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HARMONIC TIDAL ANALYSES OF LONG TIME SERIES

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Abstract

A harmonic tidal analysis program is developed for observational records of 18.61 years or longer. The amplitudes and phases of over five hundred astronomical and shallow water constituents are calculated using a least squares approach. The program is tested with 38 years of hourly observations at Victoria and the amplitudes and phases of satellite constituents whose amplitudes lie above the background noise level are generally found to be consistent with potential theory. Predictions based on the results of a 19-year analysis are found to be only slightly better than those based on averages from 19 one-year analyses, thereby confirming the accuracy of GODIN's [1972] satellite correction algorithm and satellite inference based on potential theory relationships. However it is demonstrated with constituents NO_1, J_1, N_2, and L_2, that results from the 38-year analysis can be used to improve the satellite inference calculations in shorter analyses. Based on the stability of the 38 one-year analyses, recommendations are also made for the inclusion of additional constituents in the standard prediction of tides at Victoria.

INTRODUCTION

The harmonic analysis of tides requires calculating the amplitudes and phases of a finite number of sinusoidal functions with known frequencies from a time series of observations. Although tidal potential theory (e.g., DOODSON, 1921; CARTWRIGHT and TAYLER, 1971) predicts hundreds of these frequencies, many of them are so close that they cannot be adequately separated in analyses of a few

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years or less. Attempts to do so result in an ill-conditioned matrix equation and large confidence limits around the computed constituent amplitudes and phases. Consequently, conventional *harmonic* tide analyses and predictions (e.g., GODIN, 1972) are actually *quasi-harmonic* (ZETLER, LONG, and KU, 1985) in the sense that they modify constituent amplitudes and phases to account for the satellite constituents that are not directly included in the analyses.

However many permanent tide stations now have records longer than 18.61 years, the period of rotation of the moon's node, and it is feasible to consider a harmonic analysis that does not require satellite adjustments. Such an analysis would directly resolve constituents differing by multiples of the 8.85-year and 18.61-year basic tidal periods and thereby provide, both a check of the accuracy of the satellite adjustment calculations in shorter analyses, and, in cases where a shallow water constituent and astronomical satellite have the same frequency, more accurate inference parameters for these adjustments.

Recently FRANCO and HARARI (1988) presented a tidal analysis technique for long time series that is based on Fourier transforms. They computed tidal harmonics for Cananéia (Brazil) from the Fourier coefficients of a series of $M > 5$ successive transforms of 2^{14} hourly observations. (2^{14} hours is approximately 1.869 years.) ZETLER *et al.* (1985) and AMIN (1976) have also calculated tidal harmonics from long time series. ZETLER *et al.* found that harmonics calculated from a Fourier analysis of 18.61 years of Seattle data gave slightly better predictions than those produced by three conventional quasi-harmonic techniques. Amin (1976), following an approach originated by CARTWRIGHT and TAYLER (1971), first band-pass filtered a long time series from Southend (England) into separate tidal groups, and then showed that the f and u nodal parameters differed from those computed using the equilibrium tide.

Though the technique presented here is much simpler than those of FRANCO and HARARI, ZETLER *et al.*, and AMIN, it has the advantages of easily handling missing data and being applicable to any record lengths greater than 18.61 years. In fact, even though the version described here is for regularly sampled data, the same approach could also be used for irregularly sampled data. In this sense, this technique is more versatile than Fourier-based approaches.

However, it must be emphasized that long harmonic analyses are only possible because of the increased speed and memory of present-day computers. A decade ago, the large matrix equations that arise from these analyses could only be solved on a select few super-computers. With the proliferation of faster and cheaper computers, it is now feasible to develop a program that can be used on a wide variety of machines. Indeed, it is planned that the program described here will become part of the widely distributed Institute of Ocean Sciences package of tidal programs and test data.

THE NUMERICAL TECHNIQUE

DOODSON (1921) showed that all tidal constituents have frequencies that are linear combinations, termed *harmonics,* of the rates of change of τ, mean lunar time, and the following five astronomical variables that uniquely specify the position of the sun and moon: s, the mean longitude of the moon; h, the mean longitude of the sun; p, the mean longitude of the lunar perigee; n', the negative of the longitude of the moon's ascending node; and p', the mean longitude of the solar perigee. The approximate periods for these six variables are 24.84 hours, 27.3 days, 365.24 days, 8.85 years, 18.61 years, and 20932 years respectively. For each constituent, the integer coefficients of these six harmonics are called the Doodson numbers.

The tidal frequencies used in this harmonic analysis are taken from the CARTWRIGHT and TAYLER (1971), and CARTWRIGHT and EDDEN (1973), update of DOODSON's calculations. Care was taken that pairs of constituents whose frequencies differed by integer multiples of p' would not both be included in an analysis. In such cases, only the constituent with the larger tidal potential amplitude is included. Each analysis included 474 astronomical constituents and 55 shallow water constituents, but the latter number could be easily increased if nonlinear interactions were thought to be more important. A criterion (see GODIN, 1972) to select constituents for the analysis has also been used to ensure that neighbouring constituents are separated by at least one cycle over the analysis period. For example, the frequencies $2n'$ and p require approximately 179 years for separation. So constituents whose last fourth and fifth Doodson numbers differ by $\pm(-1,2)$ are handled in the same manner as those differing only in the sixth Doodson number.

The least squares technique employed in the analysis is identical to that described by GODIN (1972) and FOREMAN (1977). It is briefly reviewed as follows. Assume that a selection procedure has chosen M constituents for inclusion in the analysis. We then wish to solve the system of equations

$$y_i = A_0 + \sum_{j=1}^{M} A_j \cos(\omega_j t_i - \phi_j) \tag{1}$$

for the unknowns A_j and ϕ_j , $j = 1,M$. A_j, ω_j, ϕ_j are the amplitude, frequency, and phase of constituent j; y_i , $i = 1,N$, is the observation at time t_i ; and in accordance with Doodson, each frequency ω_j can be written as

$$\omega_j = l_1 \tau + l_2 s + l_3 h + l_4 p + l_5 n' + l_6 p', \tag{2}$$

for integers l_k , $k = 1,6$. Each equation can be made linear in the new unknowns C_j and S_j by rewriting

$$A_j \cos(\omega_j t_i - \phi_j) = C_j \cos(\omega_j t_i) + S_j \sin(\omega_j t_i), \tag{3}$$

where

$$A_j = (C_j^2 + S_j^2)^{1/2} \quad \text{and} \quad \phi_j = \arctan(S_j / C_j).$$

As the number of equations, N, is greater than the number of unknowns, $2M + 1$, the system of equations is overdetermined and all the equations cannot be solved exactly. The least squares technique calculates the solution that minimizes the sum of the squares of the residuals. The original set of equations given by (1) is reformulated as the *normal equations* and solved efficiently and stably with the Cholesky algorithm (e.g. ORTEGA, 1972).

All calculations with the new program were done in single precision on a VAX-785 computer. Each analysis solved a 1057 × 1057 matrix and the 38-year analysis required approximately four hours of computer time. Were there no gaps in the hourly time series, the *normal* matrix could have been partitioned and the computation time would have been reduced by a factor of four.

Results

The harmonic analysis program was initially applied to hourly Victoria data from 0100 PST January 1, 1939, to 2400 December 31, 1976. Only 624 and 720 values are missing in October 1950 and March 1973 respectively. In addition to a 38-year analysis, the two 19-year sections of data were also analysed. A sequence of one-year analyses using GODIN's (1972) satellite adjustment technique as also done and averages and standard deviations were computed for the amplitudes and phases of all constituents.

Tables I, II, IV, and V are analogous to Table I in ZETLER *et al.*. They compare the results of the two successive 19-year analyses, the 38-year analysis, and the average values from the 38 one-year analyses. Values shown in brackets in the *averages/inferred* columns in Tables I, II, IV, and V are the *inferred* satellite amplitudes and phases that were calculated separately using potential theory and the 38-year analysis results. Values not enclosed in brackets in the same column are averages from the 38 one-year analyses. The columns entitled σ provide a measure of the stability of these yearly analysis results. For each constituent j, the standard deviation, σ_j, is defined (CRAWFORD, 1982) as

$$\sigma_j = \left(\frac{\sum_{l=1}^{38} (C_{l,j} - \bar{C}_j)^2 + (S_{l,j} - \bar{S}_j)^2}{37} \right)^{1/2}, \qquad (4)$$

where

$$\bar{C}_j = \sum_{l=1}^{38} C_{l,j} / 38, \qquad \bar{S}_j = \sum_{l=1}^{38} S_{l,j} / 38,$$

and $C_{l,j}$, $S_{l,j}$ are as defined by equation (3) for year l.

