PAPER ON ASTRONOMICAL NAVIGATION BY MEANS OF DIAGRAMS

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Addition to *Paper on Astronomical Navigation by Means of Diagrams* published in *International Hydrographic Review,* Vol. XXVII, No. 2, Monaco, November 1950, No. 48 of the Series, pages 147 to 161.

The text of the above-mentioned paper fulfilled the purpose of setting down results and conclusions arrived at by the author at the time, but did not indicate definite completion of the author's research on the subject.

IMPROVEMENTS SOUGHT AFTER

Starting from the idea stated on page 160 (1), which consisted in reconstructing in each case the spherical triangle on a flat grid enabling the 3 different data for the triangle to be obtained automatically, in a selected order, by means of networks of dimensioned curves, and through use of these same curves, enabling reading of the desired unknowns, we continued our research with regard to these grids in an attempt to discover their simplest and most practical aspect. As it has been seen we had already considered the use of Mercator, stereographic and gnomonic projections.

The controlling idea in our investigations was to find a grid (or method of representing the sphere) in which several of the necessarily considered families of curves would be represented by *parallel straight lines* (in gnomonic projection, the family of orthodromes has this characteristic). Actually, a family of lines of this type can so easily be *reproduced* by merely using an ordinary *ruler* and *scales in the margin* that they can be eliminated from the background, and the diagram made clearer, which is an interesting initial improvement. The fewer the networks in a diagram, the better its adaptation for practical use and the greater its operational accuracy ; and while this is a valid consideration in the case of office work, it is even truer in the case of air navigation, where practical and psychological difficulties of a special kind interfere with calculations.

On the other hand, a grid should be retained that enables the solution of spherical triangles with sides approaching an arc of 90°, since astronomical observations in navigation frequently occur for stars or low altitude; this requires complete representation of the half-sphere on the grid at a finite distance, which was not obtained, as we have seen, with the gnomonic grid.

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(1) Intern. Hydr. Rev., Vol. XXVII, N° 2, Nov. 1950.

