The Influence of Carrier's Attitude and the Position Reduction in Multibeam Echosounding and Airborne Laser Depth Sounding

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Having finished the development of the multibeam echosounding system, China is making a great effort to develop an airborne laser depth sounding system. According to the principle of the two kinds of sounding system mentioned above, a series of position reduction formulas and their error equations are developed in this paper. The dynamic effect of marine sounding, i.e., the influence of carrier's attitude, is taken into full account in developing the equations. Finally, the real parameters of the two sounding systems developed by China are used to calculate the position reductions and their accuracies at different heading angles. The results show that the accuracies of depths and positions in multibeam echosounding and airborne laser depth sounding are dependent not only on their own sensors but also on the additional sensors.

Introduction

With the development of modern science and technology, hydrographic surveying technology has at last advanced to a stage where many new types of sounding equipment have been taken into use in succession. The multibeam echosounding system and the airborne laser depth sounding system are representative of such new equipment. Multibeam echosounders as well as airborne laser depth sounders produce a dense pattern of depth soundings that cover a wide swath for each survey line. Such swath surveying systems have now almost become the standard approach for hydrographic surveying, especially for shallow water, due to its capability to produce maps with much higher quality, much higher resolution and a higher productivity rate [1~6].

Upgrading to a mode of swath surveying presents new challenges to surveyors. When working in such a new surveying mode, many influencing factors have to be considered in the procedures of sounding correction and position reduction. So far as the position reduction is concerned, a common factor to be considered first is the offsets of the positioning system antenna from the transducer. In earlier times, the offsets mentioned above were always ignored for hydrographic surveying in our country due to the fact that the positioning and the sounding approaches used were old, and the accuracy specifications of equipment were low, additionally, the size of the survey vessels was small and the offsets were minor. Now DGPS has found a widespread application in hydrographic survey and the positioning accuracy can fulfill the requirements required for marine engineering projects. In addition, some of the larger survey vessels have been put into use in our country and the offsets have become significant. Therefore, in order to finally obtain high quality reduced depth, it is necessary first consider the offsets above. For multibeam echosounding and the airborne laser depth sounding systems, the offsets between the positioning system antenna and the transducers are only a part of the position reduction. In order to get the coordinates of each sounding point, the equations relating the transducers and the sounding points have to be constructed and resolved. It is a procedure peculiar to the swath survey mode for the position reduction. The final coordinate offset to be dealt with is due to timing offsets between the positioning system and the transducer depth measurement.

Multibeam echosounding as well as airborne laser depth sounding is a kind of real time dynamic measurement system. This characteristic enhances the difficulty for the position reduction mentioned above. Due to the disturbance of external factors such as irregular wind, current, wave and swell, the survey vessels (or vehicles) will experience roll, pitch and yaw. It is because of these movements that the position reduction will be changed into a multi-dimension dynamic correction problem. The magnitude of position reduction depends on the carrier's attitude and it shows a great variation with time. It means that the carrier's attitude has to be considered when computing the co-ordinate offsets of sounding points.

Mathematic Models

The Definitions of Co-ordinate Systems

For convenience, a vessel-based co-ordinate system as well as a local-level one are first introduced. As shown in Figure 1, the origin of the vessel-based co-ordinate system (oxyz) is coincident with the centre of the transducer. The arrows at the end of each axis indicate the positive direction. The x-axis in the direction of the bow. The y-axis directs to port. The z-axis is perpendicular to the oxy-plane. They constitute a right-handed co-ordinate system. The vessel-based co-ordinate system moves with the vessel. The local-level co-ordinate system (OXYZ) is also a right-handed co-ordinate system: Z-axis up, Y-axis north and X-axis east (see Figure 1). The important difference between the two co-ordinate systems is that the local-level co-ordinate system does not move with the roll, pitch and yaw of vessel. Introducing the local-level vectors enables the calculation of three-dimensional co-ordinates in the global reference frame using the co-ordinates of the antenna and the co-ordinate offsets of the antenna from the transducer that will be discussed later.

