magnetic impulse were obtained and the computer analysis of the ponderomotive force, the temperature and the radial and circular stresses was done. The results of the analysis are illustrated by the graphs of time dependence of determining functions in the considered cylinder with a thin conductive coating.

Key words: *mathematical model, thermomechanics, long integrate electroconductive cylinder, pulsed electromagnetic field, bearing capacity, properties of the contact connection.*

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REGULARIZATION METHOD OF RESTORATION OF INPUT SIGNALS OF NONLINEAR DYNAMIC OBJECTS THAT DETERMINED BY INTEGRO-POWER VOLTERRA SERIES

The article offers a regularization method for solving the polynomial integral Volterra equations of the first kind while solving the problem of restoration of the input signal of a nonlinear dynamic object determined by the integro-power Volterra series. The use of integro-power Volterra series makes it possible to simplify the primary nonlinear mathematical models of nonlinear dynamic objects turning them into quasi-linear ones. Polynomial Volterra equations of the first kind are solved by introducing the additional differential regularization operator. It is offered to solve the obtained integro-differential equations using quadrature algorithms by iterative methods. This approach allows makes it possible to increase the efficiency of the process of signals restoration on the input of nonlinear dynamic objects if there is noise. The efficiency of the offered algorithm is verified for the restoration of input signal of a nonlinear dynamic object given in the form of a sequential connection of linear and nonlinear parts. At the same time, the linear part is represented by an inertial joint, while the nonlinear is represented by polynomial dependence of the second kind. There are presented the results of solving of polynomial Volterra integral equations of the first kind in the presence of different noises on the input dependencies. Based on the described method, in Matlab / Simulink, there are created simulation models and software-based methods for solving inverse problems of signal restoration on the input of nonlinear dynamic objects. The results of computational experiments demonstrated that the offered regularization method for solving the polynomial Volterra integral equations of the first kind may be effectively used to restore the input signals of nonlinear dynamical systems being described by the integro-power Volterra series.

Keywords: *input signals restoration, polynomial Volterra integral equation of the first kind, differential regularization operator, Matlab.*

Introduction. When solving the problem of the input signal of nonlinear dynamic systems restoration, as well as the problems of control, monitoring and diagnostics, serving as examples of ill-posed problem, usually it is necessary to solve the Volterra nonlinear equations of the first kind being more complex problems. The use of regularization methods is the most common approach to solving such ill-posed problems. A promising direction in solving such complex problems is the use of integro-power Volterra series, simplifying primary nonlinear mathematical models by transforming them into quasilinear ones. The use of tools of the integro-power Volterra series and regularization methods determines the need of creation of new and more effective mathematical methods and corresponding software-based means for solving inverse problems for nonlinear dynamic objects.

The universal mathematical model of nonlinear dynamical systems of the black box type is the integro-power Volterra series [3]:

$$y(t)=\int_0^t K_1(t,s)x(s)ds+\int_0^t\int_0^t K_2(t,s_1,s_2)x(s_1)x(s_2)ds_1ds_2+\dots, \qquad (1)$$

where $x(t)$, $y(t)$ — is the input and output signals of the object, accordingly, t — transition process time, $K_i(t_1,\cdots,t_i)$ — Volterra kernel.

The solution of inverse problems leads to the polynomial integral Volterra equations of the first kind (1). No effective methods and means exist to solve this class of equations, that is why it is crucial to improve and develop new methods for solving integral Volterra equations of the first kind.

The **purpose** of this paper is to develop a method for solving the inverse problems of the dynamics of nonlinear objects by solving the polynomial integral Volterra equations of the first kind.

Main part. It is offered to solve the problem by replacing the integrals in (1) to quadrature formulas, providing many advantages simpler implementation and high stability of computational algorithms thanks to the regularizing properties of the quadrature step [1].

