Maritime Sediments, Vol. 10, No. 3, December 1974, pp. 85-95

A Comparison of Air Photograph and Computer Simulated Wave Refraction Patterns in the Nearshore Area, Richibucto, Canada and Jervis Bay, Australia

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Introduction

Wave refraction diagrams constructed either by hand-drawn methods or by computer simulation have played an increasingly important role in the description of coastal processes and the prediction of coastal change. They have been qualitatively related by Shepard and Inman (1950) to measure longshore current speeds in California, and Cherry (1966) to sand transport and heavy mineral distributions in Drakes Bay. These diagrams have also been used by Davies (1958) and Bird (1961) to link beach plan and orientation to wave patterns around Tasmania and the Gippsland Lakes of Australia and by Richards and Bird (1970) to relate not only beach plan but form to the wave regime of the Barbados.

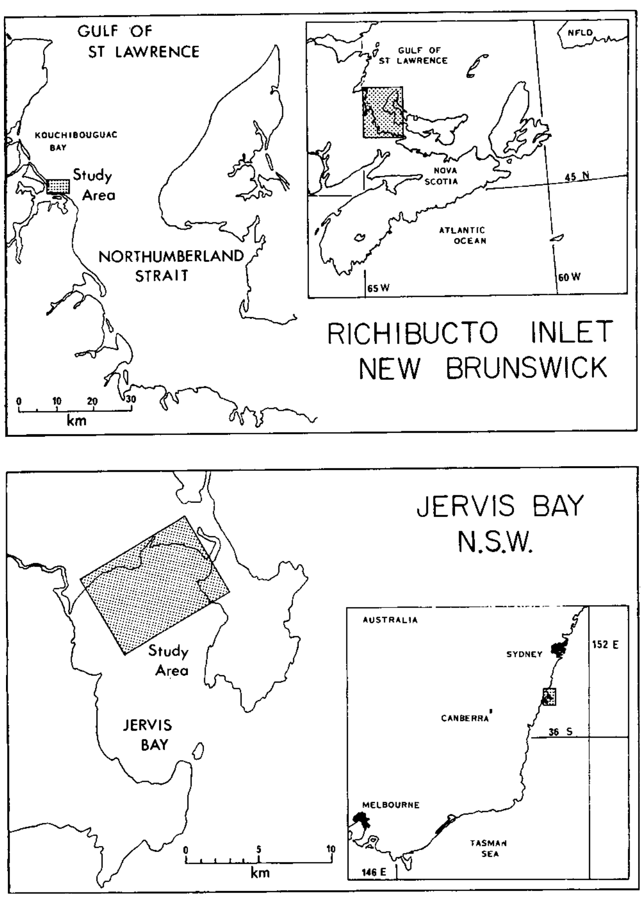

FIG. 1: Location map for study areas, Richibucto Inlet, New Brunswick, Canada and Jervis Bay, New South Wales, Australia.

Because the magnitude and direction of coastline change is partly dependent upon the frequency, amount and direction of energy input to the area and as the distribution of this energy is partly controlled spatially by wave refraction, wave refraction patterns can also delineate those coastal areas which either undergo change or remain stable. Reddy (1968) predicted the probable change in the beach configuration at Belledune Point, Chaleur Bay using such diagrams, and McCann and Bryant (1973) related areas of erosion between 1894 and 1964 for the barrier islands of Kouchibouguac Bay to wave ray diagrams for a selection of wave trains of varying periods and approaches. More intensively, May and Tanner (1973) have used wave ray diagrams to model the nearshore erosional and depositional effects of a littoral drift system along the Florida coast, and Coleman and Wright (1971) have used them to partially describe and explain sediment regimes and geomorphic changes on the Niger and São Francisco River Deltas.

In recent years, the graphical hand-drawn methods (Johnson, O'Brien, and Isaacs, 1948; and Arthur, Munk, and Isaacs, 1952) have given way to a succession of computer simulation programs (Girswold and Nagle, 1962; Mehr, 1962; Griswold, 1963; Harrison and Wilson, 1964; Wilson, 1966; Dobson, 1967; Hardy, 1968; and Orr and Herbich, 1969). Of these programs, the Wilson (1966) and Dobson (1967) ones are the most frequently used, either as published or as a basis for modification or improvement. May (1972, personal communication) has modified the Wilson program internally for the calculation of depth values. Coleman and Wright (1971) have added a bottom friction correction for wave height, and subroutines for wave energy and power calculations to the Dobson program, and Smith and Camfield (1972) have corrected the Dobs n a (h n t it t) f t breaker zone based upon work by Nakamura, Shiraishi and Sasaki (1966a, b). These programs with their modifications have been accepted as a realistic method for the simulation of wave patterns; however, though they have been compared with theoretical and hand-drawn patterns, they have rarely been compared with refraction patterns for actual waves.

This paper presents the results of an investigation examining the response of the Wilson and Dobson programs to varying water depths and wave periods for a nearshore area and, more importantly, it compares the wave refraction patterns simulated by these two frequently used programs with those constructed directly from air photograph mosaics for two nearshore areas of differing bathymetric condi..ns and wave reg'me-; -ne a R'-h'bu-t- I-l-t, New Brunswick, Canada and the other at Jervis Bay, New South Wales, Australia (Fig. 1).

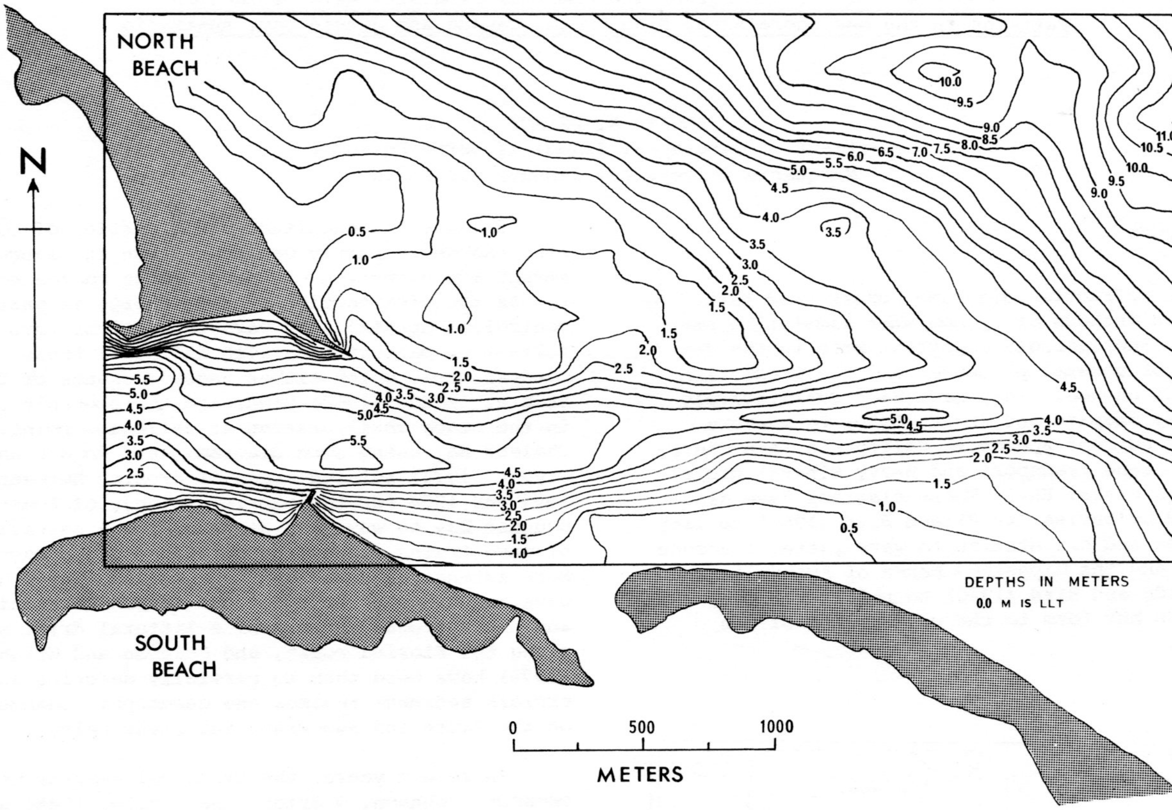

FIG. 2: Contoured nearshore bathymetry for Richibucto Inlet, Canada. Depths in meters and referenced to lowest low tide.

