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RETURN PERIODS OF EXTREME SEA LEVELS: THE EXCEEDANCE PROBABILITY METHOD

by B.V. HAMON (*) and J.F. MIDDLETON (**)

Abstract

Practical application of the Exceedance Probability Method of MIDDLETON and THOMPSON *(J. Geophys. Res.,* Vol.91, 1986, pp. 11707-11716) to the estimation of extreme sea levels is considered in detail, for ports where tidal amplitudes dominate surge. Data from Sydney (Australia) are used to illustrate the method. Direct use of the histogram of tidal residuals, instead of fitting a model to it, is discussed, and the limitations are pointed out. Graphical fitting of the sum of two normal distributions to the histogram is also discussed. A method is given for including a trend in mean sea level. A simplified method of checking if tidal amplitudes dominate the surge is given in an appendix.

INTRODUCTION

Recently, MIDDLETON and THOMPSON (1986, referred to here as MT) introduced a new method (Exceedance Probability Method, or EPM) for estimating return periods of extreme sea levels from short data records. An attractive feature of EPM is that it takes into account explicitly the deterministic nature of one component of sea level, the tide, which may be reasonably predicted decades into the future. Indeed, as will be shown below, where sea level consists only of tide, return periods of exceedance may be determined exactly.

In this paper, we give some practical details of the use of the EPM, and suggest some simplifications, using 5 years of data from Sydney, Australia, for illustration. We include a brief account of EPM, and show how return periods are obtained for the case of a single sinusoidal tidal constituent. We make frequent reference to MT for details, and where possible use the same notation.

(*) Ocean Sciences Institute, University of Sydney, Australia.
(**) School of Mathematics, University of New South Wales, Australia.

THE EXCEEDANCE PROBABILITY METHOD

Following MT, η_S will denote sea level or 'still water level' (SWL) — the sum of tide (η_T) and surge (η), with wave and seiche effects filtered out.

The probability that η_S exceeds some level η^* in a time interval $[t, t + dt]$ is denoted by

$$Q(t;\eta^*)dt \qquad (1)$$

where

$$Q(t;\eta^*) = \frac{1}{2}\{\dot{\eta}_T + |\dot{\eta}_T|\}P(\eta^* - \eta_T) \qquad (2)$$

$\dot{\eta}_T = d\eta_T/dt$ and $P(\eta)$ is the probability density function (PDF) of the surge or residual component of sea level, $\eta \equiv \eta_S(t) - \eta_T(t)$. Thus if the tide $\eta_T(t)$ and the density function $P(\eta)$ can be estimated from the sea level data available, the exceedance probability (1) may be determined. We discuss (2) in more detail later.

Now to obtain the return period T_R of sea level exceedance, we note that (1) may be interpreted as *the expected number of exceedances* in the interval $[t,t + dt]$. Thus

$$M = \int_t^{t+T} Q(\tau;\eta^*)d\tau \qquad (3)$$

represents the expected number of exceedances integrated over a time T and if equal to one, then T must equal the return period T_R. For extreme exceedance levels, the integration time may be of order 50-100 years, so the tidal components $\eta_T(t)$ must necessarily be predicted for these long time intervals. To avoid these long predictions and indeed the long integrations, MT point out that T_R may be approximated by

$$T_R \simeq T/M \qquad (4)$$

provided that T is long enough to include all significant periodicities in both the tide and surge, and that $T_R >> T$.

Before discussing in detail the practical estimation of T_R, we discuss the nature of, and restrictions associated with, the exceedance probability (2). One important restriction is that (2) will be valid only at coastal ports where tidal amplitudes dominate surge in a mean-square sense (the *strong tide limit* of MT). This strong tide limit should pertain to most ports (Appendix 1; see also MT).

To illustrate the nature of the EPM, we consider the case of a single tidal constituent, $\eta_T = a \sin \omega t$. The expected number of exceedances in one tidal period is then

$$M = \int_0^{\frac{T}{4}} \dot{\eta}_T P(\eta^* - \eta_T) dt + \int_{\frac{3T}{4}}^{T} \dot{\eta}_T P(\eta^* - \eta_T) dt$$

since $\dot{\eta}_T + |\dot{\eta}_T|$ vanishes over $[T/4, 3T/4]$ and exceedances are expected to occur only on a rising tide. With the change of variable $d\eta = -\dot{\eta}_T(t)dt$ the above may be simplified to

$$M = \int_{\eta^* - a}^{\eta^* + a} P(\eta) d\eta$$

so that the expected number of exceedances equals the probability that the surge can 'bridge' the gap between tide $\eta_T = a \sin \omega t$ and the exceedance level η^*. Note that if we allow the surge to become vanishingly small, then $P(\eta)$ is asymptotic to the delta function $\delta(\eta)$ so that

$$M = \begin{cases} 1 & |\eta^*| < a \\ 0 & |\eta^*| > a \end{cases}$$

and only one exceedance can occur in one cycle as expected for a sine wave alone: in this case the return period equals the wave period, $T_R = T$.

