La forma correcta de utilizar la ecuación de Bernoulli

The correct way to use Bernoulli´s equation

Recibido: mayo 12 de 2019 | Revisado: junio 17 de 2019 | Aceptado: julio 11 de 2019

rEsumEn Luis A. Arriola¹

> La ecuación de Bernoulli es un caso determinado de un problema del flujo de fluido. Se deben cumplir algunas restricciones para aplicar correctamente esta ecuación en particular. El flujo del fluido debe considerarse no viscoso, incompresible, estable e irrotacional. Sin embargo, si el flujo del fluido es rotacional, la ecuación de Bernoulli aún puede aplicarse siempre y cuando los puntos de interés estén en la misma línea de corriente del flujo. Aquí, nos enfocaremos en demostrar que, en el caso de un flujo de fluido rotacional, los puntos de interés deben encontrarse en la misma línea de corriente del flujo y por esta razón, sí se podría continuar con el uso de la ecuación de Bernoulli. El principio solo es aplicable a los flujos isentrópicos: cuando los efectos de los procesos irreversibles (e.g. turbulencia, fricción) y los procesos no adiabáticos (e.g. la radiación de calor, difusión de masa) son pequeños y pueden despreciarse.

> *Palabras clave:* flujo, Bernoulli, viscoso, incompresible, estable, irrotacional, línea de corriente, isentrópico, irreversible, turbulencia, fricción

aBstraCt

Bernoulli's equation is a certain case of a fluid flow problem. Some restrictions must be met in order to correctly apply this particular equation. The fluid flow must be considered inviscid, incompressible, steady and irrotational. However, if the fluid flow is rotational, Bernoulli`s equation can still be applicable as long as the points of interest are on the same streamline of the fluid flow. Here, we will focus on demonstrate that, in the case of a rotational fluid flow, the points of interest must be on the same streamline and because of that, it can be proceeding with the usage of Bernoulli`s equation. The principle is only applicable for isentropic fluid flows: when the effects of irreversible processes (e.g. turbulence, friction) and non-adiabatic processes (e.g. heat radiation, mass diffusion) are small and can be neglected.

Key words: fluid, Bernoulli, inviscid flow, incompressible, steady, irrotational, streamline, isentropic, adiabatic, turbulence, friction

https://doi.org/10.24265/campus.2019.v24n28.01

¹ Universidad de San Martín de Porres, Lima - Perú larriola@usmp.pe

Inroduction

Various forms of Bernoulli's equation can be modeled because of the existence of various types of fluid flow, and therefore Bernoulli's principle can be applied applicable-to-poi too. Bernoulli's principle states that, an increase in the speed of a fluid flow is uniform throw occurs simultaneously with a decrease in internal pressure. The principle is named all streamlines a after Daniel Bernoulli published it in his ance Danier Dernoum published it in ins formal pressure. The principle is not present to provide the principle in the principle in the principle in 1738 . Although, Bernoulli deduced that pressure decreases $\int_{P_1}^{P_2} dP_1$ when the flow speed increases, it was $\frac{1}{2}$ \frac Leonhard Euler who derived Bernoulli's $P_2 - P_1 = \frac{-1}{2}$ equation in its usual form in 1752. All $\frac{2}{1}$ $\frac{2}{1}$ in all, there is a correct way of using $P_1 + \frac{1}{2}\rho V_1^2 = P_2$ Bernoulli's equation with confidence and bernoulli s equation with confidence and
which is briefly described to continuation. $\left(\frac{P}{P} \right)$ in 1/38. Although,

The law that explained the phenomenon from the energy conservation point of view was found led us to some 1 in his Hydrodynamic work. Later, Euler deduced an equation for an inviscid equation But, fi flow (assuming that viscosity was if the flow-field i insignificant) from which Bernoulli's to exist. This i equation arises naturally when equation arises naturally when continuity equation to a conservative gravitational field.

