(0.97 m)	(0 m)	(0 m)
R_o'	5.05 m	2.80 m	3.65 m	6.06 m	3.03 m	4.03 m
R_i'	2.99 m	1.59 m	2.22 m	3.86 m	1.86 m	2.61 m
R_o''	5.39 m	3.14 m	3.65 m	6.38 m	3.40 m	4.03 m
R_i''	2.99 m	1.19 m	2.22 m	3.86 m	1.32 m	2.61 m
$R_o''-R_o'$	0.34 m	0.34 m	0 m	0.33 m	0.37 m	0 m
$R_i'-R_i''$	0 m	0.40 m	0 m	0 m	0.54 m	0 m
Breadth of minimun turning						
$B'=R_o'-R_i'$	2.07 m	1.21 m	1.42 m	2.20 m	1.17 m	1.42 m
$B''=R_o''-R_i''$	2.40 m	1.95 m	1.42 m	2.53 m	2.08 m	1.42 m
Slalom running						
$R_s = 0$ m						
α	36.1°	48.2°	40.7°	32.4°	46.4°	38.2°
W	4.80 m	3.89 m	2.84 m	5.06 m	4.17 m	2.84 m
S	4.34 m	2.68 m	3.81 m**	4.87 m	2.77 m	4.10 m**
$R_s = 0.10$ m						
α	38.7°	52.3°	43.7°	34.7°	50.4°	40.9°
W	5.00 m	4.09 m	3.04 m	5.26 m	4.37 m	3.04 m
S	4.60 m	2.85 m	4.04 m**	5.18 m	2.95 m	4.33 m**
$R_s = 0.25$ m						
α	42.3°	58.1°	47.8°	37.9°	56.0°	44.7°
W	5.30 m	4.39 m	3.34 m	5.56 m	4.67 m	3.34 m
S	4.97 m**	3.06 m	4.34 m**	5.60 m**	3.18 m	4.66 m**

Table 1. Turning radii and performance of slalom running.

Note: *, observations.

** , R_s satisfies the condition of avoiding the scrape of the outer-front corner of the vehicle against the obstacle.
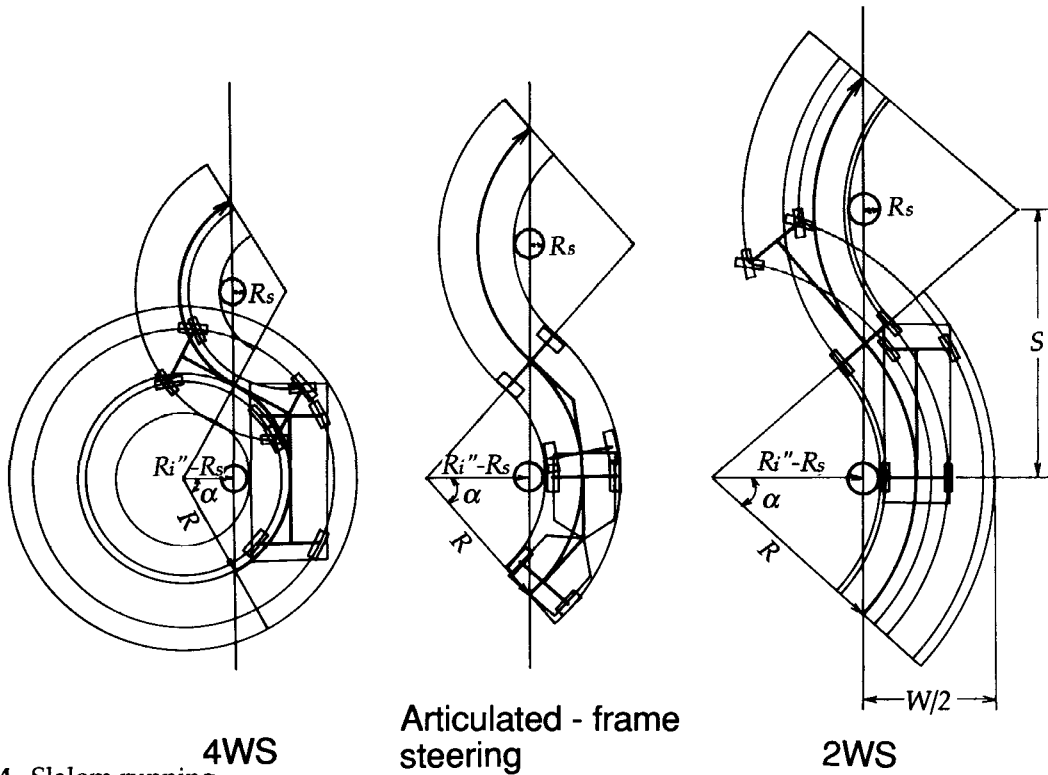
Figure 4. Slalom running.

