INVESTIGATIONS INTO THE DIP OF THE HORIZON

after H. C. FREIESLEBEN (German Hydrographic Institute).

With reference to the article entitled "Refraction" by J. E. R. Ross, which appeared on page 35 of Volume XXII of the *International Hydrographic Review*, Monaco, 1945, and to Dr. FREIESLEBEN's letter, published on page 77 of the *International Hydrographic Review*, Volume XXV, No. 2, 1948, under the title of "Studies on Refraction", an analysis of an article by Dr. FREIESLEBEN entitled "Investigations into the Dip of the Horizon" appearing in the *Journal of the Institute of Navigation*, London, Volume 3, No. 3, July 1950, pp. 270-279, is given below. In this article, which the author already alludes to in his letter to Mr. Ross, the results obtained by the latter are confirmed.

Most nautical tables use as the formula for dip of the horizon Dip $= 1'.80 \sqrt{h}$ where h is the height in metres, obtained from the formula $D = 1'.93 \sqrt{h\,(1 - k)}$, where k, the coefficient for refraction used in geodesy, is assumed as having a mean value of 0.13. The figure holds good for normal atmospheric conditions involving a uniform decrease in density of the air with increasing height. Irregularities often occur, however, and the problem is whether such abnormalities can be predicted theoretically or empirically.

After describing the practical difficulties in measuring air pressure, temperature, and humidity, on which the optical density depends, encountered in a theoretical approach to the problem, and after a brief historical outline, the author notes the empirical formula obtained by Koss and Kohlschütter :

$$D = 1'.82 \sqrt{h} - 0'.37 \,\triangle \qquad (1)$$

where $\triangle$ is the difference in degrees centigrade between the temperature of the sea surface and the temperature of the air at the height of eye.

Formula (1) was contradicted as often as not when it became possible to obtain accurate measurements of dip by using the Pulfrich instrument.

This latter feat was accomplished by Rear Admiral Dr. F. Conrad, who made observations of the dip on Heligoland and Rügen from 1934 to 1938 with the Pulfrich instrument and by theodolite, taking temperature measurements under varying atmospheric conditions.

The author shows that if a formula of the type of (1) is used and the coefficient $\triangle$ is found to depend on height of eye, then the best representations of the Heligoland observations, at mean heights of 4 m., 8 m. and 50 m., were as follows :

$$4\text{ m. Dip} = 1'.74 \sqrt{h} - 0'.47 \,\triangle$$

$$8\text{ m. Dip} = 1'.74 \sqrt{h} - 0'.23 \,\triangle$$

$$50\text{ m. Dip} = 1'.77 \sqrt{h} - 0'.04 \,\triangle$$

indicating that the influence of temperature decreases with the increase in height of eye.

The conclusion reached is that the Koss-Kohlschütter formula, although correct for a number of cases, is only approximate, and that a more precise formula must be evolved.

This the author achieves by assuming, as in the case of most Navigation Manuals, the atmosphere to be divided into concentric layers. If the depth of a layer be denoted as dh, the height above sea-level as h, and the refractive index as n, then the deflection of a ray of light passing from layer n to another layer $n + dn$ will be given by $n\beta = \left(-\dfrac{dn}{n}\right) \tan z$.

On the other hand $d\gamma = \left(\dfrac{dh}{r}\right) \tan z$ where r = the Earth's radius $+ h$.

The coefficient of refraction used in geodesy is defined as $k = \dfrac{r}{n}\left(\dfrac{dn}{dh}\right)$. In a first approximation and at moderate heights, this coefficient depends on the temperature gradient and numerical constants derived from the gas constant of the air, the refractive index at 760 mb and 0° C., and the radius of the Earth. Its value may be given as

$$k = 6.84 \left(0.034 - \frac{dT}{dh}\right)$$

Again, the refractive index n of a single layer of air depends on its height h and its temperature T_h. Integrating $d\gamma - d\beta$ for the individual layers, he gets

$$\text{Dip} = 5'.04 \sqrt{(0.1123h + T_0 - T_h)} \qquad (2)$$

where h is the height in metres and T_0 the temperature of the lowest layer in degrees centigrade.

This formula, which only uses two temperatures, is likewise an approximation, although it explains previous anomalies ; and the author points out that the use of temperature T_0 of the lowest layer of air instead of the water surface temperature appears to make a decisive difference.

A border layer of air exists over the water, and determination of the temperature of this layer is essential to knowledge of the path of light rays. It is not necessarily related to the temperature of the water itself. Only under normal atmospheric conditions can the temperature difference Δ between the water and the air be regarded as representative of the air mass structure. Only then does a relationship exist between Δ and $T_0 - T_h$, and formula (1) apply.

This conclusion corresponds with results of modern research on turbulence, and attempts have been made to determine the height above the surface where a temperature T_0 prevails. Investigations only revealed that it must certainly be below 1 m. It is therefore impossible for navigators to compute reliably the dip of the horizon from temperature measurements.

The article is a significant contribution towards the explanation of the much-debated problem of the dip of the horizon and shows that for practical purposes temperature measurements only supply approximate values and that results in individual cases will not be correct.