ASTRONOMICAL FIXES BY AZIMUTH
ASTRONOMICAL FIXES BY SIMULTANEOUS ALTITUDE AND AZIMUTH OF THE SUN
RADIOGONIOMETRIC FIXES FOR DISTANCES NEARER THE STATION

by

CAPTAIN FONTOURA DA COSTA,
NAVAL SCHOOL, LISBON.

A. ASTRONOMICAL FIXES BY AZIMUTH.

1. — The most recent improvements in the gyroscopic compass have rendered possible bearings of the sun and of the moon, with approximations to one-tenth of a degree.

In a few years the makers will provide us with instruments permitting approximation to within 1 or 2 minutes, which will lead to their use for Venus, Jupiter and probably a few bright stars, although the observations must be limited to heavenly bodies of small altitudes above the horizon (altitudes up to 30° or 40°).

Chapter I.

EQUAL AZIMUTH LINES.

1. EQUAL AZIMUTH LINES ON THE SPHERE.

2. — The locus on the earth of points from which the heavenly body is seen at the same moment under the same *azimuth,* is a *complex curve* called the *equal azimuth line.* The name has been extended, like that of *circles of altitude,* to its projection on the celestial sphere (1).

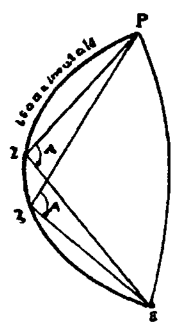

FIG. 1.

(1) By analogy with the *circles of altitude,* they should be called *azimuth curves* on the sphere.

Figure 1 shows the equal azimuth line on the sphere of the heavenly bodies E (D = 20° N) and E' (D = 20° S).

The equation of the equal azimuth lines on the sphere is easily deduced from the spherical triangle $P\ Z\ E$, Fig. 1.

$$\cot A = \cos \varphi \ \tan D \ \mathrm{cosec}\ P - \sin \varphi \ \cot P \qquad \ldots\ldots(1)$$

The equal azimuth lines on the sphere, like the altitude circles, are of no practical use.

Similar to the altitude circles, the equal azimuth lines on the sphere may also be classified in three categories :

Category I .... $A > 90° - D$
— II .... $A < 90° - D$
— III .... $A = 90° - D$

reckoning A from 0° to 180°, from the pole of the same name as D.

2. Equal Azimuth Lines on the Chart.

3. — It is possible to plot on the chart the successive points of the *arc of the equal azimuth line* in the vicinity of the *D. R. position*. For this purpose φ should be given several values, the corresponding values of P being obtained from the following (or any other) expression derived from (1).

$$\tan \frac{P}{2} = \frac{\cos D \cot A \pm \sqrt{\cos^2 D \ \mathrm{cosec}^2 A - \cos^2 \varphi}}{\sin (\varphi + D)} \qquad \ldots\ldots(2)$$

which is easy to transform for logarithmic calculation.

On the chart the values of P are reckoned from the meridian of the heavenly body (to the left if P is *E.*, to the right if P is *W.*). This, however, would be a very slow process.

4. — *Radius of Curvature.* This may be obtained by using equation (2); the expression is rather intricate. Its minimum, up to high latitudes, is given by :

$$\rho \min > \frac{3438}{2} \text{ miles} \qquad \ldots\ldots(3)$$

5. — *Distance from the equal azimuth line on the chart to its tangent.* The departure d of a plane curve from its tangent at a distance m reckoned along the tangent on each side of the point of tangency, is given with close approximation by :

$$d = \frac{m_2}{2\rho} \qquad \ldots\ldots(4)$$

Owing to (3) m min. is obtained for d = 1 mile :

$$m \min = 58.6 \text{ Miles} \qquad \ldots\ldots(5)$$

in consequence of which, as for the altitude curves of categories II and III : *the tangent may practically always be substituted for the utilisable part of the arc of the corresponding equal azimuth line on the chart.* This tangent is the equal azimuth straight line; its point of tangency is called its "computed" point.

3. Equal Azimuth Straight Lines.

6. — *Angle made by the equal azimuth straight line with the meridian.* From formula (1) and after reduction, we obtain :—

$$\frac{d\,P\cos\varphi}{d\,\varphi} = \sin h \tan E = \tan k \qquad \ldots\ldots(6)$$

where k is the angle of the equal azimuth line on the sphere with the meridian of one of its points Z, fig. 2.

On the chart, angles are preserved and consequently equation (6) is also applicable to the tangent at Z, Fig. 3 — *equal azimuth straight line.*

The calculation of k by (6) is very intricate; it may be transformed.

The supplementary angle $(180° - k)$, fig. 2 and 3, is the angle formed on the sphere by the equal azimuth straight line with the meridian Z — always reckoned from the elevated pole towards the heavenly body. We have :—

$$A + i = 180° - k \qquad \ldots\ldots(7)$$

where i is the angle between the curve, or the equal azimuth straight line, and the azimuthal bearing of the heavenly body.

From (7) we have :—

$$\tan i = -\tan (A + k) \qquad \ldots\ldots(8)$$

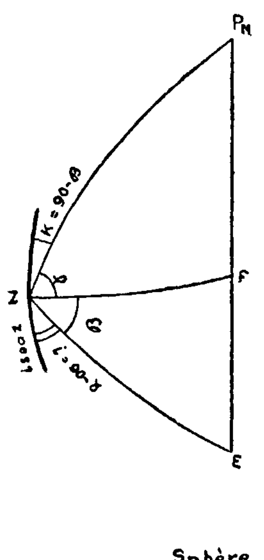

Fig. 2.

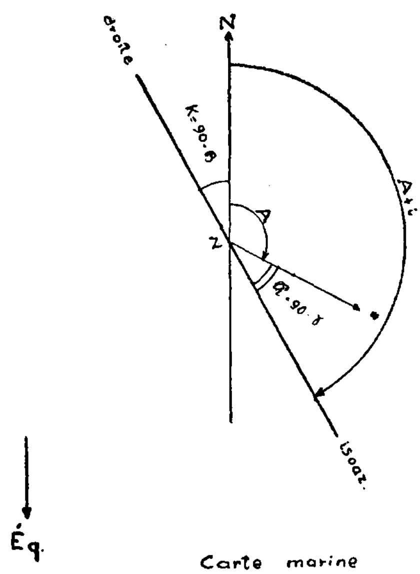

Fig. 3.

developing tan $(A + k)$ we have, by means of (6) :

$$\tan i = \sin \varphi \tan P \qquad .....(9)$$

It is seen that i depends solely on the co-ordinates of the point Z (2).

The value of i being known, the angle $(A + i)$ of the *equal azimuth straight line* with the meridian of Z, from the *elevated pole* towards the heavenly body, is known.

If $P > 90°$, $(A - i)$ is used.