As ZETLER *et al.* suggest a rejection limit (due to contamination from background noise) of 0.25 to 0.5 cm for a 19-year analysis, only constituents with amplitudes greater than 0.1 cm in the 38-year analysis are listed in Tables I and II. Due to lower and higher levels of background noise in other frequency bands, rejection limits of 0.05 cm and 0.5 cm were chosen in Tables IV and V respectively.

Figure 1 gives a visual definition of the background noise level in the low frequency band. It shows the Fourier amplitudes for a 184320 hour (21.03 year) segment of the Victoria data with no gaps. The fact that the tidal amplitudes

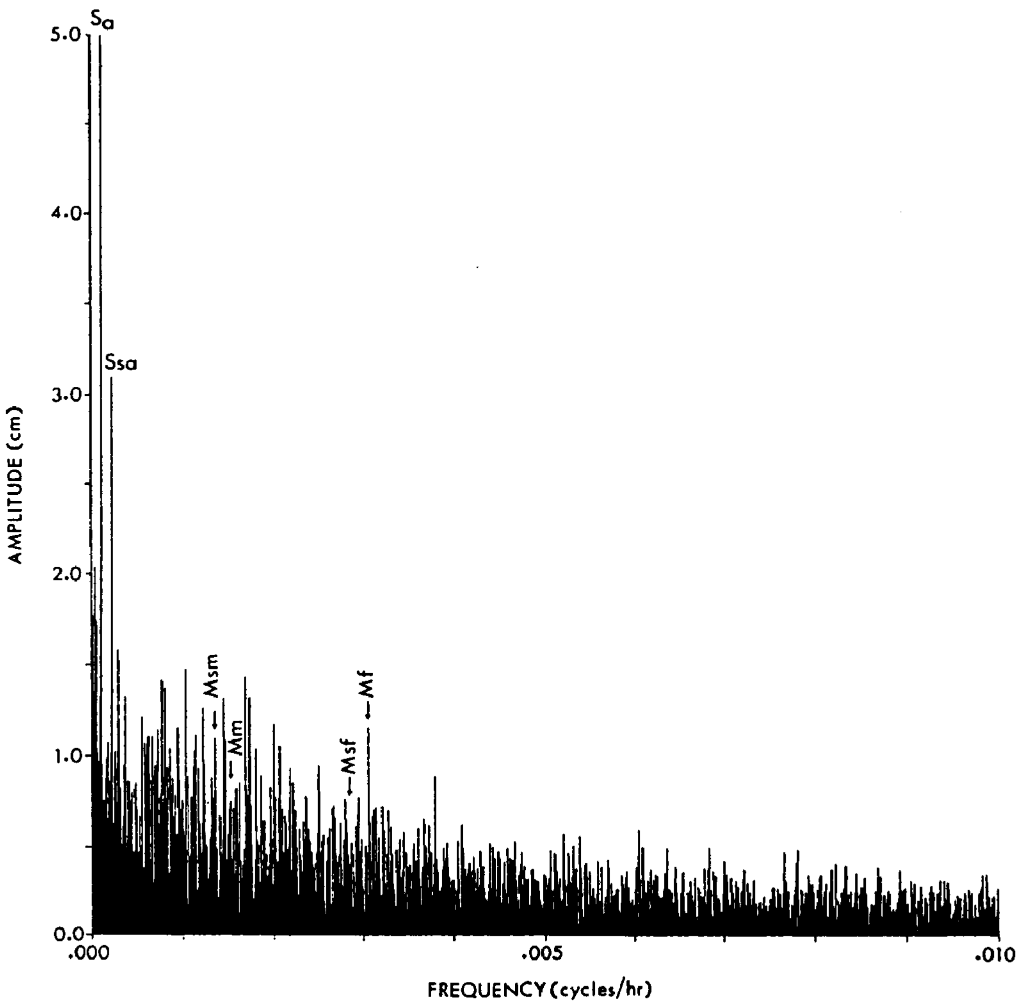

FIG. 1.— Fourier amplitudes for Victoria in the frequency range 0.0 to 0.01 cycles per hour.

barely rise above the general background level indicates that the harmonic method can be expected to have difficulty in calculating stable tidal energies. (This is confirmed by the results in Table V and will be discussed later). Better estimates of the tidal contribution to this low frequency energy could be obtained by using the CRAWFORD (1982) technique for reducing the atmospheric effects.

Figures 2, 3, 4, and 5 show the analogous plots for the diurnal, semi-diurnal, and high frequency bands. Amplitudes larger than 5 cm in Figures 1, 2 and 3, and 2 cm in Figures 4 and 5, have been cut off in order to provide a better view of the low energy spectra. Notice that many tidal amplitudes are significantly larger than the background noise in these bands. The presence of tidal cusps around the large constituents has been studied by MUNK, ZETLER, and GROVES (1965).

As pointed out by ZETLER *et al.*, Godin (1972), and Amin (1976), the presence of non-tidal energy (background noise) in the observations and variations in the tides themselves due to nonlinear interactions and changing physical regimes (e.g., silting of harbours, density variations, seasonal ice) may have the

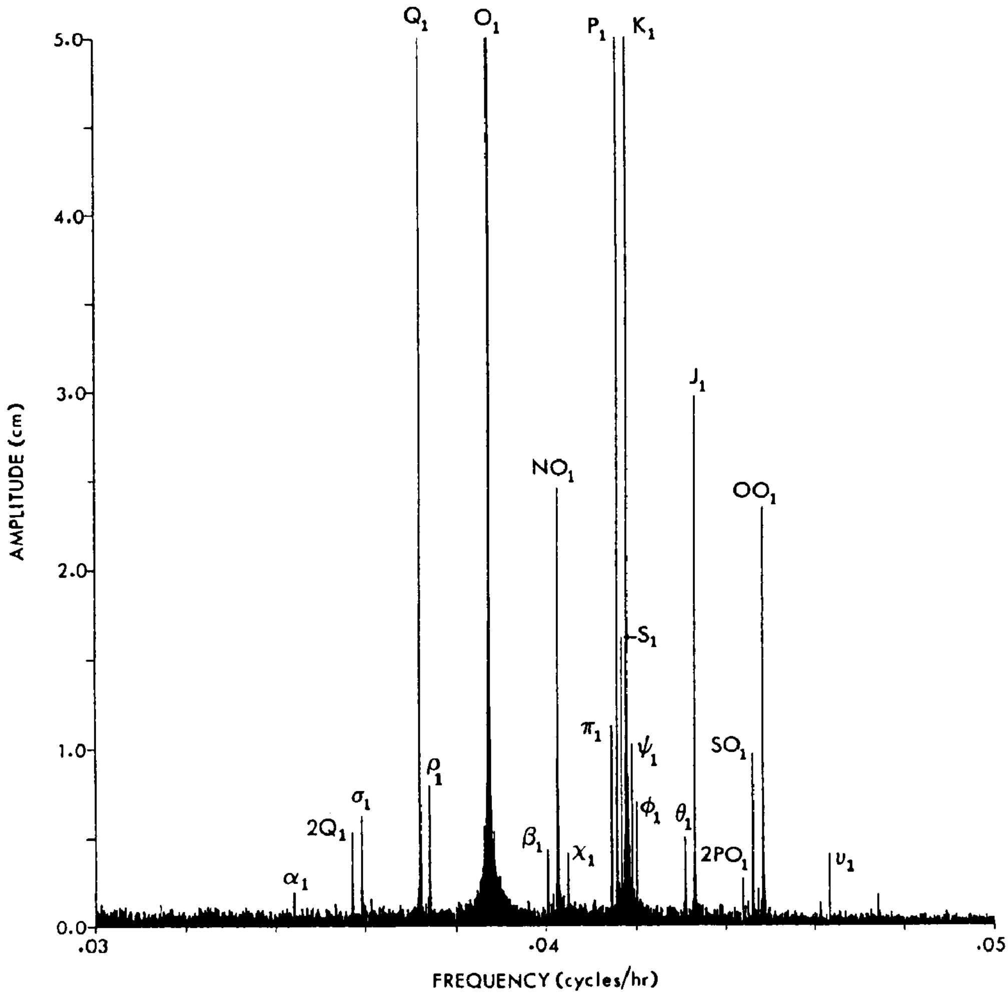

FIG. 2.— Fourier amplitudes for Victoria in the frequency range 0.03 to 0.05 cycles per hour.

effect of falsely distributing energy to neighbouring frequencies. It is therefore revealing to compare amplitude ratios obtained by the long harmonic analysis with those computed from the tidal potential. Ratios that are significantly different may denote such an energy distribution or the presence of a shallow water constituent. It should be pointed out that leakage of tidal energy to lesser constituents or satellites is not a phenomenon that is unique to harmonic analyses. Fourier techniques will exhibit the same behaviour since they are, in fact, harmonic methods with the number of unknowns equal to the number of observations and the frequencies dictated by the record length rather than tidal theory.