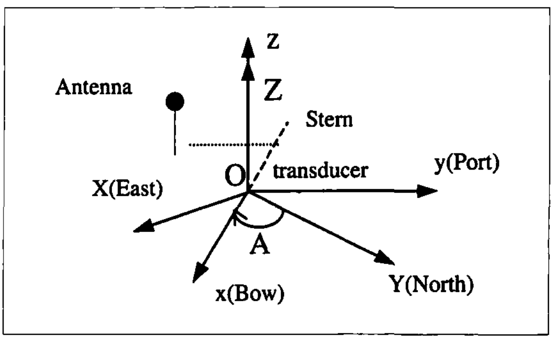

Figure 1: The vessel-based coordinate system (local-level also shown)

The Equations of the Position Reduction

As we know, the position reduction of soundings for multibeam echosounding and airborne laser depth sounding consist of three steps as following:

The Dynamic Corrections of Offsets

As shown in Figure 1, suppose to be the co-ordinates of the positioning system antenna in the vessel-based coordinate system. Then the offsets of the transducer from the positioning system antenna are: $\Delta x_{at} = -x_a, \Delta y_{at} = -y_a, \Delta z_{at} = -z_a$. As mentioned above, due to the disturbances of different external factors, the survey vessel will experience roll, pitch and yaw. Therefore, in spite of the fact that the offsets of the transducer from the positioning system antenna are invariant in the vessel-based coordinate system, they will change with the carrier's attitude in the local-level coordinate system. Suppose R to be the roll angle of the body frame, P the pitch angle of the body frame and A the gyrocompass heading of the body frame (from north). According to the principle of geodesy [7], the offsets of the transducer from the positioning system antenna in local-level coordinate system can be, using a so-called rotation matrix, expressed as:

$$\begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}_{at} = \overline{R}(A,P,R) \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{at} \quad (1)$$

where the subscript at represents the offsets from Antenna to Transducer. According to HARE (1995), the expression of the rotation matrix $\overline{R}(A,P,R)$ is

$$\overline{R}(A,P,R) = R_3(A-90°)R_2(P)R_1(-R)$$

$$= \begin{bmatrix} \sin A & -\cos A & 0 \\ \cos A & \sin A & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos P & 0 & -\sin P \\ 0 & 1 & 0 \\ \sin P & 0 & \cos P \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos R & -\sin R \\ 0 & \sin R & \cos R \end{bmatrix} \quad (2)$$

Substituting Equation (2) into Equation (1) and taking into account of the relationship: $\Delta x_{at} = -x_a$, $\Delta y_{at} = -y_a$, $\Delta z_{at} = -z_a$ □ result in Equation (3) as following:

$$\begin{cases} \Delta X_{at} = -x_a \sin A \cos P + y_a(\cos A \cos R + \sin A \sin P \sin R) - \\ z_a(\cos A \sin R - \sin A \sin P \cos R) \\ \Delta Y_{at} = -x_a \cos A \cos P - y_a(\sin A \cos R - \cos A \sin P \sin R) \\ + z_a(\sin A \sin R + \cos A \sin P \cos R) \\ \Delta Z_{at} = -x_a \sin P - y_a \cos P \sin R - z_a \cos P \cos R \end{cases} \quad (3)$$

The formula above is just the dynamic corrections of offsets between the transducer and the positioning system antenna.

The Dynamic Reduction of Sounding Point in Sounder System

Taking into account the difference in working principle between multibeam echosounding and airborne laser depth sounding, the dynamic reduction of sounding points in a sounder system is discussed in two cases. Figure 2 shows the position calculation for a multibeam echosounding system in a vessel-based coordinate system, where the range, r and beam angle, θ are the geometric distance and direction from the transducer to the sounding point P on the seafloor where the center of the beam makes contact. Using simple geometry, the cross-track distance, y and the depth below the transducer, d can be calculated by: $y_P = r\sin\theta$; $-d = z_P = -r\cos\theta$ □ The along-track co-ordinate (x) is zero if the transducer is not pitched. Referring to Figure 1, it is apparent that these co-ordinates should be corrected for roll, pitch and heading angles by rotating the vector in the body frame about the three orthogonal co-ordinate axes. This can be also written in a form of matrix equation:

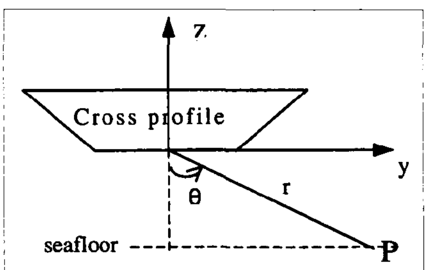

Figure 2: Position calculation for a multibeam echosounding system

$$\begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}_{tp} = \overline{R}(A,P,R)\begin{bmatrix} x_P \\ y_P \\ z_P \end{bmatrix} = \overline{R}(A,P,R)\begin{bmatrix} 0 \\ r\sin\theta \\ -r\cos\theta \end{bmatrix} \qquad (4)$$

where the subscript *tp* represents the offsets from Transducer to Point. Substituting Equation (2) into Equation (4) results in Equation (5) as follows:

$$\begin{cases} \Delta X_{tp} = -r\cos A\sin(\theta+R) + r\sin A\sin P\cos(\theta+R) \\ \Delta Y_{tp} = r\sin A\sin(\theta+R) + r\cos A\sin P\cos(\theta+R) \\ \Delta Z_{tp} = -r\cos P\cos(\theta+R) \end{cases} \qquad (5)$$

The airborne laser depth sounding system works in an automatic scan mode. Its principle is to transmit and receive laser pulses in a scan angle (which is measured for each sounding) from laser device to seafloor in succession [4,5,6,]. The combination of the aircraft' flying towards and laser's scanning in cross-track will result in an arrangement of sounding points in a zigzag pattern. For hydrographic surveying, however, one needs actually an arrangement in parallel. In order to meet the requirement above, the scientists of China have made an improvement on the scanning system. The key idea of the improved method is to add a small scan angle (about 1°) in along-track while scanning with a big angle (about 15°) in cross-track. In this case, the Equation (4) should be modified to adapt itself to the change above. As shown in Figure 3, suppose θ to be the scan angle of the laser device in y-axis direction (cross-track), β' the scan angle in x-axis direction (along-track). With a detailed derivation, it can be proved that the following Equation (6) that relates θ, β' and a transition angle β holds:

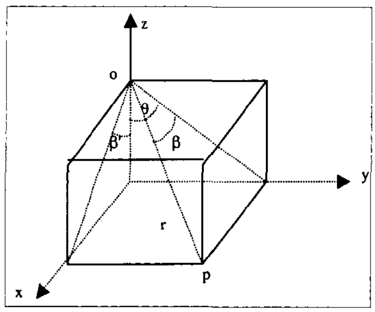

Figure 3: Position calculation for airborne laser depth sounding

$$\tan\beta = \cos\theta\tan\beta' \qquad (6)$$

and the coordinates of the sounding points in aircraft-based co-ordinate system can be expressed as

$$x_P = r\sin\beta;\ \ y_P = r\cos\beta\sin\theta;\ \ z_P = -r\cos\beta\cos\theta \qquad (7)$$

Substituting Equation (7) into Equation (4), we can obtain the dynamic reduction formula of sounding point corresponding to the airborne laser depth sounding system as follows:

$$\begin{cases} \Delta X_{tp} = r\sin\beta\sin A\cos P - r\cos\beta\cos A\sin(\theta+R) + r\cos\beta\sin A\sin P\cos(\theta+R) \\ \Delta Y_{tp} = r\sin\beta\cos A\cos P + r\cos\beta\sin A\sin(\theta+R) + r\cos\beta\cos A\sin P\cos(\theta+R) \\ \Delta Z_{tp} = r\sin\beta\sin P - r\cos\beta\cos P\cos(\theta+R) \end{cases} \quad (8)$$

The Dynamic Corrections Due to Positioning System Latency

The coordinate offset due to timing offsets between the position system and the transducer depth measurement will always result in x-axis displacement in the body frame and is calculated as the product of the time offset (or latency), Δt and the carrier's speed over the ground, V. That is

$$\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}_{\Delta t} = \begin{bmatrix} \Delta t\cdot V \\ 0 \\ 0 \end{bmatrix} \qquad (9)$$

The vector above can be rotated into the local-level coordinate system using the rotation matrix as:

$$\begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix}_{\Delta t} = \overline{R}(A,P,R) \begin{bmatrix} \Delta t \cdot V \\ 0 \\ 0 \end{bmatrix} \qquad (10)$$

Substituting $\overline{R}(A,P,R)$ given in Equation (2) into Equation (10), we get the following three dynamic coordinate offsets due to positioning system latency:

$$\begin{cases} \Delta X_{\Delta t} = \Delta t \cdot V \sin A \cos P \\ \Delta Y_{\Delta t} = \Delta t \cdot V \cos A \cos P \\ \Delta Z_{\Delta t} = \Delta t \cdot V \sin P \end{cases} \qquad (11)$$