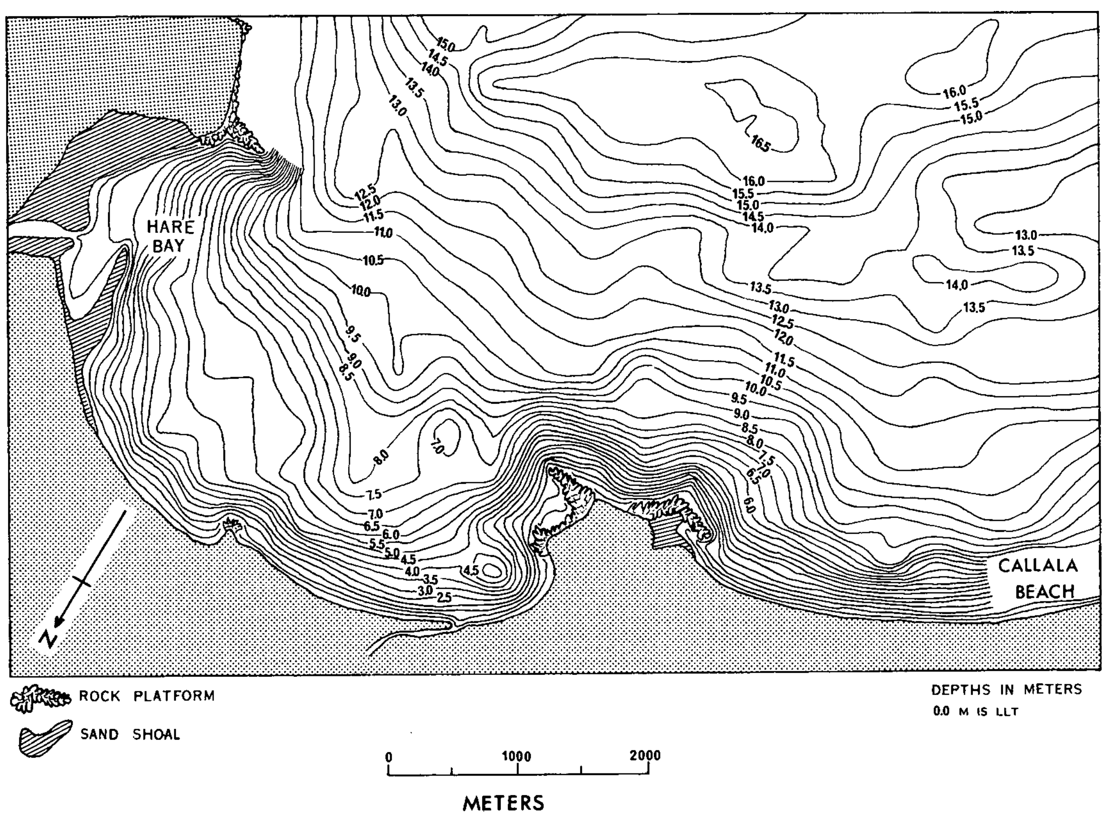

FIG. 3: Contoured nearshore bathymetry of Jervis Bay, Australia. Depths in meters and referenced to lowest low tide.

Description of the Programs

Both the Wilson and the Dobson programs use linear theory for progressive periodic waves which assumes a small wave steepness, a constant depth and a wave period which is a constant unique function of wave celerity and length. Such theory, however, has been applied successfully to small slopes and to various low amplitude waves. The programs also assume that wave refraction obeys Snell's law, that diffraction and reflection are minimal, that wave period is a constant both in time and space and that depth contours are stright and parallel over small distances. In addition, the Dobson program assumes that wave energy cannot be transmitted perpendicular to the wave orthogonals, nor lost by bottom friction and percolation to the sea bed. These assumptions limit the use of these wave refraction programs to simplified bathymetric and wave conditions and, in the shoaling and breaker zones where linear theory breaks down (Wood 1971, p. 34), render the programs ineffective.

Both programs use as input a regular, square grid of water depths, a constant wave period, the angle of approach of elements on a wave crest and the initial co-ordinate positions of these elements. They give as basic output the co-ordinate location of the wave orthogonal at calculated intervals across the grid, and the depth and the error in the interpolated depth at each location. The major difference between the programs occurs with this interpolation of depth from a regular gridded matrix. The Wilson program uses a least squares fitting based upon the four nearest grid intersects while the Dobson program uses a second degree polynomial approximation or smoothing technique based upon the twelve nearest grid intersects. The latter technique tends to smooth more complicated bathymetry and thus refracts waves less than the Wilson program in areas where depth varies considerably over small distances. The polynomial technique does allow for the calculation of wave height, energy, and refraction coefficients whereas the least squares technique does not.

The Study Areas

The two study areas, Richibucto Inlet Canada and Jervis Bay Australia, are extensively documented in the recent literature (Greenwood and Davidson-Arnott, 1972; McCann and Bryant, 1973; Bryant and McCann, 1973; Walker, 1967; and Taylor, 1970). They not only have detailed bathymetric mapping but also air photograph coverage depicting definable wave trains near the time of this mapping. The bathymetry of the Richibucto area is based upon a 1944 chart revision (Canadian Hydrographic Service, Chart No. 4438) and consists of an area of parallel offshore contours grading from 11 m shoreward into a shallow broken offshore shoal running parallel to South Beach, and of a tidal inlet and channel system which breaches this shoal (Fig. 2). The Jervis Bay bathymetry is based upon a 1954-1958 chart revision (Royal Australian Navy, Chart AUS 80, 8th ed. 1965) and consists of an irregular offshore grading from 12 m into a non-barred, evenly contoured, inshore bathymetry which is broken in one or two places by rock platforms (Fig. 3).

The photographs making up the mosaics depicting the wave trains refracting over these bathymetries are dated May 20, 1945 for Richibucto (Fig. 4, A7825 No. 55-57) and April 4, 1949 for the north-eastern part of Jervis Bay (Fig. 5, SVY562 No. 5081-5084). The Richibucto photography shows irregularly crested, broken swell typical of storm conditions with onshore winds for Kouchibouguac Bay, New Brunswick, and the photography for Jervis Bay, which is located in the east coast swell environment of eastern Australia (Davies, 1964), shows a regularly crested, unbroken swell in calm wind conditions. Waves shown on the Richibucto photographs begin in an area of linear, parallel, offshore contours, break in 2 to 3 m of water along the offshore shoals and after breaking, reform and are partially diffracted in the channel north of South Beach. These re-formed waves are partially influenced by the tidal currents in the main channel, currents which have been measured at 1.0 to 1.3 m/sec. for a spring tide situation (Bryant 1972, pp. 222-233), and ones which are similar to a type Johnson (1947) describes as affecting wave refraction patterns to some degree. In contrast, waves shown on the Jervis Bay photographs begin in an area of irregular, non-parallel contours, do not break until they reach shore, do not undergo diffraction and are not affected by tidal currents which for the most part are negligible in Jervis Bay (Taylor 1970, p. 34). The two areas thus offer air photographs showing contrasting wave conditions for differing nearshore areas of the same depth.

