

TIDAL PREDICTION AND MODERN TIME SCALES

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ABSTRACT

Modern time scales introduced since about 1950 and revised formulae for the mean lunar and solar longitudes are defined and compared with the formulae of NEWCOMB (1895) and BROWN (1919), which still form the basis of current tide prediction practice. Changes in tidal arguments of order $0^{\circ}.02$ are identified, with a tendency to increase towards the 21st century. Small changes in potential amplitude and speed of some leading harmonic constituents from AD 1900 to 2000 are also noted. While all changes are small by tidal standards, it is recommended that the modern formulae be adopted by tidal authorities before discrepancies become noticeable. The modern formulae require at least an approximate correction for the difference between Dynamic or Ephemeris Time and Civil or Universal Time, which will probably exceed 1 minute before AD 2000.

INTRODUCTION

The majority of tidal computations in current practice are based on astronomical formulae which date from the turn of the last century. Routines and techniques of analysis have mostly been adapted to the modern computer, but the basic formulae refer back to the mean lunar and solar longitudes (s, h, p, N, p') as listed in say DODDSON (1954) or SCHUREMAN (1976), both of which works are copied from much older texts. (Exceptions are DHI (**) (1967) and the author's own programs described in CARTWRIGHT & TAYLER (1971), based on formulae in 'Ephemeris Time'). The older formulae are derived from the classic works of NEWCOMB (1895) and BROWN (1919), and were expressed in Universal Time (UT), equivalent to Greenwich Mean Time (GMT), which is geared to the Earth's rotation, assumed in those days to be practically uniform.

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Since the 1950's, astronomers have radically revised their concepts of time, as everyone concerned with precise time-keeping will know. The Earth's rotation having proved to be irregular and unpredictable, a new time-scale known as Ephemeris Time (ET), geared to the Earth's orbit round the Sun, was defined as the basis for all astronomical calculations including the mean lunar and solar longitudes. UT was equal to ET about the year 1903 but has since increasingly lagged behind ET, currently by nearly a minute. In 1971 Ephemeris Time was itself replaced by International Atomic Time (TAI) as the standard time-scale for general scientific and technical purposes. At the beginning of 1972 the system of Coordinated Universal Time (UTC), which is the basis of radio time signals, was modified so that it differs from TAI by an integral number of seconds and from UT by less than 1 second. Leap-seconds are inserted into UTC at roughly yearly intervals for this purpose, and so UTC provides an adequate basis for the computation of hour angle of celestial bodies for use in navigation and tidal computations.

In recent years the International Astronomical Union has introduced a whole new set of astronomical constants and new time systems that are appropriate for use with relativistic theories. (Ephemeris time is defined only for Newtonian theories). New ephemerides derived by numerical integration and using these new constants and time scales were introduced in the USA/UK Astronomical Almanac for 1984 (henceforward denoted here by AA84).

The lunar and solar ephemerides are no longer computed as harmonic perturbations about the mean longitudes. However, luckily for tidalists, newly revised formulae for the classical mean longitudes are given for the primary purpose of calculating the Earth's nutation. These formulae refer to the epoch 2000.0 as time origin, whereas the older formulae refer to the epoch 1900.0.

Faced with all these changes, the tidal practitioner may well wonder how precise his old formulae in UT are in comparison with modern standards, and whether he ought to substitute new ones. The object of this paper is to set out the latest available formulae in a form useful to the tidal practitioner and with his required level of precision in mind; also, to compare numerical values for the mean longitudes as derived from the old and new formulae, and to note any systematic secular changes in the amplitudes and speeds of the major tidal harmonic constituents.

THE MODERN TIME SCALES

It is necessary to define the new concepts of the time and their terminology, as a basis for precise discussion. The following definitions are essentially derived from the "Supplement" pages in AA84.

International Atomic Time (TAI) is the uniform standard to which all other time scales are related. It is derived, in arrears, by the Bureau International de l'Heure (BIH) in Paris from data supplied by many national time services.

Terrestrial Dynamical Time (TDT) is the equivalent of Ephemeris Time in respect of observations made from the Earth; it is defined in such a way that for practical purposes

$$\text{TDT} = \text{ET} = \text{TAI} + 32.184 \text{ s}$$

TDT is the time scale used for all ephemerides of phenomena observed from the Earth.

