

SUMMARY OF PAPER ON ASTRONOMICAL NAVIGATION BY MEANS OF DIAGRAMS.

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(Read at "Congrès des Sociétés Savantes").

- I.—Introduction : Various Methods used in Astronomical Navigation for solution of Position Triangle.
- II.—Direct Distance Measurement or Plotting Method on Navigational Chart :-
 — Weems System (Astrograph);
 — Kahn System;
 — Development of new system using special navigational charts in gnomonic projection equipped with a diagram for reading distances and angles;
 — Comparison with Kahn System.
- III.—Review of Principal Graphic Calculating Systems suggested to date, namely :-
 — Nomograms for spherical right-angled triangles;
 — Nomograms for triangles in general;
 — Two-sided diagram used in transference and rotation.
 Discussion as to efficiency : Convenience, simplicity, degree of application, accuracy.
- IV.—Description of New Diagram based on principle similar to that of Distance and Angle-reading Diagram in Gnomonic Projection mentioned above. No graphical operation required, only readings. Comments on efficiency.
- V.—Bibliography.

THE CALCULATION OF ASTRONOMICAL NAVIGATION BY DIAGRAMS.

The practical work involved in position-fixing at sea, since the development of wireless for solving the problem of longitude, now seems to follow a standard method. Its main characteristic is the use of the sextant, by means of which a fairly constant degree of accuracy is obtained, approaching the sexagesimal minute. As to utilization of observations, two principal systems are applied :-

- I.—Dividing the position triangles into right-angled triangles (Bertin Aquino, Ageton Tables, etc.).
- II.—Utilization of *haversines*, enabling convenient semi-logarithmic calculation.

Both these processes are considered satisfactory by marine navigators, and are at present used by them, apparently in preference to any other system.

Astronomy is likewise extremely helpful in air navigation, especially in long-range flying; at such times astronomical positions are determined at relatively frequent intervals, speed of execution and of calculation then becoming a preponderant factor. As a result, there has been a natural tendency towards a continual search for new improvements in an attempt to accelerate and simplify the task of the navigator, whether it be in connection with observation instruments (... automatic averaging bubble sextants) or calculation procedures : mechanical instruments (Star globe, Spherans, Hagner or Willis machines, Bastien system), graphical methods, or precomputed tables.

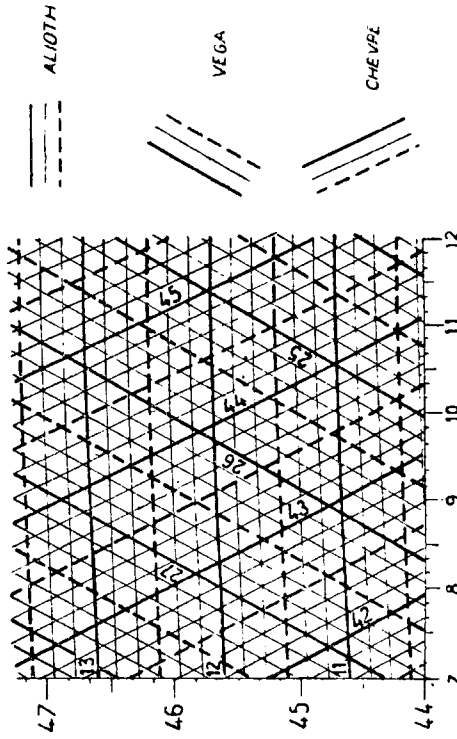


FIG. 2
Weems Method—Portion of Page Diagram.

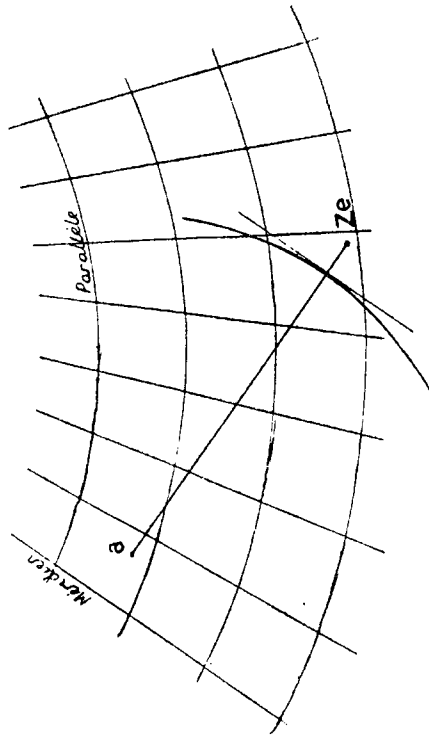
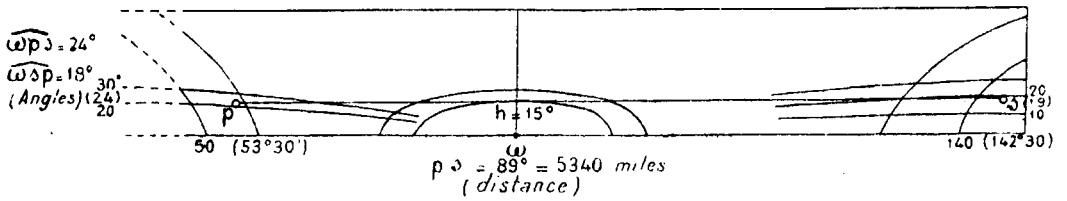
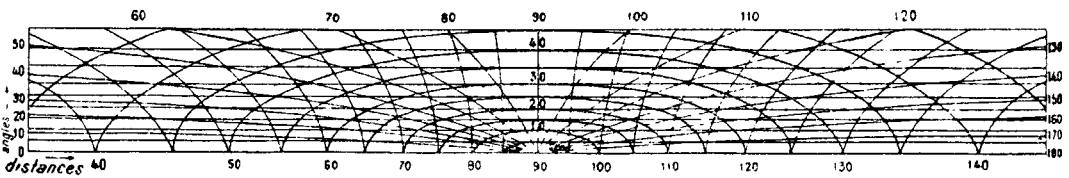
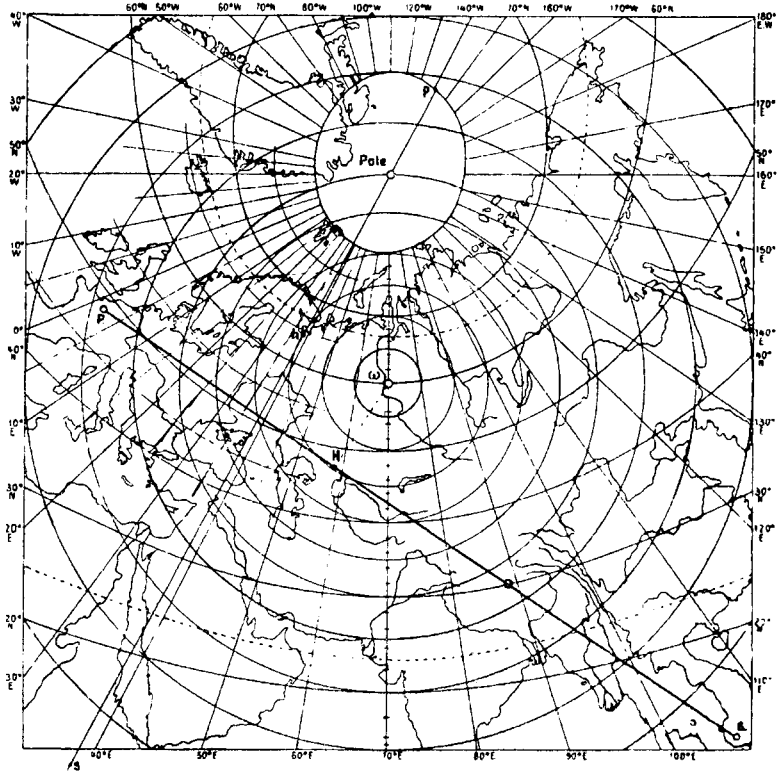