Finally it should be noted that (2) was derived by MT only under the assumption that surge is normally distributed or has a distribution corresponding to the sum of two normal distributions. However, the illustrative example above suggests that (2) might be justified heuristically and we shall assume below that P may be estimated directly from the data without the fitting procedures used in MT.

PRACTICAL ESTIMATION OF T_R

Under the assumed strong tide limit approximation, a practical scheme for estimation of T_R is based on calculating M from (2) and (3) for a set of assumed values of η^*. Assuming the prediction time T is greater than 1 year (see below), and anticipating the need to allow for a trend, M should be computed separately for each year as well as for each η^*. Let M_{ij} be the value of M for η_j^* and the *ith* year of prediction, and let $M_j = \sum_{i=1}^{T} M_{ij}$. We usually want the level η_Y^* which has a return period of Y years (typically $Y = 50$ or 100), so we need to interpolate. M_j is a non linear function of η_j^*, but with $\Delta T = 1$ year the 'reduced variate'

$$y_j = -ln(-ln(1 - M_j \Delta T/T))$$

is a nearly linear function of η_j^* from which the required value η_Y^* corresponding to

$$y = -\ ln(-\ ln(1 - \Delta T/Y))$$

can be interpolated.

The calculations require the predicted tide and its derivative, a choice of integration time T, and the surge probability density function $P(\eta)$. These will now be considered in more detail.

The Predicted Tide

The predicted tide is the easiest. Most users will have a program available for predicting hourly tide heights, for example the Canadian program (FOREMAN, 1977). But MT recommend using a half-hour time interval, since $Q(t;\ \eta^*)$ varies widely with time at large exceedance levels. Interpolation of the hourly predictions will usually be easier than modifying the prediction program. If $\eta_T(t)$ is the predicted height (cm) at time t in hours, a suitable interpolation is

$$\eta_T(t + 0.5) = 0.5625(\eta_T(t) + \eta_T(t + 1)) - 0.0625(\eta(t - 1) + \eta(t + 2)),$$

and the time derivative, at time t, in cm/hr, is then $\eta_T(t + 0.5) - \eta_T(t - 0.5)$.

Mean sea level for the full period of the data set should be included in the predictions. For example, if you have 5 years of data but analyse only one year to get the tide constants, the mean sea level obtained in the analysis should be altered to the mean for the 5 year period, before prediction. Note that hourly predictions will also be needed to get the surge PDF (see below).

Integration Time, T

The time interval T should be long enough to include at least one cycle of any important periodic variation in the extreme values of predicted tides. Unfortunately, no general statement can be made at present about the optimum value of T, nor is there any simple way of working it out from the tidal constants. There will almost certainly be an important variation with period about 4.4 years (half the period of lunar perigee (CARTWRIGHT, 1974)). This period is clear for the U.K. ports of Avonmouth and Southend (AMIN, 1979, Fig. 2). This same figure also shows 19 year variations, which are particularly marked at Southend, although the amplitude is only about 6 cm. At Sydney (Australia), monthly maxima of predicted tides 1981-1999 showed clear variation at about 4.4 year period, with amplitude 6 cm, but the 19 year variation was negligible. Graphs for Halifax and Victoria (Canada) are given in Figure 4 of MT (but note that these are from observed rather than predicted values). Neither the 4.4 nor the 19 year variation is evident at Halifax, but there is a strong 19 year variation at Victoria, which was taken into account by using $T = 19$ years for tests on this station (MT, Fig. 10). Variations with much longer periods are possible (CARTWRIGHT, 1974), but are poorly documented. We conclude tentatively that T should not be less than 4.4 years; users of the EPM are advised to do their own predictions for at least 19 years, and examine plots of monthly extreme high waters. Alternatively the integration can be carried on for the full period of interest (usually 50 or 100 years); this might seem wasteful, but the computer time needed would not be excessive.

The Surge Probability Density Function (PDF), $P(\eta)$

Estimating the PDF is the more difficult part. It has to be estimated from observations, and the difficulty comes from the need to represent as accurately as possible the extreme positive tail of the PDF. As a severe test of EPM, MT reported reasonable results from a PDF based on only one year of hourly observations, but a longer data series should be used if it is available.