Barycentric Dynamic Time (TDB) is a recent concept for motions referred to the barycentre of the Solar System. It differs from TDT only by quasi-periodic terms of amplitude less than 0.002 s.

Universal Time (UT or UT1) is determined retrospectively from observations of the Earth's rotation, in such a way that 0h UT always coincides with transits of the "apparent mean Sun" over the 180° meridian. Its variable offset from TDT

$$\Delta T = \text{TDT} - \text{UT1}$$

is recorded to 10^{-4} s at 5-day intervals in the Annual Reports of the BIH, and at lower precision and frequency in the USA/UK Astronomical Almanacs. AA84 (page K7) lists values of ΔT back to AD 1621. Values from 1900 to the present are plotted in Figure 1 herewith. UT2 (now rarely used) is a smoothed version of UT1, formed by subtracting an annually varying term of amplitude 0.03 s. UT is important in the present context because it provides the basis for civil time-keeping, including the setting of tide-gauges.

Coordinated Universal Time (UTC) is an arbitrary approximation to UT which (since 1972) is constrained to differ from TAI by an integral number of seconds and to be always within ± 0.7 s of UT1. Leap-seconds are introduced when required on January 1 or July 1. The offsets of UTC and UT from TAI from 1972.0 to 1982.0 are shown in Figure 2. (Offsets from TDT or ET may of course be obtained by adding 32.184 s to the ordinate). UTC is precisely the basis for all radio time-signals.

Greenwich Mean Sidereal Time (GMST), alternatively described as "Greenwich hour angle of the mean equinox of date", is the sidereal equivalent of UT1. The current formula (AA84) for the GMST of 0h UT1 in seconds,

$$24110.54841 + 8640184.812866 T_u + 0.093104 T_u^2 - 6.2 \times 10^{-6} T_u^3, \quad (1)$$

where T_u is the Universal Time in units of 36525 days from the epoch 2000 Jan 1.5, essentially defines the right ascension of the Greenwich meridian in this time scale.

THE MEAN LUNAR AND SOLAR LONGITUDES

The mean longitudes customarily used in the Darwinian harmonic expansion of the tide-generating potential, namely

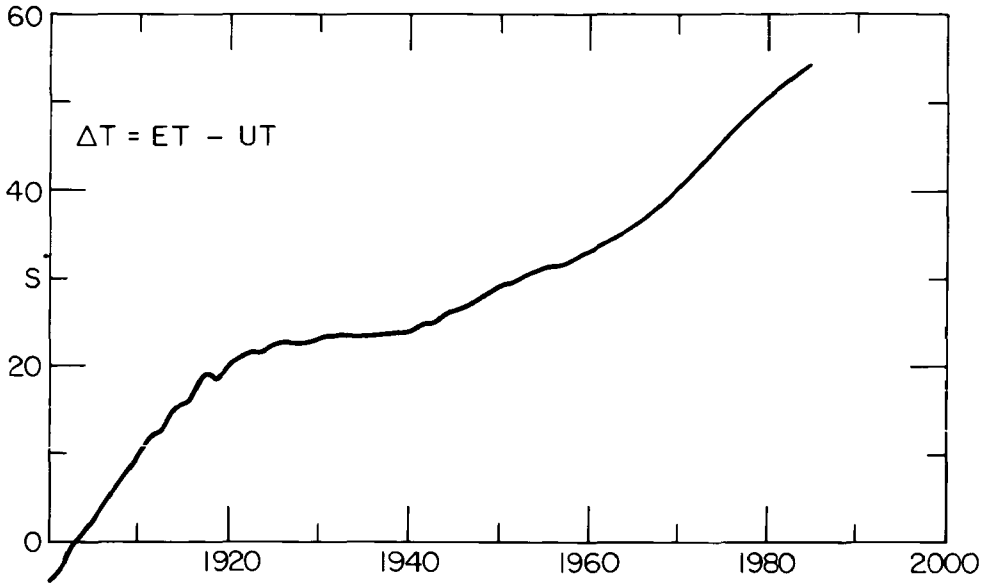


FIG. 1. — Smoothed curve of (Ephemeris Time — Universal Time) in seconds for 20th century, reflecting the variability of the Earth's rotation.