7. — *Selection of the computed point of the equal azimuth straight line* :— Practically this point may be selected :—

1) on the D. R. meridian (P_e) :— *Latitude point*

2) on the D. R. parallel (φ_e) :— *Longitude point*

3) at the intersection of the equal azimuth line of the great circle perpendicular to the same, from the D. R. position; this point corresponds to the Marcq St.-Hilaire point of the altitude line; it may be called the *Azimuth point.*

On the chart, to these various points correspond their images on the *equal azimuth straight line.*

The first two require favourable circumstances :—

1) *latitude* .... $A + i > 45°$ and $< 135°$
2) *longitude*...... $A + i < 45°$ or $> 135°$

The third does not require them; and would be selected if the calculations were not so lengthy; consequently the first two will be used.

8. — *Plotting of the equal azimuth straight line.* The co-ordinates of the computed point should be determined, the latter depending on the method used. First an approximate value of i should be calculated for the D.R. position. With an approximate $(A + i)$ the method to be used is selected. It should be noted that there must always be adopted for the following formulae :—

A — reckoned from 0° to 180°, from the pole of the same name as D. The other rules for each calculation should be strictly followed.

1) CALCULATION OF LATITUDE φ.

$$\left.\begin{aligned} \tan k &= \cos P_e \cot D \\ \cos (k + \varphi) &= \tan P_e \cot A \sin k \end{aligned}\right\} \qquad .....(10)$$

Rules :

k of the same name as D; contrary name if $P_e > 90°$
$(k + \varphi)$ of the same name as D; $k > 90°$ if $A > 90°$

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(2) In Fig. 2 the relation between k or i and the angles γ and β, formed at Z by the great circle ZF perpendicular to the meridian PE of the heavenly body with the meridian PZ, of Z, and with the vertical ZE respectively, is shown.

2) CALCULATION OF THE LONGITUDE G.

$$\left.\begin{array}{l}\cot M = \sin \varphi_e \tan A \\ \cos (M - P) = \cot \varphi_e \tan D \cos M\end{array}\right\} \quad \ldots\ldots(11)$$

Rules :

$M > 90°$ and φ_e of contrary name to D;

$(M - P)$ is (—) if $P > 90°$ (which is indicated by P_e).

After the latitude φ, or the longitude G, has been determined, i is calculated by formula (9):—

1) *latitude* .... $\tan i = \sin \varphi \tan P_e$ .....(12)

2) *longitude*...... $\tan i = \sin \varphi_e \tan P$ .....(13)

3) *Plotting of the equal azimuth straight line.* This line is traced through its *computed point.*

1) *latitude* .... φ, G_e

2) *longitude*........ φ_e, G

in the direction $(A + i)$ if $P < 90°$, or $(A - i)$ if $P > 90°$ (the azimuth in such a case being reckoned from the elevated pole).

The angle i is thus reckoned in the azimuthal direction from the heavenly body towards the Equator, Fig. 3 (3).

9. — *Shift of the equal azimuth straight line.* This carrying forward of an *equal azimuth straight line* may be made over a short interval of time, like the *Marcq. St.-Hilaire straight line.*

II. *PRACTICAL USE OF THE EQUAL AZIMUTH STRAIGHT LINES — FIX BY EQUAL AZIMUTH STRAIGHT LINES.*

10. — *Error of Position of an Equal Azimuth Straight Line.* The effect of the error $\pm \Delta \varphi$ on φ (method of the latitude), or of $\pm \Delta G$ on G (method of the longitude) is the displacement of the equal azimuth straight line parallel to its direction, thus creating a zone of uncertainty of the ship's position, of width :—

latitude .... $\varepsilon = 2 \Delta \varphi \sin (A + i)$ .....(14)

longitude........ $\varepsilon = 2 \Delta G \cos \varphi \cos (A + i)$ .....(15)

which are very similar to the errors for *lines of altitude.*

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(3) The azimuth being reckoned from 0° to 360°, from the N. clockwise, the following signs are adopted for i :

φ \ A	0° to 180°	to 360°	A / φ
N	+	—	*N*
S	—	+	*S*

With $P > 90°$ the signs should be changed.

However, the most important error is due to the error $\pm \Delta A$ on the bearing A, the influence of which is to turn the equal azimuth straight line at an angle $\pm \Delta A$ about its computed point.

11. — *Practical use of equal azimuth straight lines.* The only use that may be made is for the determination of the *ship's position*.

12. — *Fix by two equal azimuth straight lines.* Under present conditions the fix may be made using no more than two *equal azimuth straight lines,* one observed of the sun and the other of the moon simultaneously, (or almost so).

Favourable conditions are obtained if the intersecting angle of the *equal azimuth straight lines* is comprised between 30° and 90°; the best *fix* is obtained with 90°.

The great difficulty of obtaining this *fix,* independent of the difficulty involved in the determination of the azimuthal bearings, is precisely in the selection of the instant to take these bearings, in order to obtain a favourable intersection of the respective *equal azimuth straight lines.*

With the *Marcq St.-Hilaire* straight lines, the intersecting angle is $\Delta A = A_2 - A_1$, the latter always easy to foresee. In the present case, however, the angle of the *equal azimuth straight lines* is :

$$\Delta k = k_2 - k_1 = (A_1 + i) - (A_2 + i)$$

which is very difficult, if not almost impossible, to determine in advance.

Perhaps this problem may be solved — that of selection of the favourable instant — by using graphs of the equal azimuth lines on the chart, traced on tracing paper, for heavenly bodies of declination varying by 10° to 10° or from 5° to 5° from the equator up to 30° N. or S.

The superposition of the suitable graphs might give an approximate solution.

When the bearings have been taken with the gyro-compass and when the data of the two equal azimuth straight lines have been obtained, the *straight lines* are traced; a very simple graph allows the co-ordinates of the *fix* to be obtained.

This method of obtaining the fix should be used only under exceptional navigational conditions, for instance, when it is necessary to obtain the *ship's position* when the azimuth of the sun and moon do not allow an accurate *fix* by use of the *Marcq St.-Hilaire straight lines,* or when the horizon does not permit the measurement of reliable altitudes.

However, in order that this method may be usefully employed, it is necessary that it should be practiced.

13. — *Error of the fix determined by two equal azimuth straight lines.* It is not easy to determine this error, which depends on errors of φ (method of latitude) or of G (method of longitude) and of the azimuths. The area of uncertainty of the *ship's position* is an irregular quadrilateral.

B. ASTRONOMICAL FIXES BY SIMULTANEOUS ALTITUDE AND AZIMUTH OF THE SUN.

14. — By means of simultaneous observations of the sun, a fix may be obtained by a Marcq St.-Hilaire position line and by an equal azimuth straight line. Figures 4 and 5 show, respectively, the *altitude circle* and the *equal azimuth* of the sun, on the sphere, and the corresponding *straight lines*, on the chart. The latter intersect at an angle (90° — i).

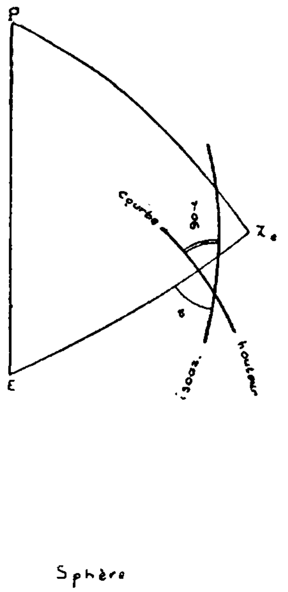

FIG. 4.