The PDF is based on a series of *tidal residuals* obtained from the available data by subtracting predicted hourly heights from observed hourly heights. It is assumed that any short period effects (seiches, for example) have been filtered out in forming the series of observed hourly heights. This series of residuals should be plotted and examined carefully, as is usual for tidal data quality control. Particular attention should be given to the highest values, as these will strongly affect the final critical levels; in fact, only these highest values are of any importance at the large exceedance levels of interest. At this stage, check for possible seasonal variation of surge statistics by plotting the surge variance for each month, as in Figures 5 and 6 of MT. At Sydney, the mean surge variance for 1980-84 (Fig. 1) shows no marked seasonal pattern. The peak in July is due

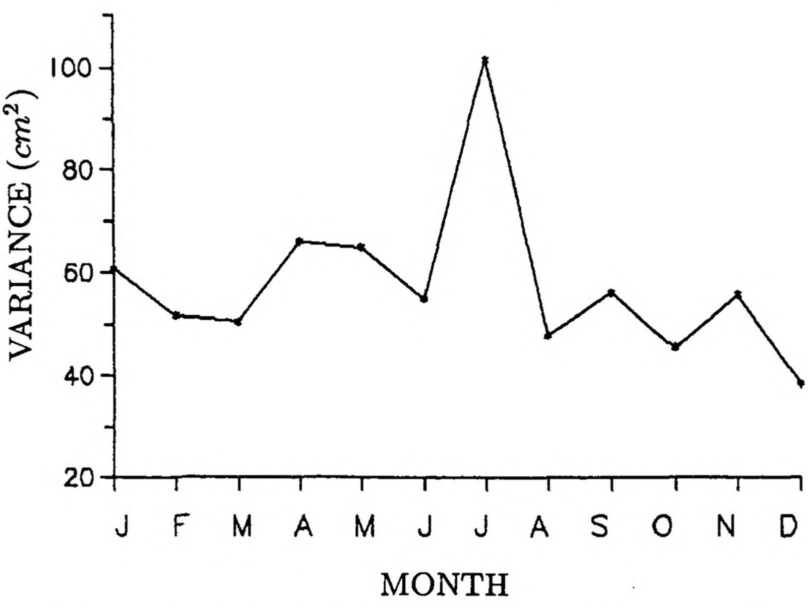

FIG. 1.— Average monthly variance of hourly tidal residuals for Sydney, 1980-1984.

to high values in 3 of the 5 years, but is probably atypical, as it did not appear in a separate 5 year data set (1936-1940).

The required PDF can be represented as the histogram of these residuals, or as a mathematical function fitted to the histogram. Reliable estimation of the PDF at large exceedance levels is difficult, however, for two reasons. The first is simply that since extreme events are by definition rare, few can be expected to have occurred during the 4-5 years of sea level monitoring. The second reason is that the observations of residual surge are drawn from two distinct populations. Moderate surge events (~ 10 cm) may arise from direct effects of atmospheric pressure, and from large scale coastal trapped waves. Extreme surges (~ 50 to 100 cm) on the other hand will be associated with intense cyclones. Thus a reliable estimation of the surge PDF at moderate exceedance levels will not in general allow the tails to be reliably estimated through the fitting of say a single normal distribution. For this reason, the sum of two normals will be fitted to the surge PDF below in an attempt to allow for both populations. However, again since a short record will not in general represent the relative importance of each population, estimation of the extremal PDF tail is at best subjective. MT used a fairly complex fitted function (their 'contaminated normal model', MT pp. 11713-4, with 7 parameters). The complexity is partly due to seasonal variation at the stations they studied, and partly to the need to give extra weight in the model to the extreme positive tail of the distribution of residuals.

The histogram can be used directly, and if seasonal variation is ignored, leads to

$$P(\eta) = f(j)/(sb) \tag{5}$$

where $f(j)$ is the number of observations ('frequency' in the statistical sense) in the j*th* bin of the histogram of residuals, s is the total number of observations, b is the bin width (cm) and η (cm) is the tidal residual at the centre of the j*th* bin. If there is a seasonal variation in surge statistics, separate histograms should be prepared for each month, and the function P becomes

$$P(m,\eta) = f(m,j)/(s(m)b) \tag{6}$$

where $s(m)$ is now the number of observations of residuals in month m.