Methodology

The comparison of wave refraction patterns drawn from the air photograph mosaics to those simulated by the computer programs involved drawing the patterns for the mosaics, measuring the variables needed for input into the Wilson and Dobson programs from these photographs, and then using this input and depth grids taken from the bathymetric maps to construct compariable, computer simulated, wave refraction diagrams. Because the wave patterns on the Richibucto photographs are complex inside the breaker zone and because linear wave theory breaks down in this area, simulation was carried out only for those waves offshore from the breaker zone.

No attempt was made in the study to correct the mosaics for tilt distortions nor the depth grids for storm surge or spatial distortions resulting from the use of bathymetric maps based upon non-orthodromic map projections (Hardy, 1968). The tilt distortions for the mosaics appear to be minimal and since the study areas are relatively small (3 to 5 km), corrections for projection distortions in the grids are negligible. Onshore wind set-up for waves on the Jervis Bay photographs was judged unimportant as winds appeared non-existent and its calculation for the Richibucto photographs, using weather maps supplied by the Atmospheric Environmental Service of Canada and a technique outlined by the U.S. Army Coastal Engineering Research Center (1966, p. 116-142), gave a value of only 0.13 m.

Both depth grids were corrected for tide height at the time of photography. This correction for

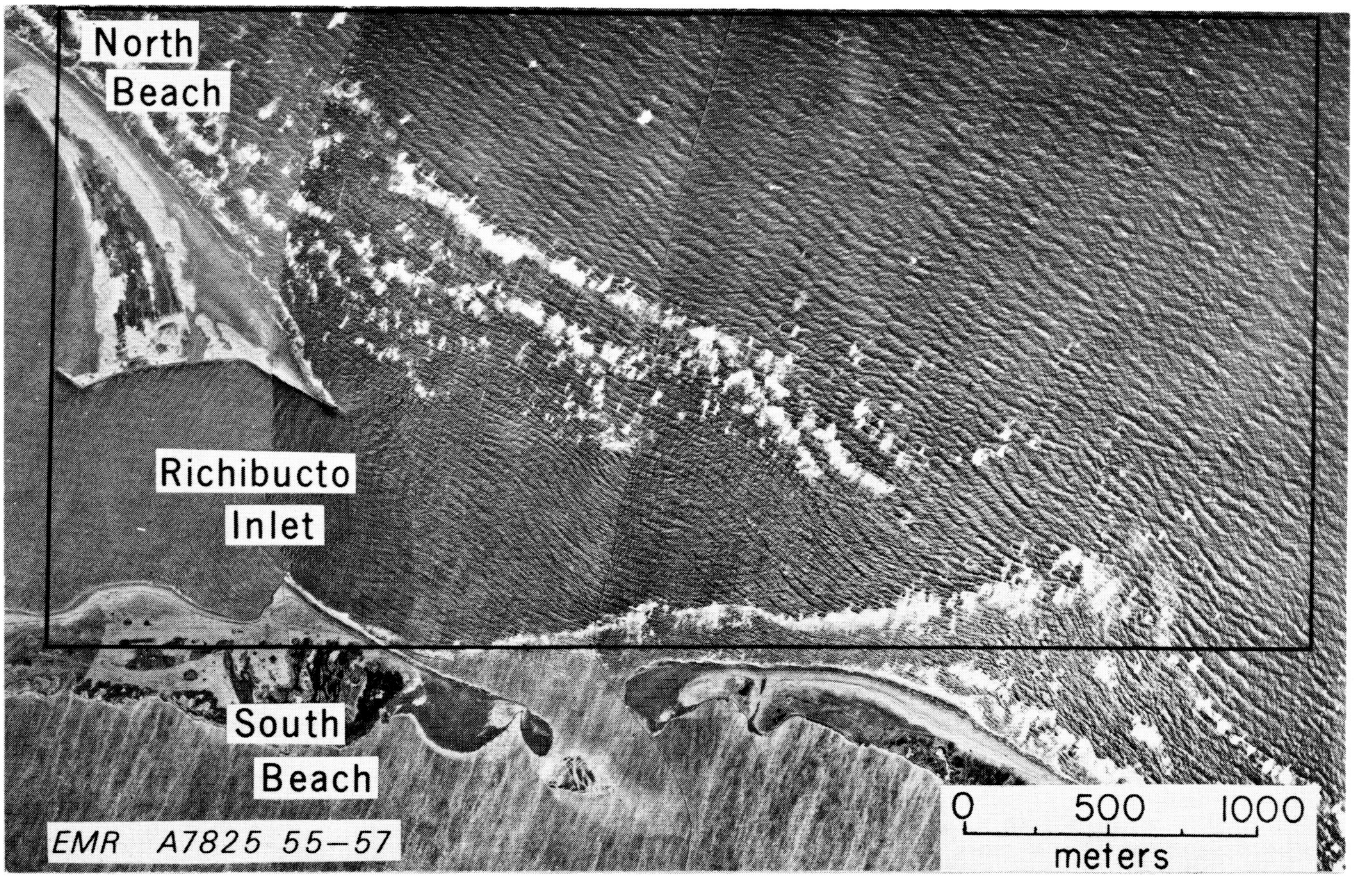

FIG. 4: Air photograph mosaic of 6.5 second waves for Richibucto Inlet, Canada, May 20, 1945.