FIG. 1
Principle of Position Line Plotting by the Cartographic Method.

FIG. 3

Small Model of Gnomonic Navigation Chart. Principle of the Reading Diagram.



These latter methods of determining the fix "in the air" will be dealt with in the present paper.

A comparison of the *respective* merits of the three systems having been made several times by experts (cf. 1, 2, 10, 11, 12... in the bibliography appended hereto), we shall not touch upon the subject here, apart from mentioning the necessarily high cost of "mechanical" or "electrical" computers, and the considerable bulkiness of precomputed tables, consisting of several large volumes. A more significant evaluation is arrived at through the result of experimentation by airmen, which shows that *at present* preference is given to precomputed tables; very large use, on the other hand, has been made of graphical processes, while electrical and mechanical computers seem to have had no great practical success.

The purpose of the present paper is to examine in detail the calculation of the fix by means of diagrams, in order to show that they are capable of attaining a degree of convenience and accuracy comparable to that of tabular methods, while preferable from the point of view of cost and bulk.

We shall examine several known methods, and shall complete the discussion by proposing several new systems that appear of advantage in air navigation as well as under other conditions where speed of calculation is desired in exchange for average accuracy.

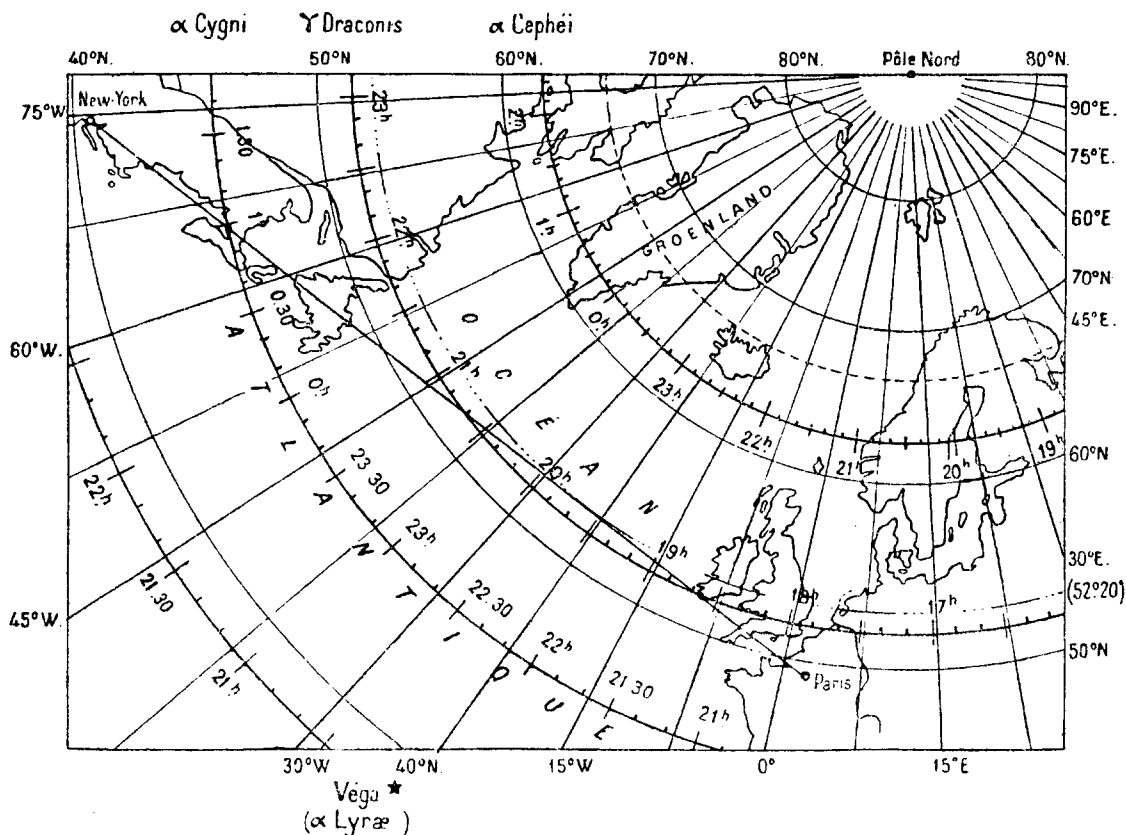


FIG. 4

Small Model of Gnomonic Navigation Chart, fitted for Astronomical Navigation.