GRAPHIQUE REPRESENTATIF DES TRIANGLES SPHERIQUES RECTANGLES

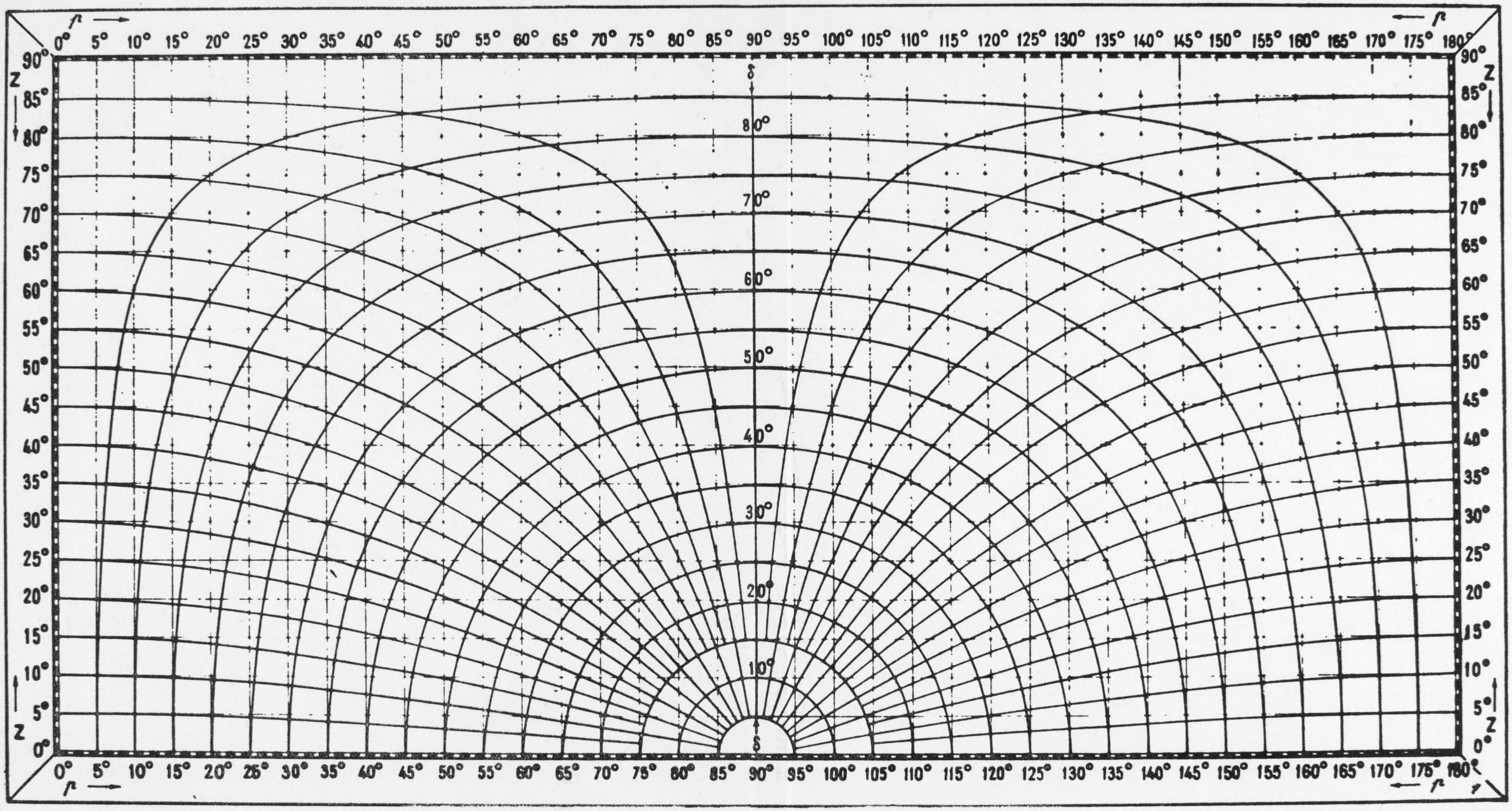


Fig. 13.

Le présent abaque permet la résolution générale des triangles sphériques.

Exemple. — Résolution du triangle : Pôle Nord - New-York - Paris.

Données

Entrée sur l'abaque : Centre P.

P.N. { Z = 76°3 (*AM. New-York - Paris*).
δ = 90° - 48°8 = 41°2 (*colatitude de Paris*).

Δp = 90° - 40°8 = 49°2 (*colatitude de New-York*).

Lectures intermédiaires (facultatives)

δ = 39°0 (*y. de la ligne P.N. - N.Y.*).

$p_{P.N.}$ = 78°0

$p_{N.Y.}$ = 127°2 Δp = 49°2

Résultats

Lectures...

N.-Y. { Z = 53°8 (*Azimut vers P.*).
δ = 52°5 (*Distance à P.*).

ADVANTAGES OF « CARTE PLATE » GRID

In order to improve the result obtained with the gnomonic grid, an attempt may be made to make the family of *orthodromes* as well as the family of their *lines of equal distance coverage* rectilinear, and in this way a return is made to the so-called *carte plate* system, known since earliest times (where latitude and longitude are represented by equidistant rectangular co-ordinates).

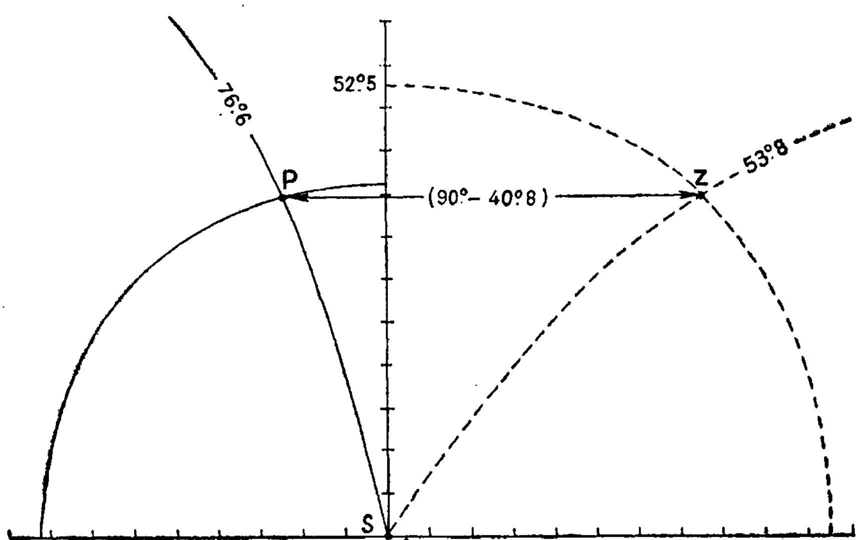
Two families of curves remain to be drawn on this grid :