2WS, but is wider than that of the frame-steering (Figure 3) because its values of $(R_o'' - R_o')$ and $(R_i' - R_i'')$ reach 0.34- 0.54, whereas those of the frame-steering equal zero.

Slalom running

The distance between obstacles in slalom running with a maximum θ_{fo} is an index of estimating the performance in avoiding obstacles [2].

The breadth of the slalom running, W, and the interval between obstacles, S, are as follows if $b_f = b_r$ and $t_f = t_r$ (Figure 4):

$$W = 2(B'' + R_s), \tag{20}$$

$$S = 2R \times \sin \alpha$$

$$= 2\{R^2 - (R_i'' - R_s)^2\}^{1/2}, \text{ and} \tag{21}$$

$$\alpha = \cos^{-1}\{(R_i'' - R_s) / R\}, \tag{22}$$

where R_s is the radius of the obstacles, and R is the minimum distance between the center of the body and the center of the turn, that is,

$$R = R_{ri} + b_r / 2 \tag{23}$$

(when the 2WS and the frame - steering), and

$$R = R_{fi} \cos \theta_{fi} + b_f / 2 \text{ (when the 4WS)}.$$

But in order to avoid the scrape of the outer-front corner of the vehicle against the next obstacle, R_s must satisfy the following condition,

$$\{(R_i'' - R_s)^2 + S^2\}^{1/2} > R_o'' + R_s. \tag{24}$$

In the case of the 4WS vehicle we tested, S is so short and R_o'' becomes relatively large that it cannot satisfy the above condition in a minimum turning, whereas the same-sized articulated-frame steered vehicle satisfies it. As shown in Table 1, the same-sized 2WS also can not satisfy the above condition when R_s is small or too large.