Note. — Parallel 52°20' Lat. N. is shown by a dotted line. It corresponds to the Vertex of Paris-New York track (in connection with the Distance and Angle measurement system by means of the Reading Diagram).

DIRECT SOLUTION OF POSITION LINE PROBLEM ON THE CHART ITSELF: "CARTOGRAPHIC" METHOD.

The substitution of a graphical method for calculation, in obtaining a fix in air navigation, can be justified for the same general reasons of acceleration and simplification mentioned in connection with mechanical computers and pre-computed tables.

But there is another important argument specifically applicable to graphical processes.

As astronomical observations made by the aviator are used exclusively on the navigational chart, a document of determinate accuracy, it is clear that calculation of the intercept (for the position line) with the same degree of accuracy is desirable, thus avoiding useless complications and loss of time.

This fact leads initially to the testing, among possible graphical methods, of that involving the following simple problem: the direct evaluation on the nautical chart of the "estimated" zenith distance corresponding to the observation, at the given instant. This method might be termed the "cartographic" method. (See fig. 1.)

In order that it may be applied, it must be possible to represent nadir *S* of the star being observed within the scope of the chart; and that on the other hand it be possible to measure the exact distance between *S* and *ZE* ("estimated" zenith) in order to obtain the intercept. Both these conditions must be satisfied separately.

Let us examine these two provisions relating to the application of the method separately.

1.—The second condition mentioned requires that the chart allow for the exact measurement of distance *S ZE* :-

Weems' Curves.—The difficulty involved in this operation on ordinary world maps is well known, where the desired degree of accuracy cannot be obtained at distances approximating 300 kms. unless precomputed curves are available—which brings us to Commander Weems' method (see fig. 2) and the Astrograph.

By means of Weems' method, the fix is obtained from the intersection of the altitude curves, thus avoiding any assumption in connection with dead reckoning.

This advantage is worthy of note, since it may become of great importance when weather conditions (wind) are inadequately known.

The Favé-Brill method which was the French Army Air Force regulation method in 1939, is based on the same idea, and is likewise connected with the Kahn method, described below. The projection used is a "local" stereographic projection, with a range of 15° or 7°.5. The development in air navigation later made the range inadequate, but extension of the method was not to our knowledge undertaken.

If an easy system of measuring the distance is desired, Mercator's map should therefore be left aside, and more suitable grids from this point of view considered.

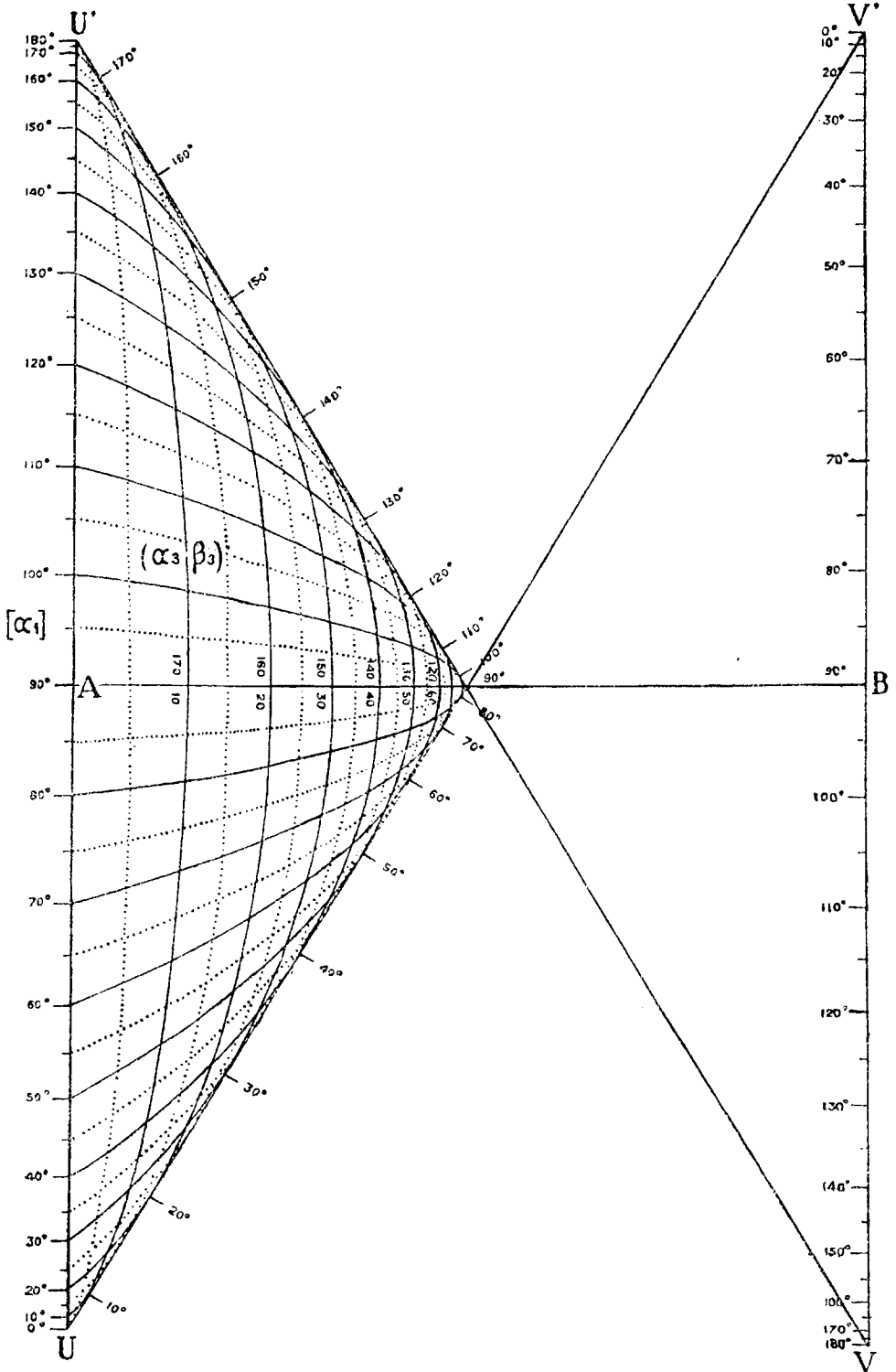
This condition is fulfilled by :-

1. *Oblique* Mercator projections, used by Mr. Kahn, making correct approximate evaluation of spherical distances possible.
2. Gnomonic projections, supplying a theoretically precise solution.