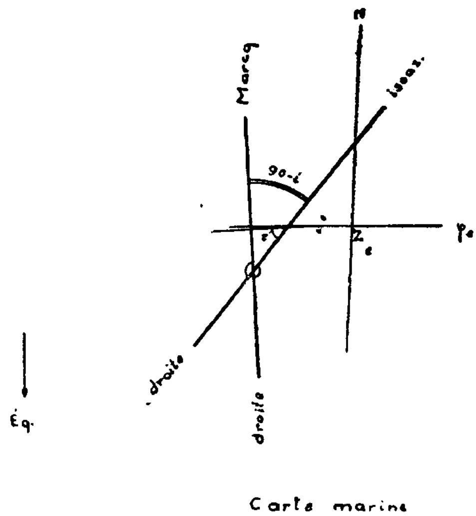

FIG. 5.

Observations should be made when $h \leqslant 40°$ and (90° — i) are comprised between 30° and 90°. The best intersect corresponds to 90° — i = 90° or $i = 0°$.

TABLE 1.

Value of the angle 90° — i between a Marcq St.-Hilaire line and an equal azimuth line.

P_e \ φ_e	0°	10°	20°	30°	40°	50°	60°	φ_o / P_e
0°	90°	90°	90°	90°	90°	90°	90°	180°
25	90	85	81	77	73	70	68	155
50	90	78	68	59	53	48	44	130
60	90	73	59	45	42	37	34	120
70	90	64	47	36	30	25	23	110
80	90	45	27	19	15	13	12	100
90	0	0	0	0	0	0	0	90

The angle $(90° - i)$ is determined by formula (9) referred to the D. R. position :—

$$\text{Cot } (90° - i) = \sin \varphi_e \ \tan P_e \qquad \ldots\ldots(16)$$

given by Table 1. This table shows that observations should not be made in the vicinity of $P_e = 90°$ (4).

15. — *Calculation of the data for the straight lines.* The method of the latitude or of the longitude is employed for obtaining the *equal azimuth straight line* (N° 7) according to the value of i, with the respective formulae given in para. 8.

With regard to the *Marcq St.-Hilaire position line,* the following may be used, which in our opinion are very simple :—

$$\left.\begin{aligned} \tan k &= \cos P_e \ \cot D \\ \tan A &= \tan P_e \ \sin k \ \sec (k + \varphi_e) \\ \tan h_e &= \tan (k + \varphi_e) \ \cos A_e \end{aligned}\right\} \qquad \ldots\ldots(17)$$

Rules :

k same name as D; contrary name if $P_e > 90°$

$$k \text{ and } \varphi_e \begin{Bmatrix} \text{add} \\ \text{sub.} \end{Bmatrix} \text{ if of } \begin{Bmatrix} \text{same} \\ \text{contrary} \end{Bmatrix} \text{ name}$$

A_e reckoned from the pole D, from 0° to 180°.

$A_e < 90^0$ if $(k + \varphi_e) < 90^0$; $> 90^0$ if $(k + \varphi_e) > 90^0$

16. — *Fix.* Once the data for the *straight lines* are obtained, a simple graphic operation allows the determination of the co-ordinates of this attractive *fix.*

We will not go into the question of the error of this *fix,* due to the errors on each of the *straight lines, Marcq St.-Hilaire* and *equal azimuth;* it suffices to indicate that their influence involves a certain area of uncertainty for the ship's position, represented by an irregular quadrilateral two sides of which are parallel (those corresponding to the *Marcq St.-Hilaire straight line*).

C. RADIOGONIOMETRIC FIXES FOR NEARER DISTANCES.

17. — The radio-compass bearings are taken : ω from a land station to the ship, or A from the ship to a land station. For each one of these procedures we have a distinct locus for the position of the ship.

The corresponding *curves* are called:—

a) *Orthodromes* (from land the ship bears in azimuth ω);

b) *Equal azimuth* (from the ship the land station bears in azimuth A); consequently the straight lines, which are tangent to these curves in the vicinity of the D. R. position are called :—

a) *Orthodromic straight lines;*

b) *Equal azimuth straight lines.*

(4) Commander MARGUET reached the same conclusion.

Under present conditions bearings are obtainable to within 2 or 3 minutes by limiting the distances, from the ship to the land station, to about 1,000 nautical miles; these are distances *nearer the station* (distances *circumstation*). In such cases approximate — but sufficiently accurate and rapidly plotted — methods are used for tracing the *orthodromic* and the *equal azimuth straight lines;* these methods become still simpler as the ship nears the station or the land transmitting station. It is even possible to plot *orthodromic* and *equal azimuth straight lines* without any calculations, exactly as for bearings from visual controls, when the distance separating the ship from the station does not exceed fixed limits.

I. *ORTHODROMES AND EQUAL AZIMUTH CURVES.*

18. — The orthodromes and equal azimuth curves are quite different.

In the following discussion, let φ_s and G_s be the co-ordinates of the land station, m the distance from the ship to the station and $g = G - G_s$,

1. Orthodromes and equal Azimuth curves on the Sphere.

19. — Let us examine the *orthodromes* and *equal azimuth curves* as projected on the sphere.

20. — *Orthodromes.* The locus of the ship's position obtained by means of the radiogoniometric bearing ω, taken from the shore station to the ship, is the *great circle* passing through the latter station at the initial azimuth ω.

This great circle may be considered as an altitude circle of radius 90°, for a heavenly body the declination of which is (90° — ψ), ψ being given by the following formula :—

$$\cos \psi = \cos \varphi_s \sin \omega$$

The equation of this great circle — *orthodrome* — is :—

$$\cot \omega = \cos \varphi_s \tan \varphi \operatorname{cosec} g - \sin \varphi_s \cot g \qquad \ldots\ldots(18)$$

21. — *Equal azimuth curves.* The statements developed in N° 2 also apply to radiogoniometric bearings from the ship to the station; however, m should be used for (90° — h), φ_s s for D, g for P and (360 — ω or 180 — As) for E.

2. Orthodromes and equal Azimuth curves on the Chart.

a) *Orthodromes*

22. On the Mercator chart the *orthodrome* corresponding to the *altitude curve* of a heavenly body at an altitude of 90° of the IId. species is always concave towards the equator.

The same method is applied to them as to the altitude curves of category II: *the tangent, in the vicinity of the D. R. position, is always substituted for the useful part of the arc of the orthodrome.* This *tangent* is the *orthodromic straight line.*

Also :

from the land station to a minimum distance of 1043 miles $\times$ (cos φ m)$\frac{2}{3}$ *the orthodrome may be practically considered as identified with the arc of the osculating circle at its middle point.*

We are still within those limits when adopting for a minimum distance the following important equation:—

$$\text{min. eq.)} \;\ldots\; m_{\min} = 1000' \qquad \ldots\ldots(19)$$

which will be referred to further on.

The loxodrome corresponding to the orthodrome is always comprised between the latter curve and the equator.

23. — *The Givry Correction.* Several expressions are known for obtaining the Givry correction; i. e. a *variation of the orthodromic azimuth when a small distance along the orthodrome is traversed;* the same applies to the difference between the orthodromic and loxodromic azimuths (5), Fig. 6.

