ON THE ULTRA-SONIC MIRAGE

by

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The problem which I have had the occasion to solve and about which I would like to say a few words, concerns one particular case of detection by echo by means of ultra-sonic waves. The greater part of its interest lies in the circumstances under which it came to my notice.

The particular case is that of the effects of mirage which occur during the propagation, in sea water, of sonic or ultra-sonic waves. You are aware that we use the echo method to detect the presence of obtacles and in order to fix their positions. We emit, by means of a piezo-electric projector, a pencil of ultra-sonic waves which is nominally rectilinear and which can be made as narrow as desired. The reflection of these waves by an obstacle gives rise to an echo which can be received by the emitting appliance acting as a receiver. The direction in which this appliance is set determines the direction of propagation of the pencil and, thus, gives the direction in which the obstacle lies and, further, the time taken by the waves to go to the obstacle and return determines the distance thereof. The very first trials and, moreover, other experiments in submarine acoustics, show that serious error may be made if the pencil be assumed to be projected in a straight line. As a matter of fact mirage effects come into action and thus deflect the general direction of the pencil; they are due to variations in velocity of propagation in the water from one point to another. The velocity depends, in fact, on temperature, salinity and pressure and is particularly affected by differences in temperature. In pure water, at temperatures ranging from 0° to 30° C. (32° to 81° F.), it varies from 1400 to 1500 metres (4593 to 4921 feet) per second, and the variation per degree near 0° C. (32° F.) is 1/300. These effects are such that the pencil, particularly when its direction is near the horizontal, undergoes considerable deflection.

As we go down from the surface, velocity of propagation is first reduced on account of the fact that the temperature of the medium is diminishing with the depth, and velocity varies with temperature. In greater depths the influence of the increase of pressure, which diminishes compressibility and consequently increases the velocity, commences to take effect. The result is that, in depths varying from 1000 to 2000 metres (3280 to 6560 feet) depending on local conditions, a minimum velocity is experienced. Down to 1000 metres there is usually a decrease in velocity as the depth increases. This gives a downward deflection of sonic and ultra-sonic rays and this is due to an action the principle of which I venture to recall to your memory, taking the particular case of a ray or pencil which was originally horizontal.

The waves corresponding to the ray or pencil may be considered as plane, within an area which is small with reference to their distance from the source, and these planes are vertical at the outset. As the upper part of these waves is propagated more rapidly than the lower part, owing to the manner in which velocity varies with depth, the direction of the planes of the waves inclines gradually during the propagation, hence there is a downward deflection of the rays which represent the orthogonal trajectories of these wave planes.

A pencil, in the form of a narrow cone, emitted horizontally thus gradually goes towards the bottom instead of running along parallel to the surface and, when the variations of the temperature are somewhat rapid, some inconvenience results, as we have noted ever since we began experiments for the detection of an obstacle lying near the surface. The range obtained by the echo method varies according to circumstances and is greater in the northern seas and in the ocean, where there is a gentle gradient in the temperature, than in seas which have a higher surface temperature, such as the Mediterranean.

I was led to examine the question more deeply and in a more quantitative manner in connection with the trials undertaken by Mr. Georges CLAUDE to determine the profile of the bottom in the vicinity of Havana where he is now setting up a tube for the purpose of bringing cold water up from great depths.

Mirage effects are not experienced when we are making a section of the bottom in the usual manner by means of soundings obtained with vertical ultra-sonic pencils. The velocity of propagation, which is generally but a function of the depth, is always the same throughout the plane of the horizontal waves which occur in a vertical pencil and thus, being symetrically affected, this pencil is not deflected.

The making of a series of vertical soundings along the line for which the profile of the bottom is required is done somewhat after the fashion of cartesian coordinates, *i.e.* the movement of the sounding vessel gives the abscissa and the observed depth gives the ordinate (see fig. 1).

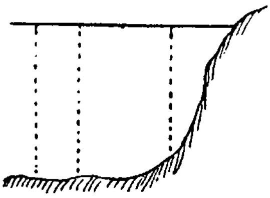

FIG. 1.

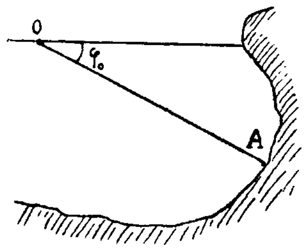

FIG. 2

But this method is not suitable in the case of a profile such as is shown in fig. 2, which Mr. CLAUDE hoped to find in order to set up his tube in a boring through the earth into the sea, thus avoiding the movement near the surface of the water. Such profiles may be obtained, by a process which I call that of "Polar Coordinates", from a fixed station of the vessel (O, fig. 2) by emitting an ultra-sonic pencil in various directions in a vertical plane, beginning with the horizontal, and determining the radius vector OA for each direction by the echo method, the vertical angle φ_0 being given by the inclination of the emitting appliance.

We have constructed an apparatus which enables us to make the necessary changes in inclination about a horizontal axis, but it is necessary to go into the question of how far the effects of mirage and of deflection of the pencil affect the application of this method, and as to how the necessary corrections can be applied in the simplest manner.

The pencil being emitted in an initial direction which lies at an angle φ_0 to the horizontal and the observed interval until the echo returns being t and, further, the velocity of propagation at the surface, deduced from local temperature and salinity conditions, being represented by V_0 the *apparent* distance l to the point on the bottom reached by the pencil is :—

$$l = \frac{V_0 t}{2}$$

and the apparent position of this point is B which lies in the direction φ_0 and at a distance $OB = l$.

The real trajectory of the pencil is, in fact, a curve OA deflected towards the bottom and tangent to the direction OB at O. The fact that the velocity of propagation depends on depth only gives the result that the trajectory OA of the pencil is a plane curve lying in the vertical plane which passes through OB. This plane is that of figure 3 which see.