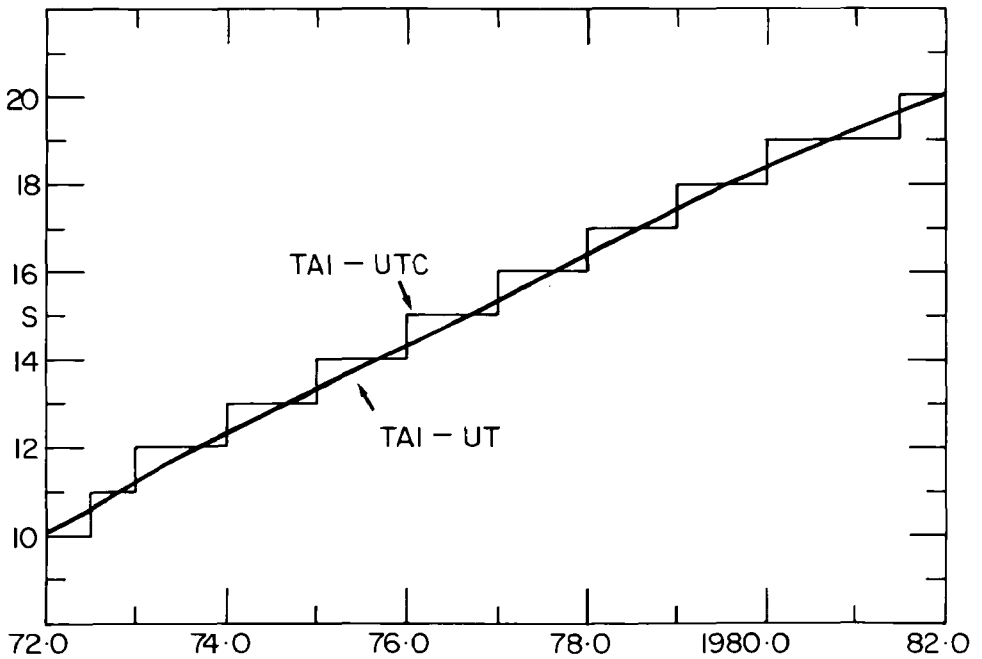


FIG. 2. — (Smooth line) : Smoothed curve of (Atomic Time — Universal Time (UT2)) for the decade 1972-1982. (Stepped line) : The same for Coordinated Universal Time, showing "leap-seconds".

- s : the mean longitude of the Moon,
 h : the mean longitude of the Sun,
 p : the mean longitude of lunar perigee,
 N : the mean longitude of the lunar node,
 p' : the mean longitude of perihelion,

have for many decades been expressed in the form

$$a + bT + cT^2 + dT^3 \quad (2)$$

where a, b, c, d are specified constants for each element, and T is the time in units of a Julian century (36525 days) from a standard epoch.

In the early formulae of NEWCOMB (1895) and BROWN (1919), T was reckoned in UT, and the standard Epoch was 1900 January 0.5, that is 1899 December 31 noon, or Julian Day 2415020.0. DOODSON (1954) quoted practically the same formulae with the Epoch advanced by 12h to January 1.0. Another version of the same formulae in terms of year- and day-numbers is given in DOODSON (1928); FRANCO (1981) gives yet another variant.

The first radical change to the formulae came with the introduction of Ephemeris Time and other improvements described in the USA/UK publication of 1954, "Improved Lunar Ephemeris 1952-59" (henceforth abbreviated to ILE (1954)). The revised formulae for the above longitudes and several others are given on page 288 of ILE, with T expressed in *days of ET*, still from the 1900 Epoch of NEWCOMB. ILE (1954) also makes extensive use of Brown's arguments

$$l = s - p, \text{ the lunar anomaly,}$$

$$l' = h - p', \text{ the solar anomaly,}$$

$$F = s - N, \text{ mean lunar extension from the node,}$$

$$D = s - h, \text{ mean lunar extension from the mean Sun,}$$

which with N, (Ω in Brown's notation), occur naturally in analytical expansions of the Moon's motion and of the Earth's nutation.