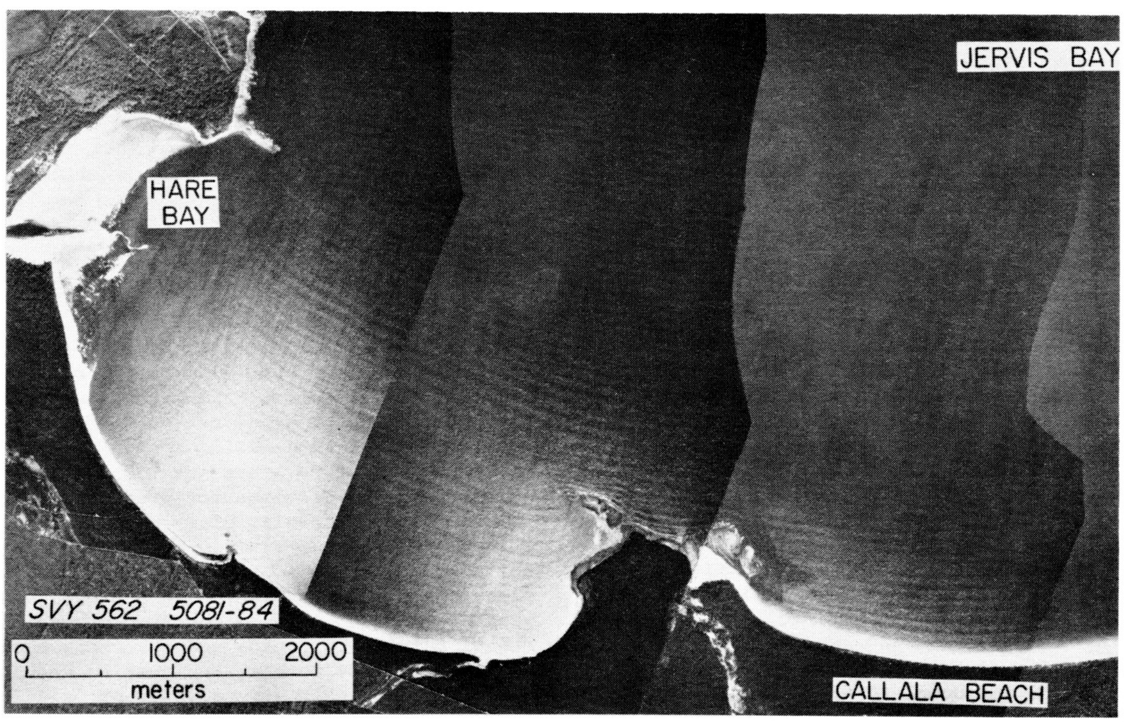

FIG. 5: Air photograph mosaic of 10.45 second swell waves for Jervis Bay, Australia, April 4, 1949.

the Richibucto grid was calculated by the Division of Tides and Water Levels, Canadian Marine Sciences Branch as 0.73 m (±0.06 m for a four hour period around the time of photography) above the zero reference level of the grid. The tide height for the Jervis Bay area at the time of photography had to be estimated from the air photographs as no exact calculation was available. Because the rock platforms in Jervis Bay undergo very slow change (Walker, 1967) and are progressively covered or uncovered during the tidal cycle which has a maximum range of 2 m (Australian National Tide Tables, 1974), a comparison of the degree of exposure of these platforms on the air photographs to observations made during a reconnaissance of the area was used to estimate a tide height of 0.6 m for the corresponding depth grid.

Wave periods required by the programs were calculated from the air photographs using the following linear theory for progressive periodic waves:

$$C^2 = \frac{gL}{2\pi} \tanh\left(\frac{2\pi d}{L}\right)$$

$$T = \frac{L}{C} \qquad \text{(Wiegel 1964, pp. 13-15)}$$

T = wave period
L = wave length
C = wave velocity
d = water depth
g = acceleration due to gravity

Wavelengths were measured at 7 locations on each photographic mosaic and the corresponding water depths (6 to 7 m for Richibucto, 10 to 12 m for Jervis Bay) were taken from the bathymetric charts and corrected for tide height. The wave period was then calculated and averaged to produce a representative period for each mosaic. This wave period averaged 6.5 seconds with a range of 6.1 to 6.7 seconds for Richibucto and 10.45 seconds with a range of 10.0 to 11.2 seconds for Jervis Bay.

Since depth values on any bathymetric chart often have a measurement error on the order of ±0.25 m and can change substantially between the time of bathymetric surveying and photography and since there was also a range of wave periods calculated from the mosaics, the effect of small variations in depth and wave period on the resulting simulated wave ray patterns was investigated. Taylor (1970, p. 128) has shown through low sand to mud ratios that the main area of Jervis Bay used in the study is one of low energy and is thus relatively stable over time. For Richibucto, Bryant (1972, p. 189) has shown that, though some of the shallow areas can change over the short term, most of the nearshore areas outside the breaker zone have not changed substantially over the past century. Thus for these two mosaics, errors in the depth grids rather than changes between the time of photography and bathymetric mapping would appear to be more crucial.

The effect of small surveying errors in the bathymetry on the wave ray patterns was evaluated by incrementing depths for values between 0.5 and 1.25 m across the Richibucto grid and constructing wave ray diagrams for 6.5 second period waves at each increment using the Wilson and Dobson programs. The diagrams were then superimposed to produce one wave ray diagram which showed the variation due to these small depth changes (Fig. 6 for the Wilson program, Fig. 7 for the Dobson program). A comparison of the diagrams shows that the Wilson program, for most rays, responds slightly more to changes in depth then does the Dobson program, a fact which is due to the different depth interpolation methods used by both programs. The two programs show that, where wave refraction is not severe, the variation in the paths of wave rays is small and that the interpretation of the wave ray patterns is not severely affected by small changes in the depth grid. The accuracy of the depth grid does become a crucial factor when wave refraction is severe (Ray 46). Similar diagrams were also constructed using the Richibucto grid for a depth of 0.78 m and increments of wave period between 5.5 and 7.5 seconds, a range which includes the wave periods calculated from the mosaic. The variation in wave ray paths computed for these periods was within the same range as that computed for the depth values used in Figures 6 and 7. These results imply that both programs can tolerate some consistent error both in the depth grid and in the wave period without seriously affecting the interpretation of the diagrams.

The direct comparison of wave refraction patterns on the air photograph mosaics to ones simulated by the computer first involved spacing wave rays at intervals on each of the photographs and then drawing lines at right angles to the initial wave crests shoreward to the next crest. This process was continued until the wave crests either became indiscernible on the photograph or else reached the breaker zone or the shoreline (Richibucto, Fig. 8 and Jervis Bay, Fig. 9). The repeatability of this method was evaluated by taking a random selection of orthogonals on each mosaic and re-drawing them. In each case the resulting ray paths were quite similar to the original drawings. The initial co-ordinate positions of the wave rays on the photographs were then calculated for the corresponding depth grids and the angle of approach of these rays measured from a standard reference. This information, along with the calculated wave periods and corrected grids, was used as input to the Wilson and Dobson programs to produce computer constructed diagrams at the same scale as those drawn from the photographs. The diagrams produced by the Wilson and Dobson programs are presented in Figures 10 and 11 respectively for Richibucto and Figures 12 and 13 respectively for Jervis Bay.