Discussion \mathbf{S} surrounding the volume must be equal to the decrease of mass with the volume \mathbf{S}

To arrive at Bernoulli`s equation, certain assumptions had to be made whe surface s which limit us the level of applicability. According to Euler equation Eq. 1, within the vol defined by Anderson, Jr. (1989), it gives the variation of pressure with respect to speed in motion, it is speed variation, ignoring shear forces question, ignoring shear forces and that has been (inviscid fluid flow) and body forces in Eq. 3 define (weight of the air fluid particle is ignored). Only pressure forces were considered. quation eq. $\frac{1}{200}$

$$
dp = -\rho v dv \qquad (1) \qquad \frac{\partial}{\partial t} + V(\rho V) =
$$

Integrating Eq. (1) by using a limit integration and considering an incompressible fluid flow (change in density is very small because of low therefore speed), will give us Bernoulli's equation can be applied applicable to points 1 and 2 which are on nciple states that, the same streamline. However, if the flow eed of a fluid flow is uniform throughout the field, then α is uniform throughout the field, then α is the same for α is the same for all the constant in Eq. (2) is the same for eprinciple is named all streamlines as defined by Anderson, Jr. (1989). s_{r} where a decrease m and m and m and m and m ned all streamlines as defined by Anderson,

\nAlthough,
\ndecreases
\n
$$
\int_{P_1}^{P_2} dp = -\rho \int_{V_1}^{V_2} V dv
$$
\ns, it was
\nfernoulli's
\n
$$
P_2 - P_1 = \frac{-1}{2} \rho (V_2^2 - V_1^2)
$$
\n1752. All
\nof using
\n
$$
P_1 + \frac{1}{2} \rho V_1^2 = P_2 + \frac{1}{2} \rho V_2^2 = \text{const along
\nstreamline
\nstreamline
\nituation. (2)\n
$$

explained the The assumptions that were made during the derivation of this equation of view was found led us to some restrictions that must be c work. Later, Euler himplemented in order to use Bernoulli`s on for an inviscid equation. But, first of all, we must verify if the flow field in question is possible restriction is possible which Bernoulli's to exist. This is done by verifying if continuity equation is fulfilled. restrictions that must be in order to use Bernaulting in order to use Bernaulting in the use Bernoulling of **1.Continuity Equation in its Vector Form** T_{t} assumptions that we remain the during the derivation of the derivation led us to some this equation led us to some the derivation of the derivation led us to some the derivation of this equation led us to some t restriction of this equation.
 $\frac{1}{2}$ and $\frac{1}{2}$ and $\frac{1}{2}$ are constrained by the strength of the st **naturally** when continuity equation is fulfilled.

ational field. **1. Continuity Equation in its vector form** \mathbf{form} states that, \mathbf{form} T surrounding the volume must be equal to the decrease of mass with the volume \mathcal{L}

The continuity equation states that, $\text{nulli's}\quad \text{equation,}\qquad \text{``the}\quad \text{net}\quad \text{outflow} \quad \text{of}\quad \text{mass}\quad \text{through}$ the surface surrounding the volume $\log \frac{1}{n}$ of applicability. Thus is equal to the decrease of mass is $\log n$ within the volume" (Bertin and Smith s 1998 , p. 24). This is, when a fluid is in motion, it must move in such a way in motion, it must move in such a way
that mass is conserved as it is stated in Eq. 3 defined by Bertin and Smith $(1998):$ ount's equation, the fit outhow of mass imough
ad to be made the surface surrounding the volume such a way that mass is conserved as it is stated in Eq. 3 defined by Bertin and Bertin an s that mass is conserved as it is stated \mathfrak{m} , the net outflow of mass through is the volume must be equal to the decrease of mass with α

(1)
$$
\frac{\partial \rho}{\partial t} + \nabla (\rho \vec{V}) = 0
$$
 (3)

Where V is the fluid density, t is the $\frac{1}{2}$ time, is the flow velocity vector field. \vec{v} is the fluid density, t is the Then, so far, the