The Computation of Sounding Point Position

Combining the three steps of position reduction mentioned above gives the following final computation formula of the sounding point position:

$$\begin{cases} \varphi = \varphi_0 + (\Delta Y_{at} + \Delta Y_{tp} + \Delta Y_{\Delta t}) / M \\ \lambda = \lambda_0 + (\Delta X_{at} + \Delta X_{tp} + \Delta X_{\Delta t}) / (N \cos(\varphi)) \end{cases} \qquad (12)$$

where φ and λ are the latitude and longitude of the sounding, φ_0 and λ_0 are the latitude and longitude of the positioning system antenna, *M* is the radius of curvature of the ellipsoid in the prime meridian and *N* is the radius of curvature of the ellipsoid in the prime vertical. Using such approximations, instead of rigorous geodetic formulae will cause errors of less than 1cm for x and y offset distances of up to a kilometre [2]. Apparently, it is negligible. The advantage of using Equation (12) is that we do not have to consider the problem of change in co-ordinate band.

The Accuracy Estimate of the Sounding Position

In hydrographic survey, it is normal to express the accuracy estimate formula of the sounding position as [2,3,8]:

$$\sigma_P^2 = \sigma_{yp}^2 + \sigma_{xp}^2 \qquad (13)$$

where σ_P^2 represents the variance of position, σ_{yP}^2 and σ_{xP}^2 are the variances in each coordinate. According to Equation (12), the accuracy estimate formula of the sounding position corresponding to the multibeam echosounding and the airborne laser depth sounding can be written as:

$$\sigma_P^2 = \sigma_0^2 + \sigma_{\Delta x}^2 + \sigma_{\Delta y}^2 \qquad (14)$$

where $\sigma_{\Delta x}$ indicates the accuracy of positioning system, $\sigma_{\Delta x}$ and $\sigma_{\Delta y}$ are the accuracy estimates of the three steps of position reduction in each coordinate. σ_0 is determined and need not to be discussed. Only the computation formulae of and $\sigma_{\Delta y}$ are described as follows:

According to Equation (12), both ΔX and ΔY consist of three parts. That is

$$\begin{cases} \Delta X = \Delta X_{at} + \Delta X_{tp} + \Delta X_{\Delta t} \\ \Delta Y = \Delta Y_{at} + \Delta Y_{tp} + \Delta Y_{\Delta t} \end{cases} \qquad (15)$$

Furthermore, it can be seen, from Equation (3), (5), (8) and (11), that each of three parts of position reduction is related to the heading, roll and pitch angle of the body frame. It means that the three parts of position reduction are not independent. Therefore, they have to be dealt with in an integrated way. With regard to the multibeam echosounding system, let:

$$\begin{cases} \sigma_{\Delta x}^2 = a_1^2\sigma_{xa}^2 + a_2^2\sigma_{ya}^2 + a_3^2\sigma_{za}^2 + a_4^2\sigma_A^2 + a_5^2\sigma_R^2 + a_6^2\sigma_P^2 + a_7^2\sigma_r^2 + a_8^2\sigma_\theta^2 \\ \quad + a_9^2\sigma_t^2 + a_{10}^2\sigma_v^2 \\ \sigma_{\Delta y}^2 = b_1^2\sigma_{xa}^2 + b_2^2\sigma_{ya}^2 + b_3^2\sigma_{za}^2 + b_4^2\sigma_A^2 + b_5^2\sigma_R^2 + b_6^2\sigma_P^2 + b_7^2\sigma_r^2 + b_8^2\sigma_\theta^2 \\ \quad + b_9^2\sigma_t^2 + b_{10}^2\sigma_v^2 \end{cases} \tag{16}$$

According to Equation (3), (5) and (11), we can make a series of derivations and finally get the following expressions:

$$\begin{cases} a_1 = -\sin A\cos P \\ a_2 = \cos A\cos R + \sin A\sin P\sin R \\ a_3 = -\cos A\sin R + \sin A\sin P\cos R \\ a_4 = -x_a\cos A\cos P + y_a(-\sin A\cos R + \cos A\sin P\sin R) + z_a(\sin A\sin R \\ \quad + \cos A\sin P\cos R) + r\sin A\sin(\theta + R) + r\cos A\sin P\cos(\theta + R) \\ \quad + \Delta tV\cos A\cos P \\ a_5 = y_a(-\cos A\sin R + \sin A\sin P\cos R) - z_a(\cos A\cos R + \sin A\sin P\sin R) \\ \quad - r\cos A\cos(\theta + R) - r\sin A\sin P\sin(\theta + R) \\ a_6 = x_a\sin A\sin P + y_a\sin A\cos P\sin R + z_a\sin A\cos P\cos R \\ \quad + r\sin A\cos P\cos(\theta + R) - \Delta t\cdot V\sin A\sin P \\ a_7 = -\cos A\sin(\theta + R) + \sin A\sin P\cos(\theta + R) \\ a_8 = -r\cos A\cos(\theta + R) - r\sin A\sin P\sin(\theta + R) \\ a_9 = V\sin A\cos P \\ a_{10} = \Delta t\sin A\cos P \end{cases} \tag{17}$$