The two 19-year analyses are included in Tables I, II, IV, and V to evaluate stability of the satellite amplitudes and phases. Notice that the harmonic constants for many of the smaller satellite constituents change dramatically from one analysis to the next. This suggests that the tidal signal has been significantly masked by the background noise and the calculated amplitudes and phases are not primarily of tidal origin. However most of the harmonic constants for the major constituents and satellites are relatively constant.

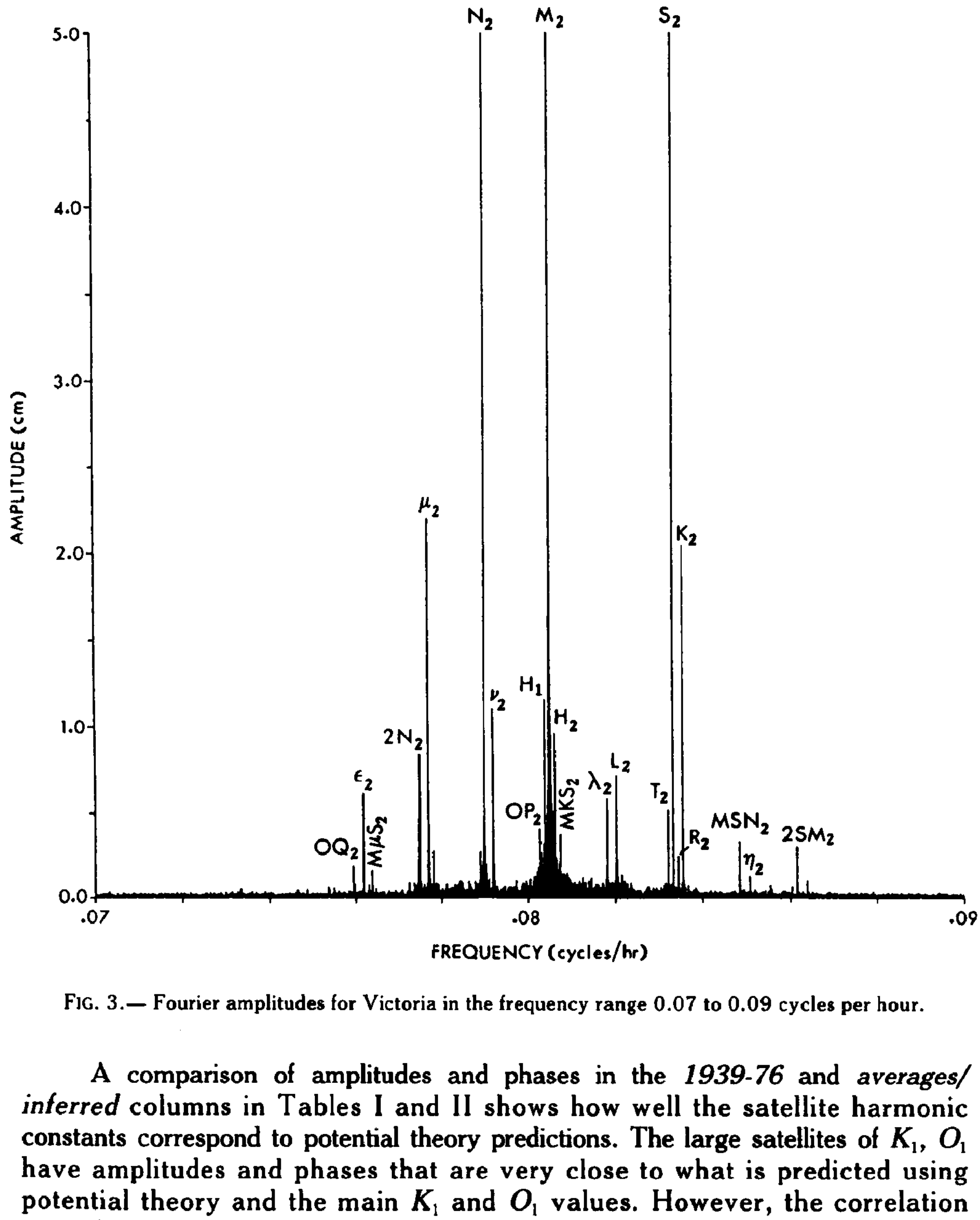

FIG. 3.— Fourier amplitudes for Victoria in the frequency range 0.07 to 0.09 cycles per hour.

A comparison of amplitudes and phases in the *1939-76* and *averages/inferred* columns in Tables I and II shows how well the satellite harmonic constants correspond to potential theory predictions. The large satellites of K_1, O_1 have amplitudes and phases that are very close to what is predicted using potential theory and the main K_1 and O_1 values. However, the correlation generally deteriorates as the amplitude of the smaller satellites approaches the background noise level.

The satellite constituents marked with an asterisk in Tables I and II denote third-order terms in the tidal potential expansion. They deserve special mention as they often do not seem to occur as predicted by theory, and are consequently omitted from nodal correction calculations (e.g. FOREMAN, 1977). The Q_1, O_1, K_1, N_2, M_2, and L_2, third order satellites exhibit high variability and should probably not be included in any predictions. As seen in Figures 2 and 3 their amplitudes are not sufficiently large to emerge from the background spectra. Consequently, the energy attributed to them has a large non-tidal component. The M_1 and J_1 satellites are relatively consistent for the two 19-year analyses, however their

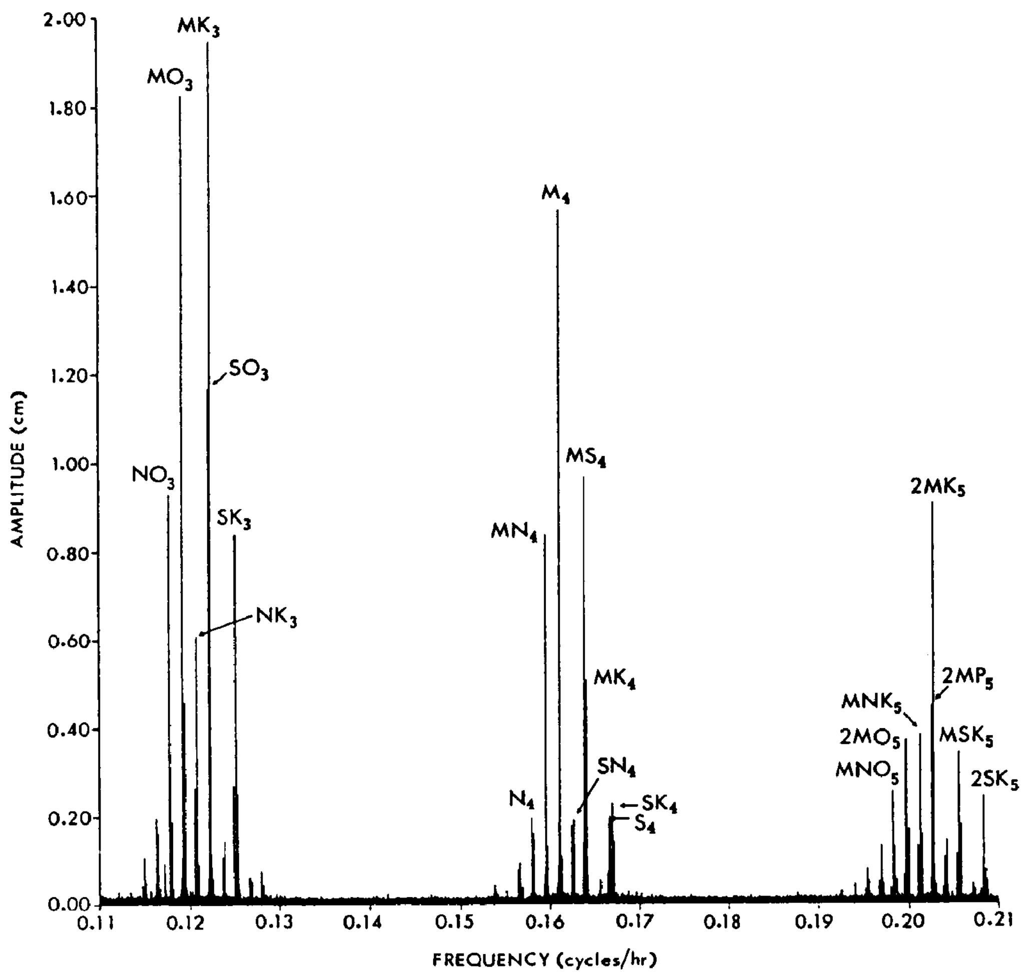

FIG. 4.— Fourier amplitudes for Victoria in the frequency range 0.11 to 0.21 cycles per hour.

amplitudes and especially phases, are quite different than predicted by potential theory.