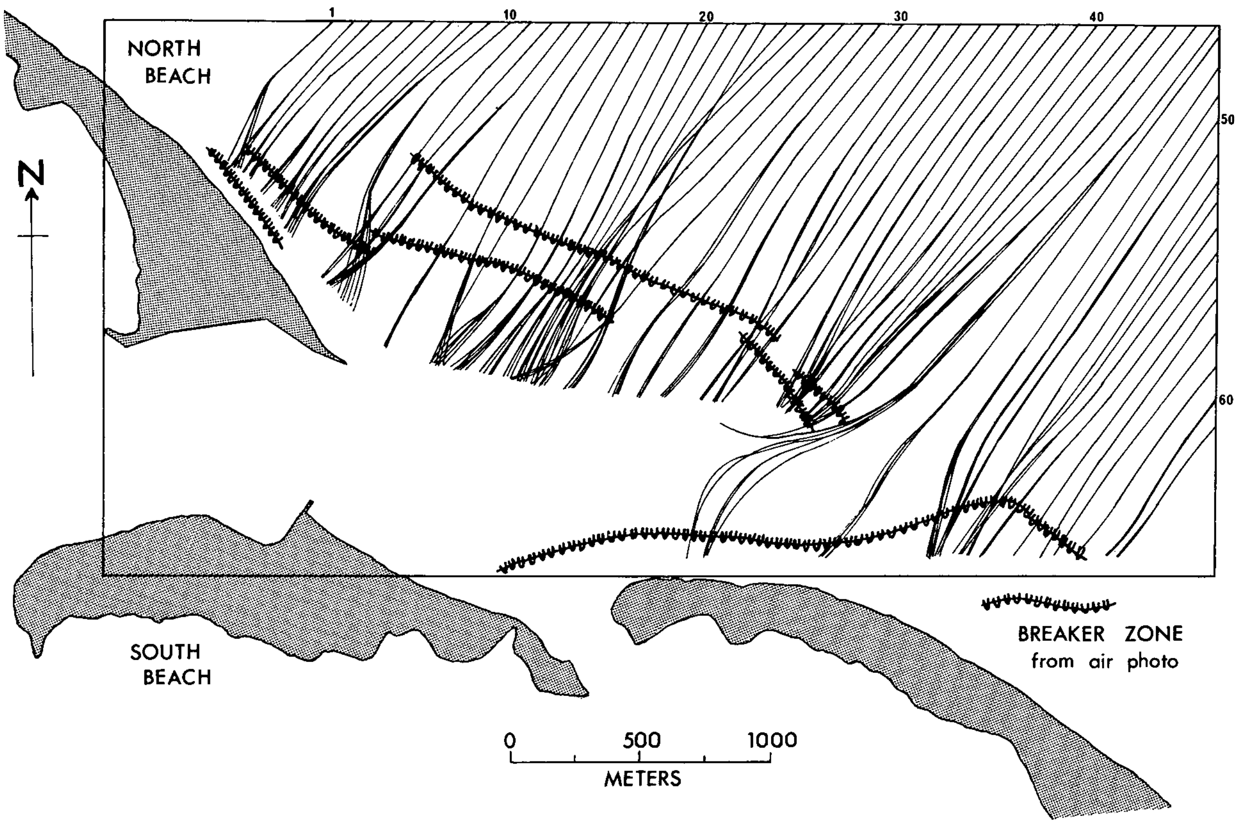

FIG. 6: Variation in wave ray paths simulated using the Wilson program for 6.5 second waves and grid depths between 0.5 and 1.25 m for Richibucto, Canada.

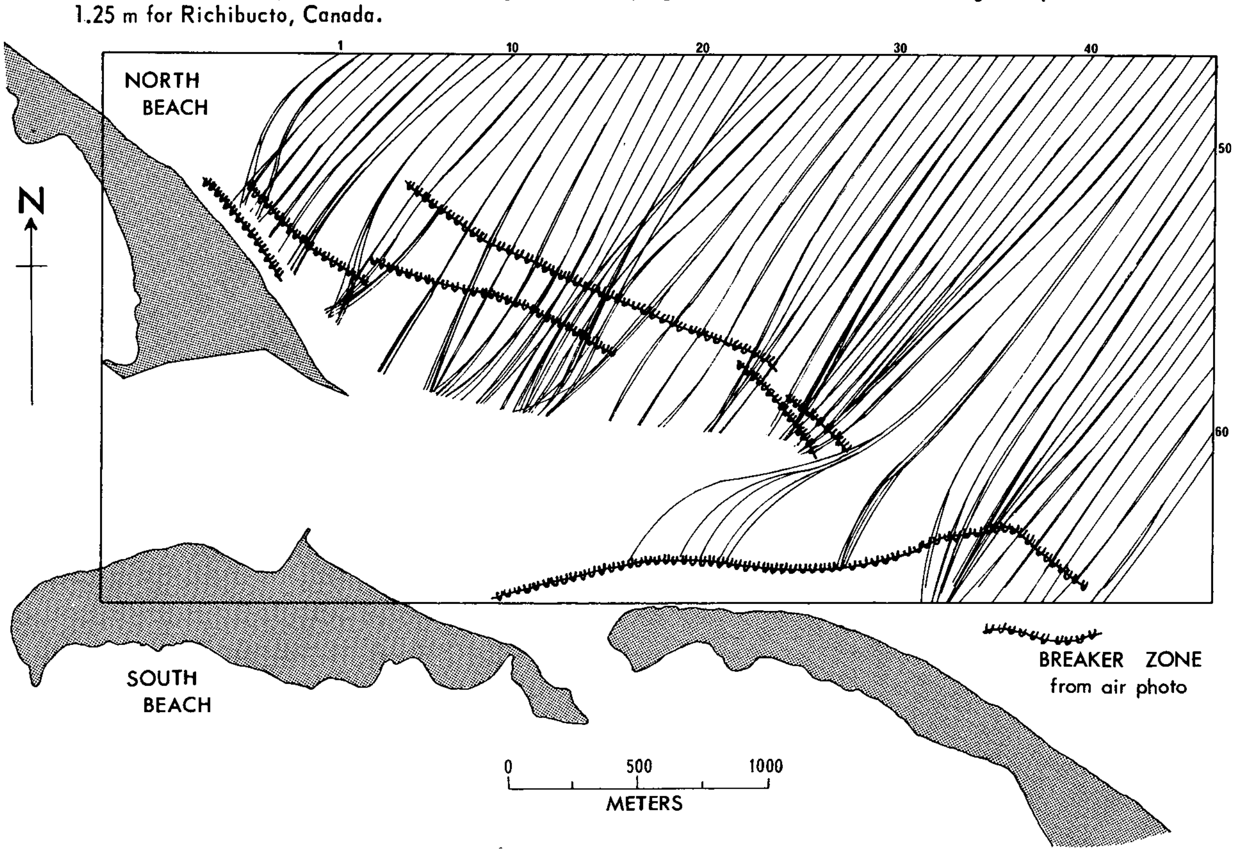

FIG. 7: Variation in wave ray paths simulated using the Dobson program for 6.5 second waves and grid depths between 0.5 and 1.25 m for Richibucto, Canada.