If this scrape can be ignored as a slalom running without real obstacles, W and S of the vehicle with a minimum turning radius are calculated theoretically from R_{fo} (Table 1). The W values are affected by B'', and those of the frame-steered vehicle are the narrowest. The S values of the 4WS are 57-62 % and 68-71 % of those of the 2WS and the frame-steering of the same size, respectively.
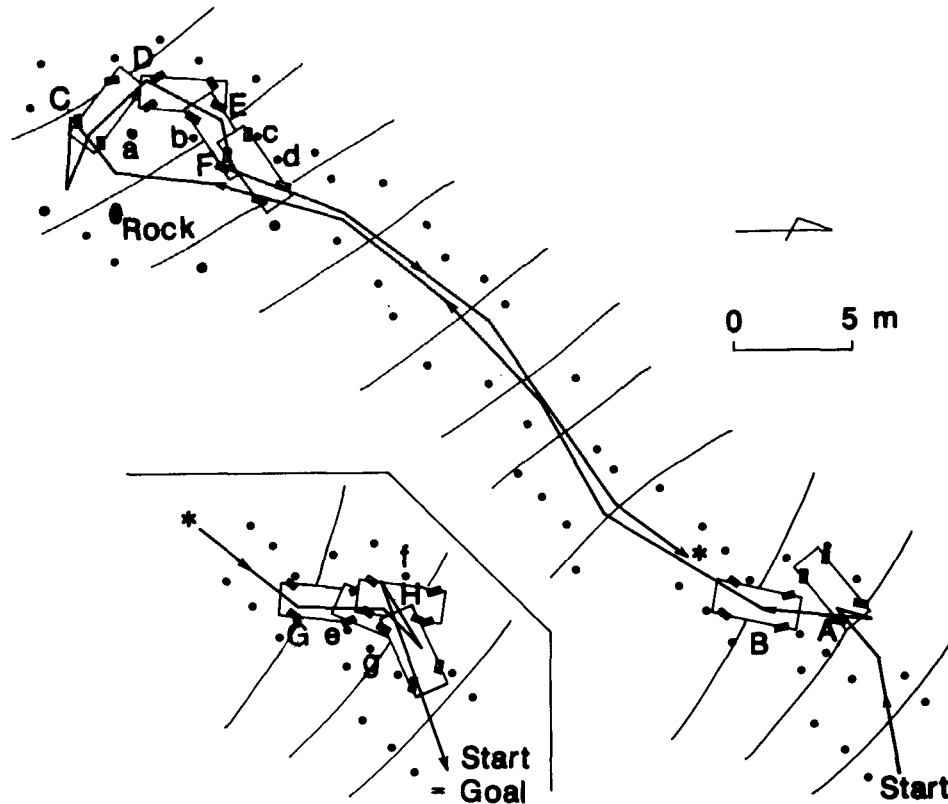
Figure 5. Routes of the 4WS vehicle on the expermental site. (Note: Contour interval is 50 cm.)

An experiment in passing through a forest

The actual movement of the vehicle in avoiding obstacles with the 4WS mechanism was experimented in a stand of the Utsunomiya University Forest at Funyu in Tochigi Prefecture (Figure 5). The experimental site was a 48-year-old plantation of hinoki (*Chamaecyparis obtusa* (Sieb.and Zucc.) Endl.) on a slope averaging 7° which the same-sized 2WS could not penetrate because of its longer turning radius and wider practical breadth of turning than those of the 4WS.

The operator chose on the site the shortest course of going straight up and down the slope and the use of the 4WS mechanism. He used the 4WS mechanism at Points A to H in Figure 4 and could avoid all trees. Especially at Points A, B, and H, the vehicle had to turn at right angles. The 4WS could turn with so small a radius and change the direction of its body so much in a narrow space that it could pass efficiently through the forest. The operations of the 4WS mechanism were as follows:

At Point A, the 4WS backed with the different steering phase of the front and rear wheels, returned

the rear wheels to a neutral position, and then went ahead. At difficult Point B, it turned with the different steering phase without switchbacks, and straightened the rear wheels after turning. At Point C, it backed and changed the direction with the different steering phase. At Point D, it circled Trees a and b with the different steering phase, steered the front wheels with the same steering phase (at Point E), and avoided Tree b. Then it circled Trees c and d with the different steering phase (at Point F), returned the rear wheels to a neutral position, and then went ahead to Point G where it turned with the different steering phase. At Point H, it circled Tree e with the different steering phase, backed a little, returned the rear wheels to the opposite direction, and went around Tree f with the different steering phase. Then it returned the rear wheels to the opposite direction again, went around Tree g with the different steering phase of a large radius, and returned the rear wheels. Finally, it moved backwards and forward twice without the 4WS mechanism and completed passing through the forest.

Running speed, including backing and changing directions, was 0.8 km/h both when going and returning, that is, 51 m per 222 seconds and 62 m per 269 seconds, respectively.

CONCLUSIONS

The 4WS mechanism is of great advantage to move in forests. Furthermore, it is useful for shunting on narrow forest-roads, loading logs at roadsides, and turning on sharp curves.

ACKNOWLEDGEMENT

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