These we shall successively examine.

Kahn Charts.—A French naval architect (now *Ingénieur général* of the *Génie Maritime*) named Louis Kahn, was the first to advocate the application of oblique Mercator grids in the construction of air route charts⁽¹⁾; as mentioned above, position lines can be plotted directly on these charts, and they moreover have the diurnal (or hour) circles of the principal stars drawn upon them.

(1) In 1928: the example of L. Kahn's "Itinéraires Transcontinentaux Orthodromiques Aéronautiques" was imitated, more than 15 years later, by the Americans in their Chicago-Gander air route chart to the scale of 1:2 000 000, and in their project for an "International Series of Air Route Charts" proposed at the I.C.A.O. in 1948 at the fourth meeting of the MAP Division,



ABAQUE 100

FIG. 5

D'Ocagne Nomogram for the fundamental formula of Spherical Trigonometry.

graduated in sidereal hours—Universal Time, which shortens and simplifies the navigator's task even further.

From the point of view that concerns us, then, the Kahn method constitutes a satisfactory solution, as all specific difficulties relating to the problem are reduced to those involved in a typical problem in cartography—the determination of distance, avoiding all recourse to calculation and to expensive and cumbersome special tables and instruments.

2.—Discussion. The first condition mentioned is equivalent to introducing a limitation to star selection, imposing observations relatively close to the zenith. This restriction, while theoretically serious (since it involves *completion* of the method so that other cases may be solved⁽²⁾) is not impossible to overcome in practice, as there are always an adequate number of brilliant stars near the zenith (up to $z \leq 30^\circ$ or so) making correct determination of position possible: high flying altitudes, increasingly used at present, are favorable in this respect as the aviator is then provided with clear skies, where choice of stars is not hampered by cloudiness.

If observation near the zenith presents some relative difficulty, owing to types of sextants now in use (in connection with which observations are preferably made at average altitudes), the difficulty is surely temporary, shortly to be solved by the use of "periscopic" sextants enabling observation at any zenith distance.

Whatever the circumstances, the particular advantages of the direct method of plotting positions on the chart appear alone sufficient in justifying this slight change in the air navigator's acquired habits.

We mentioned above the recent proposals made by qualified representatives of International Commercial Aviation in connection with the construction of a large number of air route charts: the going into effect of these, coming after Kahn's itineraries, would strongly influence astronomical navigation and we at least think would throw new light on all other proposed solutions to the problem.

USE OF THE "CARTOGRAPHIC" METHOD IN UTILIZING THE FIX ON SPECIAL NAVIGATIONAL CHARTS OF CIRCULAR RANGE AND IN GNOMONIC PROJECTION.

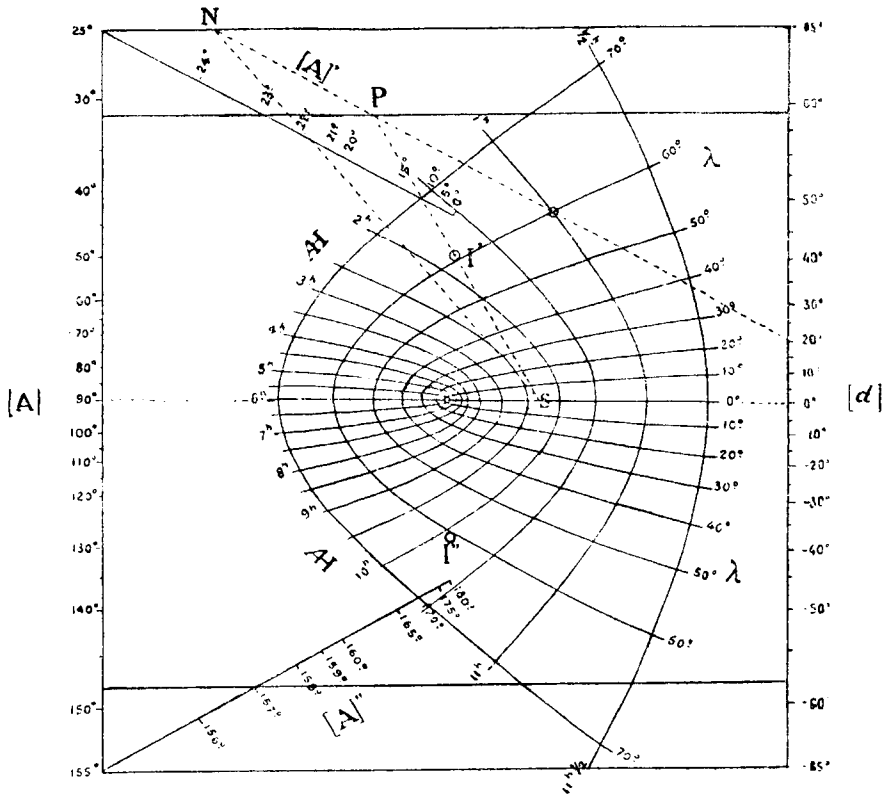
A special diagram of a new type (see fig. 3) has recently enabled us to improve the solution to the problem of distance and angle measurements in gnomonic projection.

A 2 to 5-mile degree of accuracy is obtained for distances, $0^\circ 2$ for angles, and the method of operation is extremely simple. (See fig. 3.)

The great advantage here is that unlike the Kahn system, values (distances or angles) corresponding to the orthodromic lines *themselves* are measured, with no necessity arising for a Givry or other correction, which use of the Kahn chart diminishes but does not entirely eliminate. The gnomonic chart then appears to be the only one enabling orthodromic distances corresponding to the range of zenith distances of stars observed from aircraft to be measured exactly. (See fig. 4.)

Under the circumstances, one is naturally led to suggest for navigation purposes a system of charts in gnomonic projection having a radial scope of approximately 30° of arc, a scale of about 1:1 000 000 at the centre, and equipped with a diagram enabling distances and angles to be measured directly and accurately. These new charts would present two important advantages as compared with air route charts in oblique Mercator projection. In the first place, their *wider range* would ensure better coverage of areas of interest and would make it possible to use a greater number of stars; secondly, as already mentioned, the greater *precision* of the method, the *true* images of the orthodromic lines being dealt with, while representation on the Kahn charts is only *approximately rectilinear*; no lack of accuracy need therefore be feared even in evaluating the longest distances.