$$\begin{cases} b_1 = -\cos A\cos P \\ b_2 = -\sin A\cos R + \cos A\sin P\sin R \\ b_3 = \sin A\sin R + \cos A\sin P\cos R \\ b_4 = x_a\sin A\cos P - y_a(\cos A\cos R + \sin A\sin P\sin R) + z_a(\cos A\sin R \\ \quad - \sin A\sin P\cos R) + r\cos A\sin(\theta + R) - r\sin A\sin P\cos(\theta + R) \\ \quad - \Delta t\cdot V\sin A\cos P \\ b_5 = y_a(\sin A\sin R + \cos A\sin P\cos R) + z_a(\sin A\cos R - \cos A\sin P\sin R) \\ \quad + r\sin A\cos(\theta + R) - r\cos A\sin P\sin(\theta + R) \\ b_6 = x_a\cos A\sin P + y_a\cos A\cos P\sin R + z_a\cos A\cos P\cos R \\ \quad + r\cos A\cos P\cos(\theta + R) - \Delta t\cdot V\cos A\sin P \\ b_7 = \sin A\sin(\theta + R) + \cos A\sin P\cos(\theta + R) \\ b_8 = r\sin A\cos(\theta + R) - r\cos A\sin P\sin(\theta + R) \\ b_9 = V\cos A\cos P \\ b_{10} = \Delta t\cdot\cos A\cos P \end{cases} \tag{18}$$

Substituting Equation (16) into Equation (14) results in Equation (19):

$$\sigma_P^2 = \sigma_0^2 + \sum_{i=1}^{10}(a_i^2 + b_i^2)\sigma_i^2 \tag{19}$$

Let: $$c_i^2 = a_i^2 + b_i^2 \tag{20}$$

Then, from Equation (17) and (18), we can get:

$$\left\{\begin{aligned}
c_1^2 &= \cos^2 P\\
c_2^2 &= 1-\sin^2 R\cos^2 P\\
c_3^2 &= 1-\cos^2 R\cos^2 P\\
c_4^2 &= x_a^2\cos^2 P + y_a^2(1-\sin^2 R\cos^2 P) + z_a^2(1-\cos^2 R\cos^2 P) - x_a y_a \sin 2P\sin R\\
&\quad - x_a z_a \sin 2P\cos R - y_a z_a \cos^2 P\sin 2R + r^2[1-\cos^2 P\cos^2(\theta+R)]\\
&\quad + (\Delta t\cdot V\cdot\cos P)^2 - x_a r\sin 2P\cos(\theta+R) - 2x_a\Delta tV\cos^2 P\\
&\quad - 2y_a r\cos R\sin(\theta+R) + 2y_a r\sin^2 P\sin R\cos(\theta+R) + y_a\Delta tV\sin 2P\sin R\\
&\quad + 2z_a r\sin R\sin(\theta+R) + 2z_a r\sin^2 P\cos R\cos(\theta+R)\\
&\quad + z_a\Delta tV\sin 2P\cos R + r\Delta tV\sin 2P\cos(\theta+R)\\
c_5^2 &= y_a^2(1-\cos^2 R\cos^2 P) + z_a^2(1-\sin^2 R\cos^2 P) + y_a z_a\sin 2R\cos^2 P\\
&\quad + r^2[1-\cos^2 P\sin^2(\theta+R)] + 2y_a r\sin R\cos(\theta+R) - 2y_a r\sin^2 P\\
&\quad \cos R\sin(\theta+R) + 2z_a r\cos R\cos(\theta+R) + 2z_a r\sin^2 P\sin R\sin(\theta+R)\\
c_6^2 &= (x_a\sin P + y_a\cos P\sin R + z_a\cos P\cos R)^2 + r^2\cos^2 P\cos^2(\theta+R)\\
&\quad + (\Delta t\cdot V\sin P)^2 + x_a r\sin 2P\cos(\theta+R) - 2x_a\Delta tV\sin^2 P\\
&\quad + 2y_a r\cos^2 P\sin R\cos(\theta+R) - y_a\Delta tV\sin 2P\sin R\\
&\quad + 2z_a r\cos^2 P\cos R\cos(\theta+R) - z_a\Delta tV\sin 2P\cos R\\
&\quad - r\Delta tV\sin 2P\cos(\theta+R)\\
c_7^2 &= [1-\cos^2 P\cos^2(\theta+R)]\\
c_8^2 &= r^2[1-\cos^2 P\sin^2(\theta+R)]\\
c_9^2 &= V^2\cos^2 P\\
c_{10}^2 &= \Delta t^2\cos^2 P
\end{aligned}\right. \tag{21}$$