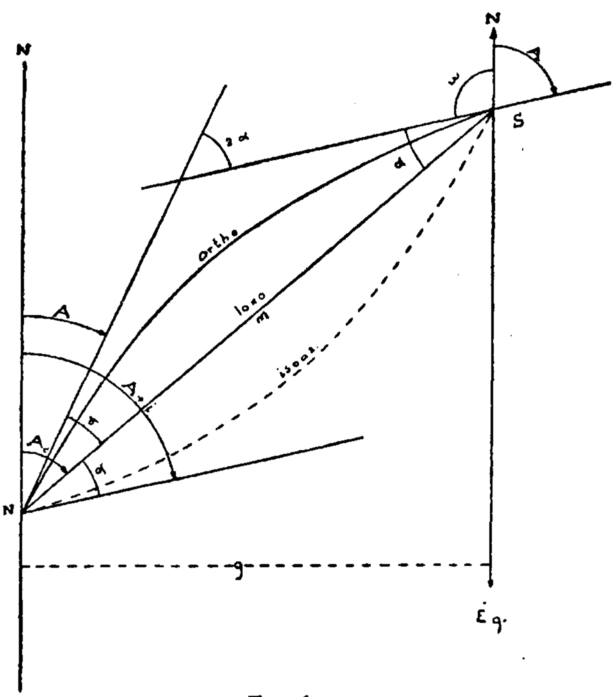

Fig. 6.

In practice for radio-compass bearings, the first-order approximation will suffice; consequently the following are used.

$$\text{min. of arc)} \ldots\ldots \alpha' = \frac{m^{mi}}{2} \tan \varphi_m \sin A_0 \qquad \ldots\ldots(20)$$

$$\text{min. of arc)} \ldots\ldots \alpha' = \frac{g'}{2} \sin \varphi_m \qquad \ldots\ldots(21)$$

(5) For these formulae see particularly :
F. Marguet — 2.
P. de Vanssay de Blavous — 8 & 9.

The latter has been developed in tables and diagrams (6); tables are also available for 2 α (Givry correction).

a) Conversion of azimuths A_s, A and A_o; from figure 6 we have :

$$\left.\begin{aligned} A &= A_s - 2\alpha && \ldots\ldots a \\ A_0 &= A + \alpha = A_s - \alpha && \ldots\ldots b \end{aligned}\right\} \qquad \ldots\ldots(23)$$

The azimuth being reckoned from 0° to 360° from the *N.* clockwise, the signs to be given to α are the following:—

Signs of α
Ship to the

φ_m	*W* of the station 0° to 180°	*E* of the station 180° to 360°	φ_m
N	+	—	*N*
S	—	+	*S*

Remark :— The shore or transmitting stations do not give A_s but ω; — we have always : $A_s = \omega + 180°$.

b) *Isoazimuth curves.*

24. — *Practical symmetry of isoazimuthal curves and orthodromes for short distances from ship to station.* — From formulae (6) and (7) the following is derived :

$$\tan (A + i) = \cos m \tan A_s \qquad \ldots\ldots(24)$$

owing to the fact that m is very small the following is obtained :

min of arc) ...... $$(A + i)' = (A_s)' - \underbrace{\frac{(m')^2}{2} \sin 2A_s \text{ arc } 1'}_{c} \qquad \ldots\ldots(25)$$

For the minimum value of m (19) we obtain $c = 73'$, without any appreciable error in practice.

Consequently formula (25) becomes :—

$$A + i = A_s \qquad \ldots\ldots(26)$$

and, from (23) :—

$$(A + i) - \alpha = A + \alpha = A_o \qquad \ldots\ldots(27)$$

(6) In America the following formula, which has been drawn up in tables, is preferred:

$$\tan \alpha = \tan \frac{g}{2} \sin \varphi\, m \qquad \ldots\ldots(22)$$

consequently the directions of $(A + i)$ and of A are symmetrical with the corresponding loxodrome; in other words, the *isoazimuth curve and the orthodrome, for the same positions of the ship and station, are practically symmetrical with the corresponding loxodrome,* fig. 6, on condition that m be not greater than the minimum. (19).

3. Orthodromic and Equal Azimuth Straight Lines.

25. — Selection of the *computed point* of the orthodromic straight line must be made, and that of the equal azimuth straight line.

Various solutions of this problem of obtaining the computed point have been proposed, among which those due to Commander Marguet (7) and to Ingénieur hydrographe général de réserve P. de Vanssay de Blavous (8).

We advise the selection of the fix at the intersection of the loxodrome originating in the direction of the shore station in the direction of A_o, with :—

the D. R. Meridian) .... $A_o = 45° \Rrightarrow 135°$ or $225° \Rrightarrow 315°$
the D. R. Parallel) ...... $A_o = 315° \Rrightarrow 45°$ or $135° \Rrightarrow 225°$

Let us show how A_o is obtained, and how the respective straight line is traced through its *computed point.*

a). *Orthodromic straight lines*

26. — *Determination of A_o.* The bearing of the shore station is taken from the ship in the azimuth ω ($A_s = \omega + 180°$). Up to a distance of 1,000 miles from the ship to the station the orthodrome may: 1) *be identified,* 2) *not be identified* with the arc of its osculating circle at its middle point.

1). *The orthodrome is identified with the arc of its osculating circle.* These curves are identical with the limiting minimum of m^{mi}, starting from the shore station and as given by formula (19), which we may write:

$$\text{min. eq.)} \quad g = 1000' - \sin A_o \qquad \ldots\ldots (28)$$

i. e. *up to the limits of* $(g)'$, as given by (28), *the orthodrome may be replaced, from the shore station, by the arc of its osculating circle at its middle point.*

These limits are indicated in Table 2.

Within these limits (23b) is always verified :—

$$A_0 = A_s - \alpha$$

2). *The orthodrome is not identified with the arc of its osculating circle.* Beyond the minimum limits shown in tabulation N° 2, we obtain, (Fig. 7).

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(7) F. Marguet — 2, 3 and 5; and *Ecole Navale* — 1.

(8) P. de Vanssay de Blavous — 8.

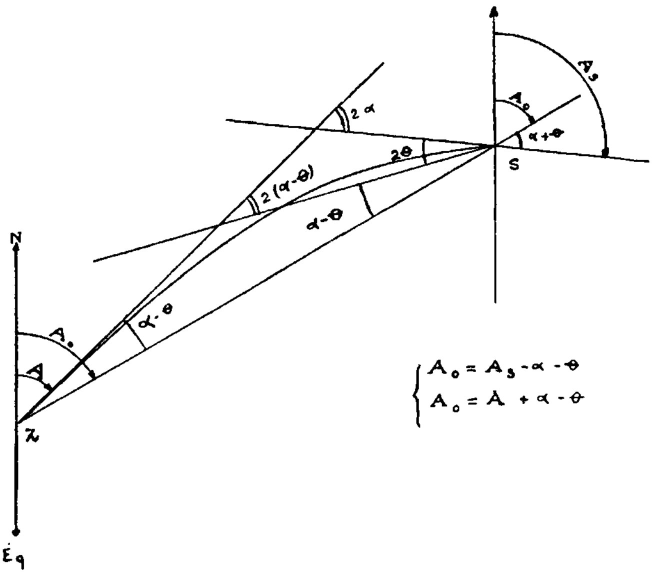


FIG. 7.