(2) Particularly with reference to observations of the sun and moon; the remark following of course applies to night.



I.—GRAPHICAL CALCULATION OF RIGHT-ANGLED TRIANGLES.

In a separate category are methods of solving the ($\sin z = \sin x \sin y$) ($\tan z = \tan y \cos x$) formulae for right-angled spherical triangles by graphical means.

Among the latter may be mentioned :-

- the Bygrave rule—logarithmic sliding scales;
- Rectilinear Nomograms of the French Hydrographic Service⁽¹⁾.

Generally speaking, we believe the latter computer to be the better of the two.

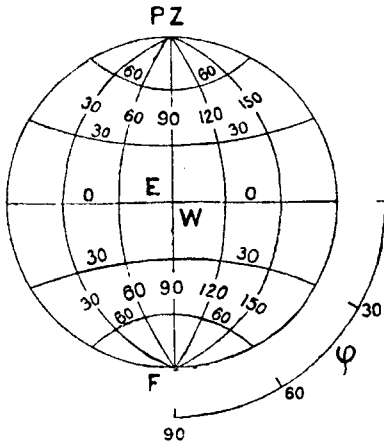


Fig. 7

Principle of solution by means of two sided cartographic diagram (Bastien special system).

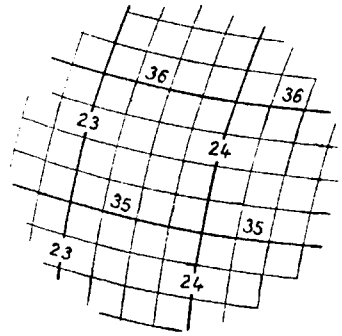


Fig. 8

Portion of Bastien-Morin network.

It measures approximately 64×39 cm., is extremely easy to read, and has a uniform rate of accuracy to within about $10'$ ($5'$ when interpolating by inspection) —corresponding to minimum 3-millimetre graduations along the linear scales.

Technically speaking, then, it is a most successful computer, although compelling the navigator to resort to splitting into right-angled triangles. This necessity carries no great intellectual effort with it, however, as in practice the operation consists in following a pre-established procedure.

Three graphical alignment operations successively supply the two unknown azimuth and estimated latitude factors, the latter being obtained by adding two intermediate λ' and λ'' auxiliary elements. (Use of the Bygrave rule entails a similar process.)

Time required depends of course upon the skill of the operator, but it seems likely that it could be shortened to a mere minute or two.

Neither this method nor the Bygrave rule, however, can be strictly considered as "graphic" means of obtaining a fix owing to the abstract element contained in the solution.

II. — NOMOGRAPHIC SOLUTIONS.

More "eloquent" solutions, in answer to similar problems, can be supplied by nomography in that the unknowns are obtained directly with no necessity of resorting to the device of splitting.

(1) Known as "Point en Ballon" nomograms.

In this connection, Soreau's "Treatise on Nomography" contains a *general diagram of spherical trigonometry*. This graphical computer, which we owe to Mr. d'Ocagne, shows the relationship of the four elements a, b, c and A and combines the principle of Cartesian diagrams and nomograms of collinear points (see fig. 5). To our knowledge no practical use has been made of this nomogram in spite of the neatness of its principle.

It may be remarked in this regard that since the air navigator also requires azimuth (even though it be with indifferent accuracy) for obtaining the direction of his position line, a supplementary graphical computer becomes necessary; nomography supplies it, in the shape of a «*Nomogram of 4 consecutive parts*» attributable to Lieut. Perret (see Soreau *loc. cit.* p. 437), which is similar in appearance to the first and used in the same way (see fig. 6).

These purely nomographic solutions⁽¹⁾ have the advantage of requiring no geometrical construction other than the alignments, of never becoming defective, and of simply needing to be read off. The degree of accuracy obtained is satisfactory, considering the necessary size.

The fact that their dissemination is not greater may well be cause for surprise.

CHANGE IN SPHERICAL CO-ORDINATES METHOD.

We shall now deal with a class of more geometrical solutions based on a plane representation of the sphere (or hemisphere), involving a simple geometric operation (rotation, transference, or symmetry).

The same principle may be applied in different ways, the properties of the computer obtained remaining approximately the same.

An important advantage of this class of computers lies in the geometric materialization of the accuracy of operation on the part of the groups of co-ordinate curves used in situating and interpreting the point. Discussion of results therefore merely resolves itself into an ordinary discussion of the working accuracy of two-sided Cartesian diagrams.

We shall refer, as to *type*, to Keller's diagram (ref. 6) in meridian stereographic projection. The instrument consists in a combination of two identical grids in meridian stereographic projection composed of orthogonal circles. Superimposed, the grids rotate freely in relation to each other (see fig. 7).

As a matter of fact, the top grid can be replaced by a sheet of tracing paper that will serve the same purpose.

The principle is well-known: triangle ABC , in perpendicular projection to plane AB , has its apexes A and B , on the plane, on the exterior great circle. Quantities b, A ; a, B , respectively, are of the same nature as the co-ordinate net,—if the apex of the net is placed in successive coincidence with A, B (whose arc distance, $A B$ equals the given quantity c).

An initial operation supplies the position of point C , b and A being known; the second operation (rotation of c on upper sheet) permits the reading of a and B .

A design of the above-mentioned computer for air navigation was executed in Germany prior to 1940 by Professor Kohlschutter. It measures 26×29 cm., and is therefore of restricted size; the grid is contained in a circle approximately 20 cm. in diameter; and there are 1° -intervals between the curves.

The Bastien-Morin computer works along the same principle, the grid here being in equidistant azimuthal projection ("Guillaume Postel" or "Hatt" projection). The diagram is small, but etched on glass, and readings are made with microscopes—whence its precision, to within $1'5$ to $1'$ —so that it really should be classed as an instrument comparing in fragility and cost to the "mechanical" calculating machines mentioned earlier (see fig. 8).