With regard to the airborne laser depth sounding system, we can deal with the accuracy estimate of position of sounding in a similar way. Let:

$$\sigma_p^2 = \sigma_0^2 + \sum_{i=1}^{11} d_i^2\sigma_i^2 \tag{22}$$

According to Equation (3), (8) and (11), we can get:

$$
\left\{
\begin{aligned}
d_1^2 &= \cos^2 P \\
d_2^2 &= 1-\sin^2 R\cos^2 P \\
d_3^2 &= 1-\cos^2 R\cos^2 P \\
d_4^2 &= x_a^2\cos^2 P + y_a^2(1-\sin^2 R\cos^2 P) + z_a^2(1-\cos^2 R\cos^2 P) - x_a y_a \sin 2P\sin R \\
&\quad - x_a z_a \sin 2P\cos R - y_a z_a \cos^2 P\sin 2R \\
&\quad + r^2[\cos^2\beta\sin^2(\theta+R) + (\sin\beta\cos P + \cos\beta\sin P\cos(\theta+R)^2] \\
&\quad + (\Delta t\cdot V\cdot\cos P)^2 - 2x_a r\sin\beta\cos^2 P - x_a r\cos\beta\sin 2P\cos(\theta+R) \\
&\quad - 2x_a\Delta tV\cos^2 P + y_a r\sin\beta\sin 2P\sin R - 2y_a r\cos\beta\cos R\sin(\theta+R) \\
&\quad + 2y_a r\cos\beta\sin^2 P\sin R\cos(\theta+R) + y_a\Delta tV\sin 2P\sin R + z_a r\sin\beta\sin 2P\cos R \\
&\quad + 2z_a r\cos\beta\sin R\sin(\theta+R) + 2z_a r\cos\beta\sin^2 P\cos R\cos(\theta+R) \\
&\quad + z_a\Delta tV\sin 2P\cos R + 2r\Delta tV\sin\beta\cos^2 P + r\Delta tV\cos\beta\sin 2P\cos(\theta+R) \\
d_5^2 &= y_a^2(1-\cos^2 R\cos^2 P) + z_a^2(1-\sin^2 R\cos^2 P) + y_a z_a\sin 2R\cos^2 P \\
&\quad + r^2\cos^2\beta[1-\cos^2 P\sin^2(\theta+R)] \\
&\quad + 2r\cos\beta\cos(\theta+R)[y_a\sin R + z_a\cos R] \\
&\quad + 2r\cos\beta\sin^2 P\sin(\theta+R)[z_a\sin R - y_a\cos R] \\
d_6^2 &= (x_a\sin P + y_a\cos P\sin R + z_a\cos P\cos R)^2 + r^2[\sin\beta\sin P \\
&\quad - \cos\beta\cos P\cos(\theta+R)]^2 + (\Delta t\cdot V\cdot\sin P)^2 - 2x_a r\sin\beta\sin^2 P \\
&\quad + x_a r\cos\beta\sin 2P\cos(\theta+R) - 2x_a\Delta tV\sin^2 P - y_a r\sin\beta\sin 2P\sin R \\
&\quad + 2y_a r\cos\beta\cos^2 P\sin R\cos(\theta+R) - y_a\Delta tV\sin 2P\sin R \\
&\quad - z_a r\sin\beta\sin 2P\cos R + 2z_a r\cos\beta\cos^2 P\cos R\cos(\theta+R) \\
&\quad - z_a\Delta tV\sin 2P\cos R + 2r\Delta tV\sin\beta\sin^2 P - r\Delta tV\sin 2P\cos(\theta+R) \\
d_7^2 &= \cos^2\beta\sin^2(\theta+R) + [\sin\beta\cos P + \cos\beta\sin P\cos(\theta+R)]^2 \\
d_8^2 &= r^2\cos^2\beta[1-\cos^2 P\sin^2(\theta+R)]^2 \\
d_9^2 &= r^2[\sin^2\beta\sin^2(\theta+R) + (\cos\beta\cos P - \sin\beta\sin P\cos(\theta+R))^2] \\
d_{10}^2 &= V^2\cos^2 P \\
d_{11}^2 &= \Delta t^2\cos^2 P
\end{aligned}
\right.
\tag{23}
$$