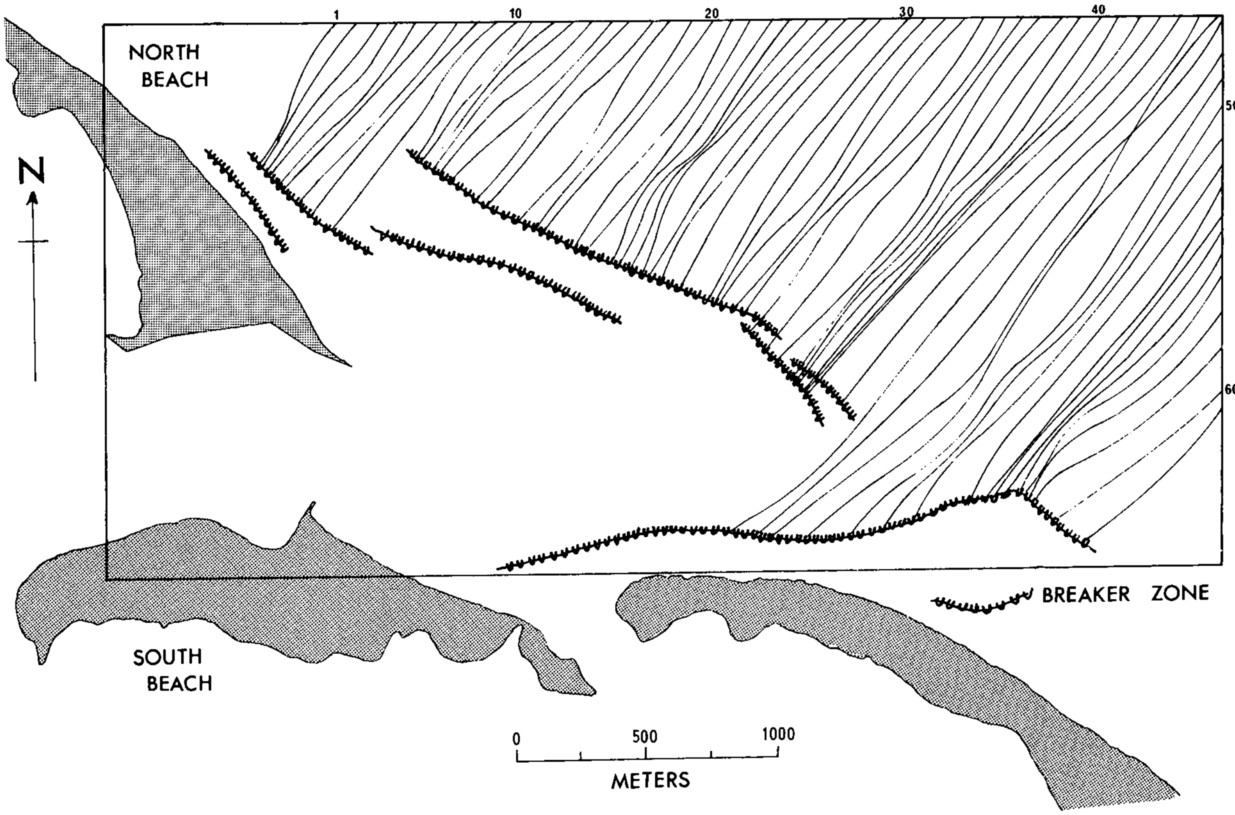

FIG. 8: Wave orthogonals drawn from waves on the Richibucto, Canada air photograph mosaic.

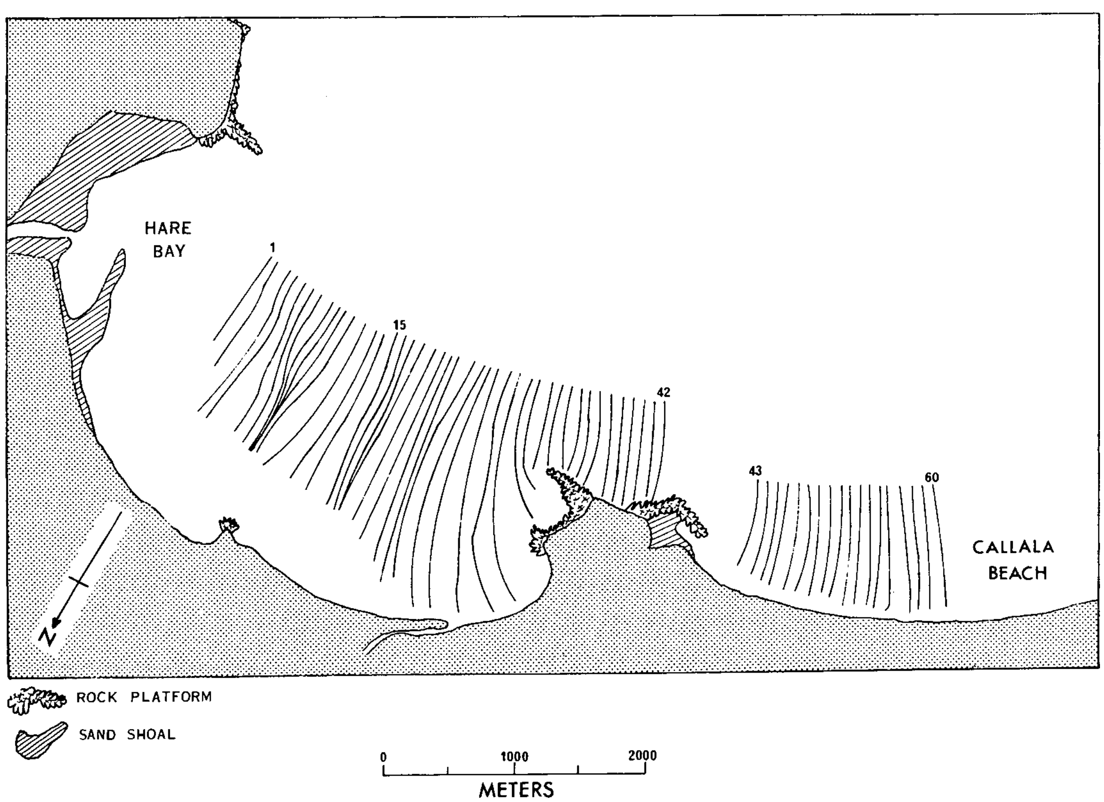

FIG. 9: Wave orthogonals drawn from waves on the Jervis Bay, Australia air photographs.

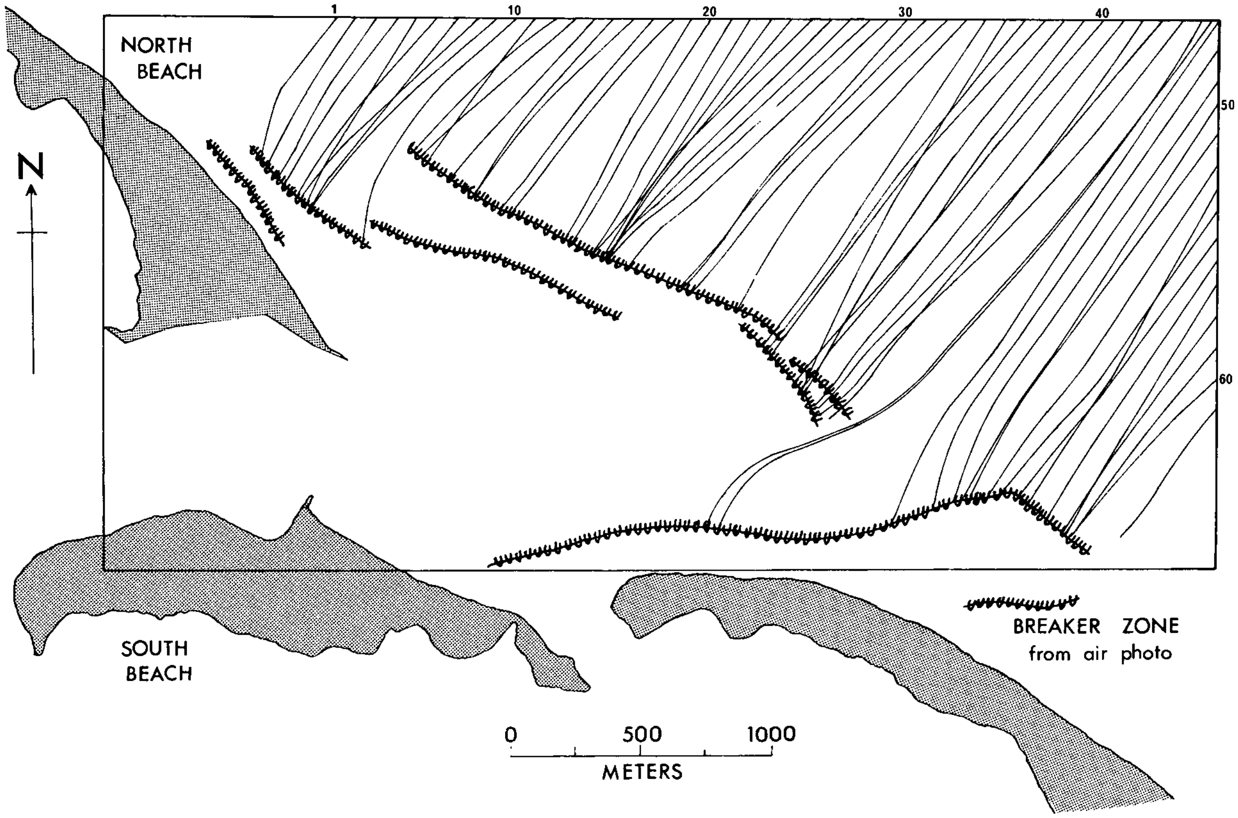

FIG. 10: Wave rays simulated by the Wilson computer program for 6.5 second waves on the Richibucto, Canada air photograph mosaic.