The difficulties in estimating the surge PDF can be illustrated by some tests using Sydney data. The histogram of all hourly residuals for 5 years (1980-1984) is slightly skewed, with the positive tail extending further than the negative tail (Fig. 2), so the histogram cannot be modelled adequately by a single normal distribution. Only the histogram values for tide anomalies greater than about 25 cm are used in the EPM calculations for values of η^* near the 50 year exceedance level. These values are uneven, showing that even 5 years of data do not lead directly to a smooth approximation to the population histogram in this critical area.

The result of using (5) to predict return periods is compared in Figure 3 to the distribution of observed annual SWL maxima for Sydney, using all available data, 1883-1975. Curve A shows the observed results, plotted using the method of Hazen (Lennon, 1963; Pugh, 1987). The ordinate is annual maximum SWL, and the abscissa is the reduced variate $y = -\ln(-\ln(K))$, where K is the mean value of $(2r-1)/(2m)$ for each value of annual maximum SWL, and r is the rank of

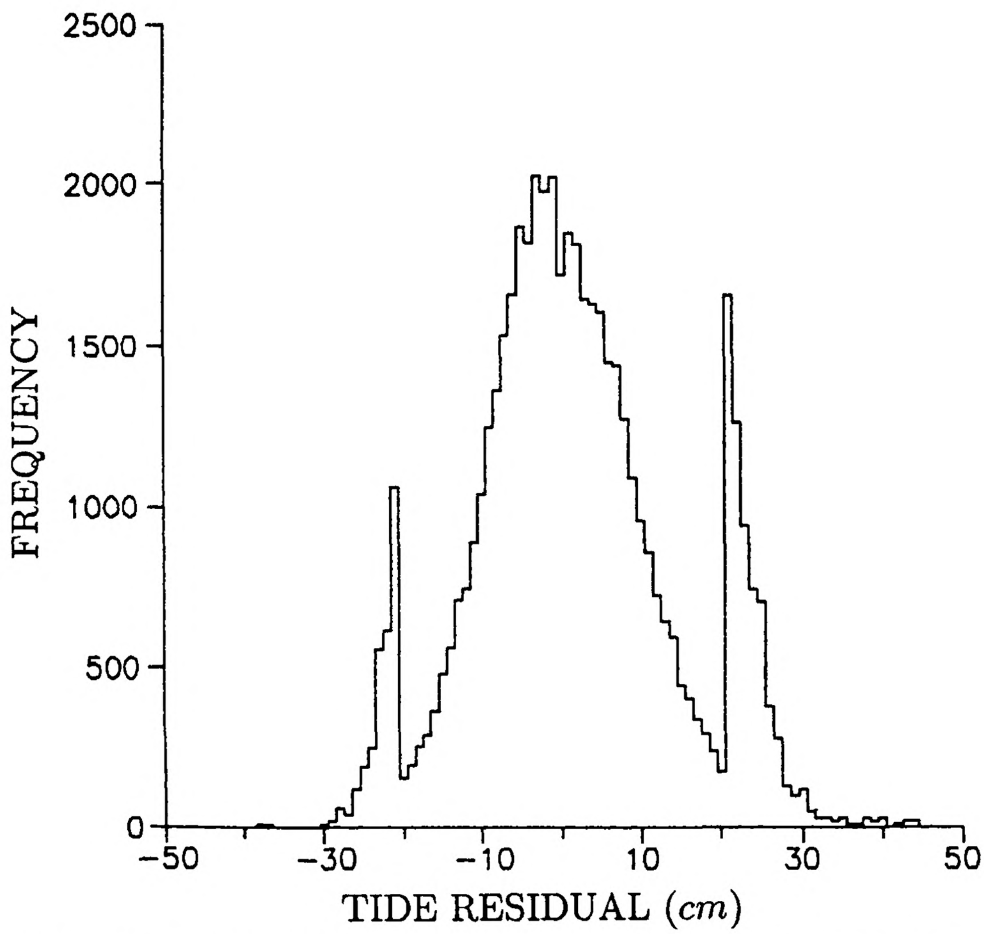

FIG. 2.— Histogram of hourly tidal residuals (observed minus predicted) for Sydney, 1980-1984. For clarity, frequencies for residuals > 20 and <-20 cm are × 10. Bin size 1 cm.

the m annual values. (This plot is similar to the GUMBEL plot used in MT, but is not quite as steep. No correction was applied for linear trend in mean sea level, which is small at Sydney (~ 5 cm/100 years)).

