HINTS TO HYDROGRAPHIC SUVEYORS

ON THE GRAPHICAL RESECTION OF A STATION.

by

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When a surveyor is occupying a station the position of which is unknown, but from which three known points are visible, he is confronted with the problem the mathematical solution of which is called the Pothenot problem.

The easiest solution of this problem is of course the mechanical one using a station-pointer, or its substitute, viz: the drawing of the angles on tracing paper. In some of the preceding numbers of this Review several notes are to be found which deal with other solutions by Euclidean geometry or by other means, and the reader is referred to this interesting discussion. *Cf. Hydrographic Review :* Vol. X, N° 2, p. 252 ; XI, 2, 201 ; XII, 1, 164 ; XI, 1, 155.

The cases mentioned are intended to be applied principally when fixing position at sea. For plane-table work they are less useful, as some are not sufficiently accurate, while others demand tedious constructions or the transportation of instruments unnecessarily cumbersome for planetabling.

There seems to be some ground for the assumption that several of these methods are accurate enough for determining an isolated point, but not for fixing a point which will serve as a departure for further plane-table work.

If there exist around the station known points at varying distances, the case may be solved by successive approximations as follows.A first position of the station is selected by eye. Then the plane table is orientated by means of this point and the most distant known point. The first orientation may be made by compass. The station is then resected from the two nearest known points ; the plane-table is re-orientated by means of this new point, and a new resection is made. A new control of the orientation will show whether a further resection is needed. Personally I have found this method more useful in plane-tabling practice than any of those mentioned above.

If the nearest known point (A) is within reach of the stadia, then the solution is obvious. The first orientation is made on the most distant known point (B), assuming as the position of the station in the first approximation either A or a point near it placed on the plane-table by eye. Then a backward shot is taken at the station with the stadia at A and, placing the straightedge over the point found and B, re-orientate the plane-table. If it be considered necessary, a second backward shot is taken at the station from A with the new orientation. In general it will be found that the station is correct within the limits of observation, as also the orientation of the plane-table. This is a fairly good and rapid method.

After not a few years of surveying in the Swedish archipelagoes this was as far as the writer had ventured in plane-tabling, when he found a reference, either in the *Geographical Journal* or in *Nature*, to a work rich in independent points of view, *viz.*, the second edition of the book "Maps and Surveys", by Mr. A.-R. HINKS, Secretary of the Royal Geographical Society (Cambridge University Press, 1923).

In this book the author gave a thorough and easily-understood description of a method of graphical resection permitting the determination of the positions of stations by successive approximation. The method was not new, as the writer found later, but its use with the surveyor's compass seems to have been introduced by the Survey of India, which in other respects also has been a source of fruitful impulses for geodesy and cartography.

Several years passed, nevertheless, before the writer was bold enough to try the new method. A surveyor of some experience easily becomes conservative and unwilling to risk the accuracy and rapidity of his results by trying new methods. Some years later the Hydrographic Manual by Captain LUNDQUIST, now Director of the Norges Sjøkartverk, appeared.

Herein it was found that Captain LUNDQUIST also was an advocate of the new method, and the writer resolved to try it, though not without misgivings. It is believed to be necessary to state this, for the method has a far more complicated look in a book than when used in the field. The method was found very easy to use and astonishingly accurate and speedy. As not all readers of the *Hydrographic Review* may have access to Mr. HINKS' or Captain LUNDQUIST's books, a short description of the method is given below.

It is a well-known fact that, if a station be made at an unknown point, orientating by the plane-table compass and then resecting from three known points, "a cocked hat" will be obtained sometimes of considerable dimensions, depending on the unknown compass error which has given to each line of resection an equally large angular error on the plane-table. (Cf. Figs. 1 and 2, full-drawn lines).