The most recent formulae appear in AA84, as part of another extensive revision of astronomical constants. The coefficients appear on page S 26 and refer to Julian Centuries of TDT from Epoch 2000 January 1.5. The new epoch is exactly one Julian Century after NEWCOMB's 1900 Epoch, so $T = 0$ in the new formulae refers to the same (Ephemeris) time as $T = 1$ in the old formulae. Only Brown's arguments are given, but the Darwinian longitudes may be derived from them, if required, by simple addition or subtraction.

The new formulae, originally based on some extensive work by Van Flandern, were first published by SEIDELMANN (1982), where they are stated to represent the best currently available values consistent with the 1976 astronomical constants agreed by the International Astronomical Union^(*) and with the latest celestial reference frame known as FK5. Although the formulae were derived primarily for the purpose of calculating the Earth's nutation, there is no doubt that their precision is more than adequate for the most advanced calculations of tides. SEIDELMANN (1982, p. 98) warns that there may be applications to very precise solar or lunar theories for which other expressions should be used, but it is certain that the most rigorous applications to ocean tides do not approach this category.

(*) The 1976 IAU system of constants are tabulated in the "Supplement" pages of AA84.

TABLE 1
Coefficients of the mean longitudes and of obliquity

Brown	Darwin	—	a	b	c	10 ⁶ b'
<i>l</i>	s — p	1	296.1094	477198.8598	0.0129	—
		2	296.1046	.8491	0.0092	151
		3	134.9630	.8674	0.0087	151
<i>l'</i>	h — p'	1	358.4758	35999.0498	0.0003	—
		2	358.4758	.0498	0.0003	11
		3	357.5277	.0503	— 0.0002	11
<i>F</i>	s — N	1	011.2549	483202.0344	0.0004	—
		2	011.2509	.0252	— 0.0032	153
		3	093.2719	.0175	— 0.0037	153
<i>D</i>	s — h	1	350.7407	445267.1231	0.0022	—
		2	350.7375	.1142	— 0.0014	141
		3	297.8504	.1115	— 0.0019	141
Ω	N	1	259.1825	—1934.1424	0.0021	—
		2	259.1833	.1420	0.0021	—1
		3	125.0445	.1363	0.0021	—1
<i>L'</i>	h	1	279.6967	36000.7689	0.0003	—
		2	279.6967	.7689	0.0003	11
		3	280.4661	.7698	0.0003	11
ϵ	ω	2	23.4523	—0.0130		
		3	23.4393	—0.0130		

Table 1 summarises the leading constants, a, b and c (equation 2) for the three generations of formulae :

- (1) NEWCOMB (1895), as quoted by SCHUREMAN (1976)
- (2) Ephemeris Time, as given in ILE (1954)
- (3) Terrestrial Dynamic Time, as given in AA84.

Units are degrees of arc to 4 decimals, as befits tidal work, and the mean longitudes listed are Brown's four principal arguments, *l*, *l'*, *F*, *D*, and the nodal longitude Ω or N. The sixth longitude, *L'* or h, is strictly redundant, since it is derivable from the identity

$$h = L' = F - D + \Omega, \quad (3)$$

but I have included its constants because of its frequent occurrence in tidal formulae as an independent variable. The quantity listed in the bottom two rows of Table 1 is the obliquity of the ecliptic, ϵ or ω . The two formulae for ϵ are essentially identical to the present level of accuracy, but the two values of "a" allow a direct comparison of the total change in obliquity from 1900 to 2000. All values of "d", (the coefficient of T^3 in equation 2), are negligible for our purpose.

Since time-scales (1) and (2) both refer to epoch 1900 (Jan 0.5), values of "a" in these two cases are fairly close, and small changes reflect genuine changes in the best estimates for the longitudes concerned. The generally large changes in "a" for

time-scale (3) are due to the change of epoch to 2000 (Jan 1.5), but values of

$$[a(2) + b(2)] \bmod (360^\circ)$$

are again of course close to values of $a(3)$, with small differences due to revised constants.