(1) These are "basic" solutions; others, not essentially distinct from them, may be obtained by anamorphosis, as with Brown and Nassau's nomographic spherical computer, *Bibl.* ref. (14), which also operates by means of collinear points. The base is defined by the co-ordinate relationships: $x = \cos d$, $y = \sin d \sin t$, which are those of orthographic projection.

We unfortunately have not been able to see any authoritative account on the practical use of this interesting instrument, several models of which were owned by the French Army Air Force by 1939.

Likewise constructed along the same lines, although extreme accuracy is no longer aimed at, is Mr. Driencourt's "diagram for preparing prismatic astrolabe observations". It resembles a Favé tracing, but is constructed on a plane-chart projection. Transference replaces rotation of P Z. Azimuth and hour angle are obtained to within a few tenths of a degree, which is sufficient for the purpose intended.

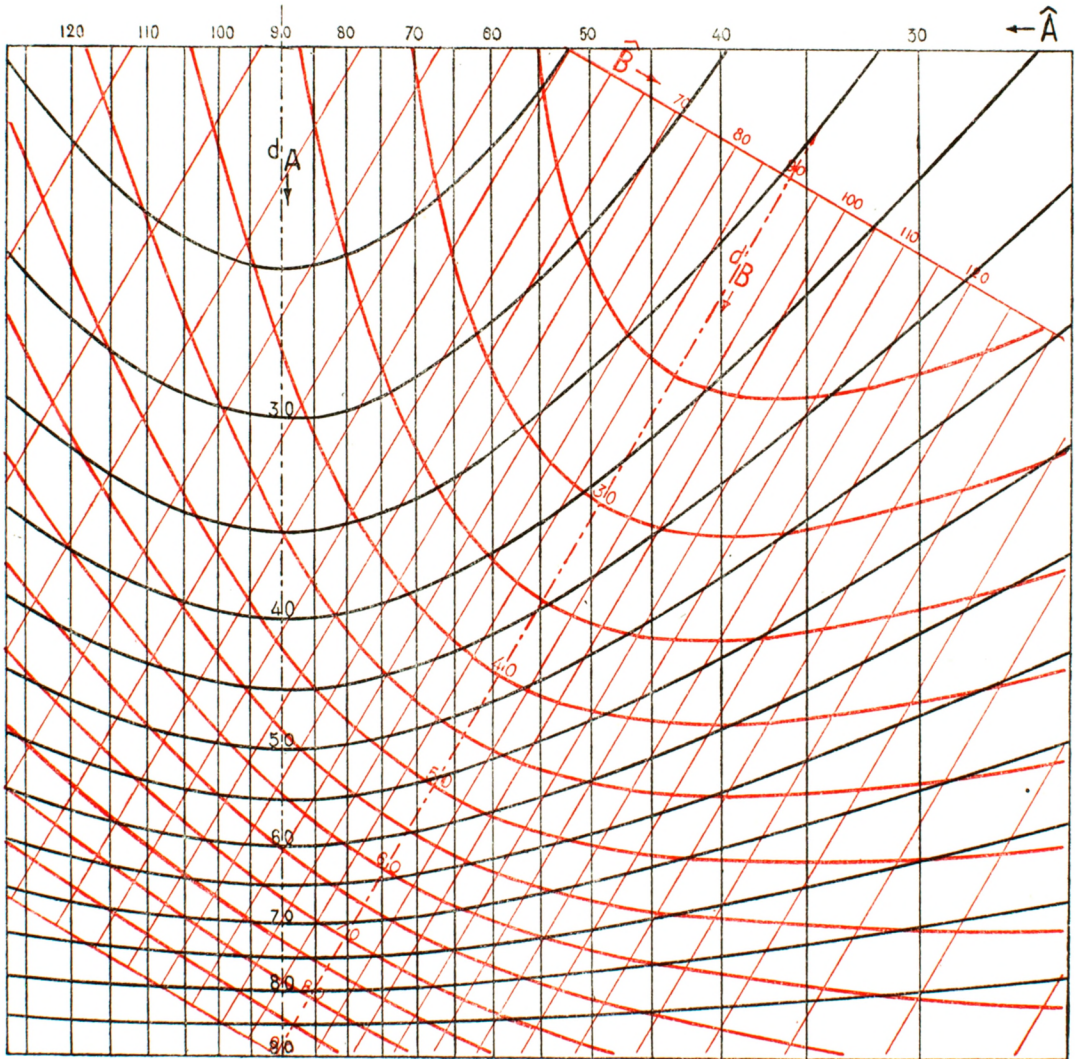


FIG. 9

Use of Spherical Co-ordinates Transfer System in Gnomonic Projection.
 In black : Point A co-ordinates network. — In red : Point D co-ordinates network.
 (Particular case $AB = 30^\circ$).

An interesting synoptic study of this type of "change in spherical co-ordinates" solution was undertaken recently by Mr. Gougenheim of the French Navy Hydrographic Service (Bibl. ref. 3), who compared the advantages of the various "conical" projections (i. e. those offering symmetry of rotation around the Pole). In the application of this general method,—Mr. Gougenheim referred to a conical

FIG. 10
Principle of the Orthodrome Diagram general method.