2. Steady flow $\mathbf v$

To see further how mass conservation **4. Remember** places restrictions on the velocity field, consider a steady fluid flow. That is, for were ignored existed a steady fluid flow. That is, for a relatively low speed flow, the pressure the case whe variations are sufficiently small, and \int forces are co because of this, the density change is $\frac{1}{2}$ also small that can be assumed to be constant and so, the density of the fluid flow does not the fluid flow does not the fluid fluid flow does not the fluid constant and so, the density of the fluid flow does not vary with time. To see further how mass conservation **4. Remember**
places restrictions on the velocity field, **(friction) and** variations are sufficiently small, and $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ of the density change is also in $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ of the density change is also in $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ of the dens also small that can be assumed to be constant and extrant erm ρ

$$
\frac{\partial \rho}{\partial t} = 0
$$

3. Incompressible

Since density change is very small for low in height "h" low velocity airflows, it can be assumed the so-small tha to be constant. One way to proof this, disregarded. is by verifying if we are dealing with $\frac{1}{1}$ $\frac{1}{1}$ low velocity airflows. As a rule of $\frac{1}{2}\rho V^2 + P +$ thumb, if its mach number is lower $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ than 0,3 or has a velocity less than 300 $\frac{2}{\mu}$ $\frac{1}{\mu}$ $\frac{1}{\mu}$ $\frac{1}{\mu}$ ft/s or 100 m/s (or approximately $200 \text{ where } t$) mph), then the velocity airflow can Static pressure mpn), tnen tne velocity airnow can
be assumed to be small and treated as incompressible Anderson, (1989) and Anderson (2003): $\frac{1}{2}$ mph), then the velocity airflow can Static pressure = P $\overline{1}$ disregarded. $\frac{1}{2}$ where

$$
M = \frac{V}{a} = \frac{V}{\sqrt{\gamma RT}}
$$

Where:

V is the flow velocity

a is the Speed of Sound

 T is the temperature of the flow field γ is the ratio of the specific heats at constant pressure and volume respectively and has a value of $\gamma = 1.4$ for dry air. $\frac{1}{2}$ is the Speed of Speed

R is the air constant for ideal gas and has a value of:

$$
R = 1716 \frac{ft \cdot lb}{slugs \cdot R} \quad \text{or} \quad R = 287 \frac{N \cdot W}{Kg \cdot K} \qquad \text{Smith (1998)}.
$$

 Then, so far, the continuity equation reduces to: . Then, so far, the continuity equation
reduces to:

$$
\nabla \cdot \vec{V} = 0 \quad \text{or:} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{4}
$$

d flow does not vary with time. equation, the change in the term pgh valion θ . Remember the field. **4. Remember that shear forces** be rattice from that she is the shear forces
places restrictions on the velocity field, (friction) and body forces (gravity) t isider a steady fluid flow. That is, for were ignored to get Eq. (1). But in α **h force h force hear force f** iations are sufficiently small, and let forces are considered like gravity, this, the density change is Bernoulli`s equation would include α small that can be assumed to be an extra term pgh as shown in Eq. (5). nd so, the density of the line many applications of Bernoulli`s In the change in potential energy) along the change in the term $\frac{1}{2}$ $\frac{\partial \rho}{\partial t} = 0$ the streamline flow is so small in ∂t and ∂t comparison to the other terms that it compassion to the other terms that it

can be ignored. For example, in the can be ignored. For example, in the case of an aircraft in flight, the change ty change is very small for in height "h" along a streamline flow is so small that the term ρgh can be disregarded. $\ddot{}$ \overline{c} but in the case where conservative body D_{new} is equation would include an extra term $\frac{1}{2}$. disponential in height θ . Bomombor that choor forces $\frac{1}{2}$ (includit) and body forces $\frac{1}{2}$ and $\frac{1}{2}$. the case where conservative body Bernoulli`s equation would include potential energy $\int f(x) f(x) dx$ is so small in comparison to the other streamline to the other small in comparison to the oth ne flow is so small in

$$
\frac{1}{2}\rho V^2 + P + \rho gh = const \quad (5)
$$

$$
\frac{1}{2}\rho V^2 + P = const = \text{Ptotal}
$$

where

Dynamic pressure = Total Pressure = Ptotal Static pressure = $\frac{1}{2}\rho V^2$
Dynamic pressure = Total Pressure = Ptotal