LIMITS OF g SO THAT THE ORTHODROME MAY BE IDENTIFIED WITH THE ARC OF ITS OSCULATING CIRCLE.

A_0	g	A_0	g	A_0	g
0°	0'	25°	420'	50°	765'
5	85	30	500	60	865
10	175	35	575	70	940
15	260	40	640	80	985
20	340	45	705	90	1,000
25	420	50	765		

$$A_0 = A_s - \alpha - \theta \qquad .....(29)$$

where θ is a very small angle the very approximate expression of which, as indicated by WEDEMAYER (9), is :

radians) .....
$$\theta = \frac{g_e \times \Delta\lambda(\varphi)}{12} \qquad .....(30)$$

in which .....
$$\Delta\lambda(\varphi) = \Delta\lambda(\varphi_s) - \Delta\lambda(\varphi_e) \qquad .....(31)$$

WEDEMAYER gave a small table which allows θ to be obtained; however, we are not acquainted with it.

(9) Giuseppe SIMEON — 6.

The value of θ may be written :—

$$\text{min of arc)} \; \ldots\ldots \theta' = 0.00146 \, (g_e)^\circ \left[\Delta \, \lambda \, (\varphi)\right]' \qquad \ldots\ldots (32)$$

$$\text{min of arc)} \; \ldots\ldots \theta' = \underbrace{\frac{(g_e)^\circ}{12} \left[\Delta \, (\varphi_s)\right]^r}_{c'_s} - \underbrace{\frac{(g_e)^\circ}{12} \left[\Delta \, (\varphi_e)\right]^r}_{c'_e} \qquad \ldots\ldots (33)$$

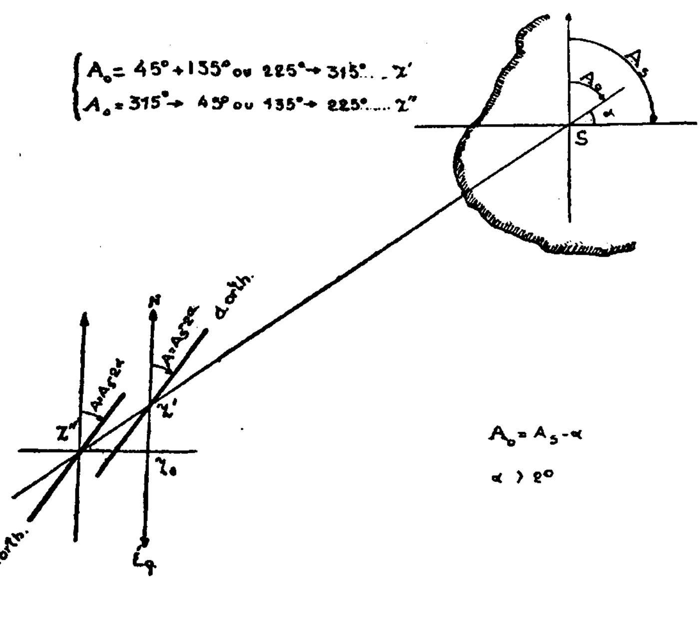
We have drawn up tables for formula (32) (See table 3) and also for formula (33) (See Table 4); the latter gives values of c' for the entries $(g_e)^\circ$ and $(\varphi)^\circ$; two entries are necessary in order to obtain θ with :—

φ_s for the first and φ_e for the second.

θ is given the signs of the following table :—

N Hemisph.	Ship to the *W* of the station (0° to 180°)	Ship to the *E* of the station (180° to 360°)	*S* Hemisph.
$\varphi_s > \varphi_e$	+	—	$\varphi_s < \varphi_e$
$\varphi_s < \varphi_e$	—	+	$\varphi_s > \varphi_e$

Remark :— If the latitude φ (or the longitude G), obtained for the computed point Z' (or Z''), figs. 8 and 9, is noticeably different from φ_e (or from G_e) α and θ are calculated anew, using φ (or g) instead of φ_e (or g_e) and consequently a new A_0 is obtained.



Droite orthodromique

FIG. 8.

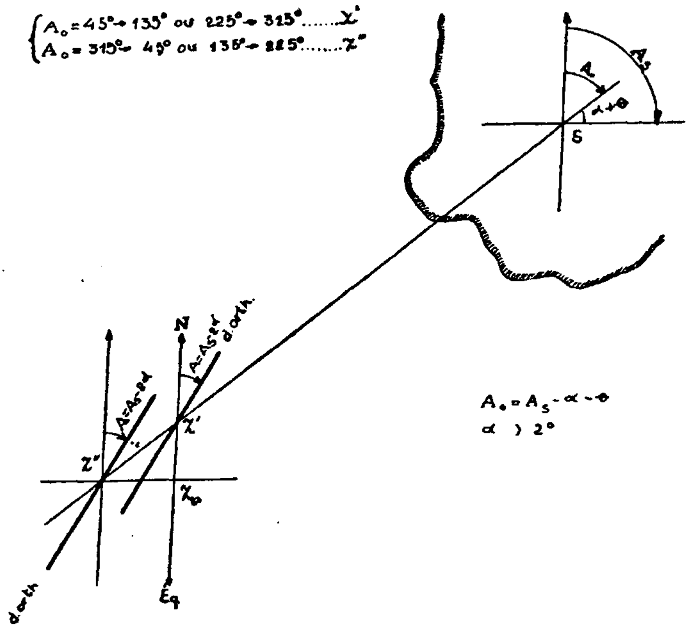


FIG. 9. Droite orthodromique

27. — *Plotting of the Orthodromic Straight Line :—* This line is plotted from the computed point in the direction :—

$$A = A_s - 2\,\alpha$$

1). *Usual case.* — It is necessary to investigate whether g_e is traced within the limits of formula N° (28) or of Table 2 because then the loxodromic azimuth is $A_o = A_s - \alpha$ (fig. 8); if it exceeds those limits we will have :—

$$A_o = A_s - \alpha - \theta \quad \text{(fig. 9)}$$

in such case the straight line is drawn from the *computed* point in the direction $A = A_s - 2\,\alpha$.

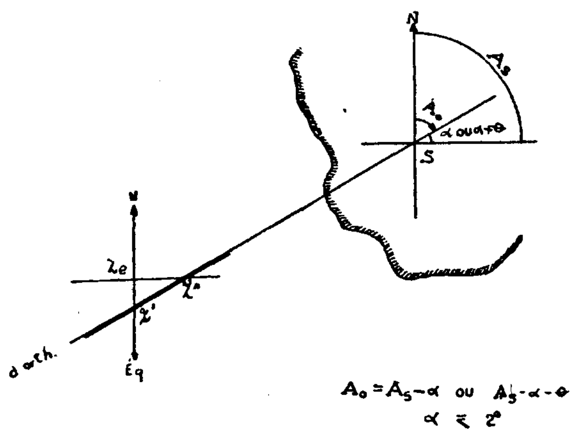


FIG. 10. Droite orthodromique

For values of α, within 2°, the orthodromic straight line is traced along the loxodrome itself, fig. 10. Since the orthodromic straight line must make the angle α with the loxodrome, the error which is involved will be from 1 to 30 miles' distance from the computed point, an error which may be neglected in practice.