Curves of *equal distance to the centre*
and of *equal azimuth of the orthodromes.*

These are standard curves (see Fig. 13 : Diagram showing right-angled spherical triangles), and we shall prove later on that they are none other than curves that represent the elements of right-angled spherical triangles on the *carte plate*. The diagram obtained, which in fact is a right-angled spherical triangle diagram, exists ; it has already been issued by the French Navy Hydrographic Office as the *Favé and Rollet de l'Isle Diagram*, a carefully produced design with 10' intervals. The same diagram appears in Bertin's *Tablette*, which supplies a kind of detailed numerical description of it': Bertin uses it in defining the small local diagrams called « inserts » used for interpolation purposes in his table.

Fig. 14

Méthode générale, appliquée sur le fond "carte plate"



Exemple : *Paris (S)* *New York (Z)* *Pôle Nord (P)*
Notations reprises d'un exemple précédent

Application of the general method is possible with the help of this diagram (see Fig. 14), and is made appreciably easier owing to the shape of the lines of equal distance coverage. After the second vertex of the spherical triangle has been placed in position through its co-ordinates (distance to centre-azimuth), the third vertex is defined by its distance from the second, reckoned along the orthodrome, and this distance can be taken directly from the x-axis graduation of the diagram, by using calipers, for instance. The spherical triangle is finally solved by reading the curvilinear co-ordinates of this third vertex. The annexed figure illustrates such a solution, and shows that since no graphical difficulty is involved, it can be arrived at with a minimum of delay and the maximum degree of accuracy set by the graduation specifications of the diagram.

Let us now compare the *carte plate* diagram with those constructed on gnomonic or stereographic grids. Where the layout on a page is concerned — which sets the conditions with reference to the graphical dimensions of the graduations — the superiority of the *carte plate* diagram again becomes evident. For the latter can be inscribed within a rectangle, the relative scale of whose two sides can be set as desired, enabling fullest use to be made of the size of paper available.

The fact that the distance curves and curves of equal azimuth no longer have a simple geometrical character causes no practical inconvenience, whether in the utilization or construction of the diagram, since the co-ordinates are tabulated.

IDENTITY OF CONSTRUCTION OBTAINED WITH ABOVE-MENTIONED STANDARD METHOD

It will be observed that the construction just described strictly coincides with that used in the method of change in pole of spherical co-ordinates, as applied on a *carte plate* grid. (See Driencourt's diagram for preparing prismatic astrolabe observations). In both methods « horizontal » transference (parallel to the edge of the diagram) occurs when proceeding from the second to the third vertex of the spherical triangle, whose first vertex is at the origin.

Further study of the reason for this identity of methods, involving *ipso facto* an identity in the network of curves used, is of interest; this is why we stated above that the « equal azimuth » and « equal distance » curves obtained coincided with those of the Favé-Rollet de l'Isle diagram.

Figures 15 and 16, showing the *carte plate* grid as used, respectively, in our general method and in the Driencourt method, demonstrate that the grids are arranged at right angles to one another.

In these two figures, the *sides* (great circle arcs) of the spherical triangles are represented by means of *thin* lines, and the *heavy* lines indicate the *loci :*

equal distances to the centre,
equal azimuths of orthodromes.

It now becomes a question of interpreting these plane images in terms of the actual spherical figures that correspond to them.