The results in Tables I and II can be used to assess the expected success of nodal corrections in the light of GODIN's (1986) recommendations. GODIN remarks that nodal corrections are effective for K_1, O_1, and K_2. We concur. Our analyses show that all these constituents have large satellites whose amplitudes and phases are both stable and in accordance with tidal potential theory. He also states that nodal corrections should be successful for Q_1, J_1, OO_1, and NO_1, where the tide is primarily linear and third order effects are minimal. Discounting the third order satellites for Q_1, and OO_1, we agree here also. Although GODIN does not recommend nodal corrections for either N_2, or L_2, we don't feel that they should be disgarded completely. Although both third order satellites seem to be unreliable and should not be included in nodal corrections, consistency of the other satellite harmonics suggest that their inclusion should help predictions. However it does not appear that one should use potential theory amplitude ratios and phase differences in the nodal correction calculations, as they are significantly different from the analysis results.

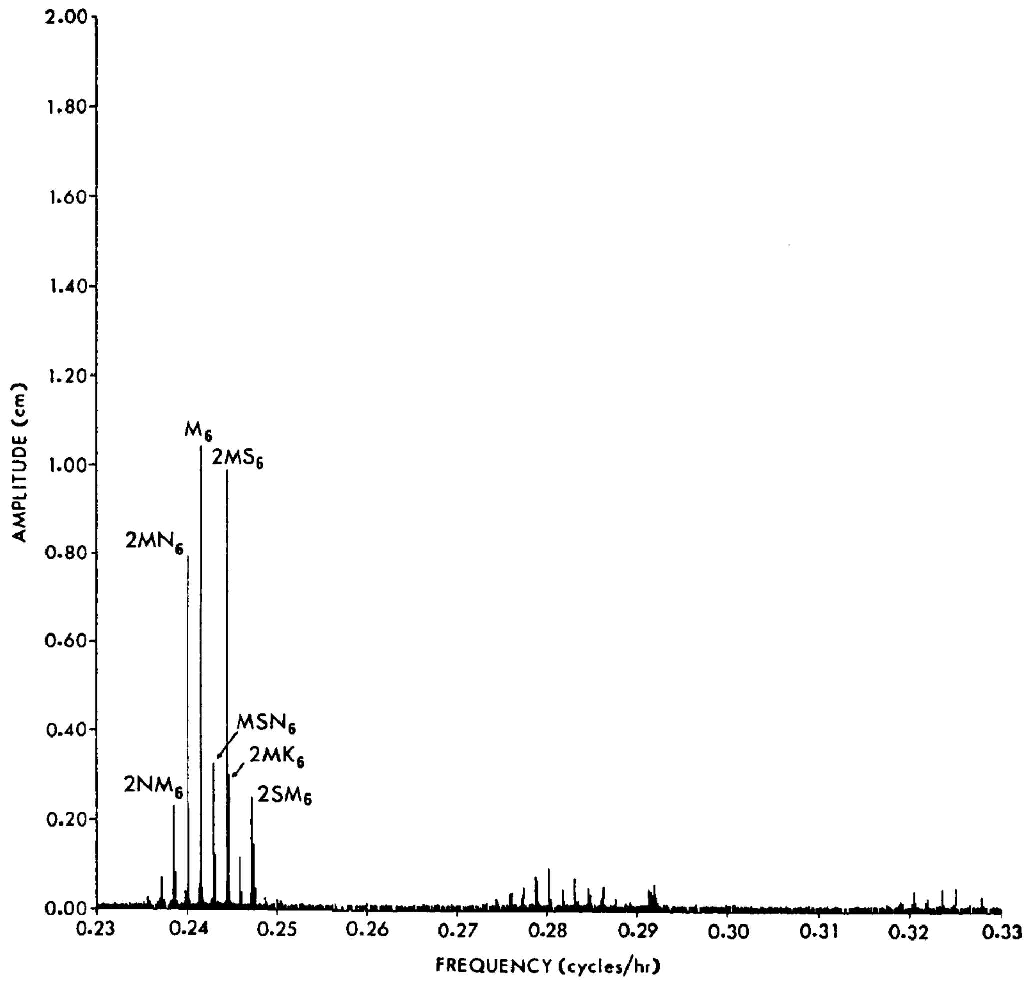

FIG. 5.— Fourier amplitudes for Victoria in the frequency range 0.23 to 0.33 cycles per hour.

Notice that some of the satellites around large constituents such as M_2, and K_1 have amplitudes that are much larger than predicted by potential theory. This probably indicates a leakage of the main constituent energy due to radiational effects, tide gauge problems, or a changing physical environment (e.g. harbour dredging).

Many shallow water constituents have the same frequency as an astronomical constituent. (Table III provides a partial list.) Consequently in cases where the harmonic analysis has computed a much larger amplitude for a major constituent (e.g., Q_1) relative to neighbour (e.g., O_1) than is predicted by potential theory, it is likely that a substantial portion of the major constituent energy is actually due to a shallow water constituent (e.g., NK_1). The precise shallow water contribution can be calculated by admittance functions along the same lines as is done by GODIN and GUTIÉRREZ (1986). A crude indication of the nonlinear activity around the diurnal frequencies is seen with the following calculations. Using K_1 as a reference and assuming a constant admittance function across the entire diurnal band, potential theory predicts amplitudes of 8.70, 45.44, 0.59, 3.57, 21.15, 3.57, and 0.59 cm respectively for Q_1, O_1, τ_1, NO_1, P_1, J_1, and SO_1. The first six of these values are 37%, 22%, 44%, 26%, 6%, and 6% larger than is calculated by the 38-year analysis whereas the last is 40% smaller. So it is likely that there are substantial nonlinear contributions to both K_1 and these seven constituents. Similar calculations could be done for some of the semi-diurnal constituents but they would be of dubious value due to the presence of several amphidromes near Victoria. In the vicinity of these amphidromes, amplitudes vary significantly and the assumption of a smooth admittance function is not valid.

In Tables I, II, IV, and V, values of σ that are relatively large with respect to the amplitude denote cases where either background noise or the satellite adjustment calculation has caused the analysed signal to be unstable. For many constituents where there are substantial nonlinear contributions, this stability can be improved significantly if the inference parameters used in GODIN's adjustment calculation are based on the results of the 38-year analysis, rather than potential theory. For example, the σ values for NO_1, J_1, N_2, and L_2, are reduced from those in Tables I and II to 0.35, 0.27, 0.26, and 0.26 respectively, when inference parameters are based on the 38-year analysis results. Consequently, an important role of long harmonic analyses will be to obtain better inference parameters for satellite adjustment calculations in shorter analyses at the same (or a nearby) location.

Table IV shows analysis results for the terdiurnal and higher frequency constituents whose amplitudes were found to be greater than 0.05 cm. The two 19-year analyses are remarkably consistent, demonstrating little contamination by background noise or tidal signals that are not included in the analysis. The inferred amplitudes were calculated using tidal potential theory and assuming just one interaction. For example, MO_3 is assumed to arise solely from $M_2 + O_1$. The extent to which these inferred values differ from the 1939-76 analysis results is an indication of the importance of other interactions. MO_3 may also arise from $M_4 - K_1$, $\mu_2 + P_1$, and $S_2 + \sigma_1$. As seen in Table IV, it is likely that at least one of these additional interactions has caused a larger than expected value (based on the size of MK_3) for MO_3 (3-10000). Notice that since the constituent selection criterion does not permit the inclusion of both M_3 and NK_3 in one-year analyses, the larger constituent, NK_3, has been chosen.

Table V shows the analysis results for the slow constituents. In this case, there is significant background noise (see Fig. 1), predominantly in the form of meteorological effects, that causes large variability in the results of the two 19-year analyses. Only the results of constituents whose amplitudes are larger than 0.5 cm are shown. Notice that even among the relatively large signals, there is significant variability in almost all the constituents. In fact, probably only *Sa* (00100-1), *Ssa* (002000), and *Msf* (02-2000) are sufficiently stable that they could be considered for inclusion in a prediction. The σ values for *Mf, Msf, Mm,* and *Msm* are quite close to those published by CRAWFORD, 1982 (Table 2) for nineteen years of Victoria data, and are all much larger than the average amplitudes.

Table VI shows the slightly improved predictive capability with harmonic constants from the new program. Amplitudes and phases from an analysis of the first 19 years of Victoria data were used to predict elevations for the second 19 years. Root mean square residuals were then computed from the actual observations for this time period. Similar predictions were also made using average amplitudes and phases from one-year analyses over the first 19 years. (For these predictions, the f and u factors were assumed constant for each month and set equal to their values on the 15^{th} day of that month.) Based on the stability of the harmonic constants in the two 19-year analyses, only a constant term and constituents with amplitudes greater than .1 cm in the diurnal and higher frequency bands were included in all the predictions.

