STUDY OF MUSLIMS IN MARITAL SYSTEM USING MARKOV CHAIN SIMPLE EXPONENTIAL SMOOTHING (MC_{SES}) TECHNIQUE

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Abstrak

Model Rantai Markov (MC) merupakan model m atik yang popular digunakan untuk melihat aliran data dalam sistem. Model ini boleh digunakan untuk meramal nilai masa depan bagi tempoh jangka pendek. Walau bagaimanapun, kebanyakan kajian lalu tidak memberi tumpuan kepada ketepatan nilai ramalan. Integrasi model MC dan teknik Purata Bergerak Mudah (SMA) diketahui dapat menghasilkan ketepatan ramalan yang lebih tinggi daripada model MC yang asal bagi kes unjuran jangka masa panjang di mana data bagi tempoh sebelumnya diketahui. Waiau bagaimanapun, teknik pelicinan eksponen mudah (SES) adalah lebih fleksibel daripada SMA kerana ia menggunakan pemalar kelicinan. Oleh yang demikian, kajian ini membangunkan langkah model bagi model MC_{SES} bagi data terhad danunjuran jangka masa pendek dengan mengintegrasikan model MC dengan SES. Model MC_{SES} hibrid digunakan untuk mengukuhkan model MC dan meningkatkan ketepatan nilai ramalan. Empat pengukur ralat yang digunakan untuk menentukan ketepatan model ini adalah min sisihan mutlak, min peratusan sisihan mutlak , min peratusan ralat mutlak dan min kuasa dua ralat. Kajian ini menggunakan sampel sebanyak 6061 pasangan Islam di daerah Pendang, Kedah yang terdapat dalam pangkalan data sistem perkahwinan bagi tahun 2013 dan 2014. Jangkaan bilangan umat Islam di Pendang berdasarkan jantina dan kategori umur bagi tahun yang berikutnya dibuat menggunakan model hibrid MC_{SES}. Perbandingan dengan model MC dan model MC_{SMA} menunjukkan bahawa model MC_{SES} kacukan menghasilkan ketepatan ramalan yang lebih baik. Oleh itu, dicadangkan model MC_{SES} adalah model yang paling sesuai dalam membuat jangkaan bilangan umat Islam yang terdapat dalam sistem perkahwinan berdasarkan jantina dan kategori umur bagi tahun 2014. Model ini boleh menyumbang dalam kes-kes yang terdapatnya data yang terhad dan untuk unjuran jangka masa pendek serta boleh juga digunakan dalam pelbagai bidang lain.

Kata kunci: Model hibrid rantai markov, Ketepatan ramalan, Pelicinan eksponen mudah

Abstract

The Markov Chain (MC) model is a popular mathematical model used to observe the flow of data in a system. It can also be used to forecast future values for short-term period. However, most previous studies do not focus on the accuracy of the forecast values. Integration of the MC model and Simple Moving Average (SMA) technique is known to produce higher forecast accuracy than the classical MC model for the case of long-term projection with known previous data. However, Simple Exponential Smoothing (SES) technique is more flexible than SMA because it uses a smoothing constant. Therefore, this study develops modeling steps for MC model in the case of limited data and short-term projection by integrating MC model with SES (MC_{SES}). The MC_{SES} hybrid model is used to enhance the MC model and improve the accuracy of the forecast values. Four error measures used to determine the accuracy of this model are mean absolute deviation, mean absolute percentage deviation, mean absolute percentage error and mean square error. This study uses a sample of 6061 Muslim couples data in Pendang, Kedah who are in the marital system for the year 2013 and 2014. The number of Muslims in subsequence year according to gender and age categories is forecasted using proposed MC_{SES} hybrid model. Comparison with MC and MC_{SMA} models indicates that the developed MC_{SES} hybrid model has better forecast accuracy. Therefore, the MC_{SES} hybrid model is the most appropriate model to forecast the number of Muslims in the marital system according to gender and age categories for the year 2014. This model can be used for short-term projection in cases with limited data and is applicable in various fields.