5. Inviscid Flow

as and has a value of term, can be ignored when compared $\frac{1}{100}$ respectively. The product of viscosity times shear velocity gradient defines the term velocity gradient dennes the term
shear stress. We must understand that, there are no real fluids for which viscosity is zero. But, there are many real cases where this product is sufficiently small that, the shear stress to other terms in the governing equations as described by Bertin and Smith (1998).

small that, the shear stress term, can be ignored when compared to other terms in ⃗⃗⃗ in Eq. (6) for irrotational flow must be zero as defined by Bertin and Smith (1998). → **6. Irrotational flow**.

then the Vorticity Vector in Eq. (6) the given flow vel for irrotational flow must be zero as defined by Bertin and Smith (1998). for irrotational flow must be zero as or irrota If the 2D flow contains no singu $\mathcal{L} = \frac{1}{2} \sum_{i=1}^{n} \frac{1}{2} \sum_{j=1}^{n} \frac{1}{2} \sum$ (6) If the 2D flow contains no singularities, ے
م defined by Bertin and Smith (1998).
Evaluating t

$$
\vec{\omega} = \nabla \times \vec{V} = \left(i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} \right) \times (i u + j v) = 0
$$
\n(6)

\nEvaluating the partial derivative equations (8) and (9) yield equa (14) and (15) respectively.

\n
$$
\nabla \times \vec{V} = \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) k = 0
$$

\n
$$
\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}
$$

\nor

\n
$$
\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0
$$

\n(7)

then, this constant is only real along a
streamline. Here, we present an example
 $\omega = -2y - (-x) = x - 2y \neq 0$ If $\omega = 0$ (irrotational flow), then the $\partial y = -x$ (1). fluid flow. But if $\omega \neq 0$ (rotational flow),
then, this constant is only real along a linto Eq. (7) yields: $rac{1}{2}$ $\frac{1}{\omega}$ constant in Eq. (2) is real in all the Substituting equation e fluid flow. But if $\omega \neq 0$ (rotational flow), substituting equations (14) and fluid flow. $\frac{1}{2}$ $\frac{1}{2}$ or the consect way of using bernouin s of the correct way of using Bernoulli's $\omega = -2y - (-x) = x - 2y \neq 0$ present an example of the correct way of using Bernoulli`s equation. S_{max} is the strength α $\frac{dy}{dt}$ is real in all the substituting given points $(1/2)$ and (15) of the correct way of using Bernoulli's $\omega = -2y - (-x) - x - 2y + 0$ equation. $\frac{1}{2}$ is only it at along a

Exercise flow the constant in Eq. (2) is real in all the flow the flow of the fluid flow) can still use Bernor field at sea level $(\rho = 1.225 \frac{\text{kg}}{\text{s}})$ and along points are only the fluid fluid fluid fluid fluid fluid f defined by: present an example of the correct way of using Bernoulli`s equation. defined by: $\int t^{3}$ field at sea level $(\rho = 1.225 \frac{kg}{ft^3})$ and given defined by: field at sea level $(\rho = 1.225 \frac{Kg}{\epsilon A^3})$ and und

$$
u = x^2 - xy
$$
 (8) us
by

$$
v = \frac{y^2}{2} - 2xy
$$
 by Bertin and Smith (1998):
(9)
$$
\frac{dx}{dx} = \frac{dy}{dx}
$$
 (16)