Notice that on average, predictions based on results of the 19 year analysis are only slightly better than those based on the average of the 19 yearly analyses. In fact, the former predictions are worse for five of the nineteen years. There are two reasons why the predictions based on the 19 year analysis are not as good as those shown by ZETLER *et al.* for Seattle. The first reason is that ZETLER *et al.* predicted for the same 19-year period that was analysed. As seen by the fact that there are differences in the two nineteen year analyses in Tables I, II, IV, and V, the values in both columns A and B of Table VI would probably decrease if the same strategy were followed here. The second reason is that Zetler compared his 19 year predictions with predictions using harmonics from Form 444 (presumably a standard set of amplitudes and phases) rather than averages of yearly analyses. It is likely that yearly-average predictions would be more accurate.

It should be pointed out that the yearly-average predictions actually infer values for all the satellites of each major constituent, whereas the predictions based on the 19 year analysis have only included those satellites whose amplitudes are larger than the threshold of 0.1 cm. However this discrepancy makes very little difference to the results. When the yearly-average predictions are repeated using the same satellites that are included in the column B predictions, the mean *rms* becomes 14.610.

The set of constituents used for the yearly-average predictions in Table VI has 31 more entries than the standard set used to produce the Canadian Tide and Current Table (1989) values for Victoria. (Constituents included in this standard set are designated with the superscript • in Tables I, II, IV, and V.) The σ values shown in Tables I, II, IV, and V demonstrate that some of these constituents — notably ϵ_2, H_1, H_2, NO_3, NK_3, $2MO_5$, and $2MP_5$, and $2MK_5$ — are both large enough and sufficiently stable to warrant inclusion in future predictions. Figures 3 and 4 show the relative importance of these constituents. Column C in Table VI shows the residuals obtained from predictions using averages of the 19 yearly analyses for only those constituents included in the Tide Table set. The residuals are consistently, though not appreciably, worse than those in column A. This indicates that slightly better Tide Table predictions could be obtained with an expanded set of constituents.

The closeness of columns A and B in Table VI confirms the accuracy of GODIN's [1972] satellite adjustment calculation and inferences based on tidal potential theory. Although approximations within these calculations are most valid when the analysis period is one year, it is not likely that predictions based on, say, an average of 6 month analyses, would be much different. Even though we have seen that the inference parameters for NO_1, J_1, N_2, and L_2 can be improved by using values from the 38-year analysis, these satellites are so small in amplitude that they scarcely make any difference to subsequent tidal predictions.

CONCLUSIONS

The preceding discussion has summarized the development and testing of a new harmonic tidal analysis program for time series of 18.61 years or longer. The method is more versatile than Fourier-based approaches in the sense that it can easily handle missing data and is applicable to all record lengths greater than 18.61 years.

Results obtained from 38 and 19-year analyses of Victoria observations demonstrate that, provided a tidal constituent amplitude lies above the background noise level, its relative amplitude and phase generally conform to potential theory predictions. However this background noise may be sufficiently large in the low frequency band that even long period analyses may not provide accurate estimates of constituents such as *Sa, Ssa, Msm, Mm, Msf,* and *Mf.* In such cases, additional measures (e.g. CRAWFORD, 1982) may be required to reduce atmospheric effects prior to the tidal analysis.

Predictions based on results from a 19-year analysis were seen to be only slightly better than conventional predictions using Godin's technique for satellite adjustment. This is an indication that: a) the harmonic constants corresponding to most of the satellite constituents that are now included directly in the long analysis are not significantly different from potential theory predictions; and b) the approximations made with Godin's satellite adjustment calculation are quite accurate.

It was demonstrated with constituents NO_1, J_1, N_2, and L_2, that an important role of long harmonic analyses will be to obtain better inference parameters for satellite adjustment calculations in shorter analyses at the same (or a nearby) location. For Victoria, the stability of these constituents is improved significantly by using satellite inference parameters from a 38-year analysis.

Differences between columns A and C in Table VI suggest that the standard set of constituents for producing the Canadian Tide and Current Tables (1989) at Victoria could be expanded. Standard deviations calculated from the 38 yearly analyses demonstrate that many of the presently omitted constituents (such as ϵ_2, H_1, H_2, NO_3, NK_3, $2MO_5$, $2MP_5$, and $2MK_5$) are both large enough and sufficiently stable to be included in future predictions. It is likely that similar improvements are possible at other sites where long records permit analyses of the type performed here.

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References

[1] AMIN, M. (1976): The fine resolution of tidal harmonics. *Geophys. J.R. astr. Soc. 44*, pp. 293-310.

[2] CANADIAN TIDE AND CURRENT TABLES, Vol. 5, Juan de Fuca and Georgia Straits (1989). Department of Fisheries and Oceans, Ottawa.

[3] CARTWRIGHT, D.E and R.J. TAYLER (1971): New computations of the tide-generating potential. *Geophys. J.R. astr. Soc.*, 23, pp. 45-74.

[4] CARTWRIGHT, D.E and A.C. EDDEN (1973): Corrected tables of tidal harmonics. *Geophys J.R. astr. Soc.*, 33, pp. 253-264.

[5] CRAWFORD, W.R. (1982): Analysis of fortnightly and monthly tides. *International Hydrographic Review*, LIX(1), pp. 131-141.

[6] DOODSON, A.T. (1921): The harmonic development of the tide-generating potential. *Proc. Roy. Soc. Series A.*, 100, pp. 306-323. Re-issued in the *International Hydrographic Review*, May 1954.

[7] FOREMAN, M.G.G. (1977): Manual for tidal heights analysis and prediction. *Pacific Marine Science Report 77-10*, Institute of Ocean Sciences, Patricia Bay, Sidney, B.C., 97 pp. Unpublished manuscript.

[8] FRANCO, A.S., and J. HARARI (1988): Tidal analysis of long series. *International Hydrographic Review*, LXV(1), pp. 141-158.

[9] GODIN, G. (1972): The Analysis of Tides. University of Toronto Press, 264 pp.

[10] GODIN, G. (1986): The use of nodal corrections in the calculation of harmonic constants. *International Hydrographic Review*, LXIII(2), pp. 143-162

[11] GODIN, G. and G. GUTIÉRREZ (1986): Non-linear effects in the tide of the Bay of Fundy. *Continental Shelf Research*, 5(3), pp. 379-402.

[12] MUNK, W.H., B. ZETLER, and G.W. GROVES (1965): Tidal cusps. *Geophys. J.*, 10, p. 211-219.

[13] ORTEGA, J.M. (1972): Numerical Analysis — A Second Course. Academic Press, New York, 193 pp.

[14] ZETLER, B.D., E.E. LONG, and L.F. KU, (1985): Tide predictions using satellite constituents. *International Hydrographic Review*, pp. 135-142.

Table Ia
Diurnal Harmonic Constants for Victoria

Constituents designated with the superscripts * and • are third order satellites, and constituents included in the standard tide table predictions, respectively. Values enclosed in brackets are inferred using results from the 1939-1976 analysis and potentiel theory relationships.

Constituent		1939-1957		1958-1976		1939-1976		averages/inferred		σ
name	*Doodson numbers*	*amp (cm)*	*phase (°PST)*	*amp (cm)*	*phase (°PST)*	*amp (cm)*	*phase (°PST)*	*amp (cm)*	*phase (°PST)*	*(cm)*
α_1	1 -4 2 1 0 0	0.19	169.9	0.21	179.6	0.20	175.0	0.19	174.5	0.24
	1 -4 4 -1 0 0	0.11	187.3	0.09	198.3	0.10	192.4			
$2Q_1$	1 -3 0 2 -1 0	0.13	141.2	0.08	108.3	0.10	128.7	(0.14)	(136.8)	
$2Q_1^{\bullet}$	1 -3 0 2 0 0	0.72	137.3	0.78	136.4	0.75	136.8	0.77	136.7	0.29
σ_1	1 -3 2 0 -1 0	0.17	160.5	0.15	166.9	0.16	163.1	(0.13)	(156.4)	
$\sigma_1^{\bullet}$	1 -3 2 0 0 0	0.76	155.4	0.67	157.4	0.71	156.4	0.67	154.4	0.27
	1 -3 3 0 0 -1	0.17	197.0	0.07	194.1	0.12	195.9			
	1 -3 4 0 0 0	0.15	68.6	0.14	42.3	0.14	56.0			
Q_1^{*}	1 -2 0 0 0 0	0.11	305.6	0.16	285.7	0.13	296.5	(0.16)	(130.9)	
Q_1	1 -2 0 1 -1 0	1.27	130.1	1.20	133.3	1.24	131.8	(1.19)	(130.9)	
$Q_1^{\bullet}$	1 -2 0 1 0 0	6.43	131.0	6.25	130.8	6.34	130.9	6.33	131.1	0.37
	1 -2 1 1 0 -1	0.12	168.0	0.11	86.8	0.14	145.9			
ρ_1	1 -2 2 -1 -1 0	0.19	128.9	0.32	114.1	0.26	120.2	(0.24)	(129.7	
$\rho_1^{\bullet}$	1 -2 2 -1 1 0	1.26	129.7	1.25	129.7	1.25	129.7	1.27	130.4	0.35
	1 -1 -2 2 0 0	0.11	140.0	0.13	132.1	0.12	136.4			
	1 -1 -1 0 0 1	0.27	240.5	0.18	245.9	0.23	242.4			