For the EPM result B, the ordinate is η^*, the critical SWL and the abscissa is $y = -\ln(-\ln(1 - 1/T_R))$. At the lower levels of y, we expect A to be less than B, as found here and in most plots in MT. The reason is that for a given SWL, an exceedance of the level can be expected in a time shorter than the return period of annual maxima at that level, since (MT p.11709) low values of annual maxima are likely to be rare.

We expect A and B to approach one another as y increases, but near the 50-100 year return periods we find B is less than A by about 5 cm. This may be due to the sharp cut-off of (5) above $\eta = 44$ cm. In using (5) or (6), we are saying in effect that at Sydney the probability of a tidal residual $\eta > 44$ cm is zero. For comparison, the residual at the time of the highest observed SWL at Sydney (237 cm, 2300 EST, 25 May 1974) was about 54 cm, rising to about

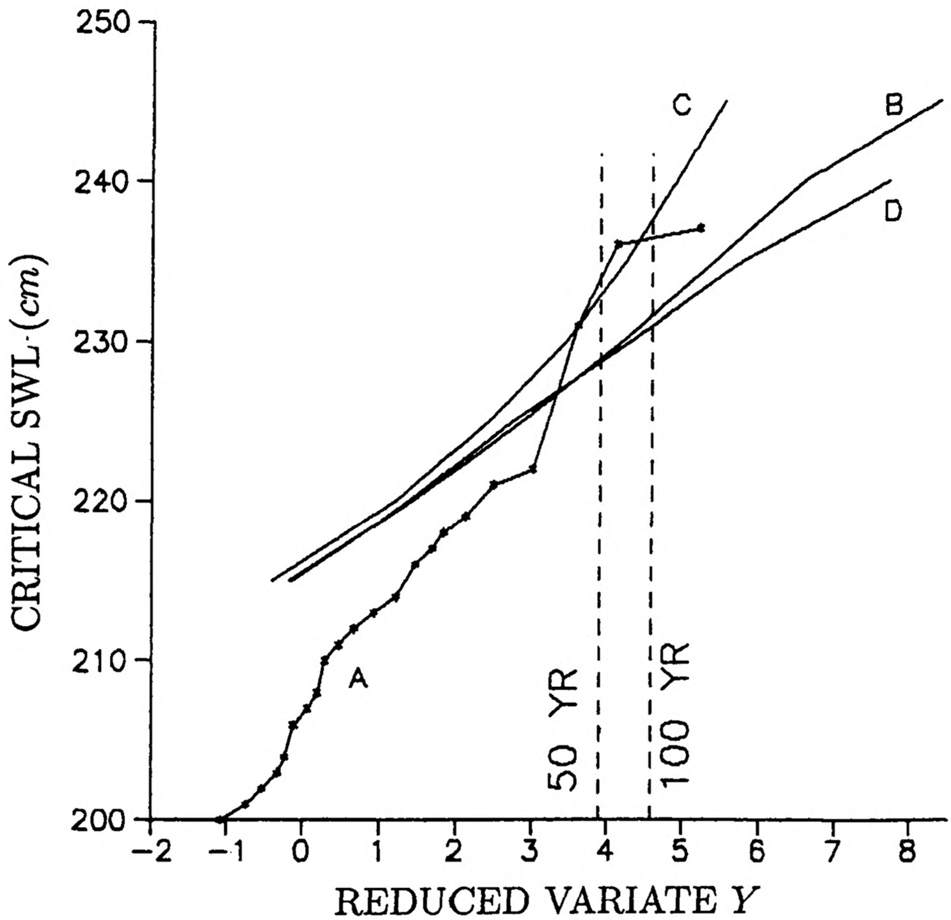

FIG. 3.— Critical (or exceedance) still water level (SWL) for Sydney, as a function of 'reduced variate' *y* (see text). *A*: observed maximum annual SWL, using data from 1883 to 1975. *B*: exceedance levels from EPM, using normalized histogram of tide anomalies. *C*: as *B*, but using double-normal distribution function, fitted graphically. *D*: as *C*, but function fitted by method of moments. The dashed lines show the values of *y* corresponding to 50 year and 100 year return periods.

69 cm a few hours later. It seems clear that we need some way of extrapolating a PDF based on an observed histogram, if we are to expect the best results from EPM.

Fitting the sum of two normal distributions, as in MT, is probably the easiest way of extrapolating and allowing for the two surge populations. The general form is

$$P(x) = \sum_{i=1}^{2} a_i \left[(2\pi)^{\frac{1}{2}} \sigma_i\right]^{-1} \exp\{-(x-\mu_i)^2/(2\sigma_i^2)\} \qquad (7)$$

with the condition: $a_1 + a_2 = 1$. Five parameters need to be estimated in (7): two means (μ_i), two standard deviations (σ_i), and a_1. MT (p.11714) used a non-linear least-squares library program, which included a provision for weighting the observations, in this case to give extra weight to the extreme positive tail. This weighting is likely to be important, but is necessarily subjective. The program may not be available to all, and in any case may not converge for all data sets.