The application of this method is considered while solving the polynomial Volterra integral equation of the second kind:

$$y(t)=\int_0^t K_1(t,s)x(s)ds+\int_0^t\int_0^t K_2(t,s_1,s_2)x(s_1)x(s_2)ds_1ds_2. \qquad (2)$$

Solving the posed problems of restoration of the signals passing through nonlinear dynamic signals is incorrect and the application of classical methods, with input signals with noises, fails to provide the required accuracy solutions. It is offered to use the first kind differential regularization operator. In such a case solving the polynomial Volterra equation is reduced to the solving of the following equation:

$$\alpha\frac{dx}{dt}+\int_0^t K_1(t,s)x(s)ds+\int_0^t\int_0^t K_2(t,s_1,s_2)x(s_1)x(s_2)ds_1ds_2=y(t),\quad (3)$$

where α – regularization parameter.

Having applied to (3) the trapezoid method [4] difference formula of the first order we will obtain:

$$\begin{aligned}y(t_i)=\alpha\frac{x(t_i)-x(t_{i-1})}{h}+\frac{1}{2}hK_1(t_i,t_i)x(t_i)+\sum_{j=1}^{i-1}hK_1(t_i,t_j)x(t_j)+\\+\frac{1}{2}hK_1(t_i,t_0)x(t_0)+\frac{1}{4}h^2K_2(t_i,t_0,t_0)x(t_0)x(t_0)+\\+\frac{1}{2}h^2\sum_{j=1}^{i-1}\left(K_2(t_i,t_{0,},t_j)+K_2(t_i,t_{j,},t_0)\right)x(t_0)x(t_j)+\\+h^2\sum_{j=1}^{i-1}\sum_{g=1}^{i-1}K_2(t_i,t_{j,},t_g)x(t_j)x(t_g)+\\+\frac{1}{4}h^2\left(K_2(t_i,t_i,t_0)+K_2(t_i,t_0,t_i)\right)x(t_0)x(t_i)+\\+\frac{1}{2}h^2\sum_{j=1}^{i-1}\left(K_2(t_i,t_{i,},t_j)+K_2(t_i,t_{j,},t_i)\right)x(t_j)x(t_i)+\frac{1}{4}h^2K_2(t_i,t_{i,},t_i)x(t_i)x(t_i),\end{aligned}\quad (4)$$

where $i=\overline{1..n}$, $h=t_i-t_{i-1}$. Let's rewrite (4) having grouped addends for desired $x(t_i)$:

$$\begin{aligned}y(t_i)=\frac{1}{4}h^2K_2(t_i,t_{i,},t_i)x(t_i)x(t_i)+\Bigg(\frac{1}{4}h^2\left(K_2(t_i,t_i,t_0)+K_2(t_i,t_0,t_i)\right)x(t_0)+\\+\frac{1}{2}h^2\sum_{j=1}^{i-1}\left(K_2(t_i,t_{i,},t_j)+K_2(t_i,t_{j,},t_i)\right)x(t_j)+\frac{1}{2}hK_1(t_i,t_i)+\frac{\alpha}{h}\Bigg)x(t_i)+\\+\sum_{j=1}^{i-1}hK_1(t_i,t_j)x(t_j)+\frac{1}{2}hK_1(t_i,t_0)x(t_0)+\frac{1}{4}h^2K_2(t_i,t_0,t_0)x(t_0)x(t_0)+\\+\frac{1}{2}h^2\sum_{j=1}^{i-1}\left(K_2(t_i,t_{0,},t_j)+K_2(t_i,t_{j,},t_0)\right)x(t_0)x(t_j)+\\+h^2\sum_{j=1}^{i-1}\sum_{g=1}^{i-1}K_2(t_i,t_{j,},t_g)x(t_j)x(t_g)-\frac{\alpha}{h}x(t_{i-1}).\end{aligned}\quad (5)$$

Let introduce:

$$A_i = \frac{1}{4} h^2 K_2 \left(t_i, t_{i,}, t_i \right), \qquad (6)$$

$$B = \frac{1}{4} h^2 \left(K_2 \left(t_i, t_i, t_0 \right) + K_2 \left(t_i, t_0, t_i \right) \right) x \left(t_0 \right) +$$
$$+ \frac{1}{2} h^2 \sum_{j=1}^{i-1} \left(K_2 \left(t_i, t_{i,}, t_j \right) + K_2 \left(t_i, t_{j,}, t_i \right) \right) x \left(t_j \right) + \frac{1}{2} h K_1 \left(t_i, t_i \right) + \frac{\alpha}{h}, \qquad (7)$$

