OPTIMUM STOCKS FOR NAUTICAL CHARTS

by Rear Admiral W. LANGERAAR
Hydrographer, Royal Netherlands Navy

The continual necessity to correct all charts before they are ready to be distributed creates the problem of deciding on the optimum size for stocks of any particular chart that is ready for reprinting.

As the labour involved in continual corrections has to be paid for, and as printing is also a costly business, very small stocks are expensive because of the need for frequent reprinting and very large stocks become a financial burden because of the accumulation of corrections to be made.

In general, the optimum size of any particular stock of charts should be such that at the time when corrections become too large a burden stocks have gone down to zero.

It is obvious that this optimum size depends on several parameters, such as :

1) printing costs;
2) correction costs;
3) average number of charts sold daily;
4) average daily increase in the number of correction units;
5) retail price of charts, and discount allowed to Sales Agents.

Printing costs

In general it will be sufficient to represent total printing costs D by the equation :

$$D = O + N \cdot b \qquad (1)$$

in which

O = overhead costs

N = number of charts printed

b = cost per chart, for paper, actual machine time, etc.

In the Netherlands O is of the order of 700 Fl. (U.S. $ 195) and b is about 0.75 Fl. (U.S. 20 cents). O is, of course, dependent on the number of colour separations and on the number of printing runs, and b varies slightly with the format of the chart.

Correction costs

If the average number of copies of a chart sold daily is designated by A, and the average daily increase in the number of correction units for that chart by C, the following equation can be derived.

The first day of the newly printed stock, prior to the distribution of the A charts, A.C correction units have to be handled. The second day there will be 2 A.C correction units, etc.

If t is the time (in number of days) that it takes to exhaust the stocks of this chart, the total number of correction units to be handled, T, can be found from :

$$\begin{aligned} T &= AC + 2AC + 3AC + \ldots + tAC \\ &= AC\,(1 + 2 + 3 + \ldots + t) \\ &= AC\,\frac{t(t+1)}{2} \end{aligned} \tag{2}$$

If U is the cost involved for handling one correction unit then the total cost, S, for correcting the stock will be found from (2) to be :

$$S = \frac{t(t+1)}{2}\,ACU \tag{3}$$

It is easy to find an equation for t as a function of N and A :

$$t = \frac{N - e}{A} \tag{4}$$

in which e is the number of charts forwarded to other hydrographic offices under the IHB's mutual exchange agreement and those that for any reason are not for sale to the general public or to the armed forces.

From (3) and (4) we find :

$$\begin{aligned} S &= \frac{\frac{N-e}{A} \times \left(\frac{N-e}{A} + 1\right)}{2}\,ACU \\ &= \frac{(N-e)\,(N-e+A)}{2A}\,CU \end{aligned}$$

which for all practical purposes may be considered to be :

$$S = \frac{(N-e)^2}{2A}\,CU \tag{5}$$

It should here be mentioned that a " correction unit " does not necessarily coincide with a " Notice to Mariners ". A correction unit is one clearly-defined manipulation such as deleting a wreck symbol or inserting a new buoy. There are Notices to Mariners that may contain more than 40 correction units — when, for example, a number of buoys have been moved.

In the Netherlands U amounts to about 0.15 Fl. (4 U.S. cents) per correction unit.

Chart sales

If the price of a chart is given by P, then the total receipts reach a maximum of $(N - e)P$. However, the Hydrographic Department does not normally sell charts over the counter (or only a very small proportion of the stock); it has a sales organization in which the Sales Agents are allowed a certain discount and this in some countries can be as much as 50 % of the retail price.

For this and other reasons the Hydrographic Department receives only a certain percentage of the total selling value of the stock. If this percentage is represented by F, the Hydrographic Department's receipts, R, are found from :

$$R = (N - e)\ P.F \tag{6}$$

In the Netherlands F is about 70 %.

Gain

If, for the time being, the cost of survey vessels and other large overhead costs are disregarded, the concept of " gain " can be defined as the difference between the sales receipts from a stock of charts and the costs directly related to creating and maintaining that stock.