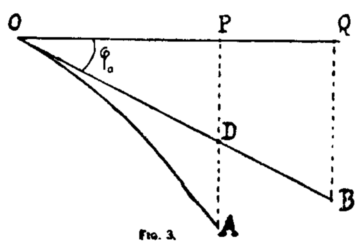

FIG. 3.

The problem consists in determining the position of the true point of mpact A from the apparent position B, taking into account the fact, which is evident from the principle of the inverse return of rays, that the echo returns from A to O along the curvilinear trajectory of the outgoing pencil but, of course, in the opposite direction. It will be seen that the mirage correction, which will give the true point of impact A from the apparent point of impact B, may become very great under the ordinary conditions which are met with in the ocean, particularly when the pencil lies in a direction which is not far from the horizontal or, in other words, when the value of the angle φ_0 is small.

After various trials and some fairly complicated calculations I have succeeded in putting these corrections into the following form :—

It is essential that, in the first place, the observers should ascertain the manner in which the velocity, at the place where they are to work, varies as a function of the depth. They must take a series of soundings (in the case of M. CLAUDE's experiments up to 1000 or even 1500 metres) to determine how temperature and salinity vary with depth.

A formula established by Mr. EKMAN, the results of which are given in numerical tables published by the British Admiralty, gives the variation in velocity V as a function of the depth z. The value of the *index* $n = \frac{V_0}{V}$, of water of depth z in ratio to surface water and then the value of the quantity $\varepsilon = n^2 - 1$ as a function of z can be deduced therefrom for given serial values of z.

This quantity ε, which would be 0 if the velocity were everywhere the same as at the surface, reached values which attained 0.1 in the conditions under which the experiments had to be made. In fig. 3 erect the perpendiculars AP and BQ to the surface from the true and apparent points of impact respectively. The perpendicular AP cuts the apparent trajectory OB at D. The position of point A will be fixed if we know the abscissa $OP = x$ and its ordinate $PA = z$. These quantities may be ascertained easily by means of the following formulas :—

$$OB = l = \int_0^z \frac{(1 + \varepsilon)\, dz}{\sqrt{\varepsilon + \sin^2 \varphi_0}}$$

$$OP = x = \cos \varphi_0 \int_0^z \frac{dz}{\sqrt{\varepsilon + \sin^2 \varphi_0}}$$

to which the following may be added :—

$$PQ = \cos \varphi_0 \int_0^z \frac{\varepsilon\, dz}{\sqrt{\varepsilon + \sin^2 \varphi_0}}$$

$$OD = \int_0^z \frac{dz}{\sqrt{\varepsilon + \sin^2 \varphi_0}}$$

$$DB = \int_0^z \frac{\varepsilon\, dz}{\sqrt{\varepsilon + \sin^2 \varphi_0}}$$

When ε has been determined as a function of z these integrations may be carried out; however, the calculation may be replaced by a graphic method which is well-known, using, to calculate the quantity $\frac{1}{\sqrt{\varepsilon + \sin^2 \varphi_0}}$, a curve which applies to all cases, and whose ordinate is equal to $\frac{1}{\sqrt{x}}$ as a function of the abscissa x.

It is easy to obtain for the value φ_0, which corresponds to each experiment, the values of $\frac{1}{\sqrt{\varepsilon + \sin^2 \varphi_0}}$ which correspond to the various depths z for which the value ε is known by means of this curve.

This makes the construction of the two curves (figs. 4 and 5) which give $OB = l$ and $OP = x$ as functions of z respectively, quite easy.

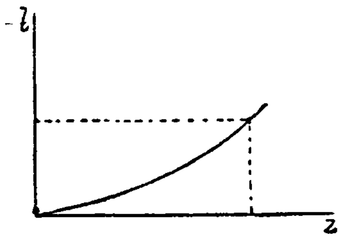

FIG. 4.

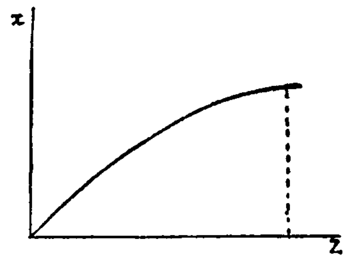

FIG. 5.

In the first curve the point on the ordinate l will give the true depth z, and on the second curve the ordinate of the point on the abscissa z will be the true value of the horizontal distance x at which the point A lies.

The application of this process consists, therefore :—

1. In determining, at the point selected as origin for the curves by means of soundings, the value of ε for the values of z at 100 metres apart, for example using tables similar to those of the British Admiralty ;

2. In utilizing the curve of $\frac{1}{\sqrt{x}}$, which applies to all cases, for carrying out the calculation (if desired graphically) of the integrals, which give l and x as functions of z for the values of φ_0 at which the polar ordinates are to be found (e.g. every 5°);

3. As each echo experiment at a given angle φ_0 results in the determination of an apparent distance $l = \frac{V_0 t}{2}$, in deducing therefrom the value of z by means of the curve which gives l as a function of z, and from this last quantity in determining x by means of the second series of curves.

The application of these formulas to the particular conditions which exist in the vicinity of Havana showed me that, for distances of about $l = 1000$ metres and with angles of from 5° to 10°, the true depth might be doubled, by the mirage effect, in relation to the apparent depth.

It is possible that these results may never be used, but I thought it advisable to point them out to you, first, in order that the somewhat extensive calculations which I have made should not be done all over again, should such a case arise, and also because this work, which I carried out owing to my friendship for Mr. Georges CLAUDE, has given me the opportunity to be associated with you in your work, for which I am very pleased.