Compared to HARE (1995), the accuracy estimate formulae of the sounding positions derived above are more reasonable due to the fact that we have taken account of the correlation of the three parts of position reduction.

Numerical Example

In order to show the magnitudes of position reduction and carrier's attitude influence, some numerical computations have been carried out using the real parameters of the two sounding systems developed by China. The technical specifications of the two sounding systems are first listed as follows:
(1) the multibeam echosounding system

The offsets of the transducer from the positioning system antenna and their measurement accuracies are:

$$\Delta x_{at} = -x_a = 19.53m \, , \sigma_{xa} = \pm 0.2m; \Delta y_{at} = -y_a = 2.50m \, , \sigma_{ya} = \pm 0.2m;$$
$$\Delta z_{at} = -z_a = -28.80m \, , \sigma_{za} = \pm 0.2m$$

The attitude of the vessel is determined by the TSS 335B heave. And the maximums of the roll and pitch angles and their measurement accuracies are:

$$R = 10°, \quad \sigma_R = \pm 0.1°; \quad P = 4°, \quad \sigma_P = \pm 0.1°$$

The system is designed to operate in water depth from 10 to 1000m below the transducer. So we can have:

$$r = 1000m, \quad \sigma_r = \pm 5m; \ 6 = 45°, \quad \sigma_\theta = \pm 0.1°.$$

The normal speed of the vessel and its measurement accuracy are:
$V = 6m/s, \quad \sigma_V = \pm 0.1m/s$. The accuracy of the gyrocompass heading of the body frame is:
$\sigma_A = \pm 0.5°$. The maximum of the time latency is: . $\Delta t = 1s, \quad \sigma_{\Delta t} = \pm 0.1s$
(2) the airborne laser depth sounding system

The offsets of the centre of the laser device from the positioning system antenna and their measurement accuracies are:

$$\Delta x_{at} = -x_a = 8.0m, \sigma_{xa} = \pm 0.2m; \Delta y_{at} = -y_a = -0.9m, \sigma_{ya} = \pm 0.2m;$$
$$\Delta z_{at} = -z_a = -1.85m, \quad \sigma_{za} = \pm 0.2m$$

The attitude and heading of the aircraft are determined by a TCM2-20 electric compass. The maxima of the roll and pitch angles and their measurement accuracies are:

$$\sigma_A = \pm 1°; \ R = 5°, \quad \sigma_R = \pm 0.2°; \quad P = 5°, \ \sigma_P = \pm 0.2°.$$

The system is designed to operate at 500m of altitude. So we can have:

$r = 520m, \sigma_r = \pm 0.25m$ The maximums of the scan angles and their measurement accuracies are: ;
$\sigma_\theta = \pm 0.1°; \beta = 1.2°, \sigma_\beta = \pm 0.1°$. The normal speed of the aircraft is:
$V = 70m/s, \ \sigma_V = \pm 1.0m/s$ The maximum of the time latency is: $\Delta t = 0.6s, \ \sigma_{\Delta t} = \pm 0.01s$.

It should be pointed out that our airborne laser depth sounding system is designed to be mounted on a gyro-stabilised platform in order to decrease the influence of the aircraft's dynamics. In this case, it should be, in Equation (8), to let: $R = P = 0$.
Using the parameters listed above and Equation (3), (5) and (11), we have finished the computations of the three parts of position reduction in five different heading. The results are listed in Table 1.

The influences of each factors have been calculated using Equation (19) ~(21) and are given in Table 2.