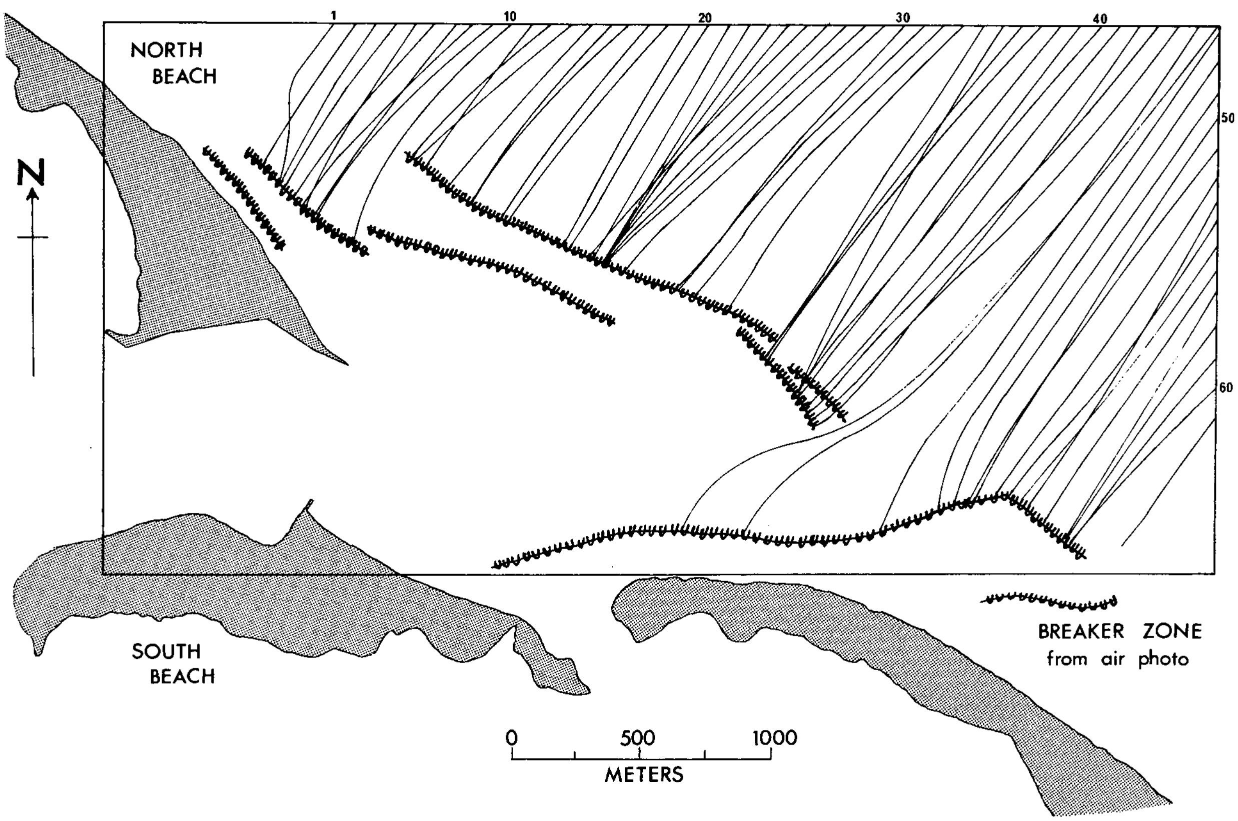

FIG. 11: Wave rays simulated by the Dobson computer program for 6.5 second waves on the Richibucto, Canada air photograph mosaic.

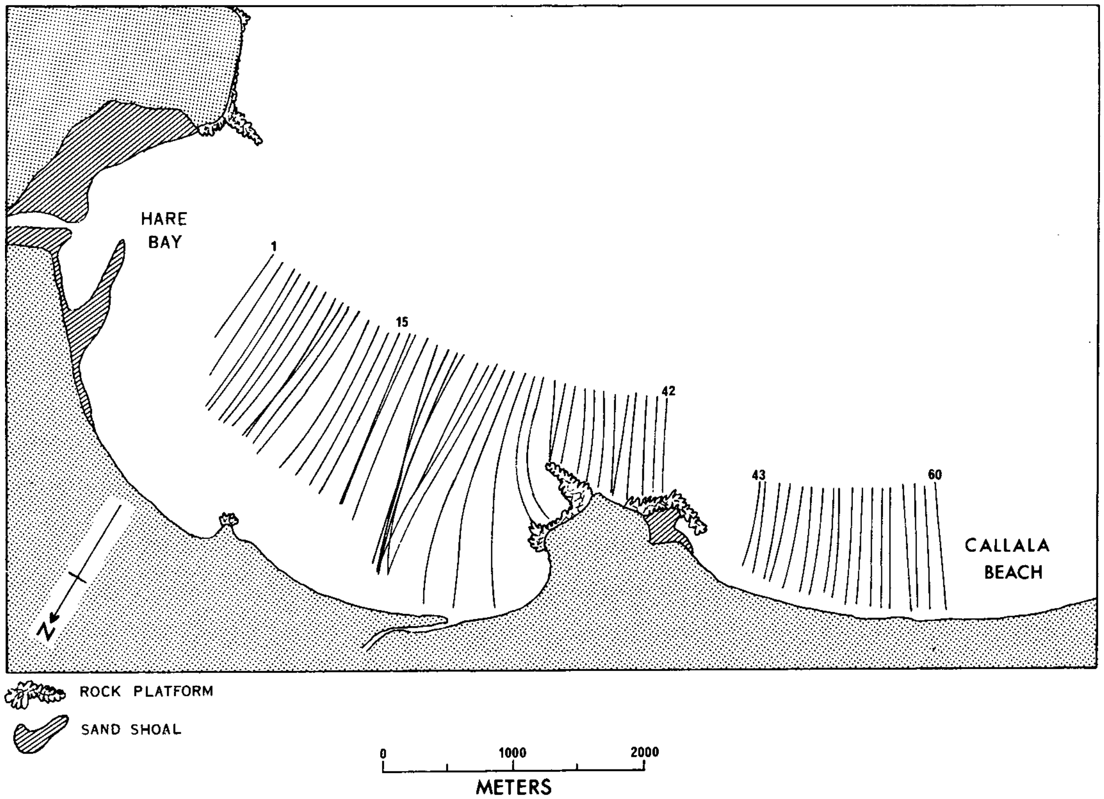

FIG. 12: Wave rays simulated by the Wilson computer program for 10.45 second waves on the Jervis Bay, Australia photographs.