$$C_i = \sum_{j=1}^{i-1} h K_1 \left(t_i, t_j \right) x \left(t_j \right) + \frac{1}{2} h K_1 \left(t_i, t_0 \right) x \left(t_0 \right) + \frac{1}{4} h^2 K_2 \left(t_i, t_0, t_0 \right) x \left(t_0 \right) x \left(t_0 \right) +$$
$$+ \frac{1}{2} h^2 \sum_{j=1}^{i-1} \left(K_2 \left(t_i, t_{0,}, t_j \right) + K_2 \left(t_i, t_{j,}, t_0 \right) \right) x \left(t_0 \right) x \left(t_j \right) + \qquad (8)$$
$$+ h^2 \sum_{j=1}^{i-1} \sum_{g=1}^{i-1} K_2 \left(t_i, t_{j,}, t_g \right) x \left(t_j \right) x \left(t_g \right) - \frac{\alpha}{h} x \left(t_i \right) - y(t_i).$$

Then (5) considering (6)–(8) will be:

$$A_i {x_i}^2 + B_i x_i + C_i = 0 \qquad (9)$$

n square equations (9) are solved sequentially based on the iterative method, the root of the previous equation is taken as the initial approximation.

Generally, we should search for the roots of a polynomial of some kind. For the computer implementation of the algorithms for solving the polynomial equation, it is offered to use the MATLAB — fzero. But, there it is necessary to separate the real and complex roots that may be solved by the roots function [2].

The essence of the roots function is that the roots of the polynomial equation

$$p\left(x \right) = a_1 x^n + a_2 x^{n-1} + \ldots + a_n x + a_{n+1} \qquad (10)$$

are the numbers of the so-called companion matrix being determined for the given polynomial (10) as follows:

$$C = \begin{pmatrix} -\frac{a_2}{a_1} & -\frac{a_3}{a_1} & \cdots & -\frac{a_n}{a_1} & -\frac{a_{n+1}}{a_1} \\ -1 & 0 & \cdots & 0 & 0 \\ 0 & -1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -1 & 0 \end{pmatrix}.$$

Computational experiments. The effectiveness of this approach was studied while the signal restoration that passes through the system given in Fig. 1, where LPM is the linear part model. In this system, the nonlinearity of the second kind is considered.

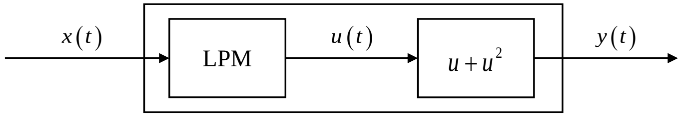

Fig. 1. *Structure diagram of the nonlinear dynamic system*

There have been considered the cases when the linear part is determined by different types of models inertial joint. In this case, we will have the following model:

$$\alpha \frac{dx(t)}{dt} + \int_0^t e^{-s} x(t-s)\, ds + \int_0^t \int_0^t e^{-(s_1+s_2)} x(t-s_1)\, x(t-s_2)\, ds_1 ds_2 = y(t)\,.$$

The results of computational experiments are given in the figures below. The input signal has the form given in Fig. 2. Simulation step in experiments — 0.1. Fig. 3 demonstrates the signal based on which the input signal is restored while applying 1% noise, Fig. 4 — restored signal, fig. 5 — the accuracy of recovery. Fig. 6 demonstrates the signal based on which the input signal is restored while applying 10% noise, Fig. 7 — the restored signal, Fig. 8 — the accuracy of recovery.