Table Ia (continued)

O_1	1 -1 0 0 -2 0	0.17	143.4	0.25	112.9	0.20	124.4	(0.22)	(136.6)	
O_1	1 -1 0 0 -1 0	7.27	137.9	6.81	137.5	7.05	137.8	(7.05)	(136.6)	
$O_1^{\bullet}$	1 -1 0 0 0 0	37.69	136.4	37.19	136.9	37.41	136.6	37.46	136.6	0.62
O_1^{*}	1 -1 0 1 -1 0	0.24	357.0	0.21	286.4	0.18	323.3	(0.01)	(136.6)	
O_1	1 -1 0 2 0 0	0.19	132.0	0.31	174.0	0.25	157.0	(0.24)	(136.6)	
	1 -1 1 0 0 -1	0.34	145.7	0.29	138.8	0.32	143.1			
τ_1	1 -1 2 0 0 0	0.40	239.4	0.42	233.4	0.41	236.3	0.43	232.0	0.46
τ_1	1 -1 2 0 1 0	0.25	103.3	0.17	121.9	0.21	110.7	(0.08)	(236.3)	
	1 -1 3 0 0 -1	0.15	207.8	0.14	230.9	0.14	218.6			
β_1	1 0 -2 1 -1 0	0.17	163.4	0.09	192.1	0.13	173.2	(0.10)	(128.9)	
β_1	1 0 -2 1 0 0	0.44	126.7	0.43	130.9	0.44	128.9	0.41	129.5	0.39
	1 0 -1 1 0 1	0.16	89.7	0.17	86.0	0.17	87.7			
M_1	1 0 0 -1 -1 0	0.34	143.9	0.24	125.8	0.29	136.8	(0.19)	(149.1)	
M_1	1 0 0 -1 0 0	1.28	137.0	1.28	137.9	1.28	137.4	(1.02)	(149.1)	
M_1^{*}	1 0 0 0 0 0	0.43	190.0	0.36	203.8	0.40	196.2	(0.55)	(149.1)	
$NO_1^{\bullet}$	1 0 0 1 0 0	2.92	149.1	2.76	149.1	2.84	149.1	2.80	150.0	1.04
NO_1	1 0 0 1 1 0	0.59	143.4	0.58	147.2	0.58	145.3	(0.57)	(149.1)	
	1 0 1 1 0 -1	0.12	264.5	0.12	275.6	0.12	270.2			
$\chi_1^{\bullet}$	1 0 2 -1 0 0	0.60	137.8	0.53	138.5	0.56	138.1	0.47	133.7	0.32
χ_1	1 0 2 -1 0 0	0.16	146.9	0.13	142.1	0.15	144.3	(0.12)	(138.1)	
π_1	1 1 -3 0 -1 1	0.09	116.9	0.11	138.1	0.10	128.3	(0.01)	(153.9)	
$\pi_1^{\bullet}$	1 1 -3 0 -1 1	1.23	152.2	1.19	155.7	1.21	153.9	1.21	153.9	0.33

Table Ib
Diurnal Harmonic Constants for Victoria

Constituent		1939-1957		1958-1976		1939-1976		averages/inferred		σ
name	*Doodson numbers*	*amp (cm)*	*phase (°PST)*	*amp (cm)*	*phase (°PST)*	*amp (cm)*	*phase (°PST)*	*amp (cm)*	*phase (°PST)*	*(cm)*
P_1	1 1 -2 0 -1 0	0.19	125.6	0.37	135.5	0.28	132.3	(0.22)	(147.5)	
$P_1^\bullet$	1 1 -2 0 0 0	20.01	147.4	19.77	147.6	19.86	147.5	19.90	147.5	0.47
P_1	1 1 -2 2 0 0	0.08	343.9	0.19	246.4	0.10	273.2	(0.03)	(147.5)	
$S_1^\bullet$	1 1 -1 0 0 1	1.61	114.7	1.65	118.4	1.63	116.6	2.34	128.4	1.08
S_1	1 1 -1 0 1 1	0.05	308.4	0.30	24.1	0.16	16.9	(0.58)	(116.6)	
K_1^*	1 1 0 -1 -1 0	0.02	312.2	0.23	280.4	0.12	278.0	(0.01)	(149.3)	
K_1^*	1 1 0 -1 0 0	0.10	290.1	0.29	45.9	0.12	28.6	(0.04)	(149.3)	
K_1	1 1 0 0 -1 0	1.03	143.2	1.35	140.6	1.19	142.3	(1.27)	(149.3)	
$K_1^\bullet$	1 1 0 0 0 0	64.42	149.1	63.22	149.5	63.90	149.3	63.74	149.3	0.88
K_1	1 1 0 0 1 0	8.78	147.0	8.52	149.5	8.66	148.2	(8.66)	(149.3)	
K_1	1 1 0 0 2 0	0.22	150.6	0.28	93.51	0.22	118.3	(0.19)	(149.3)	
K_1^*	1 1 0 1 1 0	0.14	152.2	0.20	257.7	0.10	214.9	(0.01)	(149.3)	
$\Psi_1^\bullet$	1 1 1 0 0 -1	0.99	159.3	0.96	165.4	0.98	162.4	0.98	162.2	0.54
$\Psi_1^\bullet$	1 1 1 0 1 -1	0.10	171.2	0.20	183.0	0.15	179.2	(0.02)	(162.4)	
$\phi_1^\bullet$	1 1 2 0 0 0	0.79	158.6	0.77	156.4	0.78	157.5	0.78	157.1	0.42
$\theta_1^\bullet$	1 2 -2 1 0 0	0.63	145.6	0.57	154.4	0.60	149.8	0.62	149.3	0.36

Table Ib (continued)

J_1	1 2 0 -1 -1 0	0.13	167.1	0.15	151.2	0.13	157.6	(0.10)	(167.1)	
$J_1^{\bullet}$	1 2 0 -1 0 0	3.39	167.2	3.39	167.0	3.38	167.1	3.36	167.4	0.49
J_1	1 2 0 -1 1 0	0.70	163.3	0.69	164.7	0.70	163.8	(0.67)	(167.1)	
J_1^{*}	1 2 0 0 0 0	0.21	223.6	0.24	225.2	0.22	224.4	(0.24)	(167.1)	
	1 2 1 -1 0 -1	0.11	186.2	0.11	156.0	0.10	171.3			
$2PO_1$	1 3 -4 0 0 0	0.31	299.8	0.28	304.2	0.29	302.2	0.30	302.0	0.18
	1 3 -3 0 0 1	0.21	304.9	0.13	333.0	0.16	315.1			
$SO_1^{\bullet}$	1 3 -2 0 0 0	0.97	265.0	1.00	260.7	0.99	262.9	0.98	263.4	0.20
SO_1	1 3 -2 0 1 0	0.18	245.0	0.18	255.0	0.18	250.0	(0.19)	(262.9)	
	1 3 -1 0 0 -1	0.16	209.4	0.14	216.0	0.15	212.6			
OO_1	1 3 0 -2 0 0	0.21	184.2	0.28	172.5	0.25	177.6	(0.33)	(182.4)	
$1OO_1$	1 3 0 0 0 0	2.31	182.6	2.13	182.0	2.22	182.4	2.21	181.6	0.30
OO_1	1 3 0 0 1 0	1.47	181.9	1.48	182.5	1.47	182.1	(1.42)	(182.4)	
OO_1	1 3 0 0 2 0	0.30	191.0	0.35	191.0	0.32	190.9	(0.30)	(182.4)	
	1 4 -2 -1 0 0	0.06	336.1	0.14	331.4	0.10	331.8			
υ_1	1 4 0 -1 0 0	0.33	200.7	0.38	207.0	0.36	204.1	0.37	203.1	0.18
υ_1	1 4 0 -1 1 0	0.16	208.8	0.21	200.3	0.18	204.1	(0.23)	(204.1)	