Heading (A)	ΔX_{at}	ΔY_{at}	ΔX_{tp}	ΔY_{tp}	$\Delta X_{\Delta t}$	$\Delta Y_{\Delta t}$
0°	-7.49	21.43	-819.15	40.01	0.0	5.99
45°	9.85	20.45	-550.94	607.52	4.23	4.23
90°	21.43	7.49	40.01	819.15	5.99	0.0
135°	20.45	-9.85	607.52	550.94	4.23	-4.23
180°	7.49	-21.43	819.15	-40.01	0.0	-5.99

Table 1: The magnitude of position reduction in mutibeam echosounding(/m)

Parameter	x_a	y_a	z_a	A	R	P	r	6	Δt	V
σ (m)	0.20	0.20	0.04	7.24	1.06	1.04	4.10	1.01	0.60	0.10

Table 2: The influences of individual parameter on the position

According to Table 2, the total influence of the parameters to the multibeam echosounding system is:

$$\sigma_P = \pm\sqrt{\sigma_0^2 + \sigma_{\Delta x}^2 + \sigma_{\Delta y}^2} = \pm 13.15(m)$$

The calculated results for the airborne laser depth sounding system corresponding to Tables 1 and 2 are given in Tables 3 and 4.

Heading (A)	ΔX_{at}	ΔY_{at}	ΔX_{tp}	ΔY_{tp}	$\Delta X_{\Delta t}$	$\Delta Y_{\Delta t}$
0°	0.74	8.14	-134.56	10.89	0.0	41.84
45°	6.27	5.23	-87.45	102.85	29.59	29.59
90°	8.14	-0.74	10.89	134.56	41.84	0.0
135°	5.23	-6.27	102.85	87.45	29.59	-29.59
180°	-0.74	-8.14	134.56	-10.89	0.0	-41.84

Table 3: The magnitude of position reduction in airborne laser depth sounding (/m)

Parameter	x_a	y_a	z_a	A	R	P	r	6	β	Δt	V
σ (m)	0.20	0.20	0.03	3.58	1.71	1.69	0.09	0.85	0.90	0.70	0.60

Table 4: The influences of individual parameter on the position

According to Table 4, the total influence of the parameters to the airborne laser depth sounding system is:

$$\sigma(total) = \pm\sqrt{\sum_{i=1}^{11} d_i^2 \sigma_i^2} = \pm 4.589 \text{ (m)}$$

The results given in Table 1 and 3 have shown the significance of position reduction in both of the two systems. The magnitude of each of the three parts can affect the quality of the depth. Therefore, they should not be ignored.

As shown in Table 2, the measurement accuracies of the range and the heading have significant influence on the accuracy of sounding position in the multibeam echosounding system. While in the airborne laser depth sounding system, as shown in Table 3, the measurement accuracies of the heading and the roll and pitch angles are the main factors affecting the accuracy of sounding position. These facts show that the accuracies of depths and positions in multibeam echosounding and airborne laser depth sounding are dependent not only on their own sensors but also on the additional sensors.

It is estimated that the accuracy of the DGPS positioning system used for multibeam echosounding in our country is about: σ_0=±10m. Then, according to Table 2, the final accuracy of sounding position is:

$$\sigma_P = \pm\sqrt{\sigma_0^2 + \sigma_{\Delta x}^2 + \sigma_{\Delta y}^2} = \pm 13.15(m)$$

The accuracy of the DGPS positioning system used for airborne laser depth sounding in our country is about: σ_0=±5m. Then, according to Table 4, the final accuracy of sounding position is:

$$\sigma_P = \pm\sqrt{\sigma_0^2 + \sigma_{\Delta x}^2 + \sigma_{\Delta y}^2} = \pm 6.79(m)$$

The calculated results above show that using the equipment now available and the method of position reduction proposed here for depth measurement can fulfill the requirement of mapping the seafloor.

Conclusions

In summary, we think that the position reduction is an important procedure of data processing in both of multibeam echosounding and airborne laser depth sounding. Also the dynamic effect of marine sounding, i.e., the influence of carrier's attitude, has to be taken into full account in the procedure of position reduction. According to the requirement for accuracy to the depth, the formulae provided in this paper can be, inversely, used to demonstrate the requirement for accuracy to the additional sensors.

Finally, it should be pointed out that not all the factors influencing the position reduction are described in this paper. Apparently, the transducer misalignment angles for roll, pitch and heading are an error source that is independent of the measurement component of roll, pitch and heading. The errors resulting from sound speed error source are another potentially large source of position error in the outer beams. The factors mentioned above have not been discussed in the paper due to the fact space forbids. Additionally, the range and beam angle measurement error components are of course far more complicated than we represent. In fact, they are functions of beam angle, pulse length, range sampling frequency and detection mode. The simplification made in this paper is a limitation on the analysis.

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