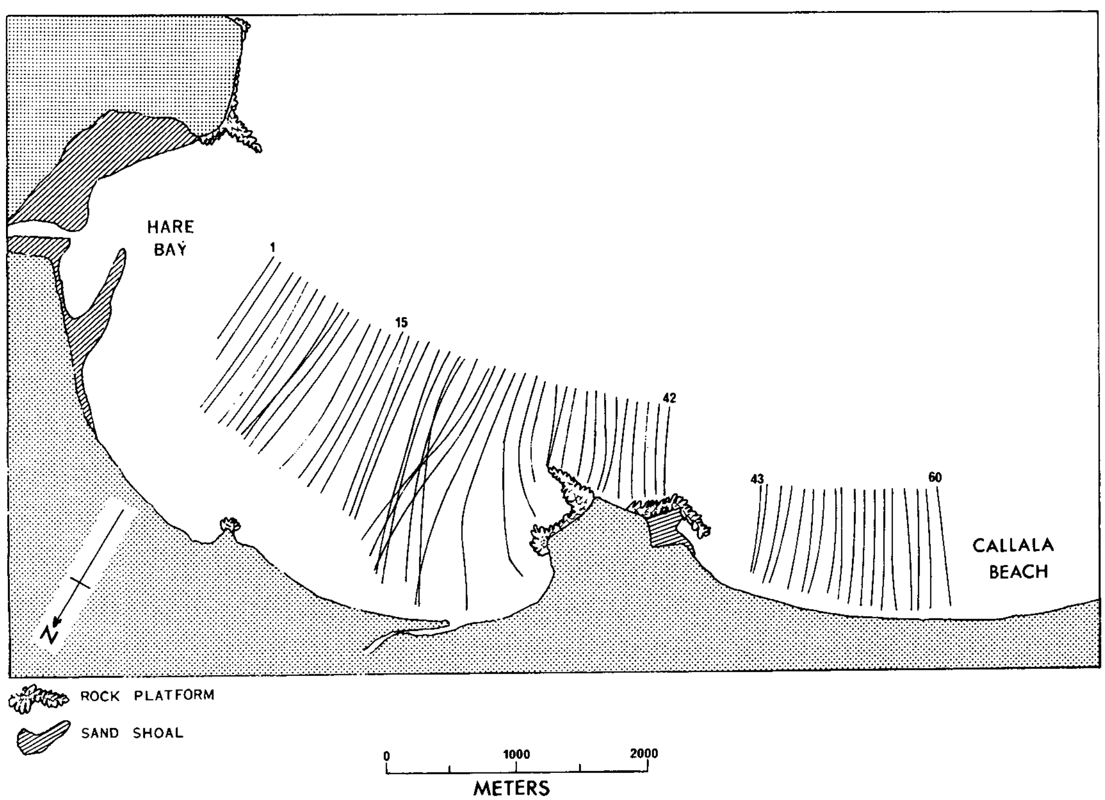

FIG. 13: Wave rays simulated by the Dobson computer program for 10.45 second waves on the Jervis Bay, Australia photographs.

Comparisons

The wave rays constructed from the Richibucto Inlet air photographs (Fig. 8) are concentrated in four main areas along the shoals (Rays 10 to 14, 23 to 28, 38 to 43 and 53 to 59) with slight divergence between these areas and around the offshore mouth of the channel (Ray 45 to 51). The overall pattern, rather than the individual rays, simulated by the Wilson and Dobson programs agrees with this pattern on the air photographs for rays outside the breaker zone; however, both programs tend to over concentrate and diverge the wave rays to some extent. (Fig. 10 and 11 respectively). This over refraction is evident for the concentration of rays 21 to 29 and 38 to 43 and the divergence of rays around these areas and the mouth of the offshore channel. Both programs, while concentrating rays in similar areas depicted on the photographs miss the precise concentration of rays 10 to 14 and 53 to 59 (Fig. 6). These rays are located in shallow areas which can undergo short term changes in bathymetry between the time of surveying and photography (Bryant 1972, p. 189). Both programs, while similar in the patterns they produce, do differ in that the Dobson program tends to refract waves slightly more and, although not depicted, both programs break down completely over the main shoal along North Beach. For the short-period, broken-crested waves depicted on the Richibucto photographs offshore from the breaker zone, the Wilson and Dobson programs simulate wave refraction patterns satisfactorily.

The wave rays constructed from the Jervis Bay mosaic were grouped for each photograph as follows: rays 1 to 14, 15 to 42, and 43 to 60 (Fig. 9). These wave rays diverge gently as they approach the beaches and converge in two areas of slightly irregular bathymetry (Rays 5 to 11 and 15 to 18). The individual rays and the overall pattern simulated by the Wilson and Dobson programs (Fig. 12 and 13 respectively) agree with the air photograph ones in many respects, but there are discrepancies. The programs do not delineate the two areas of convergence shown on the air photographs and simulate one main area of caustics which is not shown (Rays 19 to 26). The rays generated by both programs fit those drawn from the air photographs very well along Callala Beach; but, in the areas outside those where the discrepancies occur, the programs tend to under-refract. The rays generated by the Dobson program fit the air photograph pattern better than those constructed by the Wilson program, a fact which is due to the different depth interpolation methods used by the programs for a larger spaced grid. For these long-period, unbroken swell patterns of Jervis Bay both the Wilson and Dobson programs also produce a reasonable fit.

Conclusions

Computer programs, because of their underlying theory and assumptions, are limited to areas where wave regimes consist of a single wave period from a single direction at any one time and where currents, diffraction and reflection are minimal. The existing programs, such as the Wilson and Dobson ones, have not yet been sufficiently developed to take into account wave refraction in the breaker or surf zones or to adequately simulate wave refraction over small, shallow areas with complex and varying bar-and-channel topography. The programs are also sensitive to small changes in the angle of wave approach if wave incidence to the bottom contours is low, but they can tolerate errors in the depth grids on the order of 0.4 and in wave period input on the order of 1.0 second, for low period waves, without the interpretation of the resulting diagrams being seriously affected.

The comparison of simulated diagrams generated by the Wilson and Dobson programs to those constructed from air photographs depicting two areas (Richibucto Inlet, Canada and Jervis Bay, Australia) with differing swell patterns and nearshore bathymetries, but with similar depths, shows that such programs can produce an adequate representation of a wave refraction pattern in these areas. This study may be limited by the fact that only one wave pattern for each of the nearshore areas was compared and the conclusions may not be applicable for different wave patterns in the same areas or to different nearshore areas. Such results may also be different for the inner continental shelf where mild wave refraction can occur over long distances. Yet, if such programs are used within their limitations, they should provide an accessible means for the sound interpretation of coastal processes and the productive prediction of coastal change and stability for large areas of coastline, for which long term observations are unobtainable but wave data and detailed bathymetry are available.

Acknowledgements

The National Research Council of Canada through Dr. S.B. McCann, and the Australian National University funded the computer expenses for this study. The Canadian National Air Photo Library and the Australian Department of Air supplied the various air photographs, while the Canadian Hydrographic Service supplied chart No. 4438 along with comments on its accuracy and the Australian Department of the Navy supplied chart AUS 80. The Canadian Marine Sciences Branch provided tidal information for the Richibucto area and the Canadian Atmospheric Environment Service supplied weather charts for a ten-day period around May 20, 1945 for the northeastern part of North America. The manuscript was critically read by Dr. S.B. McCann, Dr. R.F. McLean, Mr. Bill Bruce and Mrs Doug Guppy.

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