Notes

External Alignment between Two Beacons at Different Elevation for Use in Bathymetric Operations

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Given two fixed beacons or two removable surveyor's rods, we can establish an alignment ashore to use for navigable channels, as well as for bathymetric operations in the vicinity of the same coastline. This paper proposes algorithms and problem solutions, even if the inland beacon or rod is at higher elevation compared to the shoreline beacon.

The alignment depends on the resolving power (or separation power) of the human eye, indicated as ρ. Some authors (Melchior 1971) propose a $\rho = 72''$ (of arc), whilst others (Nicolini 1963) consider, for more security, a resolving power $\rho = 90''$. Employing an optical device with a lens of diameter δ (in cm), the resolving power of such a device is defined as follows:

$$\rho'' = \frac{12}{\delta}$$

A typical external alignment used in bathymetry is shown at Figure 1, where N is the vessel, A and B are the surveyor's rods, separated by distance c. In Figure 1, position N' is defined as the ship's location at the maximum allowed distance s from N, within which the ship is still supposed to be in alignment, and $\overline{NB} = d$ is the maximum distance between the vessel and B.

Figure 2 describes the interesting case of a ship which moves in such a way that resolving power ρ' is maintained constant, i.e. $\rho' = 50''$. In such conditions, the ship can describe a circle having the following radius:

$$r = \frac{c}{2\sin\rho'}$$

while maintaining alignment with respect to $\overline{AB}$.

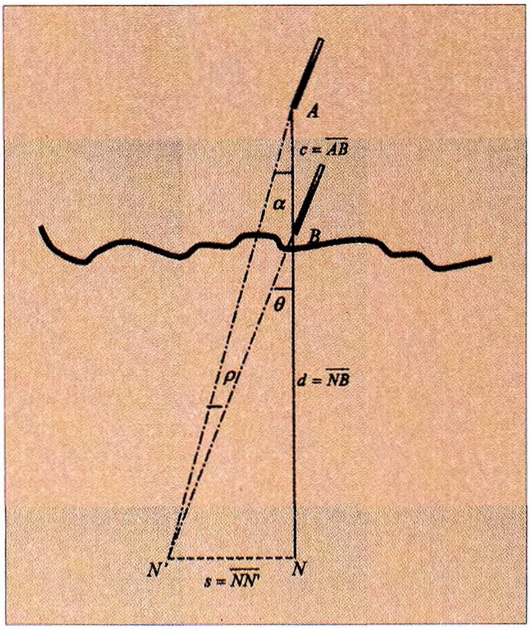

Figure 1: Alignment seen by a ship located at position N

Alignment with Two Beacons at the Same Elevation

In accurate bathymetry, the distance $s = \overline{NN'}$ is not to exceed 2m[1] at *d = 1000m*, given $\rho \geq 90''$. At this point, the following question can be posed: For a vessel at *1,000m* from *B*, what is the maximum beacon separation *c* that ensures $s \leq 2m$? To answer this question, the use of the weakest resolving power, $\rho = 90''$ (Nicolini 1963), is strongly suggested. With reference to Figure 1, using a precise instrument capable of measuring small angles, we obtain the following equations (in radians):

$$\alpha = \frac{s}{d+c} \quad \text{(with } \tan\alpha \cong \alpha \text{ for small angles)}$$

and

$$\vartheta = \rho + \alpha\,; \quad \vartheta = \rho + \frac{s}{d+c}\,; \quad \vartheta = \frac{s}{d}$$

(with $\tan\vartheta \cong \vartheta$ for small angles)

and at last:

$$\frac{s}{d} = \rho + \frac{s}{d+c} \text{ and } s(d+c) = d\rho(d+c) + sd$$

Solving for c yields:

$$c = \frac{d^2 \cdot \rho}{s - d\rho} \qquad (1)$$

Substituting the borderline values, we obtain the following results:

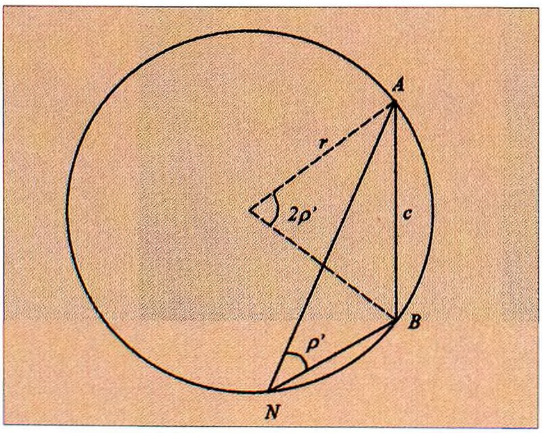

Figure 2: Circle described by a ship seeing base c under angle ρ'

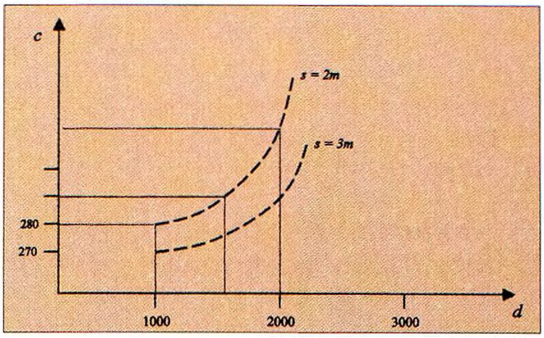

Figure 3: Variation of c versus d (values in metres)

$$c = \frac{(1000)^2 \cdot \rho}{2 - 1000\rho} \text{ (with } \rho = \frac{90''}{206264''.8} = 90''\sin 1'')$$

$$c = 279m \ (\overline{AB})$$

This value is greater than the values presented by references (Mannoia 1949).