2). *Special case :* $\alpha \leqslant 0°.5$. — In this case the loxodromic azimuth A_0 is practically identified with the azimuth A_s ; at a distance of 120 miles from the shore station, the distance of the two straight lines originating from A_0 and from A_s, is only 1 mile. The orthodromic straight line is also identified with the direction A_s.

Let us see the limits of g in this special case.

We have, roughly :—

$$\text{min. eq.)} \quad g' = 120^{mi} \sin A_0 \cos \varphi_m \qquad \ldots\ldots(34)$$

$$\text{min. eq.)} \quad g' = 120' \sin A_0$$

however, for $\alpha = 0°.5$, formula (21) gives :

$$\text{min. eq.)} \quad g' = 60' \operatorname{cosec} \varphi_m \qquad \ldots\ldots(35)$$

These two expressions for g' (34 and 35) permit the compilation of Table 5, where limits of g' are found for use in this special case.

The straight line is traced on the loxodrome (azimuth $A_0 = A_s$) in the vicinity of the D. R. position, fig. 11.

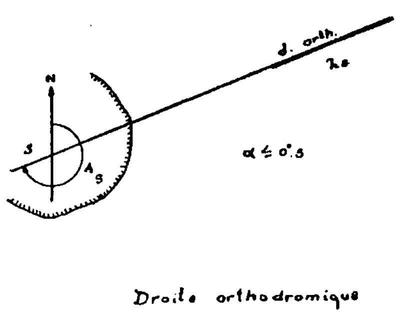


FIG. 11.

28. — *On the Use of Meridional Parts in the Usual Case.* Lack of a chart to sufficient scale, or in cases where the distance from the ship to the station is several degrees, or, again, in order to avoid errors of graphical construction, the latitude φ of Z', or the longitude G'' of Z'' (figs. 8 and 9) should be determined by calculation.

In such a case the meridional parts are used :—

$$\text{min. of arc)} \ \ldots\ldots \ [\lambda(\varphi')]' = [\lambda(\varphi_s)]' + g'_e \cot A_0 \qquad \ldots\ldots(36)$$

whence φ' for the computed point Z' (φ', G_e);

or :—

min. of arc) (10) ..... $G'' = G_e + [\lambda(\varphi_s) - \lambda(\varphi_e)] \tan A_0$ .....(37)

The solution given for the tracing of the orthodromic straight line gives the best results; it is perhaps the only really practical one with A_0 in the vicinity of 0° or 180°.

b) *Equal azimuth straight lines*

29. — In this case the bearing of the land station is taken from the ship at the azimuth A. We will have the same solution of the orthodromic straight lines in order to obtain the *computed point.*

30. — *Determination of* A_0. Up to the limits of $(g)'$ as given by the formula (28) or by Table 2, we have for the loxodromic azimuth : $A_0 = A + \alpha$; beyond these limits, from Fig. 7 we must use the following formula :—

$$A_0 = A + \alpha - \theta \qquad .....(40)$$

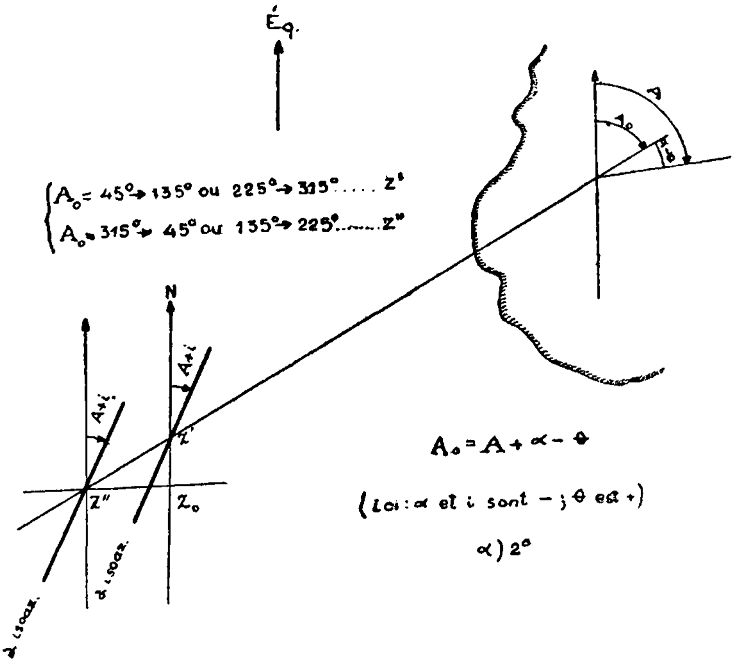


FIG. 12.

Droite isoazimutale

(10) In the triangle $P\ Z''\ S$ on the sphere, we have, according to Neper's equation :

$$\tan \frac{g''}{2} = \sec \varphi_m \sin \frac{\Delta\varphi}{2} \tan \frac{1}{2}(A_s + A) \qquad .....(38)$$

where $\Delta\varphi = \varphi_s - \varphi_e$. On the other hand we have :—

$$A_s + A = 2\,\alpha$$

Consequently formula (38) may be written :—

$$(g'')^0 = (\alpha)^0 \sec \varphi_m \sin \frac{\Delta\varphi}{2} \qquad .....(39)$$

which allows the calculation of G'' without using meridional parts.

31. — *Plotting of the Equal Azimuth Straight Line.* Through the position S of the station, a loxodrome is traced on the chart in the direction A_0, and the computed point Z' (or Z''), fig. 12, is obtained as for the orthodromic straight lines.

1. — *Usual Case.* The straight line is traced through the computed point in the direction $A + i$. Let us see how angle i is obtained. Referring to (9):—

$$\tan i = \tan g \sin \varphi$$

in which i is the angle between the equal azimuth straight line and the direction of the azimuth A, φ being the latitude of the computed point. When g is very small we have :—

$$\text{min. of arc)} \; \ldots\ldots \quad i' = (g)' \sin \varphi + \frac{(g')^3}{6} \sin 2\varphi \cos \varphi \operatorname{arc}^2 1' \quad \ldots\ldots (41)$$

Up to distances from ship to station approximately equal to 1,000 miles, the term of the third order is less than 30'; consequently we can suppress it because only a length of about 30 miles of the isoazimuth straight line is used on each side of the *computed point.* Consequently we have, roughly :

$$\text{min. of arc)} \; \ldots\ldots \quad i' = (g)' \sin \varphi \quad \ldots\ldots (42)$$

By comparing the latter expression of i with that of α (21) the following is obtained :—

$$\text{min. of arc)} \; \ldots\ldots \quad i'' = 2\alpha \quad \ldots\ldots (43)$$

however, α should be calculated with φ of the computed point instead of of φ_m.

From formula (43) it is seen that i should be given the same sign as α (N° 23).