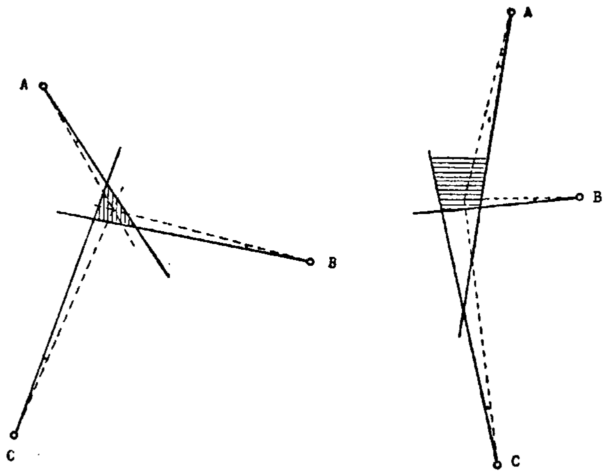

Fig. 1. Fig. 2.

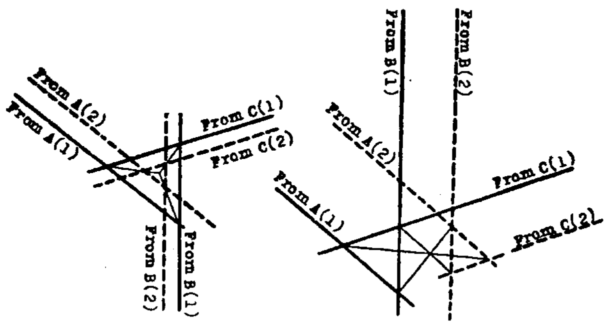

Fig. 3.

It is often assumed that the correct position of the station is the centre of the circle inscribed in the triangle. This is generally not correct. The position is frequently outside the cocked hat; and its distances from the resection-lines are proportional to the lengths of these lines, as they all suffer from the same angular error and, by turning the plane-table through that angle exactly, the three lines will intersect at the right point.

The method gives complete rules for determining the correct position of the point with reference to the lines of intersection. This position is marked by pricking a hole very lightly in the paper, and the lines of intersection are erased around the point.

Thereafter the plane-table is re-orientated, using the new point and (if possible) a fourth known and distant point.

Then the second resections are made from the three other known points. In most cases these lines will intersect at the point just determined, thus providing a check of its correctness.

If a small cocked hat should still exist, then the process is repeated.

The time needed to obtain the exact position of the point is, in general, 10 to 15 minutes.

The advantages of the method are (1) better means for checking the accuracy and (2) more rapid working. If it be found (1) that the intersections do not give a point but a triangle, then an error must have been committed in the position just occupied and not at another station, to return to which would demand perhaps an hour or more. Here an error can be corrected at once. As regards (2), the gain in time, the method generally makes it unnecessary to visit triangulation stations outside the field of work or points difficult of access.

DETAILS OF PROCEDURE. It has already been mentioned that it is generally not correct to assume that the final position is in the centre of the cocked hat and that it often occurs that the point is outside the triangle of error. If the error lies in the orientation exclusively, then the following rules are valid:–

First rule. If the station lies within the triangle formed by the three known points used for resection, then the correct position of the station on the board is within the triangle of error, and *vice-versa.* (Cf. Figs. 1 and 2).

Second rule. The correct point always lies to the right or always to the left of the three lines of resection, looking along a line from the known point from which it is drawn.

Third rule. The perpendicular distances from the correct point to each of the three resection-lines are directly proportional to the distances from the station occupied to the known points, respectively.

As Mr. HINKS points out, rules 1 and 2 are not independent of each other, as No 1 is really included in No 2, but they are stated separately for the sake of clearness.

With aid of these rules the correct position of the point is marked by a small needle-prick, and the cocked hat is erased. The re-orientation of the plane-table and, if necessary, a second resection, is made as described above and according to the three rules.

It may be pointed out that the compass is used only for the first orientation, but it is of course not necessary even then. In the absence of a compass, or in very broken country, the first orientation may be made by marking the position of the station by eye with a pencilmark. In such case, however, the cocked hat will probably be larger and one or two extra re-orientations will be necessary.

It is very important to bear in mind that, if there is any error in the known points, either they are wrongly laid down or a mistake has been made in identifying the points, then the method will operate as if all were well and give a final position which is all wrong. This is the case also when using the station-pointer or other graphical or numerical methods. It is for this reason that it is recommended above always, if possible, to make the resection from three nearby points and the re-orientation from a fourth more distant one. If this is not possible, a check may be obtained by pointing on another resected station. Of course it is not always necessary to use triangulated points as known points; on the contrary, one or more of these may be replaced by fully checked secondary points.

