# **Perfiles aerodinámicos Joukovsky notas suplementarias de aerodinámica**

**Abstract**

Joukowski airfoils supplementary notes to aerodynamics

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Mapping a cylinder into a Joukowski airfoil, takes several geometric steps that have to be carefully considered. By doing this, the velocity around the cylinder has to be related to the velocity at the surface of the airfoil. Once the velocity at the surface of the airfoil has been determined, the pressure distribution on the airfoil can be obtained using the pressure coefficient. Moreover, the value of the Circulation Γ must be obtained from Kutta´s condition. According to this condition, the flow must leave the trailing edge smoothly and because of this, the velocity components on the trailing edge which are a function of Γ are equal to zero. Solving this equation equal to zero becomes easy when we determine the circulation Γ. Hence, by using the Kutta-Joukowski theorem and the circulation that has been previously found, the lift can be determined. Then, the lift coefficient can be found as well.

*Keywords:* Joukowski airfoil, airfoil, pressure distribution, pressure coefficient, Circulation, Kutta´s condition, trailing edge, Kutta-Joukowski theorem, lift, lift coefficient

### **Resumen**

Levantando el plano de un cilindro al plano de un perfil aerodinámico Joukowski, lleva varios pasos geométricos que tienen que ser considerados cuidadosamente. Al realizar esto, la velocidad alrededor del cilindro tiene que estar relacionada con la velocidad en la superficie de sustentación del perfil aerodinámico. Una vez que la velocidad de este último haya sido determinada, la distribución de presión sobre la superficie de sustentación se puede obtener por medio del coeficiente de presión. Además, el valor de la circulación Γ se debe obtener de la condición de Kutta. De acuerdo con esta condición, el flujo debe dejar el borde de fuga suavemente, y debido a esto, los componentes de la velocidad en el borde de fuga que son una función de Γ son cero. Resolver esta ecuación que es igual a cero resulta fácil al determinar la circulación Γ. Por lo tanto, considerando el teorema de Kutta-Joukowski y la circulación previamente determinada, la sustentación puede ser encontrada y, entonces también, el coeficiente de sustentación Cl puede ser hallado.

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*Palabras clave:* perfil aerodinámico, Joukowski, superficie de sustentación, distribución de presión, coeficiente de presión, circulación, condición de Kutta, borde de fuga, teorema de Kutta-Joukowski, sustentación, coeficiente de sustentación

### **Introduction**

One of the major results obtained in Aerodynamics is the calculation of the ideal flow past a cylinder. Because we do not see flying cylinders, you may wonder why this result is so important. Two reasons will be given here. The cylinder has a simple shape and can be easily tested in a wind tunnel to determine the limitations of ideal fluid theory. Moreover, the cylinder can be mapped into arbitrary shapes by using appropriate mapping functions. This allows us to determine the velocity and the pressure distribution on a body of arbitrary shape.

### Discussion

These notes will describe the procedure that maps a cylinder into an airfoil. As an an We can write t illustration, a Joukowski airfoil will be masturitien, a Joursville airform will be considered. Before we do this, we need to The result can b review some results relevant to flow past cylinders. cylinders.  $\Psi = U_{\infty} | 1 - \frac{\Omega}{\Omega}$ 

If the origin of the coordinates coincides with the origin of a cylinder of radius R, then the stream function, Ψ, or potential determine the function,  $\Phi$ , as described by Bertin and  $y'$ ) and  $(x, y)$  $Smit (1998)$  for a lifting cylinder can be obtained by superimposing a uniform flow in the x-direction, a doublet at the origin  $\qquad$  the cylinder both whose strength is:

$$
\hat{A} = R^2 U_{\infty} \tag{1}
$$

and a vortex at the origin of strength  $\Gamma$ 

(gamma). The result can be written as:  
\n
$$
\Psi = U_{\infty} \left[ 1 - \frac{R^2}{r^2} \right] r \sin \theta + \frac{\Gamma}{2\pi} \ln \left( \frac{r}{R} \right)
$$
\n(2)

and

$$
\phi = U_{\infty} \left[ 1 + \frac{R^2}{r^2} \right] r \cos \theta - \frac{\Gamma \theta}{2\pi}
$$
 (3)  
It is noted that

As a first step in achieving our goal, Let us determine the stream function for a  $x = x'' - a_c$ cylinder whose center is not at the origin of  $\mathcal{L}$  first step in achieving our goal, let us determine the stream function for a cylinder st

coordinates and whose angle of attack  $\alpha$  is different from zero.