We considered two alternatives. The first is based on the first five moments of the (normalized) histogram; these are sufficient to determine the five parameters in (7) (see Table 1), but the set of equations is still non-linear so con-

Table 1

Parameter	Method of Fitting	
	Moments	Graphical
a_1	0.806	0.9919
μ_1(cm)	−1.506	0
μ_2(cm)	6.318	0
σ_1(cm)	8.0	9.4
σ_2(cm)	9.46	22.0

vergence is not assured. No weighting was used although the positive extreme tail could be inflated so as to force a better subjective fit at that end (see Fig. 4). The second alternative is a graphical fit to a suitable plot of the normalized histogram (5). Specifically, we plot log $(P(\eta))$ against η^2, and assume $\mu_1 = \mu_2 = 0$, and $\sigma_2^2 >> \sigma_1^2$. In this case, the two normals in (7) appear as straight lines with slopes σ_i^2, which can be fitted separately (Table 1). In the graphical fitting for Sydney data, σ_1 is easily determined to within a few percent, but fitting for a_1 and σ_2 is much more subjective. The crosses in Figure 4 are the right side of the normalized histogram (5) for Sydney, plotted as above. The dashed lines represent the two normal curves, fitted by eye to the main part of the graph and the 'tail', respectively. The continuous curve C is their sum, while D is a plot of (7) for the parameter values found by the method of moments.

Judgment will always be needed at this stage, since the tail of the observed distribution is dependent on the occurrence of rare events. For example, if the 5 year data period happened to contain a single unusually severe sea level event, we would expect a nearly constant 'tail' for the PDF, at a level near 1 in 43800 hrs or 2.3×10^{-5}. This would grossly over-estimate the probabilities of the higher residuals. On the other hand, if the residuals were due to normal weather patterns for the whole 5 year period, we would have no data at all to use in fitting a_1 and σ_2.

Assuming the double-normal model for the PDF, the above discussion suggests an inherent difficulty in determining a_1, and more particularly σ_2, from a short set of data. We can see no easy way round this difficulty. A PDF based on a much longer record at a nearby port could be used as a guide; presumably the weather statistics would be similar enough, but different local sea-floor topographies might lead to quite different surge results. This problem might not be serious in practice. At Sydney, omitting the σ_2-term altogether made a large change (a factor of 3) in M_j for $\eta^* = 230$ cm, but this translates into a lowering of