Fig. 15 – *Dispositif de la Carte plate, utilisation dans la présente méthode*

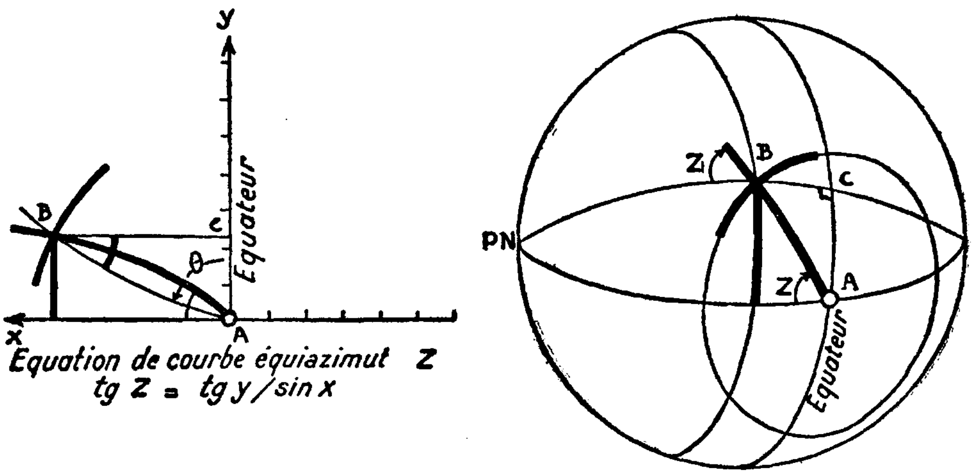
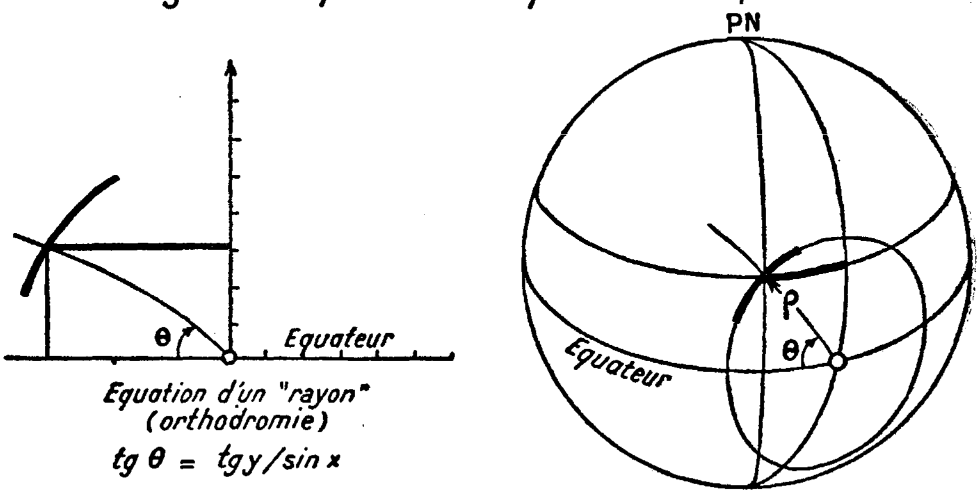


Fig. 16 – *Dispositif classique de la Carte plate.*



In the arrangement shown in Figure 15, the *equal-azimuth curves* are used, defined by the condition :

« Angle Z of right-angled spherical triangle........ constant » and their equation, according to the properties of right-angled triangles, therefore is : $\tan Z = \tan y / \sin x = \text{constant}$.

The curves of equal distance to the centre are

« Arc ρ of right-angled spherical triangle .... constant » therefore,

$$\cos \rho = \cos x \times \cos y = \text{constant}.$$

In the arrangement shown in Figure 16, the *orthodromes* radiating from the centre are used, defined by the condition :

« Angle θ of right-angled spherical triangle ...... constant »

whence, in accordance with the same spherical trigonometry relations : $\tan \theta = \tan y / \sin x =$ constant,

the same curves are found as above.

The curves of equal distance to the centre also remain unchanged, since their equation is symmetrical in relation to x and y.