It can, therefore, be said that gain G is found from :

$$G = R - S - D \tag{7}$$

Substituting in (7) the values of R given by (6), of S given by (5) and of D given by (1) we obtain :

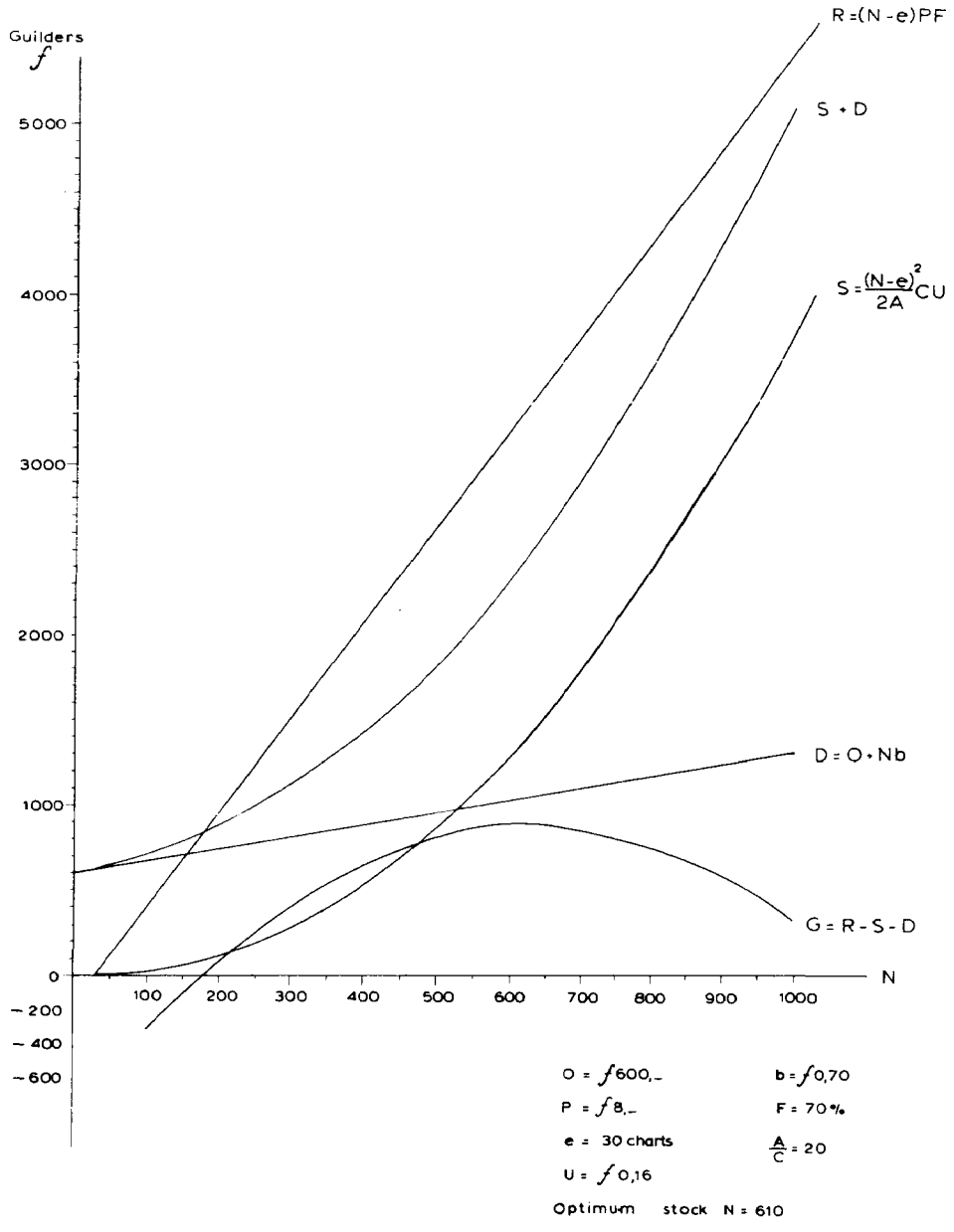
$$G = (N - e)P \cdot F - \frac{(N - e)^2}{2A} CU - O - N \cdot b \tag{8}$$

It is clear that when P and F are large, A large, C and U small, and O and b small, these all tend to increase G for a given value of N. However, P, F, A, C, U, O, b and e are all parameters of known magnitude. P, A and C are dependent on the particular chart; in the case of P, F, U and e, on the country being considered; for O and b, on the printing office.