For values of i up to 2°, the straight line is traced on the loxodrome itself.

2. *Special case : $\alpha \leqslant 0°.5$.* For such values of α, the loxodromic azimuth A_0 is practically identified with the azimuth A as is the equal azimuth straight line. These special cases should be limited to the value of g shown in Table N° 5.

32. — *On the Use of Meridional Parts in the Usual Case.* The procedure is the same as that indicated for the orthodromic straight lines (N° 28).

The following are some of the results obtained :—

φ_s	φ_e	g_e	A	φ correct	φ calculated
60° N	49° N	— 20°	40°	49° 35′ N	49° 36′ N
—	—	—	—	g correct	g calculated
60 N	50 N	— 4	12	— 4° 16′	— 4° 16′
60 N	50 N	— 5	15	— 5 24	— 5 23

II. *PRACTICAL USE OF ORTHODROMIC STRAIGHT LINES AND EQUAL AZIMUTH STRAIGHT LINES: RADIO-COMPASS FIX.*

1. Practical use of an Orthodromic Straight Line or of an Equal Azimuth Straight Line.

33. — *Error of Position of an Orthodromic or of an Equal Azimuth Straight Line.* This error derives almost exclusively from the errors in the radiogoniometric bearings, which are of an occasional or of a systematic nature, due :— (11).

a) *Occasional*

1) To fading;

2) To refraction from the coast;

3) To unequal periodical ionisation of air at twilight and dawn;

4) To uncertainty as to radio-compass deviation due to the conductors on the ship.

5) To the fact that the compass course is not read at the same instant as that at which the radiogoniometric signals are heard.

b) *Systematic*

1) To the inaccurate position of the index of the radiogoniometric bearings;

3) To inaccuracy of orientation of the line 0° — 180° of the dial of the above-mentioned index;

3) To an inaccurate knowledge of the magnetic declination.

These errors are transmitted directly to A_s (or to A) and consequently, to A_0. This involves the displacement of the loxodrome about the position of the station on the chart, which further involves the displacement of the computed point— in latitude (or in longitude); and also the shift of the orthodromic or equal azimuth straight line respectively, about its computed point.

34. — *Utilisation of an orthodromic straight line or of an equal azimuth straight line.* The only practical utilisation of one of these lines is in the case of landfalls or difficult channels; their direction, however, should always be carefully checked as the ship approaches land or those channels.

2. Radiogoniometric Fix.

35. The fix is obtained at the intersection of two straight lines, either orthodromic or equal azimuth. In order to obtain favourable conditions it is necessary that the angle between these two lines be comprised between 30° and 90°; the best fix corresponds to the latter value. With three straight

(11) Domenico Spanò — 7.

lines a small triangle is rarely obtainable unless use is made of only those lines corresponding to special cases (N^{os} 27 and 31). With a large triangle it is dangerous to select a *fix* for the ship.

When more than three straight lines are used, the determination of the ship's position is still more uncertain.

For this reason we must limit our choice to two straight lines; the third will be always of use for navigators' practice in radiogoniometric methods, and also as a check for the intersection of the first two lines.

36. — *Error of the Radiogoniometric Fix.* We will refer only to the radiogoniometric fix obtained by two straight lines.

The position error of each one of those straight lines (N° 33) involves an irregular quadrilateral of uncertainty in the ship's position, the size of which may reach rather large dimensions.

TABLE 3.

Values of Θ in minutes of arc.

(arguments : $\Delta\lambda(\varphi)$ and g_e)

g_e / $\Delta\lambda(\varphi)$	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°	20°	g_e / $\Delta\lambda(\varphi)$
0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'
100	0	0	0	0	1	1	1	1	1	1	1	2	2	2	2	2	2	2	3	3	3	100
200	0	0	1	1	1	1	2	2	2	3	3	3	4	4	4	4	5	5	5	6	6	200
300	0	0	1	1	2	2	3	3	4	4	4	5	5	6	6	7	7	7	8	8	9	300
400	0	1	1	2	2	3	4	4	5	5	6	6	7	8	8	9	9	10	11	11	12	400
500	0	1	1	2	3	4	4	5	6	7	7	8	9	9	10	11	12	12	13	14	15	500
600	0	1	2	3	4	4	5	6	7	8	9	10	11	11	12	13	14	15	16	17	18	600
700	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	700
800	0	1	2	4	5	6	7	8	9	11	12	13	14	15	16	18	19	20	21	22	23	800
900	0	1	3	4	5	7	8	9	11	12	13	14	16	17	18	20	21	22	24	25	26	900
1: 000	0	1	3	4	6	7	9	10	12	13	15	16	18	19	20	22	23	25	26	28	29	1: 000
1: 100	0	2	3	5	6	8	10	11	13	14	16	18	19	21	22	24	26	27	29	31	32	1: 100
1: 300	0	2	4	6	8	9	11	13	15	17	19	19	21	23	25	26	28	30	32	33	35	1: 200
1: 200	0	2	4	5	7	9	11	12	14	16	18	21	23	25	27	28	30	32	34	36	38	1: 300
1: 400	0	2	4	6	8	10	12	14	16	18	20	22	25	27	29	31	33	35	37	39	41	1: 400
1: 500	0	2	4	7	9	11	13	15	18	20	22	24	26	28	31	33	35	37	39	42	44	1: 500
1: 600	0	2	5	7	9	12	14	16	19	21	23	26	28	30	33	35	37	40	42	44	47	1: 600
1: 700	0	2	5	7	10	12	15	17	20	22	25	27	30	32	35	37	40	42	45	47	50	1: 700
g_e / $\Delta\lambda(\varphi)$	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°	20°	$\Delta\lambda(\varphi)$ / g_e

NOTE. — Above 20° resolve g_e into two parts.

TABLE 4.

Values of Θ in minutes of arc.

(arguments : φ_s and g_e, φ_e and g_e)