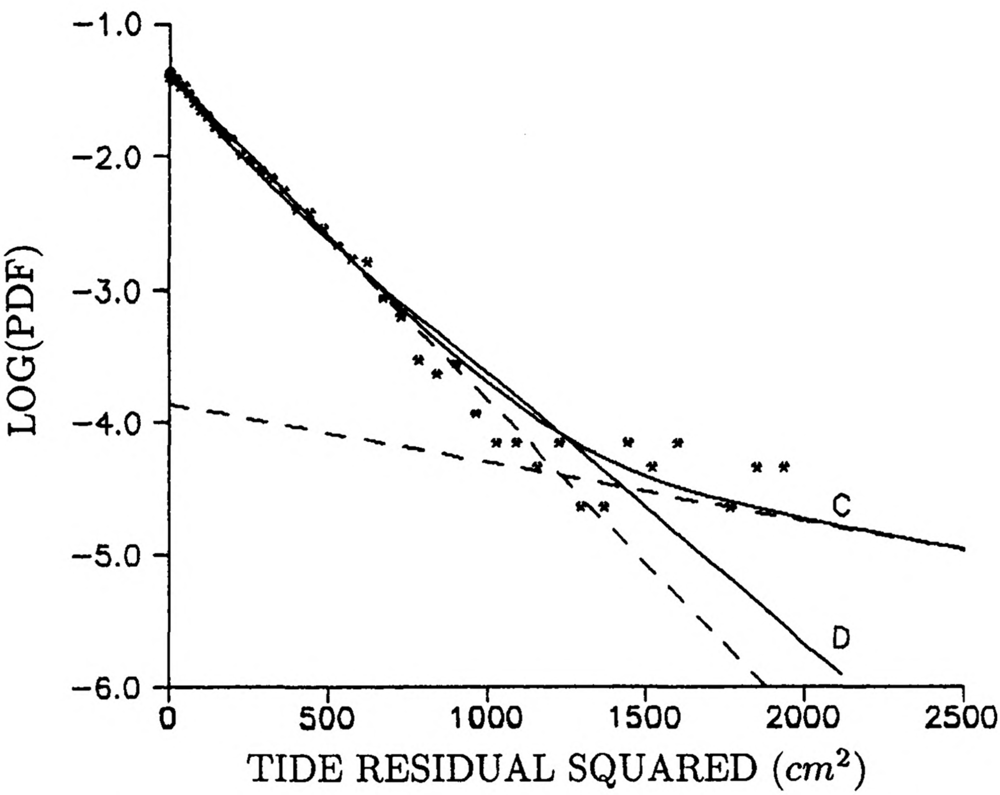

FIG. 4.— Log ($P(\eta)$) as a function of η^2, where η is the hourly tide anomaly, and $P(\eta)$ its probability density function. The crosses represent $P(\eta)$ derived from the positive side of the histogram in Figure 2. *C*: double-normal distribution, fitted graphically. *D*: double-normal distribution, fitted by method of moments. Dashed lines: components of *C*.

50 or 100 year critical levels of only 5 and 7 cm respectively. Any reasonable estimates of a_1 and σ_2 might be expected to affect these critical levels by only about 1 or 2 cm.

The results of using these double-normal fits in the EPM are shown as *C* (graphical fit) and *D* (method of moments) in Figure 3. Using the graphical fit, which best represents the extreme tail, gives a marked improvement near the 50 and 100 year return periods; differences between *A* and *C* are reduced here to about 1 cm. The result from the method of moments is not significantly different to that from using the histogram directly.

EFFECT OF A TREND

So far, we have assumed that mean sea level, used in calculating the predicted tide η_T, is constant. In the simplest case it will have been determined as $\overline{\eta}_S$: the mean of the observed tide over the period of the available data. If there is an appreciable regional trend in mean sea level, determined from longer series at other ports in the region, or based on assumptions about the 'greenhouse effect', the trend should be taken into account. The calculated return periods for a given critical level η^* will then depend on the starting or 'base' year for the computations, i.e. the return period might be 50 years from 1988, but only 40 years from some later base year.

Let ζ_i be the mean sea level in year i. Assume a linear trend

$$\zeta_i = \zeta_0 + k(i-1)$$

where ζ_0 is an estimate of mean sea level in the base year ($i = 1$), and k is the trend in cm/yr. Estimation of k and ζ_0 will depend on what additional data are available, and will not be considered here. We note only that ζ_0 may differ from the mean $\bar{\eta}_S$, even if the base year is the central year of the data period, since regional sea levels for the data period may have been above or below the regional trend.

The difference $\zeta_0 - \bar{\eta}_S$ should simply be added to any estimates of critical level. To account for the trend, the EPM can be modified as follows, assuming the M_{ij} have been calculated as above for the years $i = 1, T$ and for the trial values of critical level η^*_j.

Extend the upper limit of i to 50 or 100 years for each j by repeating each set of M_{ij}. A positive trend decreases the gap between tide (η_T) and trial critical level (η^*), leading to increases in the M_{ij}.

The effect is equivalent to reducing each η^*_j by $(\zeta_i - \zeta_0)$. The new (increased) values, $M_{ij}(\zeta_i)$, can be estimated by interpolation. For Sydney, with η^*_j at 5 cm intervals, piecewise linear interpolation of $ln(M_{ij})$ was found satisfactory.

For large trends, the $ln(M_{ij})$ may need to be extrapolated first to higher values of j, unless the original calculations of M_{ij} have been made to higher values of η^* than would be needed in the absence of trend. Linear extrapolation appeared satisfactory.

The value of η^* for which $\sum_{i=1}^{T_R} M_{ij}(\zeta_i) = 1$ can now be interpolated, and is the maximum SWL expected to be reached in T_R years after the base year.

For Sydney, maximum SWL in the next 50 years was found to increase from 233 cm with no trend to 271 cm for a trend of 1 cm/yr. The increase is not quite linear, being more rapid for the higher levels. The estimates were made using the graphically-fitted double-normal model.