Where " u " and " v " are defined in m/s.
First, we need to identify if continuity Where "u" and "v" are defined in m/s. $\frac{u}{u} - \frac{v}{v}$ $\frac{1}{2}$ solving the given velocity components shown in equation $\frac{1}{2}$ and $\frac{1}{2}$ and

equation is satisfied, or in other words, if veloci equation is satisfied, of in other words, in electry components show the velocity flow field is possible to exist. (8) and (9) one finds the where α and ν are defined in m/s. First, we need to identify if continuity Solving this \overline{a} in a 2D form is:

Then, which is a strong of the state of $\frac{\partial u}{\partial y}$ at $\frac{\partial u}{\partial x}$ Continuity equation in a $2D$ form is:

$$
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{10}
$$

 $\frac{\partial u}{\partial x}$ σ *x* $\frac{\partial u}{\partial x} = 2x - y$ \overline{a} $2x - y$ Then, $\frac{\partial u}{\partial x} = 2x - y$ (11)

and $\frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial y} = y - 2x$ (12)

into Eq. (10), yields: $=$ $\frac{1}{2}$ Substituting equations (11) and (12) $\frac{\text{Since this equation is}}{\text{then}}$ ∂y

$$
2x - y + y - 2x = 0 \qquad (13) \qquad d\varphi = \frac{\partial F}{\partial x} dx - \frac{\partial F}{\partial y}
$$

Eq. (13) shows that the given flow $Eq. (13)$ shows that the given flow
velocity field satisfy the continuity equation. Second, we need to find out if $\text{ctor} \quad$ in Eq. (6) and the given flow velocity field is rotational or irrotational. $\frac{1}{2}$ 6.**Irrotational flow**. If the 2D flow contains no singularities, then the Vorticity Vector Γ = $(1$ **onal flow.** If the singular contains the 2D flow contains no singularities or in the 2D flow contains no singularities, then the 2D flow contains no singularities, then the 2D flow contains no singularities, then the 2D f cond, we need to find out if velocity field satisfy the continuity
 $\frac{1}{2}$ and Γ (19) in Fig. irrotational flow must be zero as defined by Bertin and Smith (1998). velocity field satisfy the continuity
ow contains no singularities, equation. Second, we need to find out if \mathbf{L}

Evaluating the partial derivatives of equations (8) and (9) yield equations (14) and (15) respectively.
 $\frac{\partial v}{\partial u}$) ⁼ **⁰** $\sum_{n=1}^{\infty}$ and $\sum_{n=1}^{\infty}$ respectively. luatin. $\lim_{x \to 0}$ the partial der \mathbb{R}^2

$$
(\frac{\partial u}{\partial y})k = \mathbf{0}
$$
\n
$$
\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}
$$
\nr\n
$$
(\frac{\partial v}{\partial x}) = -2y
$$
\n(14)

and flow), then the

\nis real in all the

\n
$$
\frac{\partial u}{\partial y} = -x
$$
\n(15)

Eq. (2) is real in all the
ut if $\omega \neq 0$ (rotational flow) Substituting equations (14) and (15) $\lim_{n \to \infty} \frac{1}{n} \int_{\text{total}}^{\text{in}} \frac{1}{n} \cdot \frac{1}{n} \cdot$ $\lim_{\epsilon \to 0}$

$$
\omega = -2y - (-x) = x - 2y \neq 0
$$

It is clear that, the given velocity flow $\frac{1}{2}$ field is rotational. So, that means that we high velocity flow field is rotational. So, that means that we Solution and the sea letter two field at still use Bernoulli's eq. only if the two well $\alpha = 1.225 \frac{kg}{m}$ and can still use Bernoulli s eq. only if the two
given points are on the same streamline. $50₁₀$ given points are on the same streamline. $u = x^2 - xy$ (8) using the 2D streamline Eq. (16) defined If is clear that, the constant in Eq. (2) is real in Eq. (2) is real in all the flow. But if α is real in all the fluid flow. Let' me consider a 2D velocity flow
field at sea level $(a - 1.225 \frac{kg}{g})$ and It is clear that, the given velocity flow field is rotational. So, that means that we Let`me consider a 2D velocity flow ϵ can still use Bernoulli`s eq. only if the two $\frac{1}{f}$ $\frac{1}{f}$ $\frac{1}{f}$ $\frac{1}{f}$ $\frac{1}{f}$ and $\frac{1}{f}$ given points are on the same streamline. $\frac{e}{g}$ $L_{\nu} = 1.225 f_{t3}$ and given points are on the same streamline $\frac{1}{2}$ by:

Where "u" and "v" are defined in m/s.
$$
\frac{dx}{u} = \frac{dy}{v}
$$
 (16)

Solving this equation for the given or in other words, if velocity components shown in equations
is possible to exist. (8) and (9) one finds that (8) and (9) one finds that $\frac{1}{2}$. $\sum_{n=1}^{\infty}$ this equation

on in a 2D form is:
\n
$$
\frac{\partial v}{\partial y} = 0
$$
\n
$$
2x - y
$$
\n(11)\n
$$
\frac{y^2}{2} - 2xy
$$
\n
$$
y - 2x
$$
\n(12)\n
$$
y = 2y
$$
\n(13)

Since this equation is a point function, then ² [−] 2 \overline{a} $2 \text{ (11) and } (12)$ $\text{2} \text{ (12) the point function}$

$$
d\varphi = \frac{\partial F}{\partial x} dx - \frac{\partial F}{\partial y} dy = 0
$$

where
\n
$$
\frac{\partial F}{\partial x} = v = \frac{y^2}{2} - 2xy
$$
\n(17)
\n
$$
\frac{\partial F}{\partial y} = \frac{y^2}{2} - 2xy
$$
\n(17)
\n
$$
\frac{\partial F}{\partial z} = \frac{y^2}{2} - 2xy
$$
\n(17)
\n
$$
\frac{\partial F}{\partial x} = \frac{\partial F}{\partial y} = \frac{y^2}{2} - 2xy
$$
\n(17)

 $\frac{1}{2}$ with respect to $\frac{1}{2}$ and (9)

and

$$
\frac{\partial F}{\partial y} = -u = -(x^2 - xy) \quad (18)
$$

Integrating Eq. (17) with respect to even tho
'sidde " x " yields Integrating Eq. (17) with resp Γ Γ (17) \therefore 1 − 2 + (19)
− 2 + (19) $\int \frac{d\theta}{dt} \int \frac{d\theta}{dt}$ integrating Eq. (1/) with

$$
\varphi = F = \frac{y^2}{2}x - x^2y + f_{(y)} \quad (19)
$$
\n
$$
u = x^2 - xy
$$
\n
$$
v = \frac{y^2}{2} - 2xy
$$

Where $\varphi = F$ is the respective stream $v =$ function $\frac{1}{\sqrt{2}}$ re $\varphi = F$ is the respective stream Where $\varphi = F$ is the respective stream
function the derivation the derivative of Eq. (19) with respect to \mathcal{L} yields to \mathcal{L}

 π \cdots \cdots

$$
v = \frac{(y)^2}{2} - 2(x)(y) = \frac{(-1)^2}{2} - 2(x)(y)
$$

$$
\frac{\partial F}{\partial y} = yx - x^2 + f'(y)
$$
 (20)

$$
\frac{1}{2} - 2(-2) = \frac{1}{2} + 4 = \frac{9}{2} = \frac{1}{2}
$$

$$
-x2 + xy = yx - x2 + f'(y)
$$

$$
f'(y) = 0
$$

$$
V_1 = \sqrt{3}
$$

For point t

Therefore; $f_{(y)} = const$ and the streamline is: \mathbf{S} : and the streamline is: $\frac{2}{\pi}$ and the $\frac{2}{\pi}$ $\frac{1}{2}$ \mathcal{L} const and the $\mu = (2)$