There is however a fundamental change in values of “ b ” from time-scale (1) to time-scales (2) or (3), due to the growing difference between UT and ET (or TDT). Tidalists will always want to refer to Universal Time, because this, (or more strictly UTC), is the time-scale on which tide-gauges are ideally maintained. Newcomb’s formulae (1) purported to give UT directly, but scales (2) or (3) require correcting to

$$a + bT + cT^2 + b' \Delta T \quad (4)$$

to give the exact value in UT, where $b' = 0.317 b \times 10^{-9}$ and ΔT is in seconds. Numerical values of b' are tabulated in the last column of Table 1. For example, by the year 2000 the value of ΔT is reasonably expected to be about 65 s, (Figure 1), for which the corresponding increment in the Moon’s anomaly l for prediction in Universal Time is

$$151 \times 65 \times 10^{-6} = 0^\circ.0098$$

CHANGES IN TIDAL ARGUMENTS

The arguments of the principal harmonic tidal constituents are simple linear combinations of the basic longitudes detailed in Table 1. In order to assess numerically the changes in the tidal arguments resulting from the new formulations, Table 2 shows, for 7 major diurnal tides and 9 major semi-diurnal tides, values in degrees to 4 decimals of their angular arguments at January 1 at 0h UT of the years 1980 and 2000, as computed from the old formulation (1) — upper figure, and from the most recent formulation (3) — lower figure. Integral degrees are omitted from the lower figures, for convenience.

The first columns of Table 2 give the common Darwinian symbol of each tidal constituent and the coefficients of the Brownian arguments which define its argument at 0h. In some cases, the latter combinations are simpler than the more familiar expressions in Darwinian arguments. For example,

$$M_2 : -2D \text{ instead of } -2s + 2h,$$

$$N_2 : -2D - l \text{ instead of } -3s + 2h + p.$$

The 5th column indicates whether $\pm \cos$ or $\pm \sin$ is applied to the argument, according to the convention based on the harmonic expansion of the tide-generating potential. If one wishes to convert to the common convention in which only cosines are used, then $-\cos$, $+\sin$, $-\sin$ in column 5 are equivalent to addition of 180° , 270° , 90° to the argument, respectively. DOODSON (1928) adds these angles, and also a constant angle covering the occasional references to perihelion p' , where necessary. In the latter case he assumed $p' = 282^\circ$, which was adequate for say 1930-1970, but for 1980-2020 $p' = 283^\circ$ is more nearly correct. In the present context, p' is accommodated by reference to the solar anomaly l .

The angular arguments in the final columns correspond to the central argument, E in the notation of the Admiralty Tide Tables, V_0 in the notation of

DHI (1967). The Admiralty Tide Tables list daily values of $(E + u)$ to the nearest degree, where u is an increment which varies periodically, usually with the 18.6y period of the lunar node N . I have made no allowance for u in Table 2; its values will not be substantially altered by the new formulation. DHI (1967) lists values of v , equal to the *average* value of u for each year, but it also lists $V_0 + v$ to the nearest $0^\circ.1$, from which values of V_0 at January 1.0 may be deduced for direct comparison with Table 2, at least for 1980.0.

TABLE 2
Tidal arguments from formulae (1) and (3)

	D	h	l	l'	sin	1980.0	2000.0
Q1	-2	-1	-1		+	034.8174 .8335	088.0361 .0573
O1	-2	-1	0		+	123.4904 .5005	216.4857 .4976
*M1	-1	0	0		+	201.6548 .6602	068.2294 .2358
P1	0	-1	0		+	080.1808 .1801	080.0269 .0260
K1	0	1	0		-	279.8192 .8199	279.9731 .9740
ψ 1	0	1	0	1	-	277.0422 .0453	277.0062 .0096
J1	0	1	1		-	008.4922 .4849	048.4227 .4143
					cos		
2N2	-2	0	-2		+	225.9636 .9904	239.5596 .5910
μ^2	-4	0	0		+	086.6192 .6408	272.9176 .9432
N2	-2	0	-1		+	314.6366 .6554	008.0092 .0313
ν 2	-4	0	1		+	175.2922 .3058	041.3672 .3835
M2	-2	0	0		+	043.3096 .3204	136.4588 .4716
L2	-2	0	1		-	131.9826 .9854	264.9804 .9119
T2	0	0	0	-1	+	002.7770 .7746	002.9669 .9644
S2	0	0	0		+	000.0000 .0000	000.0000 .0000
K2	0	2	0		+	199.6384 .6398	199.9462 .9480

* The argument assigned to M1 is that used by Doodson (1928); see text.