*Figure 1*. Schematic of a Cylinder

These notes will describe the procedure coordinate axes x  $x'$  is the angle of attack  $\alpha$ . Figure 1 shows such a cylinder. Note that of arbitrary shape.  $\begin{array}{r} \text{the } x' \text{-coordinate is the direction of the} \end{array}$ freestream velocity and y' is normal to the  $f_{\text{reconram}}$  velocity and  $f_{\text{reconram}}$  is normal to the freestream direction. The angle between the We can write the stream function in terms which an anton. As an we can write the stream runction in terms<br>wski airfoil will be of the coordinate  $(x', y')$  using equation 2. The result can be written as:

$$
\Psi = U_{\infty} \left[ 1 - \frac{R^2}{r'^2} \right] r' \sin \theta' + \frac{\Gamma}{2\pi} \ln \left( \frac{r'}{R} \right) \tag{4}
$$

 $\mathbf{y} = \nabla \cdot \mathbf{y}$  To determine  $\mathbf{y} = \Psi(x, y)$ , we have to ", or potential determine the relationship between  $(x',$  $y'$ ) and  $(x, y)$ . To accomplish this, let us  $x^2$  introduce the coordinate system  $(x'', y'')$ **)** whose origin coincides with the center of loublet at the origin  $\qquad$  the cylinder but is parallel to  $(x, y)$ .



*Figure 2.* Relationship between (x',y') and  $(x,y)$ 

It is noted from Figure 2 that It is noted from Figure 2 that

$$
x = x'' - ac , \qquad y = y'' + bc (5)
$$

*x*<sup>0</sup> where are the coordinates at the center of the cylinder. Again, we see from Figure 2 that: *x*′ = *x*′′cos<sup>α</sup> + *y*′′sin<sup>α</sup> *y*<sup> $\overline{y}$ </sup> written as:

$$
x' = x'' \cos \alpha + y'' \sin \alpha
$$
  
\n
$$
y' = |-x'' \sin \alpha| + y'' \cos \alpha
$$
 (6)  
\nThe resulting shape is

Combining (5) and (6) yields the desired in Figure 4. The actual *x*<sup>2</sup>  $\frac{1}{2}$   $\frac{1$  $\sum_{i=1}^{n}$  and  $\sum_{i=1}^{n}$  yields the desired relationship, i.e. ho dosiro

$$
x' = (x + a_c) \cos \alpha + (y - b_c) \sin \alpha
$$
 The point A (Figure  
coordinate  

$$
y' = \begin{vmatrix} -(x + a_c) \sin \alpha \\ + (y - b_c) \cos \alpha \\ (z + b_c) \cos \alpha \\ r = a \end{vmatrix}
$$

Moreover, we can relate  $\theta$  and  $\theta'$  by writing the coordinate of any arbitrary point maps into the trail!<br>B on the cylinder ax: Substituting equatio B on the cylinder as:

$$
R\sin(\theta' + \alpha) = r\sin\theta - b_c
$$
 yields the coordinate

equation 4, we can determine  $\Psi = \Psi(x, y)$ .  $T(\omega)$ , if we substitute for  $\chi$   $\chi$  must are  $\chi$  flow part at the original  $\Pi$ and whose angle of attack is α.

This gives the stream function for a flow past a cylinder whose center is not at the This gives the stream function for a flow parameter is α. and whose angle of attack is angle of attack is  $\alpha$ . This **Joukowski Transformation**  $\Omega$  origin and whose angle of attack is  $\alpha$ .