$$
\varphi = \frac{y^2}{2}x - x^2y + const = 0 \quad (21)
$$

Now, if we intend to use Bernoulli's equation, for example to find the static $V_2 = \sqrt{(0)^2 + (-6)^2} = 6 \frac{m}{s}$ pressure difference between two points in the flow, we must be sure to have Using Bernoulli's
these two points on the same streamline these two points on the same streamline.
Consider these two points to be: (.1. 2) $P_1 + \frac{1}{2}\rho V_1^2 =$ Consider these two points to be: (-1, 2) $P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$ and (2, 2). The coordinates of these two
p. $\frac{1}{p_1 + (-\frac{1}{2})^2}$ $\frac{1}{p_2 + (-\frac{1}{2})^2}$ $\frac{1}{p_3 + (-\frac{1}{2})^2}$ $\frac{1}{p_4 + (-\frac{1}{2})^2}$ $\frac{1}{p_5 + (-\frac{1}{2})^2}$ on the streamline. Now, if we intend to use Bernoulli`s \overline{a} \overline{a} $\frac{1}{P_1 + - \ast 1}$ and (2, 2). The coordinates of these two points are defined in meters tend to us
mple to fi negative sign means opposite direction)
w, if we intend to use Bernoulli`s ire defined in meters. ² 1 U_s came at α matrix α

For point one (-1, 2), Eq. (21) results $P_1 - P_2 = 4.123 Pa$ in -4. t one $(-1, 2)$, Eq. (21) res For point one $(-1, 2)$, Eq. (21) results $\begin{bmatrix} 1 & 1 & 2 \end{bmatrix}$ For point one (-1, 2). Eq. (21) results $P_1 - P_2 =$ are defined in meters. are defined in meters. $\ln -4$. $\frac{1}{2}$ of point one $(-1, 2)$, $U = \frac{1}{2}$ is $E = \frac{1}{2}$ $(-1, 2)$. Eq. (21) results

$$
\frac{(2)^2}{2}(-1) - (-1)^2(2) = -4
$$

ere For point two (2, 2), Eq. (21) also $\frac{1}{2}$ are $\frac{1}{2}$ results in -4 ere For point two $(2, 2)$, Eq. (21) also T_{max} two points are on the same streamline, so we can use Bernoulline, so we can $results in -4$ T_{Sults} are on the same streamline, so we can use Bernoulline, so we can use T_{Sults} in -4

$$
\frac{1}{\partial x} - v - \frac{1}{2} - 2xy
$$
 (17)
and

$$
\frac{(2)^2}{2}(2) - (2)^2(2) = -4
$$

 $au = -(x^2 - xv)$ (18) Then, these two points are on the same $\frac{\partial F}{\partial y} = -u = -(x^2 - xy)$ (18) Inten, these two points are on the same
streamline, so we can use Bernoulli's equation only between these two points are on the same streamline, so we can use Bernoulli's equation only equation only between these two points
even though the flow is rotational. Now, using Eq. (8) and (9) ϵ q α $using Eq. (8) and (9)$ 1 respect to even though the flow is rotational. Now using Eq. (8) and (9)
 $u = x^2 - xy$
 $u = \frac{y^2}{2xy}$

$$
-x2y + f(y) (19)
$$

$$
u = x2 - xy
$$

$$
v = \frac{y2}{2} - 2xy
$$

Therefore stream

For point one $(-1, 2)$ $\frac{1}{2}$ For point one $(-1, 2)$

Then, taking the derivative of Eq. (19)
$$
u = (-1)^2 - (-1)(2) = 1 - (-2) = 3 \text{ m/s}
$$

\nwith respect to "y" yields
\n $v = \frac{(y)^2}{2} - 2(x)(y) = \frac{(-1)^2}{2} - 2(-1)(2) = \frac{3\pi}{2}$
\n $\frac{\partial F}{\partial y} = yx - x^2 + f'(y)$ (20)
\n $\frac{1}{2} - 2(-2) = \frac{1}{2} + 4 = \frac{9}{2} = 4.5 \text{ m/s}$
\nReplacing Eq. (20) into Eq. (18) yields
\n $V_1 = \sqrt{(3)^2 + (4.5)^2} = 5.41 \text{ m/s}$
\n $f'(y) = 0$ For point two (2, 2)