The method described above is not new, and this is to be expected since the plane-table is about 350 years old. The present perfection of the plane-tabling method seems to have been brought about by Major LEHMANN, a Saxon geodesist, born in 1765, as has been pointed out to the writer by Professor Tryggve RUBIN of the Stockholm Technical University. A description of the procedure is given by EGGERT (*).

A description is also given by EGGERT of another geometrically interesting modification of the method, proposed by BOHNENBERGER, a Wurttemberg astronomer, born in 1765. This modification is included in the Norwegian manual also.

BOHNENBERGER proves the following:– If the position of a station be resected twice with different errors in orientation, this will produce two triangles of error. If homologous points in these triangles, *e. g.*, corresponding angles, be joined by straight lines, these straight lines will intersect at the same point, which is the point sought or the exact position of the station (cf. fig. 3).

(*) *Einführung in die Geodäsie, Leipzig* 1907, *pp.* 179-181.

Professor EGGERT states as his opinion that the procedure of LEHMANN is more accurate than that of BOHNENBERGER, but that the latter is more rapid. Consequently he recommends that the approximate position of the station be determined by the two-triangle method and the accurate position by the approximation procedure.

The experiences of the present writer do not point quite in the same direction. Using the Survey of India method proper, *i. e.*, the approximate point procedure with a first orientation by compass, a surveyor, in most cases, will find the quality of this first orientation so good and the consequent triangle of error so small that with but small experience he will be able to approximate the point so well that its position, checked by the second orientation and resection, will be practically exact.

The two-triangle method demands always a lengthy geometrical construction, but may be of advantage when a compass cannot be used or for surveyors who find it difficult to acquire the habit of judging the position of the point with regard to the triangle of error, or who may not find successive approximations quite to their taste.

It should be repeated that the problem dealt with above is the same as the general station-pointer or Pothenot problem on a plane or in space and, accordingly, that it is subject to the same limitations. Thus, if the circle through the three known points passes through the station or near it, then the solution will be indeterminate or weak. In such a case another set of known points or another method must be chosen.

The procedure described above seems to have been used for many years by the Survey of India. The writer has not been able to ascertain whether the methods have been used in practice in Germany. In Norway it seems to have been among the standard methods for some ten years. In Sweden the writer has used it several seasons and now he would not think of making a survey without it, as its use sometimes allows twice the number of points to be fixed per day as compared with older methods. It was accepted with some diffidence by other surveyors, as originally by the writer, but it is now introduced as a standard method in the new Swedish Manual for Hydrographic Surveyors issued this year. The purpose of this article is merely to direct attention to the procedure, as it does not seem to have been mentioned before in this Review. When the writer had the honour, some fifteen years ago, to attend the courses of the General Hydrographic Service in Paris, where a great many methods were brought under review, this one was not then among them. In general it seems that this procedure is all too little known, although anyone who has used it will undoubtedly subscribe to the testimony of Mr. HINKS, viz., that it is the method that has brought perfection to plane-tabling. Thus it may perhaps be deemed pardonable to have taken so many of these pages to describe a method which has not yet come into such universal use as it undoubtedly deserves.

THREE POINT FIX: GRAPHICAL.

by

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Topographers have various methods of approximating the true position from the first attempt when a plane-table is set up for a three point position at an unknown point. A note in the Engineering News Record of January 10, 1935, by Professor J. MAUGHS BROWN of the University of South Dakota mentions a method which has been in use by some field engineers of the Coast and Geodetic Survey for many years.

Briefly, the method is the determination of the true point by finding the intersection of the circles through the control stations and point sought. The table is oriented by magnetic needle, or by estimation, and the cuts are drawn from the control stations forming the usual "triangle of error", *a, b, c*. The table is then changed in orientation slightly and additional cuts are drawn from the control stations forming another triangle, a_2, b_2, c_2. If both triangles are on the same side of the point sought, "*P*", they will be similar, and if the second triangle is nearer to the point *P*, it will be smaller as indi-