This explains the strict analogy observed between the method of *change in spherical co-ordinates* and the one described by us in this paper. An additional note as to the properties of the *carte plate* will be found at the end of the article, resulting from comparison of the two figures.

Let us first, however, complete investigation of the practical consequences of these properties.

THE BASTIEN COMPUTER APPLIED TO THE CARTE PLATE GRID : THE SAME GRAPHICAL METHOD IS USED

On page 156 of Vol. XXVII, N° 2 of the *International Hydrographic Review* a brief description was given of the principle of the Bastien Computer : the 1938 model constituted application of the principle of change in spherical co-ordinates on a grid in azimuthal projection, change in co-ordinates being effected by rotation about a centre. The originality of the computer — from the measurement and accuracy aspect — consisted in the use of a diagram based on two dimensioned nets, very finely etched on glass and viewed through lenses.

It is worth noting that in carrying forward his research for the purpose of improving his computer, Professor Bastien reached the point where he substituted a transference operation for one of rotation, with the result that he too was led back to the *carte plate* grid, Favé-Rollet de l'Isle diagram and Driencourt construction discussed above.

Professor Bastien has conferred upon this standard diagram the neatness of appearance and fine-reading possibilities (viewing of a film through lenses), together with the accuracy approaching the sexagesimal minute, that characterized his first computer. The same degree of accuracy has been preserved in the new model, which is at present of an altogether portable type and is much easier and faster to operate. The diagram in the new Bastien computer winds on rollers like a roll of camera film, the winding providing the transference required by the method.

There is reason for believing, therefore, that this new instrument will prove extremely useful to navigators in astronomical navigation; it is our personal opinion that it will simplify their task to such an extent that it may notably contribute to the more wide-spread application of this type of navigation by airmen, who make comparatively little use of it. Safety in flying would certainly be benefited.

The same category of users desiring a degree of accuracy close to the sexagesimal minute may find the same diagram set up in book form, the general method being applied through use of a simplified ground-glass slide. This book was designed by M. Bastien for the fastest and easiest possible reference, and great care has gone into its preparation. It may be looked upon as a most convenient kind of graphical version of Bertin's *Tablettes,* a basic reference work in spherical trigonometry.

If a lesser degree of accuracy (5 to 6 sexagesimal minutes) is desired, which is the case for the majority of airmen using astronomical navigation, a second reduced version is available, likewise according to Professor Bastien's design and representing the *carte plate* grid on the scale of 3 mm per degree, the curves being drawn every degree. It has been printed by the French National Geographic Institute on plastic material, its size is 54 × 27 cm., and it appears to be well suited for use in the air. Distance circles corresponding to 12 principal stars have been shown in advance to speed up even further and simplify the solving of the spherical triangles in these cases.

From the theoretical point of view, which was the initial aspect of this work, it appears to be of particular interest to stress the identity of the conclusions reached by Driencourt, Professor Bastien and ourselves (1) with regard to the best graphical method for solving spherical triangles. The oldest and simplest of right-angled triangle diagrams seems to supply, as we have seen, the most practical results.

REMARK ON INTERPRETATION OF FIGURES (RIGHT-ANGLED SPHERICAL TRIANGLES) DRAWN ON CARTE PLATE

Figure 15 contains interesting peculiarities that deserve consideration. From the following figure (16), the shape of the line representing side AB of the right-angled spherical triangle ABC is known.

This line, as we observed, has the same equation as the equal-azimuth curve, but with y exchanged for x. It therefore belongs to the set that is symmetrical to the set of equal-azimuth lines in relation to the first bisector.

The angle at the base of the equal-azimuth line at the centre point represents the actual size of the angle at B. Furthermore, since the *carte plate* is conformal at the centre point, angle θ represents, likewise according to its true size, angle A of the right-angled spherical triangle. Consequently the small angle comprised between the two curves represents the *spherical excess* of the right-angled triangle under consideration.

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(1) A design fairly close to Bastien's, but of smaller size and lesser accuracy, has been brought to our attention as being attributable to M. Lecoq, Inspector General of Hydrography, as of 1948 : « Planisphère Sphérique », by Lecoq.