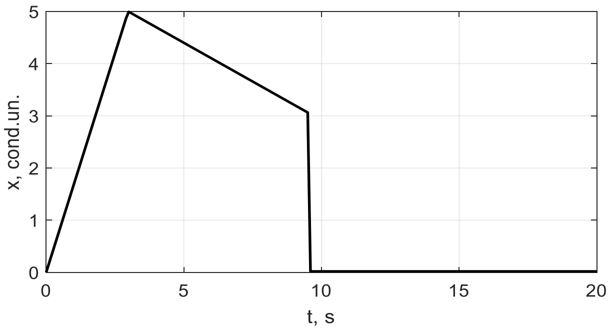

Fig. 2. *Input signal*

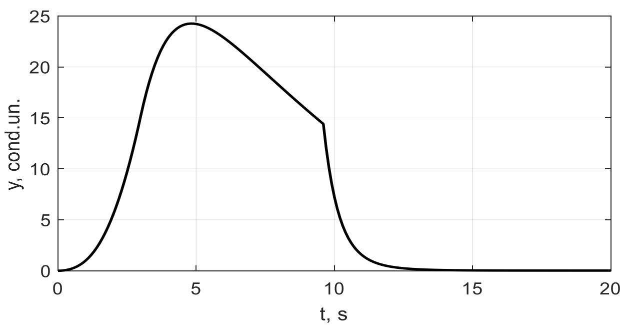

Fig. 3. *Signal on the output of the nonlinear system (the case when the linear part is determined by the inertial joint) applying 1% noise*

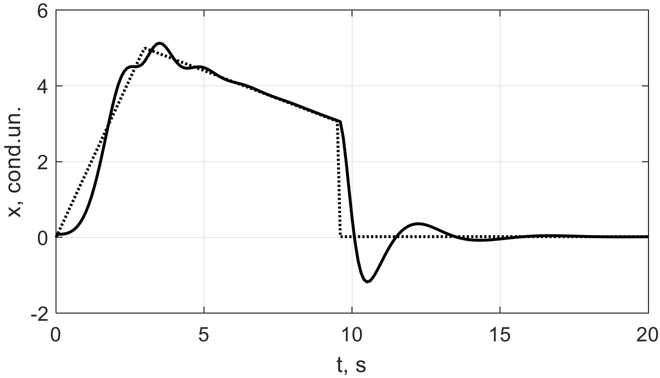

Fig. 4. *Restored signal — accurate and approximate*

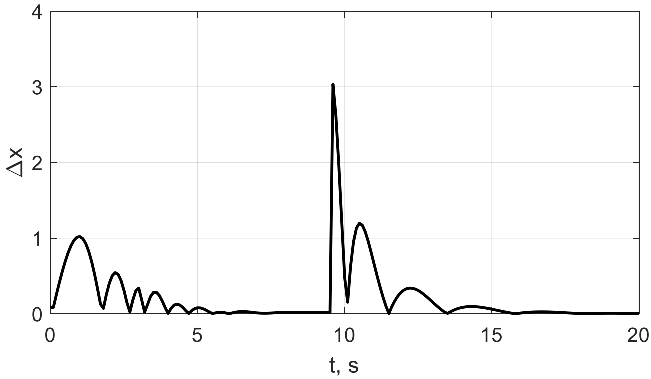

Fig. 5. *The accuracy of recovery*

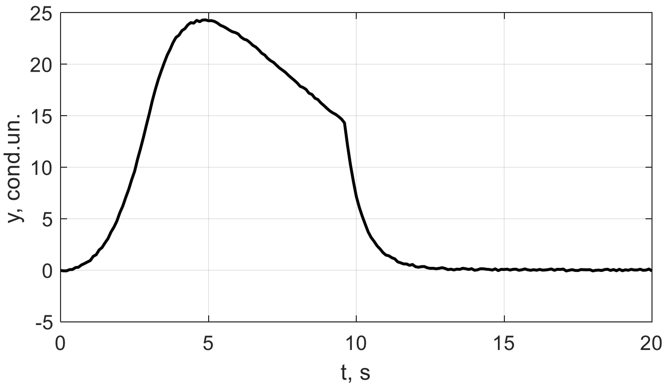

Fig. 6. *Signal on the output of the nonlinear system (the case when the linear part is determined by the inertial joint) applying 10% noise*