Optimum size for stocks

From (8) it is clear that G will reach a maximum for a certain value of N and will diminish for either larger or smaller values of N. For this optimum value of N the gain G will be maximum (or in cases where G is negative, the loss will be minimum).

It seems a quite straightforward principle to stockpile in such a way that the gain, established in equation (8), is maximized. Larger stocks will

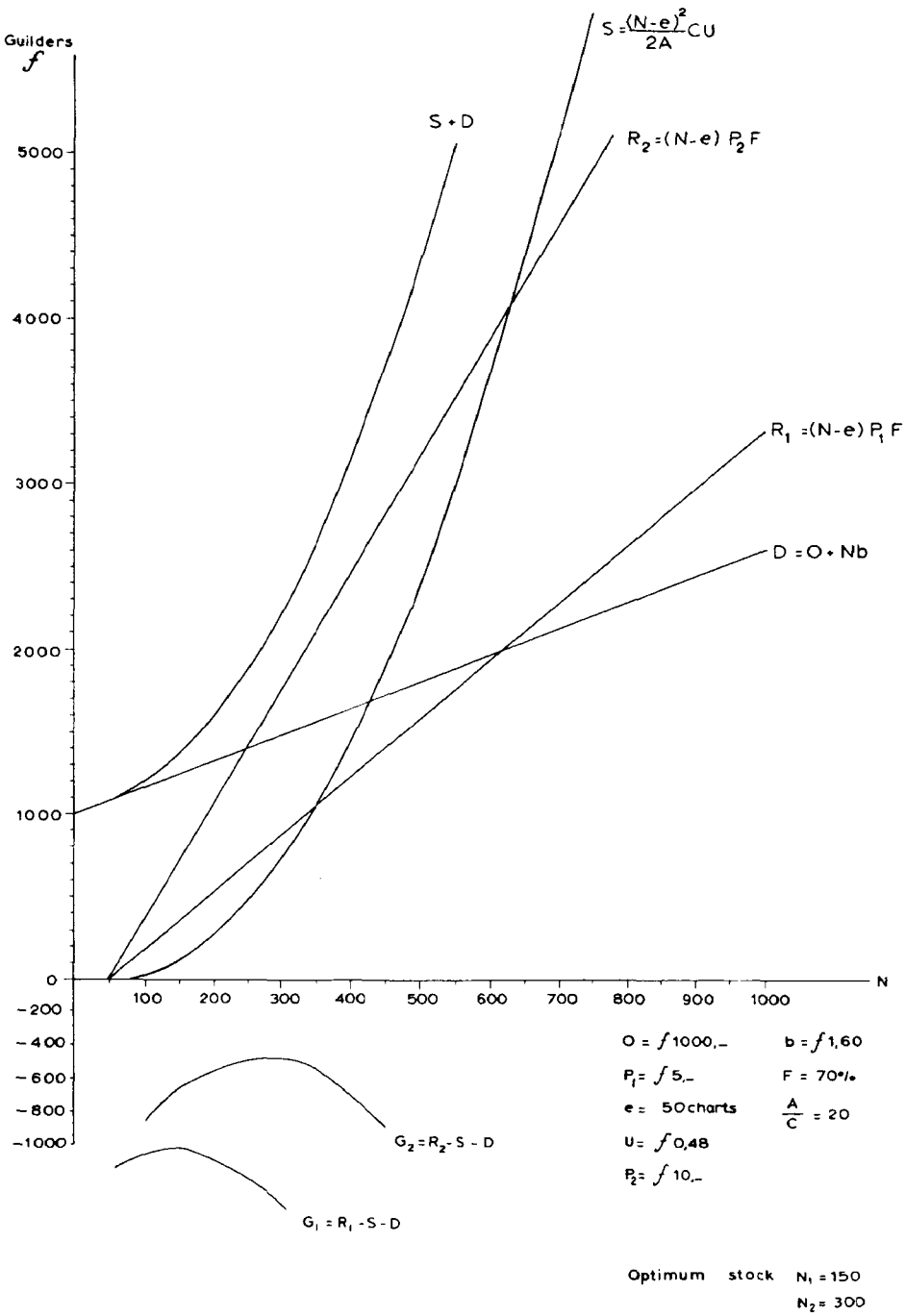


GRAPH No. 1

mean diminishing gain, because of the larger burden of corrections to be made : smaller stocks will also mean diminishing gain, this time on account of too frequent reprinting.

The first derivative of G in relation to N is written, according to (8) :

$$\frac{dG}{dN} = P \cdot F - \frac{N - e}{A} CU - b \qquad (9)$$


Guilders
f
S = (N-e)²/2A CU
S + D
R2 = (N-e) P2 F
R1 = (N-e) P1 F
D = O + Nb
5000
4000
3000
2000
1000
0
-200
-400
-600
-800
-1000
100
200
300
400
500
600
700
800
900
1000
N
G2 = R2 - S - D
G1 = R1 - S - D
O = f 1000,-
b = f 1,60
P1 = f 5,-
F = 70%
e = 50 charts
A/C = 20
U = f 0,48
P2 = f 10,-
Optimum stock N1 = 150
N2 = 300

GRAPH No. 2

Maximum gain is found when $\frac{dG}{dN} = 0$, consequently :

$$\frac{N - e}{A} CU = PF - b$$

or

$$N - e = \frac{A(PF - b)}{CU}$$

and

$$N = \frac{A(PF - b)}{CU} + e \qquad (10)$$

Apparently the optimum value for N is invariant in relation to overhead costs O.

Examples

In graph No. 1 circumstances are considered as they prevail in the Netherlands for certain charts. Overhead, printing and correcting costs are medium, the chart price is from medium to high, and the average quantity of charts sold daily is 20 times as large as the average daily increase in the number of correction units. It is seen that the optimum number, N, to be printed each time is 610, in which case gain G = 880 Fl.