DISCUSSION

Operationally, 50 or 100 year return period levels are not likely to be significant to better than 10 cm accuracy. With this in mind, any of the three results *B*, *C* or *D* in Figure 3 can be considered satisfactory, especially if compared to a smoothed version of the observed curve *A* (for example, smoothed by fitting a Generalised Extreme Value distribution (PUGH, 1986, p.273)). Since the

PDF (5), based directly on the histogram of residuals, is the simplest, its use should be further explored. Its main disadvantage is that it cuts off sharply at the highest residual observed in the available data. This cut-off is likely to lead to underestimating the critical levels.

The graphical fitting of two normal distributions, as curve *C* of Figure 4, offers the simplest way of extrapolating the histogram data, but introduces an appreciable subjective element. If this method is used, you should check that the fitted PDF is reasonable at large values of residual. For example, using the parameter values under 'Graphical' in Table 1 gives $P = 1.07 \times 10^{-6}$ for the probability of observing an hourly residual of 69 cm (the maximum recorded at Sydney in 93 years). The value of 1.07×10^{-6} is equivalent to one observation in 115 years, which seems reasonable. It will usually be necessary to use results from other stations in the same area, as well as practical judgment, in deciding what is reasonable. (The 'moments' parameters in Table 1 lead to $P=6.8\times10^{-22}$ for a 69 cm residual, which appears much too low.)

It may be noted that the better fits to observed annual maxima at Halifax and Victoria were obtained by fitting the sum of two normal distributions (MT, Fig. 10, 11; Table 2), with σ_1 and σ_2 not too different from the values estimated graphically for Sydney. In particular, the ratio σ_2/σ_1 was about 1.8, compared to 2.3 for Sydney. These admittedly very limited results suggest that $\sigma_2/\sigma_1 \approx 2$ might be a useful guide in the graphical fitting.

Effects of very rare events such as tsunamis are difficult to include in any estimate using EPM, since they will not be adequately sampled in the assumed 5 years of data.

The development of EPM in MT and here assumes the surge PDF is independent of predicted tide height. For ports bordered by large areas of shallow water, non-linear tide/surge interaction may invalidate this assumption (PUGH, 1986), so that the PDF might need to be a function of tide height as well as season and residual. We have not considered this for Sydney, which is bordered by a narrow continental shelf.

Acknowledgments

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APPENDIX 1

Checking the Strong Tide Limit

The relevant criterion is that $\Gamma >> 1$, where $\Gamma = \sigma_T \omega \lambda / \sigma$ (MT, eq.18). σ_T and σ are the standard deviations of the tide and 'surge' (non-tidal or residual sea level) respectively. ω is the frequency of the dominant tidal constituent, while λ is a time scale of surge variability. MT estimate λ via the normalized power spectrum (eqs.27 and 28), but it is easier to use their eq.15, i.e.:

$$\lambda = \sigma / \nu$$

where ν is defined (eq.12 of MT) as the standard deviation of the time-derivative of the surge. Anticipating that the strong tide limit will apply, it is not necessary to aim for high accuracy in λ or ν, since their values do not appear in the strong tide limit formula (MT, eq.22). The easiest method of estimating ν is to start with the hourly tide residuals, i.e. hourly values of observed minus predicted tide, since these are needed in any case to get the surge statistics. Estimating time derivatives between every 12th hourly value seems to be good enough; this avoids the high-frequency noise you would get by straight differencing of the hourly values, and the 12 hour span reduces the effects of any residual semi-diurnal tide, caused perhaps by tide gauge errors. (Note that this simple method of estimating ν and λ may not be suitable if initial tests throw doubt on the strong tide limit assumption, since the actual value of λ will then be needed for the full expression for Q (MT eq. 16) or for the strong surge limit (MT eq. 19).)

σ_T can be estimated with sufficient accuracy as $\sigma_T = \left(\frac{1}{2} \sum_{i=1}^{5} h_i^2\right)^{\frac{1}{2}}$ where the h_i are the amplitudes of the 5 main tidal constituents (*M2*, *S2*, *N2*, *K1*, *O1* at Sydney).

For Sydney, we found $\sigma = 0.1$ m, $\nu = 0.004$ m/hr, giving $\lambda = 25$ hr. With $\sigma_T = 0.4$ m and $\omega = 2\pi/12$, we get $\Gamma = 52$, so there is no doubt that the strong tide limit applies here. Since surges are mainly due to weather effects, we expect λ to vary little from port to port. The product $\omega\lambda$ will then be about 13 and 6 at ports with mainly semi-diurnal and diurnal tides respectively. The criterion $\Gamma >> 1$ will then be satisfied in either case (diurnal or semi-diurnal tide) if the standard deviation of the tide (σ_T) equals or exceeds that of the surge (σ). This will be the case at most ports, unless the tides are particularly weak.