Table II
Semi-Diurnal Harmonic Constants for Victoria

Constituent		1939-1957		1958-1976		1939-1976		averages/inferred		σ
name	*Doodson numbers*	*amp (cm)*	*phase (°PST)*	*amp (cm)*	*phase (°PST)*	*amp (cm)*	*phase (°PST)*	*amp (cm)*	*phase (°PST)*	*(cm)*
OQ_2	2 -3 0 3 0 0	0.11	6.0	0.09	22.0	0.10	13.6	0.10	13.1	0.22
ϵ_2	2 -3 2 1 0 0	0.62	348.3	0.61	350.3	0.61	349.4	0.62	350.7	0.13
$M\mu S_2$	2 -3 4 1 0 0	0.17	7.4	0.18	2.0	0.17	4.4			
$2N_2^\bullet$	2 -2 0 2 0 0	1.15	37.2	1.16	36.9	1.16	37.1	1.16	36.6	0.62
μ_2	2 -2 2 0 -1 0	0.09	3.1	0.12	10.0	0.11	8.1	(0.10)	(18.5)	
$\mu_2^\bullet$	2 -2 2 0 0 0	2.59	18.3	2.49	18.6	2.54	18.5	2.56	19.5	0.18
	2 -2 3 0 0 -1	0.28	29.0	0.30	24.5	0.29	26.8			
	2 -1 -1 1 0 1	0.21	42.7	0.27	30.9	0.24	36.1			
N_2^*	2 -1 0 0 0 0	0.07	301.9	0.17	331.6	0.12	326.0	(0.56)	(62.2)	
N_2	2 -1 0 1 -1 0	0.22	64.9	0.28	49.9	0.25	57.1	(0.33)	(62.2)	
$N_2^\bullet$	2 -1 0 1 0 0	8.95	61.8	8.76	62.6	8.85	62.2	8.88	62.0	0.66
$\nu_2^\bullet$	2 -1 2 -1 0 0	1.56	70.8	1.54	70.5	1.55	70.7	1.54	70.3	0.24
γ_2	2 0 -2 0 -2 0	0.08	48.5	0.13	54.0	0.10	50.5			
OP_2	2 0 -2 0 -1 0	0.19	101.5	0.11	24.3	0.12	72.5	(0.08)	(20.7)	
OP_2	2 0 -2 0 0 0	0.41	15.8	0.41	25.8	0.41	20.7	0.40	21.6	0.41
γ_2	2 0 -2 2 0 0	0.13	92.8	0.21	73.5	0.17	83.5	(0.03)		
H_1	2 0 -1 0 -1 1	0.12	239.4	0.11	278.3	0.11	259.5		(19.1)	
H_1	2 0 -1 0 0 1	1.26	16.4	1.36	21.4	1.31	19.1	1.30	19.3	0.39

Table II (continued)

M_2^*	2 0 0 -1 -1 0	0.05	318.5	0.26	188.4	0.12	200.0	(0.00)	(85.7)	
M_2	2 0 0 0 -2 0	0.21	100.9	0.26	168.7	0.19	137.6	(0.02)	(85.7)	
M_2	2 0 0 0 -1 0	1.14	87.8	1.45	72.7	1.28	78.6	(1.39)	(85.7)	
$M_2^\bullet$	2 0 0 0 0 0	37.57	85.5	37.10	85.9	37.35	85.7	37.29	85.7	0.70
M_2^*	2 0 0 1 0 0	0.15	44.5	0.19	344.7	0.17	12.4	(0.07)	(85.7)	
H_2	2 0 1 0 0 -1	1.09	257.8	1.12	252.4	1.10	254.9	1.10	255.1	0.37
$MKS_2^\bullet$	2 0 2 0 0 0	0.38	305.2	0.45	299.6	0.42	301.8	0.42	301.2	0.35
	2 0 3 0 0 -1	0.10	329.3	0.15	335.9	0.12	331.9			
	2 1 -3 1 0 1	0.05	208.4	0.15	202.7	0.10	204.3			
$\lambda_2^\bullet$	2 1 -2 1 0 0	0.69	201.8	0.62	196.8	0.66	199.4	0.65	198.1	0.22
	2 1 -1 -1 0 1	0.13	238.0	0.16	226.5	0.14	231.0			
$L_2^\bullet$	2 1 0 -1 0 0	1.05	185.0	1.00	182.6	1.02	183.5	1.03	185.9	0.50
L_2^*	2 1 0 0 0 0	0.14	250.2	0.10	198.4	0.11	228.7	(0.40)	(183.5)	
L_2	2 1 0 1 0 0	0.16	111.0	0.16	98.5	0.16	104.2	(0.26)	(183.5)	
$T_2^\bullet$	2 2 -3 0 0 1	0.59	114.7	0.54	115.5	0.56	115.0	0.56	114.9	0.20
S_2	2 2 -2 0 -1 0	0.08	134.8	0.17	210.0	0.10	189.4	(0.02)	(93.7)	
$S_2^\bullet$	2 2 -2 0 0 0	10.43	93.6	10.38	93.8	10.40	93.7	10.41	93.6	0.23
R_2	2 2 -1 0 0 -1	0.24	98.0	0.24	98.4	0.24	98.0	0.19	103.6	0.15
$K_2^\bullet$	2 2 0 0 0 0	2.00	99.0	1.96	100.2	1.98	99.6	1.96	99.3	0.22
K_2	2 2 0 0 1 0	0.68	101.0	0.54	106.9	0.61	103.5	(0.59)	(99.6)	
$MSN_2^\bullet$	2 3 -2 -1 0 0	0.38	68.9	0.39	65.8	0.39	67.3	0.39	67.1	0.12
η_2	2 3 0 -1 0 0	0.13	107.3	0.15	107.3	0.14	107.7	0.14	107.7	0.12
$2SM_2$	2 4 -4 0 0 0	0.32	70.6	0.31	67.9	0.32	69.3	0.32	69.2	0.09
	2 4 -2 0 0 0	0.10	343.1	0122	349.3	0.11	246.7			

Table III
Astronomical and shallow water constituents with the same frequency

Astronomical Constituent							Shallow Water Constituent	
Name	Doodson numbers						Name	Composition
Q_1	1	-2	0	1	0	0	NK_1	$N_2 - K_1$
O_1	1	-1	0	0	0	0	MK_1	$M_2 - K_1$
τ_1	1	-1	2	0	0	0	MP_1	$M_2 - P_1$
NO_1	1	0	0	1	0	0	NO_1	$N_2 - O_1$
P_1	1	1	-2	0	0	0	SK_1	$S_2 - K_1$
K_1	1	1	0	0	0	0	MO_1	$M_2 - O_1$
J_1	1	2	0	-1	0	0	MQ_1	$M_2 - Q_1$
SO_1	1	3	-2	0	0	0	SO_1	$S_2 - O_1$
ϵ_2	2	-3	2	1	0	0	MNS_2	$M_2 + N_2 - S_2$
μ_2	2	-2	2	0	0	0	$2MS_2$	$M_2 + M_2 - S_2$
N_2	2	-1	0	1	0	0	KQ_2	$K_1 + Q_1$
M_2	2	0	0	0	0	0	KO_2	$K_1 + O_1$
λ_2	2	1	-2	1	0	0	SNM_2	$S_2 + N_2 - M_2$
L_2	2	1	0	-1	0	0	$2MN_2$	$M_2 + M_2 - N_2$
S_2	2	2	-2	0	0	0	KP_2	$K_1 + P_1$
K_2	2	2	0	0	0	0	K_2	$K_1 + K_1$
η_2	2	3	0	-1	0	0	KJ_2	$K_1 + J_1$

Table IV
High Frequency Harmonic Constants for Victoria

Constituent		1939-1957		1958-1976		1939-1976		averages/inferred		σ
name	*Doodson numbers*	*amp (cm)*	*phase (°PST)*	*amp (cm)*	*phase (°PST)*	*amp (cm)*	*phase (°PST)*	*amp (cm)*	*phase (°PST)*	*(cm)*
NO_3	3 -2 0 1 -1 0	0.12	19.3	0.10	15.3	0.11	17.5	(0.17)	(20.4)	
NO_3	3 -2 0 1 0 0	0.93	20.4	0.92	20.4	0.93	20.4	0.92	20.5	0.09
MO_3	3 -1 0 0 -1 0	0.25	28.2	0.19	27.9	0.22	28.2	(0.40)	(41.1)	
$MO_3^\bullet$	3 -1 0 0 0 0	2.12	41.4	2.09	40.9	2.11	41.1	2.09	41.1	0.18
NP_3	3 0 -2 1 0 0	0.28	10.7	0.26	9.0	0.27	9.9	0.27	9.6	0.08
M_3	3 0 0 0 0 0	0.05	17.6	0.05	4.1	0.05	11.3			
NK_3	3 0 0 1 0 0	0.61	2.2	0.61	2.9	0.61	2.6	0.61	2.9	0.12
NK_3	3 0 0 1 1 0	0.09	359.2	0.10	8.9	0.09	4.2	(0.08)	(2.6)	
$SO_3^\bullet$	3 1 -2 0 0 0	1.21	44.3	1.23	44.8	1.22	44.5	1.21	44.2	0.17
$MK_3^\bullet$	3 1 0 0 0 0	2.04	25.4	2.00	26.0	2.02	25.7	2.03	25.7	0.10
$MK_3^\bullet$	3 1 0 0 1 0	0.28	18.1	0.32	29.5	0.30	24.3	(0.27)	(25.7)	
SP_3	3 3 -4 0 0 0	0.28	37.1	0.28	38.1	0.28	37.6	0.28	37.6	0.06
$SK_3^\bullet$	3 3 -2 0 0 0	0.84	61.2	0.83	61.0	0.83	61.1	0.83	61.3	0.08
SK_3	3 3 -2 0 1 0	0.13	68.2	0.14	76.8	0.13	72.7	(0.11)	(61.0)	
N_4	4 -2 0 2 0 0	0.21	305.5	0.20	308.8	0.20	307.1	0.20	306.5	0.15
$MN_4^\bullet$	4 -1 0 1 0 0	0.85	329.6	0.84	332.4	0.84	331.0	0.84)	330.8	0.09
$M_4^\bullet$	4 0 0 0 0 0	1.84	355.8	1.82	358.1	1.83	357.0	1.83	356.9	0.13
$SN_4^\bullet$	4 1 -2 1 0 0	0.21	341.9	0.20	342.2	0.20	342.1	0.20	341.7	0.07
$MS_4^\bullet$	4 2 -2 0 0 0	1.02	11.3	1.03	12.7	1.02	12.0	1.02	11.9	0.09
$MK_4^\bullet$	4 2 0 0 0 0	0.53	26.7	0.48	24.4	0.51	25.6	0.53	25.4	0.10
MK_4	4 2 0 0 1 0	0.15	24.7	0.12	32.3	0.13	28.1	(0.15)	(25.6)	
S_4	4 4 -4 0 0 0	0.19	357.0	0.20	1.3	0.20	359.2	0.20	359.2	0.05