= const and the
$$
u = (2)^2 - (2)(2) = 4 - 4 = 0
$$
 m/s
\n $v = \frac{(2)^2}{2} - 2(2)(2) = 2 - 8 = -6$ m/s
\n $v + const = 0$ (21)

(negative sign means opposite direction) \mathcal{L} , \mathcal{L} , \mathcal{L} , \mathcal{L} , \mathcal{L} $= 0$ (21) (negative sign means opposite direction) $= 0$ (21)
(negative sign means opposite direction) 2 = √(0)2 + (0)2 +

Bernoulli's
the static
$$
V_2 = \sqrt{(0)^2 + (-6)^2} = 6 \ m/s
$$

Using Bernoulli's Eq. (2): $2 \sin \theta$ = 6.1.1.0 and 0 = 9.1.2.1. 2 = √(0)² + (−6)² = 6 / Using Bernoulli`s Eq. (2): Using Bernoulli`s Eq. (2): 2 = √(0)² + (−6)² = 6 / Using Bernoulli`s Eq. (2):

points are defined in meters. Now, if we intend to use Bernoulli`s eq, for example to find the static pressure difference between two points in the flow, we must be sure to have these two points on the same streamline. Consider these two points to be: (-1, 2) and (2, 2). The coordinates of these two points − ² + = 0 difference between two points in the flow, we must be sure to have these two points Consider these two points to be: (-1, 2) and (2, 2). The coordinates of these two points difference between two points inthe flow, we must be sure to have these two points ² [−] 2(2)(2) ⁼ ² – ⁸ ⁼ - ⁶ m/s (negative sign means opposite direction) 2 = √(0)² + (−6)² = 6 / ² 1 ² ⁼ 2 ⁺ ¹ ² 1 ² ⁼ 2 ⁺ ¹ ² 2 2 1 ⁺ ¹ ² ⁼ 2 ⁺ ¹ ² 2 1 + Using Bernoulli`s Eq. (2): 1 ⁺ ¹ ² 1 ² ⁼ 2 ⁺ ¹ ² 2 1 ⁺ ¹ ² 1 ² ⁼ 2 ⁺ ¹ ² 2 2 1 ² [∗] 1.225 [∗] 5.412 ⁼ 2 ⁺ 1 ² [∗] 1.225 [∗] ⁶² 1 + ² [∗] 1.225 [∗] 5.412 ⁼ 2 ⁺ 1 ² [∗] 1.225 [∗] ⁶² 1 − 2 = 4.123

The static pressure difference between $(2)^2$ and which are located on the same $(2)^2$ $\frac{1}{2}$ (-1) – (-1)²(2) = -4 streamline in the fluid flow is Pa. $T(t) = 1$ and which are located on the same The static pressure difference between
and which are located on the same

 $\overline{1}$

Conclusion

Throughout this paper, the correct way of using Bernoulli`s equation has been shown. Initially, it has been presented some assumptions for which this equation is valid to apply. These assumptions led to a set of restrictions that must be met in order to apply correctly this equation.

However, during the process of the application of Bernoulli`s equation, the analyst has to be sure which restrictions apply for the particular case. According to this, the results must be presented in a similar way as it was done in this paper like the pressure difference between the two points on the same streamline.

References

- Anderson, Jr, J.D. (1989). *Introduction to Flight*. 3rd ed. New York: McGraw-Hill.
- Anderson, Jr. J.D. (2003). *Modern Compressible Flow with Historical*

Perspective. 2nd ed. New York: McGraw-Hill.

Bertin, J.J. and Smith, M.L. (1998). Aerodynamics for Engineer. 3rd ed. New Jersey: Prentice Hall.