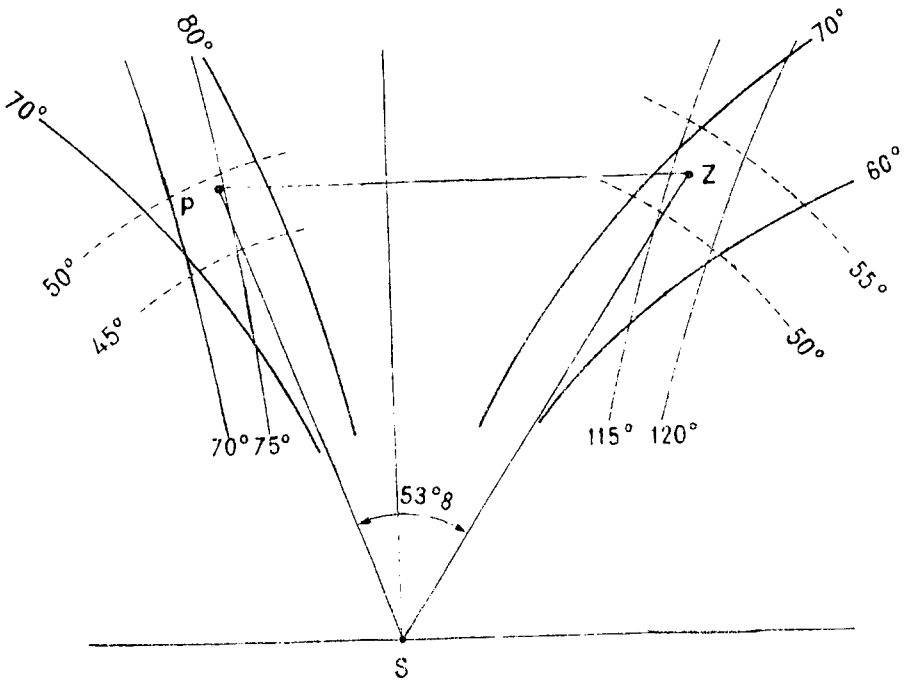
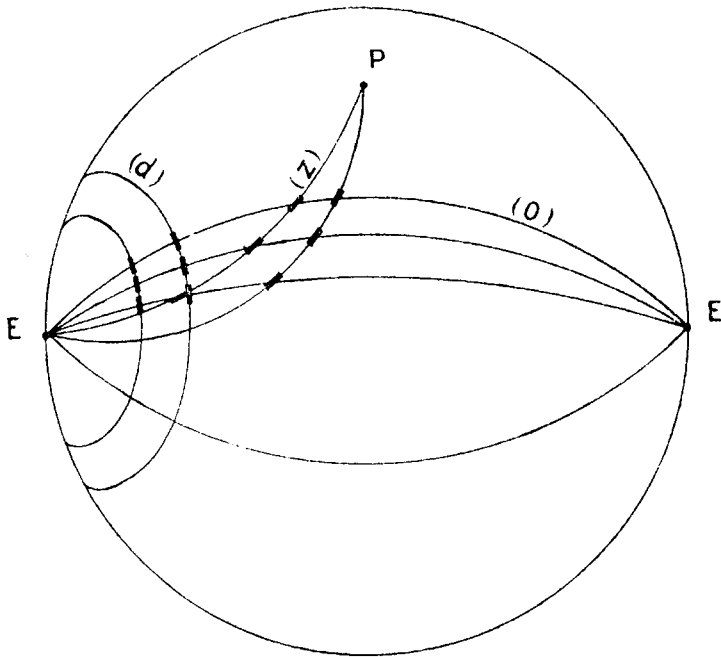


FIG. 12
Use of Gnomonic Orthodrome Diagram for the complete solution of Position Triangle.

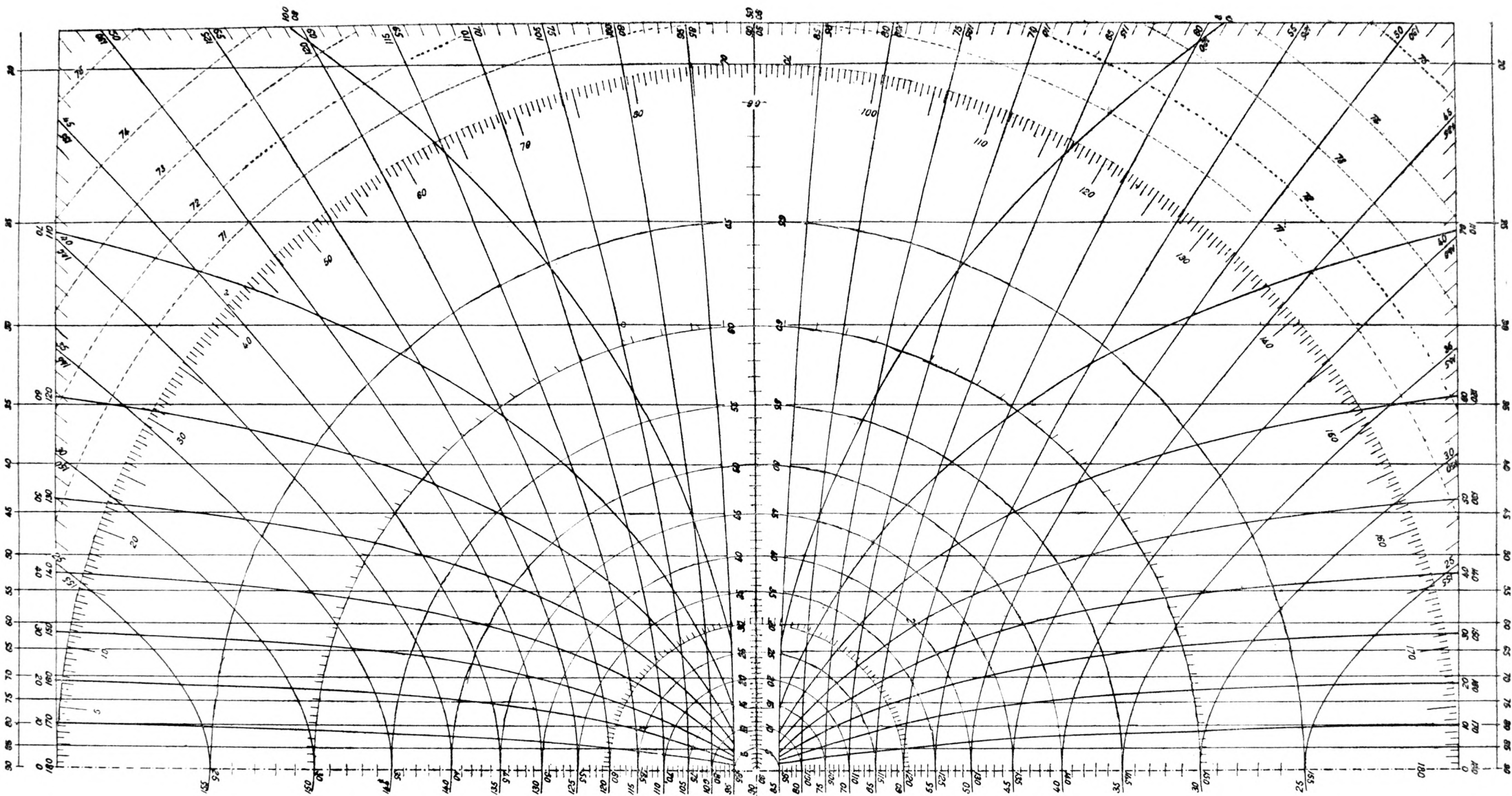


FIG. 11
Gnomonic Orthodrome Diagram.

conformal projection of the second power as also being capable of supplying the value of the 6th unknown of the spherical triangle, but for the practical needs of air navigation, he rather inclined towards stereographic or Postel azimuthal grids.