φ \ g_e	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°	20°	g_e / φ
0°	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0'	0°
1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	1
2	0	0	0	1	1	1	1	1	1	2	2	2	2	2	2	3	3	3	3	3	3	2
3	0	0	1	1	1	1	2	2	2	2	3	3	3	3	4	4	4	4	5	5	5	3
4	0	0	1	1	1	2	2	2	3	3	3	4	4	5	5	5	6	6	6	7	7	4
5	0	0	1	1	2	2	3	3	4	4	4	5	5	6	6	7	7	7	8	8	9	5
6	0	1	1	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10	11	6
7	0	1	1	2	2	3	4	4	5	6	6	7	7	8	9	9	10	10	11	12	12	7
8	0	1	1	2	3	4	4	5	6	6	7	8	8	9	10	11	11	12	13	13	14	8
9	0	1	2	2	3	4	5	6	6	7	8	9	9	10	11	12	13	13	14	15	16	9
10	0	1	2	3	4	4	5	6	7	8	9	10	11	11	12	13	14	15	16	17	18	10
11	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	14	15	16	17	18	19	11
12	0	1	2	3	4	5	6	7	8	9	11	12	13	14	15	16	17	18	19	20	21	12
13	0	1	2	3	5	6	7	8	9	10	11	13	14	15	16	17	18	19	21	22	23	13
14	0	1	2	4	5	6	7	9	10	11	12	14	15	16	17	19	20	21	22	23	25	14
15	0	1	3	4	5	7	8	9	11	12	13	15	16	17	19	20	21	23	24	25	27	15
16	0	1	3	4	6	7	8	10	11	13	14	16	17	18	20	21	23	24	25	27	28	16
17	0	2	3	5	6	8	9	11	12	14	15	17	18	20	21	23	24	26	27	29	30	17
18	0	2	3	5	6	8	10	11	13	14	16	18	19	21	22	24	26	27	29	30	32	18
19	0	2	3	5	7	8	10	12	14	15	17	19	20	22	24	25	27	29	30	32	34	19
20	0	2	4	5	7	9	11	12	14	16	18	20	21	23	25	27	29	30	32	34	36	20
21	0	2	4	6	8	9	11	13	15	17	19	21	23	24	26	28	30	32	34	36	38	21
22	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	31	33	35	37	39	22
23	0	2	4	6	8	10	12	14	17	19	21	23	25	27	29	31	33	35	37	39	41	23
24	0	2	4	6	9	11	13	15	17	19	22	24	26	28	30	32	35	37	39	41	43	24
25	0	2	5	7	9	11	14	16	18	20	23	25	27	29	32	34	36	38	41	43	45	25
26	0	2	5	7	9	12	14	16	19	21	24	26	28	31	33	35	38	40	42	45	47	26
27	0	2	5	7	10	12	15	17	20	22	24	27	29	32	34	37	39	42	44	47	49	27
28	0	3	5	8	10	13	15	18	20	23	25	28	31	33	36	38	41	43	46	48	51	28
29	0	3	5	8	11	13	16	19	21	24	26	29	32	34	37	40	42	45	48	50	53	29
30	0	3	5	8	11	14	16	19	22	25	27	30	33	36	38	41	44	47	49	52	55	30
φ / g_e	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°	20°	g_e \ φ

NOTE. — Above 20° resolve g_e into two parts.

TABLE 4 (continued).

Values of Θ in minutes of arc.

(arguments : φ_s and g_e, φ_e and g_e)

φ \ g_e	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°	20°	g_e / φ
30°	0'	3'	5'	8'	11'	14'	16'	19'	22'	25'	27'	30'	33'	36'	38'	41'	44'	47'	49'	52'	55'	30°
31	0	3	6	9	11	14	17	20	23	26	28	31	34	37	40	43	46	48	51	54	57	31
32	0	3	6	9	12	15	18	21	24	27	30	32	35	38	41	44	47	50	53	56	59	32
33	0	3	6	9	12	15	18	21	24	27	31	34	37	40	43	46	49	52	55	58	61	33
34	0	3	6	9	13	16	19	22	25	28	32	35	38	41	44	47	51	54	57	60	63	34
35	0	3	7	10	13	16	20	23	26	29	33	36	39	42	45	49	52	55	59	62	65	35
36	0	3	7	10	14	17	20	24	27	30	34	37	40	44	47	51	54	57	61	64	67	36
37	0	3	7	10	14	17	21	24	28	31	35	38	42	45	49	52	56	59	63	66	70	37
38	0	4	7	11	14	18	22	25	29	32	36	39	43	47	50	54	57	61	65	68	72	38
39	0	4	7	11	15	19	22	26	30	33	37	41	44	48	52	56	59	63	67	70	74	39
40	0	4	8	11	15	19	23	27	31	34	38	42	46	50	53	57	61	65	69	72	76	40
41	0	4	8	12	16	20	24	28	31	35	39	43	47	51	55	59	63	67	71	75	79	41
42	0	4	8	12	16	20	24	28	32	36	40	45	49	53	57	61	65	69	73	77	81	42
43	0	4	8	12	17	21	25	29	33	37	42	46	50	54	58	62	67	71	75	79	83	43
44	0	4	9	13	17	21	26	30	34	39	43	47	51	56	60	64	69	73	77	81	86	44
45	0	4	9	13	18	22	26	31	35	40	44	48	53	57	62	66	71	75	79	84	88	45
46	0	5	9	14	18	23	27	32	36	41	45	50	54	59	63	68	73	77	82	86	91	46
47	0	5	9	14	19	23	28	33	37	42	47	51	56	61	65	70	75	79	84	88	93	47
48	0	5	10	14	19	24	29	34	38	43	48	53	57	62	67	72	77	81	86	91	96	48
49	0	5	10	15	20	25	30	34	39	44	49	54	59	64	69	74	79	84	89	93	98	49
50	0	5	10	15	20	25	30	35	40	45	51	56	61	66	71	76	81	86	91	96	101	50
51	0	5	10	16	21	26	31	36	42	47	52	57	62	67	73	78	83	88	93	99	104	51
52	0	5	11	16	21	27	32	37	43	48	53	59	64	69	75	80	85	91	96	101	107	52
53	0	5	11	16	22	27	33	38	44	49	55	60	66	71	77	82	88	93	99	104	109	53
54	0	6	11	17	22	28	34	39	45	51	56	62	67	73	79	84	90	96	101	107	112	54
55	0	6	12	17	23	29	35	40	46	52	58	63	69	75	81	86	92	98	104	110	115	55
56	0	6	12	18	24	30	36	41	47	53	59	65	71	77	83	89	95	101	107	113	119	56
57	0	6	12	18	24	30	36	43	49	55	61	67	73	79	85	91	97	103	109	116	122	57
58	0	6	12	19	25	31	37	44	50	56	62	69	75	81	87	94	100	106	112	119	125	58
59	0	6	13	19	26	32	38	45	51	57	64	71	77	83	90	96	103	109	115	122	128	59
60	0	7	13	20	26	33	39	46	53	59	66	72	79	86	92	99	105	112	119	125	132	60
φ / g_e	0°	1°	2°	3°	4°	5°	6°	7°	8°	9°	10°	11°	12°	13°	14°	15°	16°	17°	18°	19°	20°	φ \ g_e

NOTE. — Above 20° resolve g_e into two parts.

TABLE 5.

Limits of g for $\alpha \leqslant 0°.5$.

A_0 \ φ_m	0°	30°	35°	40°	45°	50°	60°	φ_m / A_0
0°	0'	0'	0'	0'	0'	0'	0'	0°
5	10	10	10	10	10	10	10	5
10	20	20	20	20	20	20	20	10
15	30	30	30	30	30	30	30	15
20	40	40	40	40	40	40	40	20
25	50	50	50	50	50	50	50	25
30	60	60	60	60	60	60	60	30
35	70	70	70	70	70	70	70	35
40	75	75	75	75	75	75	70	40
45	85	85	85	85	85	75	70	45
50	90	90	90	90	85	75	70	50
60	105	105	105	90	85	75	70	60
70	115	115	105	90	85	75	70	70
80	120	120	105	90	85	75	70	80
90	120	120	105	90	85	75	70	90
A_0 / φ_m	0°	30°	35°	40°	45°	50°	60°	A_0 \ φ_m

NOTE. — The azimuth A_0 is identified with A and A_s.

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