#### **Joukowski Transformation** and whose angle of attack is angle of attack is angle of attack is a.  $e^{i\theta}$  which we can be written as:

Now we are ready to talk about mapping. Figure 4. Schematic As can be seen from Figure 3, the equation of the circle under consideration can be Next (theta) written as:

$$
R^{2} = l^{2} + r^{2} + (2lr)\cos(\theta + \delta)
$$
 (9)

with equation of the circle under consideration can be written as:

$$
l^2 = b_c^2 + a_c^2
$$
,  $\tan \delta = b_c / a_c$  (10)



*Figure 3.* Schematic of a Circle

The Joukowski transformation that transforms equation 9 into an airfoil can be<br>written as: written as: *x* youxowski and transformation that  $\frac{d}{dx}$  are  $\frac{d}{dx}$  and  $\frac{d}{dx}$   $\frac{d}{$ 

$$
x_1 = x(1 + \frac{a^2}{r^2}), \qquad y_1 = y(1 - \frac{a^2}{r^2}) \quad (11)
$$

The resulting shape is shown schematically in Figure 4. The actual shape is calculated as follows:  $T_{\text{total}}$  (b) yields the desired  $\frac{6}{\text{follows}}$ .  $\frac{1}{1}$  shown scheme is shown scheme is shown scheme in  $\frac{1}{1}$ 

> $\ln \alpha$  The point A (Figure 3), which has the coordinate

$$
r = a, \qquad \theta = 0 \qquad (12)
$$

*y*ields the coordinate of the trailing edge as:<br> $R\sin(\theta' + \alpha) = r\sin\theta - b$  (8) maps into the trailing edge of the airfoil. Substituting equation 12 into equation 11 maps into the trailing edge of the trailing edge as:



bout mapping. *Figure 4.* Schematic of a Joukowski Airfoil

where the set of a  $r = r(\theta)$  and since  $\theta$  and since Next (theta) is stepped from 0 to  $2\pi$ using, say, 5° intervals. For a given theta,

$$
x = r \cos \theta, \qquad y = r \sin \theta
$$

 $l^2 = b_c^2 + a_c^2$ ,  $\tan \delta = b_c/a_c$  (10) the process will allow us to generate the  $x_1, y_1$  follow from Equation 11. Repeating  $\alpha$  desired shape.

**2** and the free stream velocity, nothing Although we were able to relate the strength of the doublet to the radius of the **A** was said, so far, about Γ. Obviously, we need to determine Γ if we hope to calculate the *x* = *r*  $\checkmark$  we ocity and the pressure distribution over the airfoil. The value of  $\Gamma$  is determined to determine from Kutta´s condition. According to this determine the velocity and the pressure over the pressure o condition, the flow must leave the trailing  $\overline{\phantom{a}}$  was said, so rad, about T. Obviously, we need

edge smoothly as described by Anderson, Jr *r*′ = *R* , <sup>θ</sup> ′ = −(<sup>α</sup> + β ) (14) stagnation point. This condition prevents the flow from turning around the trailing edge (1989). This, in turn, requires the point A on the cylinder, which maps into the trailing angle if the edge, to be a stagnation point. This condition This allows prevents the flow from turning around the trailing edge and insures the continuity of the pressure at the trailing edge. The and as a result we have implications of A being a stagnation point follows from calculating the velocities at  $C_l = 2\pi(\alpha + \beta)$ <br>A. If we consider the  $(x', y')$  coordinate A. If we consider the  $(x', y')$  coordinate<br>system (see Figure 3), then the point A has For symmetric airfoils, A. If we consider the  $(x', y')$  coordinate<br>system (see Figure 3), then the point A has For sy the coordinates: or the pressure at the training edge. The and as edge smoothly as described by Anderson, Jr Usua The velocity components are: no ine training and and **Point A has** For symmetric airf

$$
r' = R
$$
,  $\theta' = -(\alpha + \beta)$  (14)  
which gives

 $\left( \begin{array}{c} a & b \\ c & d \end{array} \right)$ <sup>(11)</sup> when gives<br>velocity components are:  $\theta = 0$ velocity components are *D* <sup>2</sup> F <sup>2</sup> ts are:

$$
D = 0
$$
  

$$
U'(\theta) = -\frac{\partial \Psi}{\partial r'} = -[U_{\infty}(1 + \frac{R^2}{r'^2})\sin\theta' + \frac{\Gamma}{2\pi r'}]
$$
 (15) and the lift coefficient for a symmetric airfoil

$$
U'(r) = \frac{1\partial \Psi}{r'\partial \theta'} = U_{\infty} (1 - \frac{R^2}{r'^2}) \cos \theta' \qquad (16)
$$
 reduces to:

For the conditions indicated in equation<br>(1*0*) acception (1*5*) and acception (1*0*) For the conditions indicated in equation<br>(14), equation (15) and equation (16)<br>become The pressure distribution become ∂ ′ *r <i>r*  $\overline{r}$  *r*  $\overline{r}$  *r*  $\overline{r}$  *r*  $\overline{r}$  *r*  $\overline{r}$  *r*  $\overline{r}$  *r*  $\overline{r}$  *<i>r*  $\overline{r}$  *r*  $\overline{r}$  *r*  $\overline{r}$  *r*  $\overline{r}$  *r*  $\overline{r}$  *<i>r*  $\overline{r}$  *r*  $\overline{r}$  *r*  $\over$ indicated in equation<br>and equation (16) **Pressure Distribution on the Airfoil** *r 00*<br>For the conditions indicated in equation

$$
U'_{r} = 0, \quad U'_{\theta} = 0 = -[-2U_{\infty}\sin(\alpha + \beta) + \frac{\Gamma}{2\pi R}] \tag{17}
$$

7) yields Γ = 2(2<sup>π</sup> sin(<sup>α</sup> + β )) *RU*∞ (18)

equation (17) yields  
\n
$$
\Gamma = 2(2\pi R U_{\infty} \sin(\alpha + \beta))
$$
\n(18)

Now, using the Kutta-Joukowski theorem <sup>where</sup> sub described in Bertin and Smith (1998), the pressure coefficient defined lift can be determined as:  $\sum_{i=1}^{n} a_i$  the Kutta-Joukowski theorem described in Bertin and Smith (1998), the life smith (1998), the can be determined as:  $S_{\rm tot}$  is a set of  $S_{\rm tot}$  (1000)

$$
L = \rho U_{\infty} \Gamma = 4\pi \rho (U_{\infty})^2 \text{Rsin}(\alpha + \beta) \quad (19) \qquad \qquad C_p = \frac{P_a - P_a}{\frac{1}{2}\rho U}
$$

For typical airfoils as,  $a_c$ ,  $b_c$  are small. Because of this, β is a small angle and the  $\frac{1}{2}$  and  $\frac{1}{2}$  $\frac{1}{\text{Area}}$  of this,  $\frac{1}{\text{Area}}$  is a small algebra and the  $\frac{1}{\text{base}}$ [see equation 13]: Eq. (17) yields For typical airfoils as,  $a_c, b_c$  are small.  $2'$   $2'$  (2)

$$
c \approx 4R \tag{20}
$$

As a result, the lift coefficient  $C<sub>l</sub>$  defined by Anderson, Jr. (1989) can be expressed as:  $\Delta S$ , using the Kutta-Joukowski theorem described in  $I$  $\alpha$  the Kutta-Joukowski theorem described in Bertin and Smith (1998), the lift (1998), the lift (1998), the lift (1998), the lift (1998), the life (1998 ) can be expressed

$$
C_{l} = \frac{L}{\frac{1}{2}\rho U_{\infty}^{2}c} = 2\pi \sin(\alpha + \beta)
$$
\n
$$
U
$$
\n(21)

d by Anderson, Jr Usually, the flow separates at high angles of attack. Therefore, α should remain a small be into the trailing angle if the above theory is to remain valid. This allows as setting: by Anderson, Ir **Example 1** Usually, the flow separates at high angles  $=$  2  $\frac{1}{2}$   $\$ 

$$
\sin(\alpha + \beta) \approx (\alpha + \beta)
$$

and as a result we have

$$
C_1 = 2\pi(\alpha + \beta) \tag{22}
$$

] <sup>2</sup> )sin For symmetric airfoils, = 2<sup>π</sup> (<sup>α</sup> + β ) *Cl* (22)  $\ddot{o}$   $\ddot{o}$ 

$$
b_c = 0
$$

**Pressure 2 Pressure Distribution on the Air Section** of the Air Section of the Air Sec  $T(14)$  which gives

 $\beta = 0$ 

and the lift coefficient for a symmetric airfoil<br>reduces to: z<sub>ar</sub> (22) and the lift coefficient for a symmetric a<br> **Pressure Distribution on the Airfor**lian on the Air symmetric air and the Air coefficient for a symmetric and  $\sqrt[r]{1-\frac{1}{2m'}}$  (15) and the lift coefficient for a symmetric a  $\Gamma = 2\pi\alpha$ **1 Pa**  $\alpha$  + *Pa* 