A diagram, Figure 3, can be developed to describe the effects of varying geometry of rod placement for fixed ρ (i.e. ρ = 90'').

In case of fixed masonry beacons, normally used to identify an alignment of a navigable channel, the problem can be differently posed. In this case, and in relation to the width of the navigable channel, the problem is resolved to find out the tolerance s at the different distances of the ship from the beacons. If *l* is the width of the navigable channel, we have the following equation:

$$s = \frac{l}{2}$$

which is a constant as well as the base c (distance between the two beacons); the variable is now the distance $d = \overline{NB}$.

From the formula (1), we can extract the variable *d* and obtain:

$$cs - cd\rho = d^2\rho \text{ and so } d^2\rho + cd\rho - cs = 0$$

Solving for d, the following formula is obtained:

$$d = \frac{-c\rho \pm \sqrt{c^2\rho^2 + 4\rho cs}}{2\rho} \qquad (2)$$

If we substitute *l* = 20m, $s = \frac{l}{2} = 10m$ and *c = 1000m* in the formula (2), the sign + of the radical proposes the following result:

[1] *As per the IHO S-44 Publication, hydrographic surveys Order 1 allow maximum ±2m of horizontal accuracy at 95 per cent confidence level*

$d = 4313m \cong 2.3Nm$ with (*1 Nm = 1852m*)

Alignment with Two Beacons at Different Elevation (Case of 'A' At Elevation H Above 'B')

In case of presence of a very steep coastline, the surveyor's rod *A* (at Figure 4 identified as *A'*) can be at an elevation h above rod *B* (with *B* approximately at mean sea level). This case is frequent and necessary for the beacons. In bathymetric operations, the spherical conditions of figure 4 are to be taken in consideration.
Assuming $\beta = ANA'$, and ρ enough small, we can write the following equations:

$$\tan\beta = \frac{h}{d+c} \quad \text{and} \quad \rho' = \rho\cos\beta$$

The base $\overline{AB} = c$ (with *A* at the same elevation of. *B*) is now seen under the angle $\rho' < \rho$ vice angle ρ. Even if the difference exceeds the assumed *s* value, the ship does not realise this and believes it is in alignment.
To account for elevation differences between *A* and *B*, a new resolving power ρ_o is developed:

$$\rho_o = \frac{\rho}{\cos\beta} \tag{3}$$

Conclusions Including Numerical Evaluations

In the presence of a very steep coastline, for example, with *h = 180m* over mean sea level, we are forced to consider a base *c = 150m*, as a minimum. Solving for ρ_o (in seconds) in equation (3), given *d = 1000m* the result *cosβ = 0.98797105* is obtained. It finally yields (in seconds):

$$\rho''_o = \frac{\rho''}{\cos\beta} = \frac{90''}{\cos\beta} = 91''.096 \cong 91''.1$$

ρ''_o points out that as *h* rises, so the resolving power apparently decreases. In such a situation it is interesting to understand the distance *d* required to restore $s \le 2m$. Using ρ_o in radians, and substituting it in formula (2), we obtain the following result:

$$d_0 = \frac{-c\rho_0 + \sqrt{c^2\rho_0^{\,2} + 4\rho_0 cs}}{2\rho_0} = 752.6m$$

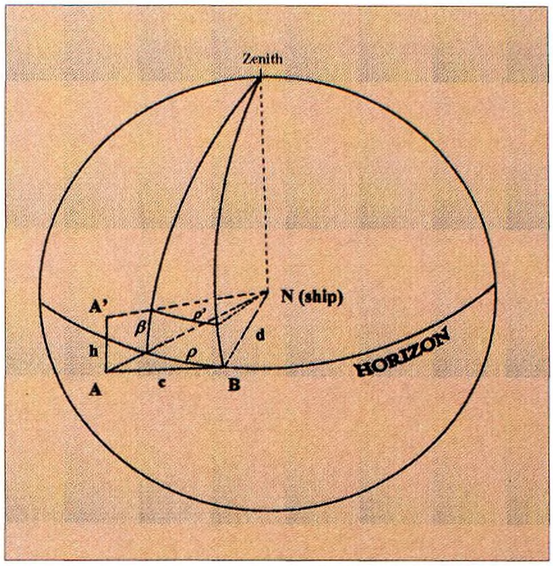

Figure 4: On the celestial sphere, beacons and ship are on the plane of the horizon

Using again formula (2) for *d = f(ρ)* and *h = 0*, it proposes *d = 757.6m*. It is evident that if *h* rises, the distance *d* has to decrease in order to maintain $s \le 2m$.
In hydrographic surveys, the presence of *h* is negligible when a vessel carries out soundings following an alignment made of mansory beacons or removable rods at different elevation. Such alignment ensures a horizontal accuracy within 2m, as requested by IHO Hydrographic Surveys Order 1. But, for scientific precision, it is worthy to underline the influence of *h* on the external alignment, which makes *d* vary to maintain $s \le 2m$.
The results shown above come from a unique non empirical formulation of the problem not presented in references, and represent a rigorous solution based on the resolving power of the human eye.

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