Keywords: Hybrid markov chain model, Forecast accuracy, Simple exponential smoothing

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List of Abbreviations

ARMA	autoregressive moving average
ARRES	adaptive response rate exponential smoothing
MAD	mean absolute deviation
MAE	mean absolute error
MAPD	mean absolute percentage deviation
MAPE	mean absolute percentage error
MSE	mean square error
RMSE	root mean square error
SSE	sum of squared errors

CHAPTER ONE

INTRODUCTION

1.1 Background of the Study

A stochastic process is a random process which can be used to predict a set of possible outcomes weighted by their probabilities. It considers some indeterminacy in the future evolution which can be described by probability distributions. A stochastic process is the opposite of a deterministic process which deals with only possible reality of how the process might evolve overtime (Adeleke, Oguntuase, & Ogunsakin, 2014).

A Markov process can also be defined as a stochastic process since it involves probabilities. This process is only concerned with the current data without references to historical data. Markov process deals with state space and a Markov process that is in a discrete state is known as a Markov Chain.

A Markov Chain (MC) model, which is named after Andrei Andreyevich Markov, a Russian mathematician, is a mathematical system that undergoes transitions from one state to another. It involves a finite or countable number of possible states (Borah & Kakaty, 2012). It is modeled using current data which is in discrete state where the historical data are not necessary in order to predict the future values (Taylor & Karlin, 1998).

MC model is a popular mathematical model that is used to examine the flow of data using the stochastic processes and also forecasting future values for long-term and

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Appendix A

Smoothing of Values

Table 1

Interpolated Values of 34 Transient Cases for the Previous 4 Periods, $V_{tran}^{c-1}, V_{tran}^{c-2}, V_{tran}^{c-3}$, and V_{tran}^{c-4}

Coding of Transient Cases	V_{tran}^{c-1}	V_{tran}^{c-2}	V_{tran}^{c-3}	V_{tran}^{c-4}
$C_{1,1}$	612	522	583	499
$C_{1,2}$	243	224	230	213
$C_{2,2}$	903	718	844	679
$C_{2,3}$	114	108	107	101
$C_{3,3}$	241	223	228	212
$C_{3,4}$	31	29	28	27
$C_{4,4}$	82	77	76	73
$C_{4,5}$	11	10	9	9
$C_{5,5}$	29	27	26	25
$C_{5,6}$	2	2	2	2
$C_{6,6}$	22	20	19	19
$C_{6,7}$	3	3	3	3
$C_{7,7}$	17	15	15	14
$C_{7,8}$	2	2	2	2
$C_{8,8}$	19	17	17	16
$C_{8,9}$	4	3	3	3
$C_{9,9}$	40	38	37	36
$C_{10,10}$	977	763	907	718
$C_{10,11}$	290	266	276	254
$C_{11,11}$	666	562	633	536
$C_{11,12}$	60	57	56	54
$C_{12,12}$	145	136	136	129
$C_{12,13}$	19	17	17	16
$C_{13,13}$	63	60	58	56
$C_{13,14}$	9	8	7	7
$C_{14,14}$	28	26	25	24
$C_{14,15}$	4	7	6	6
$C_{15,15}$	20	18	18	17

Table 1 continued.

$C_{15,16}$	1	0	0	0
$C_{16,16}$	18	16	16	15
$C_{16,17}$	1	1	1	1
$C_{17,17}$	8	7	6	6
$C_{17,18}$	2	2	2	2
$C_{18,18}$	14	12	12	12

Table 2

Interpolated Values of 18 Absorbing Cases for the Previous 4 Periods, $V_{abs}^{c-1}, V_{abs}^{c-2}, V_{abs}^{c-3}$, and V_{abs}^{c-4}