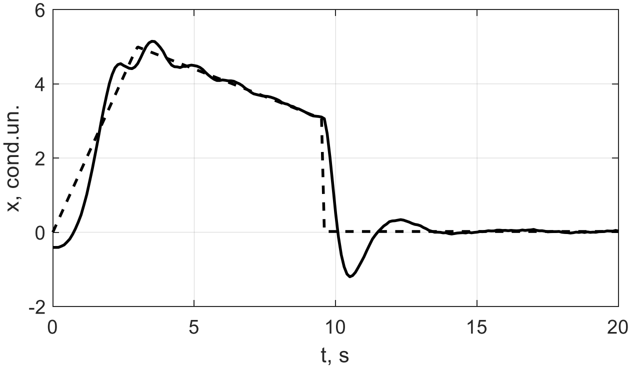

Fig. 7. *Restored signal — accurate and approximate*

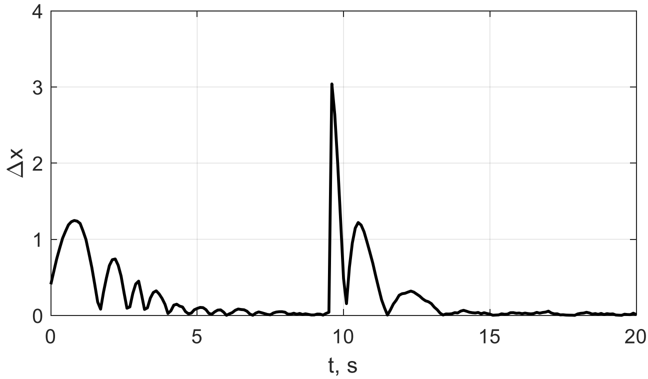

Fig. 8. *The accuracy of recovery*

Conclusion. There has been developed the regularization method for solving polynomial Volterra integral equations of the first kind based on the introduction of the differential regularization operator making it possible to improve the efficiency of the process of restoring signals on the input of nonlinear dynamic objects if there is noise.

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РЕГУЛЯРИЗАЦІЙНИЙ МЕТОД ВІДНОВЛЕННЯ СИГНАЛІВ НА ВХОДІ НЕЛІНІЙНИХ ДИНАМІЧНИХ ОБ'ЄКТІВ, ЩО ЗАДАНІ ІНТЕГРО-СТЕПЕНЕВИМИ РЯДАМИ ВОЛЬТЕРРИ.

У статті пропонується регуляризаційний метод розв'язування поліноміальних інтегральних рівнянь Вольтерри I-го роду при розв'язуванні задачі відновлення вхідного сигналу нелінійного динамічного об'єкта, що поданий інтегро-степеневим рядом Вольтерри. Застосування інтегро-степеневих рядів Вольтерри дає змогу спростити первинні нелінійні математичні моделі нелінійних динамічних об'єктів перетворивши їх до квазілінійного вигляду. Розв'язування поліноміальних інтегральних рівнянь Вольтерри I-го роду здійснюється шляхом введення додаткового диференціального регуляризаційного оператора. Отримані інтегро-диференціальні рівняння пропонується розв'язувати за допомогою квадратурних алгоритмів шляхом використання ітераційних методів. Такий підхід дозволяє підвищити ефективність процесу відновлення сигналів на вході нелінійних динамічних об'єктів при наявності шумових завад. Ефективність запропонованого алгоритму, перевірено для відновлення вхідного сигналу нелінійного динамічного об'єкту, що поданий у вигляді послідовного з'єднання лінійної та нелінійної частин. При цьому лінійна частина представлена інерційною ланкою, а нелінійна – поліноміальною залежністю другого порядку. Представлено результати розв'язування поліноміальних інтегральних рівнянь Вольтерри I-го роду при наявності шумових завад різного характеру у вхідних залежностях. На основі описаного методу, створено у середовищі Matlab / Simulink імітаційні моделі та програмні засоби розв'язування обернених задач відновлення сигналів на вході нелінійних динамічних об'єктів. Результати обчислювальних експериментів показали, що запропонований регуляризаційний метод розв'язування поліноміальних інтегральних рівнянь Вольтерри I-го роду може ефективно використовуватись для відновлення вхідних сигналів нелінійних динамічних систем, які описуються інтегро-степеневим рядом Вольтерри.

Ключові слова: *відновлення вхідних сигналів, поліноміальне інтегральне рівняння Вольтерри I-го роду, диференціальний регуляризаційний оператор, Matlab.*

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