In graph No. 2 we consider the same chart, but this time labour costs are very high, as reflected in the values of O, *b* and U. Two possibilities are shown, price $P_1 = 5$ Fl., and price $P_2 = 10$ Fl. In the first case we find an optimum for $N_1 = 150$ and, in the second case, for $N_2 = 300$. In these cases the maximum gain is, respectively, $G_1 = -1\,000$ Fl. and $G_2 = -440$ Fl.

Conclusions

Very large values of $\frac{A}{C}$, combined with medium to high prices, P, for the charts, may lead to an optimum stock to be printed being of several thousand copies. Such a stock may very well last several years with a continuing daily increase in the number of correction units. If, however, the area covered by the chart is resurveyed at regular intervals, then the term of the optimum stock should not be appreciably longer than that interval. From (4) and (10) we find :

$$t = \frac{N - e}{A} = \frac{PF - b}{CU} \qquad (11)$$

For circumstances in the Netherlands, from (11) we can derive :

$$t = \frac{1}{C} \frac{PF - b}{U} = \frac{1}{C} \times 32.6 \qquad (12)$$

Since for Netherlands charts

$$2 < \frac{1}{C} < 30, \text{ it is easy to see that } 65 < t < 1\,000 \text{ days} \qquad (13)$$

Analogous reasoning shows that (10) can be written as :

$$N = \frac{A}{C} 32.6 + e \qquad (14)$$

and since for Netherlands charts

$$5 < \frac{A}{C} < 75, \text{ we find :}$$

$$195 < N < 2\,475 \text{ charts.}$$

Since $N = 2\,475$ represents a stock that may last something like 1 000 days, it may well be that the interval between resurveys, and consequently the issue of a " large correction " or a " new edition " will render it necessary to print a smaller stock than the optimum one.