We personally were curious enough to experiment with the application of this change of co-ordinates method in *gnomonic* projection. It will be objected *a priori* that points P and Z are both at infinity on the diagram—but this is not serious, since their polar co-ordinate lines (ρ, θ) are the ones that are needed, and they are at finite distances. This grid is most convenient and easy to trace, consisting of Hilleret's equatorial gnomonic projection, where great circles equal parallel straight lines and small distance circles are equivalent to co-axial hyperbolae.

Careful investigation shows that these advantages are of great value in constructing curves accurately and in interpolating readings correctly. It may therefore provide greater precision than any other diagram.

Even if the range of this computer is limited, the limit being in the neighbourhood of 65° of arc from the centre (implying $z > 25^\circ$, $D > 65^\circ$) a thorough examination of the possibilities of this idea is of definite interest. Here are the results of our own investigations on this subject.

We stress the fact that the basic advantage lies in the equations of co-ordinates :-

Projections of great circles..... θ fixed $Y = \tan \theta$

Projections of distance circles..... ρ fixed $X = \cotan \rho \sec \theta$

permitting precise plotting with the co-ordinatograph *at any scale*.

Under these conditions, it will be possible to :-

1. Exchange side PZ with either other side of the triangle, in constructing the figure;
2. Then attempt to eliminate one of the parameters by one of the following means :-

The Weems method, where D is assumed to be fixed (selected star);

The Favé-Brill hypothesis (by assuming that the approximate latitude is known);

The Driencourt method of fixed z observations (... 30° in this case).

The elimination of a parameter has the very great advantage here of doing away with the rotation process (as it can then be admitted that superimposition of the two grids is fixed, and can be printed on an identical base). Better possibilities of presentation are thus afforded.

It seems to us that the most interesting results obtainable from this idea would be through the setting-up of albums (similar to Weems') dealing separately with successive zones of latitude, having ranges in proportion to the kind of astronomical work being carried out. The advantage of easy Cartesian representation of the dimensioned curves will then be fully evident.

The double interpretation of fig. 9 shows the appearance of album pages in the two cases mentioned below :-

- (a) The zenith observation distance being constant, the readings of the parallel straight lines geometrically supply unknowns Az and S for the given D and L values (the hour-angle curves, which will not have a simple geometrical form, can be plotted by points).
- (b) The star being selected beforehand (Weems method), and *its declination assumed as being fixed*, the parallel straight line readings geometrically supply unknowns H. A. and S in terms of the given D and L values (as previously indicated, curves of equal azimuth can be drawn).

Questions of size, intervals between curves, and available drawing materials show that the present method could provide a completely satisfactory solution for surface and air navigation, as well as desert expeditions (maximum precision obtained : about 0'5). It is also an adequate solution to the problem of preparing lists for expedited star observations in the field.

However, these field running observations (where the minimum degree of accuracy desired in any case amounts to a few seconds) cannot be applied if this

or any graphical method, we think, is used; experience has shown that the acquiring of increased accuracy is a laborious process, and that one is generally disappointed in attempting the almost impossible task of gaining a decimal place by means of graphical methods.

We shall now deal with a final type of solution to the position triangle problem, deriving from remarks in connection with the two principal methods discussed above.

SOLUTION OF POSITION TRIANGLE BY MEANS OF GREAT CIRCLE DIAGRAMS.

Even simpler than the change in co-ordinates system for solving the position triangle is a method where previously constructed graphical aids involving no movable parts are used; these consist of *great circle diagrams*.

The principle consists in noting the similarity between the problem of the position triangle (given by L, H.A. and D) and the problem described as the "direct geodesy problem" (considered on the sphere); i. e., in determining point B on the sphere, its polar co-ordinates being known with reference to a known point A.

The relationship between the two questions becomes apparent as soon as it is observed that H. A. corresponds to the azimuth at A, \overline{PN} corresponding to A.

Great circle diagrams of the Favé tracing type—comprising the two "equi-distance" and "equal azimuth" tracings—supply the answer to the problem in cartography involved. The Favé method usually consists in sliding the diagrams (transference) on a Mercator chart base. But there is no necessity for the base if only co-ordinates are needed, so that both the problem in cartography and the astronomical problem of the position triangle can be solved by superimposition of the two Favé tracings.

Since the argument is based on elements actually existing on the sphere (see fig. 10), namely;

- A bundle of great circles passing through points E E;
- Curves of equal length on the orthodromes;
- Curves of equal azimuth of the orthodromes;
- Parallels of latitude;

the same solution can be transferred to any kind of projection; choice of the latter, in connection with the construction and use of the diagram, should of course be governed by factors of convenience and accuracy.

Figures 11 and 12 together show the general appearance of the diagram obtained in gnomonic projection as well as the method of using the diagram in order to solve the position triangle.

A range of solutions is obtained having in common the characteristics of requiring no rotation of the diagram, of making only readings and a mental addition necessary, and of clearly showing, in geometrical form, the accuracy of the result obtained.

The possibilities of this method appear interesting, and it may provide the most satisfactory solution to the long-studied problem of calculating the fix by direct graphical means. The existence of new transparent materials such as Astralon and Vinylite, due to their excellent flexible and resistant qualities and rigidity of shape, might prove of valuable assistance in connection with this new solution.

Before concluding, it should be pointed out that "great circle" or gnomonic charts of the Kahn type already provide a very direct solution to the question, but with limited coverage.



Our own choice lies in the joint use of these two methods; but it should be stressed that this choice is mainly due to a desire for simplicity, and that one of the many other already existing solutions, which might be preferred because of its convenient use in aircraft or marked technical development, is an acceptable alternative.

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