Table IV (continued)

SK_4	4 4 -2 0 0 0	0.23	68.1	0.22	64.9	0.22	66.5	0.22	67.0	0.05
SK_4	4 4 -2 0 1 0	0.05	57.0	0.05	86.7	0.05	71.4	(0.07)	66.3	
MNO_5	5 -2 0 1 0 0	0.27	211.2	0.26	203.7	0.26	27.5	0.26	170.1	0.06
$2MO_5$	5 -1 0 0 -1 0	0.10	199.0	0.09	198.8	0.10	198.9	(0.10)	(176.4)	
$2MO_5$	5 -1 0 0 0 0	0.52	177.1	0.52	175.7	0.52	176.4	0.52	176.3	0.12
MNK_5	5 0 0 1 0 0	0.37	198.8	0.38	197.8	0.37	198.3	0.38	198.4	0.17
MNK_5	5 0 0 1 1 0	0.09	201.3	0.06	207.8	0.08	203.9	(0.05)	(198.3)	
$2MP_5$	5 1 -2 0 0 0	0.55	208.1	0.53	208.5	0.54	208.3	0.54	208.5	0.12
$2MK_5$	5 1 0 0 0 0	1.07	200.6	1.05	200.6	1.06	200.6	1.06	200.4	0.12
$2MK_5$	5 1 0 0 1 0	0.18	201.1	0.17	218.5	0.17	209.4	(0.14)	(200.7)	
MSK_5	5 3 -2 0 0 0	0.37	245.7	0.35	244.3	0.36	245.1	0.36	245.4	0.06
MSK_5	5 3 -2 0 1 0	0.05	246.9	0.07	259.1	0.06	254.0	(0.05)	(245.1)	
$2SK_5$	5 5 -4 0 0 0	0.07	295.4	0.07	306.3	0.07	301.2	0.07	301.1	0.05
$2NM_6$	6 -2 0 2 0 0	0.24	175.6	0.24	176.3	0.24	175.9	0.24	176.0	0.07
$2MN_6^{\bullet}$	6 -1 0 1 0 0	0.84	201.4	0.82	203.1	0.83	202.2	0.83	202.3	0.07
$M_6^{\bullet}$	6 0 0 0 0 0	1.49	226.8	1.46	228.6	1.47	227.7	1.47	227.7	0.11
MSN_6	6 1 -2 1 0 0	0.33	224.8	0.32	226.7	0.32	225.7	0.32	225.9	0.06
$2MS_6^{\bullet}$	6 2 -2 0 0 0	1.15	252.7	1.15	253.6	1.15	253.2	1.15	253.2	0.09
$2MK_6^{\bullet}$	6 2 0 0 0 0	0.36	229.9	0.36	230.8	0.36	230.4	0.36	229.1	0.06
$2MK_6$	6 2 0 0 1 0	0.13	226.1	0.10	243.7	0.11	233.6	(0.11)	(230.4)	
$2SM_6$	6 4 -4 0 0 0	0.26	278.6	0.27	276.0	0.26	277.4	0.26	277.4	0.05
MSK_6	6 4 -2 0 0 0	0.15	252.6	0.16	251.5	0.15	252.0	0.15	250.4	0.05
MSK_6	6 4 -2 0 1 0	0.06	252.1	0.05	274.6	0.06	262.2	(0.05)	(252.0)	
$3MK_7$	7 1 0 0 0 0	0.03	329.8	0.04	349.5	0.03	341.1	0.03	343.9	0.03
M_8	8 0 0 0 0 0	0.05	293.5	0.06	275.5	0.05	284.2	0.05	284.5	0.04

Table V
Low Frequency Harmonic Constants for Victoria

Constituent		1939-1957		1958-1976		1939-1976		averages/inferred		σ
name	*Doodson numbers*	*amp (cm)*	*phase (°PST)*	*amp (cm)*	*phase (°PST)*	*amp (cm)*	*phase (°PST)*	*amp (cm)*	*phase (°PST)*	*(cm)*
$Z_o^\bullet$	0 0 0 0 0 0	186.53	0.0	188.29	0.0	187.40	0.0	187.41	0.0	3.06
Z_o	0 0 0 0 1 0	1.18	269.2	1.43	199.9	1.07	299.5			
Z_o	0 0 0 0 2 0	1.56	14.5	1.12	293.2	1.03	342.3			
Z_o	0 0 0 2 1 0	1.10	37.9	1.11	356.9	1.11	14.8			
Sa	0 0 1 0 -1 -1	1.25	243.0	1.83	341.2	1.08	304.0	(0.07)	(356.0)	
Sa•	0 0 1 0 0 -1	7.99	357.5	9.13	354.5	8.56	356.0	8.57	355.8	4.46
Ssa	0 0 2 -2 0 0	0.32	344.3	0.77	356.1	0.52	347.6	(0.03)	(222.9)	
*Ssa**	0 0 2 -1 0 0	0.27	62.4	1.04	338.2	0.59	351.9	(0.00)	(222.9)	
Ssa•	0 0 2 0 0 0	3.00	209.0	2.64	238.0	2.73	222.9	2.75	223.4	3.62
Ssa	0 0 2 0 2 0	1.11	111.0	0.34	63.3	0.70	100.6	(0.01)	(222.9)	
	0 0 4 0 0 -2	0.77	122.5	0.64	98.3	0.67	111.3			
Msm	0 1 -2 -1 -1 0	0.96	239.9	0.65	343.2	0.49	277.2	0.35	281.3	2.37
Mm•	0 1 0 -1 0 0	1.36	165.8	0.89	181.3	1.12	171.7	1.10	168.4	2.38
Mm	0 1 0 1 1 0	1.21	34.4	0.36	11.2	0.75	29.0	(0.02)	(171.7)	
	0 1 3 -1 0 -1	0.83	325.5	0.32	327.7	0.58	327.1			
Msf	0 2 -2 0 0 0	0.86	186.7	0.79	190.1	0.81	187.7	0.85	188.2	1.80
Msf	0 2 -2 1 0 0	0.42	261.1	0.68	213.2	0.50	229.2	(0.00)	(187.7)	
	0 2 -1 0 0 1	0.35	343.3	0.73	307.6	0.51	318.3			
Mf•	0 2 0 0 0 0	1.64	144.9	0.85	164.3	1.24	150.4	1.28	150.1	1.85
Mf	0 2 0 0 1 0	0.83	168.5	0.64	180.9	0.74	175.5	(0.51)	(150.4)	

Table VI
Root Mean Square Residuals (cm) for Victoria

A: averages of yearly analyses for 1939→1957;
B: 1939→1957 nineteen year analysis;
C: as in A but with the same diurnal and higher frequency constituents used in the Canadian Tide and Current Tables (1989).

	Predictions based on		
Year	A	B	C
1958	16.447	16.408	16.578
1959	13.784	13.752	13.930
1960	13.634	13.591	13.783
1961	14.667	14.630	14.810
1962	12.983	12.972	13.125
1963	13.847	13.881	13.981
1964	15.813	15.850	15.943
1965	15.248	15.256	15.359
1966	16.010	16.009	16.146
1967	13.766	13.820	13.919
1968	15.831	15.864	15.982
1969	16.021	16.003	16.159
1970	16.259	16.230	16.421
1971	12.714	12.679	12.879
1972	14.250	14.218	14.440
1973	16.986	16.962	17.086
1974	14.788	14.762	14.950
1975	14.020	13.992	14.123
1976	10.542	10.474	10.672
Mean	14.611	14.598	14.752