$$
\text{reduces to:} \\
 C_l = 2\pi\alpha \qquad (23)
$$

## $T_{\text{test}}$  from Bernoullion on the Arrivia **Pressure Distribution on the Airfoil**

and equation (16)<br>The pressure distribution follows from  $=-[-2U_{\infty}\sin(\alpha+\beta)+\frac{\Gamma}{2\pi R}]$  (17) Bernoullis equation being defined to:<br>Anderson, Jr. (1989) as Bernoulli's equation being defined by<br>Anderson, Jr. (1989) as *Cl*  $\frac{1}{2}$  = 2πα (23) **a**  $\frac{2\pi}{1}$   $\left(1\right)$   $\frac{2\pi}{1}$   $\left(1/9\right)$  as where subscript " " *a* designation" (10*)*<br>The pressure coefficient defined by Bertin and Bertin and by Bertin and Bertin and Bertin and Bertin and Bertin and

equation (17) yields  
\n
$$
\Gamma = 2(2\pi R U_{\infty} \sin(\alpha + \beta))
$$
\n(18)

Now, using the Kutta-Joukowski theorem described in  $B$ ertin and  $S$ mith (1998), the lift (1998), the where subscript  $a^{\dagger} a^{\dagger}$  designates an airfoil. The ith (1998), the pressure coefficient defined by Bertin and Smith (1998) is  $\frac{1}{100}$   $\frac{1}{100}$   $\frac{1}{100}$   $\frac{1}{100}$ ubscript a designates an airfoil. The<br>e coefficient defined by Bertin and<br>1998) is

$$
C_p = \frac{P_a - P_\infty}{\frac{1}{2}\rho U_\infty^2} = 1 - (\frac{U_a}{U_\infty})^2
$$
  
as,  $a_c, b_c$  are small. (25)

*CO*<sup>γ</sup> cylinder. The relationship follows from<br>As a result, the lift coefficient G defined velocity on the airlon surface. This will be accomplished by relating the velocity at Thus, we need to determine the  $c \approx 4R$  (20) the airfoil surface to the velocity  $U_c$  at the *In* angle and the Thus, we need to determine the approximated as velocity on the airfoil surface. This will be  $1$  (  $1$  (  $1$  ) (  $1$ <sup>∞</sup> <sup>=</sup> <sup>−</sup> <sup>−</sup> <sup>=</sup> *<sup>U</sup> <sup>P</sup> <sup>P</sup> <sup>C</sup> <sup>a</sup> <sup>a</sup> p* ρ (25)  $\begin{bmatrix} 20 \end{bmatrix}$  are affected to the velocity  $\mathcal{C}_c$  at the cylinder. The relationship follows from

> $\int_{a}^{b} \int_{a}^{b} e^{ibx} e$  $\Gamma = \int U_a dS_a = \int U_c dS_c$

or

$$
U_a = U_c \left(\frac{dS_c}{dS_a}\right) \tag{26}
$$

where  $dS_a$  and  $dS_c$  are arc lengths around the or<br>*airfoil and the cylinder respectively*. Now where  $\omega_a$  and  $\omega_c$  are are rengins around airfoil and the cylinder respectively. Now *r x r* and the cylinder respective.  $rac{1}{2}$ 2

$$
dS_c = \sqrt{(dx)^2 + (dy)^2} ; \qquad \frac{dS_a}{dS_c} = [1 - 2\frac{a}{r^2}]
$$
  

$$
dS_a = \sqrt{(dx_1)^2 + (dy_1)^2} \qquad (27)
$$
  

$$
U_c = U'_a = -[2\frac{a}{r^2}]
$$

The relationship between  $(x_1, y_1)$  *y*  $(x, y)$  is given by equation 11. equation 11 gives equation 11. equation 11 gives with  $\theta'$  given by Eq. (8), i.e. *a* given by equation 11. equation 11 gives *r y x xy* The relationship between  $(x_1, y_1)$  *y*  $(x, y)$  is given by equation 11. equation 11 gives 2 2 2 2 2 ip between  $(x_1, y_1)$  y  $(x, y)$  is