It is worth mentioning that the argument given for M_1 in Table 2 is that of the exact subharmonic of M_2 , as adopted by DOODSON (1928). In reality, the group of lines vaguely named M_1 contains two principal component lines at $(K_1 - l)$ and $(O_1 + l)$, separated in frequency by $2p$, that is $(4.4y)^{-1}$. According to the common convention about harmonic "constituents", definition of the central frequency is arbitrary and different choices are associated with different modulating functions. SCHUREMAN's definition of M_1 would be $(h - l)$ in the scheme of Table 2. CARTWRIGHT (1975, 1976) showed that at some parts of the Atlantic Ocean, the true subharmonic frequency as adopted here is actually dominant in the observed tide, owing to local relative magnification of the 3rd degree spherical harmonic in the tidal potential. However, the 3rd degree effect would strictly have a cosine argument. The negative sine noted in Table 2 is again arbitrarily consistent with DOODSON's (1928) definition.

The mean longitudes involved were computed from the coefficients (1) and (3) in Table 1, using the appropriate time scale. Thus for 1980.0,

$$36525 T = 29219.5 \text{ for scale (1), } -7305.5 \text{ for scale (3),}$$

while for 2000.0,

$$36525 T = 36524.5 \text{ for scale (1), } -0.5 \text{ for scale (3).}$$

Scale (3) required the increments for ΔT as in equation (4). For 1980.0, ΔT is known to have been 50.5 s (AA84, p. K 9), and this value was used in Table 2, while for 2000.0, a predicted value of 65.0 s was assumed. An error of 1 s or so in ΔT clearly affects only the 4th decimal place in these results.

Differences between corresponding arguments computed from the old and new formulae vary from small quantities of order $0^\circ.001$ for K_1 and K_2 to $0^\circ.02 - 0^\circ.03$ for the lower frequency tides of species 2. The phase of S_2 is always exactly zero, by definition of UT. The phase of the most important tidal constituent M_2 has changed by $0^\circ.0108$ in the year 1980 to $0^\circ.0128$ for the year 2000. Here, as in all cases the discrepancy between arguments from the new and the old formulae increases from 1980.0 to 2000.0, as would be expected.

CHANGES IN AMPLITUDE AND SPEED

This subject has little to do with the changes in time-scale, but it is appropriate here to take stock of other, albeit slight, changes in the tide-generating potential over the 20th century. Changes in the harmonic amplitudes are principally due to the steady decrease in the obliquity of the ecliptic from $23^\circ.452$ in 1900 to $23^\circ.439$ in 2000. DOODSON's (1954) harmonic expansion used the 1900 value of the obliquity and perhaps gave an illusion of constancy of amplitude, although the algebraic dependence was implicit in his formulae. CARTWRIGHT & TAYLER (1971) re-computed the amplitudes from the revised formulae for Brown's lunar theory given in ILE (1954), using the full expressions for every element of time-dependence at three different epochs, namely 1870, 1924 and 1960. Slight secular drifts in many of the harmonic amplitudes were apparent, some positive, some negative. There is every reason to suppose that these drifts can be treated as linear in time over a period of a few centuries. Hence it is easy to interpolate or extrapolate

between the tabulated values at say 1870 and 1960 to give amplitudes appropriate to the standard epochs of 1900.0 and 2000.0.

The amplitudes of two leading diurnal constituents and three leading semi-diurnal constituents for the above two century years are given in the fourth column of Table 3, under the heading CTE (CARTWRIGHT-TAYLER-EDDEN). They were derived from the tables of CARTWRIGHT & EDDEN (1973) which result from a small correction to the computing procedure of CARTWRIGHT & TAYLER (1971). As one would expect from a decrease in obliquity, the diurnal amplitudes for AD 2000 are reduced and the semi-diurnal amplitudes are increased, although the rates of increase for N_2 and S_2 are extremely small. Units are 10^{-5} m in "equilibrium amplitude". Note that greater proportional changes in amplitude and phase may be observed in very long data series, due to slow changes in oceanic admittance and local estuarine effects, (CARTWRIGHT, 1972; AMIN, 1983).

The third column of Table 3 gives for comparison the corresponding values for AD 1900 in the scale used by DOODSON (1954). They differ from the CTE figures by a constant factor of 1.43712, (CARTWRIGHT & TAYLER, 1971, Table 2), with a change in sign for diurnal constituents, owing to a different normalisation for the spherical harmonics. (There is a further slight difference in the ratio for the solar tides, since the CTE calculations used revised values for the Sun : Moon mass ratio and the Sun's mean parallax — Doodson's computations originated in 1921).

The last two columns of Table 3 define the "speed" in degrees. h^{-1} of the same constituents at the two epochs. These are computed from the derivatives of equations of type (4), using the latest (AA84) coefficients and a simple addition for the Earth's rotation. The speeds are given in two parts, the first, labelled σ , being the main increment due to the speed

$$b + 2cT \quad (5)$$

of the orbital longitudes, and the second, labelled $\Delta\sigma$, being the variable increment

$$b' d(\Delta T)/dT \quad (6)$$

TABLE 3
Secular changes in potential amplitude and speed

	Amplitude			Speed		
	Year	Doodson	CTE	σ		$10^{10}\Delta\sigma$
O1	1900	37694	- 26229	13.94303	55923	- 382
	2000		- 26216		56003	- 291
K1	1900	- 53011	36887	15.04106	86393	11
	2000		36872		86400	15
N2	1900	17386	12098	28.43972	95453	- 563
	2000		12099		95342	- 430
M2	1900	90809	63188	28.98410	42316	- 367
	2000		63194		42403	- 280
S2	1900	42248	29398	30.00000	00000	0
	2000		29401		00000	0

due to the changes in the Earth's speed of rotation. The actual speed of each constituent is $\sigma + \Delta\sigma$. For example, in the case of M_2 ,

$$\sigma + \Delta\sigma = 30 - 2(b + 2cT) - 2b'd(\Delta T)/dT,$$

where the coefficients b , b' , c are those pertaining to the variable D .

Values of σ are given to 10 decimals, only the last 5 of which are printed for the year 2000. Equivalent figures for the years 1900, 1950 and 2000 are quoted on pp. 7-8 of DHI (1967), based on the Ephemeris Time formulae (2). The DHI figures are very close to those in Table 3 for AD 2000 but differ in the 9th decimal for AD 1900. However, DHI (1967) ignores the Earth rotation variability, $\Delta\sigma$, which evidently affects the 7th or 8th decimals in some cases.

The figures for $\Delta\sigma$ in the last column of Table 3 depend on estimates for the *rate of change* of ΔT , that is, of $(- UT)$ relative to the atomic standard. An average value for the present century would be about 0.7 s.y^{-1} (Figure 1), but values appropriate to individual years vary between about 0 and 2 s.y^{-1} . For the year 1900 I used 1.14 s.y^{-1} which is fairly precise, and for 2000 a predicted estimate of 0.87 s.y^{-1} which is in fact the recorded value for 1980. The numerical values given for $\Delta\sigma$, in units of the last decimal of the previous column, are evidently too small to be of any consequence in practical calculations involving constituent speeds, but they serve to show the magnitude of the correction.

CONCLUDING REMARKS

The changes recorded in Tables 1-3 are small by the normal standards of tidal practitioners. Some authorities may take this as an excuse to retain their antique formulae. On the other hand, the discrepancies are increasing, as the vagaries in Earth rotation accumulate and large values of T^2 and T^3 in Newcomb's formulae begin to take importance. A wiser view would therefore be to change to the modern system now, while the discontinuity is negligible, rather than be forced to do so at a later date by noticeable errors.

Use of the essentially unpredictable ΔT values to compute tidal predictions will be novel to most tidalists. However, the correction to UT is easy enough to make in analysing records from past years, and the computations presented here show that an error of a second or two in predicting ΔT a few years into the future is negligible for ordinary tidal calculations. Many cases of tide-gauge records being maintained with errors estimated in minutes rather than seconds will be cited to belittle the importance of these small corrections, but *average* errors over an analysed record are much smaller than the individual extremes, and instrumental precision is steadily increasing with the use of crystal timing mechanisms. Finally, with the growth of libraries of tide-gauge data going back to the beginning of the century or beyond, new possibilities for high precision tidal analyses are opened up, some involving the identification of small secular trends. For such analysis only the very best astronomical formulae should be employed.

ACKNOWLEDGEMENT

I am grateful to Dr George Wilkins and Dr Andrew Sinclair of the Royal Greenwich Observatory, Herstmonceux, for reading the first draft of this paper and for correcting my initial phraseology for the definitions of time.

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