$$
dx_1 = dx\left(1 + \frac{a^2}{r^2}\right) - 2x\frac{a^2}{r^4}(xdx + ydy) \qquad \theta' = dx\left[1 + \frac{a^2}{r^2}(y^2 - x^2)\right] - 2xy\frac{a^2}{r^4}dy \qquad \text{Equation} \text{follows from the following equation} \text{since} \qquad \text{range of } \theta
$$

since discussed by the contract of t *x*  $\frac{1}{2}$   $\frac{1}{2}$ 

since  
\n
$$
x = r \cos \theta
$$
,  $y = r \sin \theta$  range of  $\theta'$  is fr

$$
dx_1 = dx \left[1 - \frac{a^2}{r^2} \cos 2\theta\right] - \sin 2\theta \frac{a^2}{r^2} dy
$$
 (28) We can determine the velocity on the  
airfoil surface and the pressure distribution

*x* similarly  $\frac{1}{2}$  = *r* 

$$
dy_1 = dy[1 - \frac{a^2}{r^2} \cos 2\theta] + \sin 2\theta \frac{a^2}{r^2} dx
$$
\n(29)

Therefore

Therefore  
\n
$$
dS^{2}{}_{a} = [(1 - \frac{a^{2}}{r^{2}}cos2\theta)^{2} + (\frac{a^{2}}{r^{2}}sin2\theta)^{2}]dS_{c}^{2}
$$
\n
$$
dS^{2}{}_{a} = [(1 - \frac{a^{2}}{r^{2}}cos2\theta)^{2} + (\frac{a^{2}}{r^{2}}sin2\theta)^{2}]dS_{c}^{2}
$$
\n(30) as the usage

or

$$
\frac{dS_a}{dS_c} = [1 - 2\frac{a^2}{r^2}\cos 2\theta + (\frac{a^2}{r^2})^2]^{1/2}
$$
(31)

*Uc* follows from Eq. (4) as

$$
u_{\nu_a} - \sqrt{(u_{11} + (u_{21}) - (27))}
$$
  
The relationship between  $(x_1, y_1)$   $y(x, y)$  is 
$$
U_c = U'_{\theta} = -[2U_{\infty} \sin \theta' + \frac{\Gamma}{2\pi R}]
$$
 (32)

$$
\theta' \text{ given by Eq. (8), i.e.}
$$
  

$$
\theta' = \sin \theta^{-1} \left[ \frac{r \sin \theta - b_c}{R} \right] - a
$$
(33)

follows from equation 25. Note that the range of  $\mathcal{C}(\mathcal{A})$  is and  $\mathcal{C}(\mathcal{A})$  of  $\mathcal{C}(\mathcal{A})$  is an analyze of  $\mathcal{C}(\mathcal{A})$  is an analyze of  $\mathcal{C}(\mathcal{A})$  is a contract of  $\mathcal{C}(\mathcal{A})$  is a contract of  $\mathcal{$ Equation 26, 31 and 32 give *Ua* and *Cp* range of  $\theta'$  is from 0 to  $2\pi$ .

### **Conclusion**

 $\frac{1}{2}$  =  $\frac{1}{2}$   $\frac{1}{2}$  The method presented here, is an alternative  $r^2$  and  $r^2$  (29)  $C_l$ . However, this can be done if we map  $r^2$   $\frac{1}{28}$   $\frac{1}{28}$   $\frac{1}{28}$  airfoil surface and the pressure distribution  $= [(1 - \frac{1}{r^2} \cos 2\theta)^2 + (\frac{1}{r^2} \sin 2\theta)^2] dS_c$  complicated mathematical calculations such  $\frac{a^2}{2}$  =  $\frac{a^2}{2}$  $\alpha_1 = dy \left[1 - \frac{u}{x^2} \cos 2\theta\right] + \sin 2\theta \frac{u}{x^2} dx$  determined too and so, the lift coefficient at the cylinder. Consequently, the lift can be  $\begin{array}{lll}\n\text{(2)} & \text{C}_1. \text{ However, this can be done in we map} \\
\text{a cylinder into a Joukowski airfoil correctly.}\n\end{array}$ determined to and solutions of complex numbers.<br>
(30) as the usage of complex numbers. over the airfoil just by knowing the velocity of such a procedure that reduces other

#### 1 = References <sup>2</sup> [(1 cos2 ) ( sin 2 ) ] *<sup>c</sup> <sup>a</sup> dS* a *d* + *dS* **References**

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