Coding of Absorbing Cases	V_{abs}^{c-1}	V_{abs}^{c-2}	V_{abs}^{c-3}	V_{abs}^{c-4}
$C_{1,19}$	28	32	37	46
$C_{2,19}$	96	87	76	62
$C_{3,19}$	90	83	75	63
$C_{4,19}$	72	71	69	65
$C_{5,19}$	70	70	69	65
$C_{6,19}$	56	59	62	66
$C_{7,19}$	38	42	48	59
$C_{8,19}$	23	27	31	35
$C_{9,19}$	56	58	61	66
$C_{10,19}$	60	62	64	66
$C_{11,19}$	109	93	77	61
$C_{12,19}$	108	93	77	61
$C_{13,19}$	63	64	65	66
$C_{14,19}$	55	58	61	66
$C_{15,19}$	38	42	48	59
$C_{16,19}$	30	34	40	49
$C_{17,19}$	23	27	31	35
$C_{18,19}$	32	37	42	52

Table 3

Interpolated Values of all Cases for the Previous 4 Periods, $V^{c-1}, V^{c-2}, V^{c-3}$, and V^{c-4}

Coding of Cases	V^{c-1}	V^{c-2}	V^{c-3}	V^{c-4}
$C_{1,1}$	612	522	583	499
$C_{1,2}$	243	224	230	213
$C_{2,2}$	903	718	844	679
$C_{2,3}$	114	108	107	101
$C_{3,3}$	241	223	228	212
$C_{3,4}$	31	29	28	27
$C_{4,4}$	82	77	76	73
$C_{4,5}$	11	10	9	9
$C_{5,5}$	29	27	26	25
$C_{5,6}$	2	2	2	2
$C_{6,6}$	22	20	19	19
$C_{6,7}$	3	3	3	3
$C_{7,7}$	17	15	15	14
$C_{7,8}$	2	2	2	2
$C_{8,8}$	19	17	17	16
$C_{8,9}$	4	3	3	3
$C_{9,9}$	40	38	37	36
$C_{10,10}$	977	763	907	718
$C_{10,11}$	290	266	276	254
$C_{11,11}$	666	562	633	536
$C_{11,12}$	60	57	56	54
$C_{12,12}$	145	136	136	129
$C_{12,13}$	19	17	17	16
$C_{13,13}$	63	60	58	56
$C_{13,14}$	9	8	7	7
$C_{14,14}$	28	26	25	24
$C_{14,15}$	4	7	6	6
$C_{15,15}$	20	18	18	17
$C_{15,16}$	1	0	0	0
$C_{16,16}$	18	16	16	15
$C_{16,17}$	1	1	1	1
$C_{17,17}$	8	7	6	6
$C_{17,18}$	2	2	2	2

Table 3 continued.

$C_{18,18}$	14	12	12	12
$C_{1,19}$	28	32	37	46
$C_{2,19}$	96	87	76	62
$C_{3,19}$	90	83	75	63
$C_{4,19}$	72	71	69	65
$C_{5,19}$	70	70	69	65
$C_{6,19}$	56	59	62	66
$C_{7,19}$	38	42	48	59
$C_{8,19}$	23	27	31	35
$C_{9,19}$	56	58	61	66
$C_{10,19}$	60	62	64	66
$C_{11,19}$	109	93	77	61
$C_{12,19}$	108	93	77	61
$C_{13,19}$	63	64	65	66
$C_{14,19}$	55	58	61	66
$C_{15,19}$	38	42	48	59
$C_{16,19}$	30	34	40	49
$C_{17,19}$	23	27	31	35
$C_{18,19}$	32	37	42	52

Table 4

Fitted Values for $C_{1,1}$ *of the Previous k Periods,* $V_{SES}^{c-4}, V_{SES}^{c-3}, V_{SES}^{c-2}$ and V_{SES}^{c-1} *and the Current Period of 2013,* V_{SES}^{c} *with* $\alpha = 0.1$ until $\alpha = 0.9$

α	k	V^{c-k}	V_{SES}^{c-k}	e $(V^{c-k} - V_{SES}^{c-k})$	e^2
0.1	4	499	499	0	0
	3	583	499	83.7531	7014.59
	2	522	507	14.8796	221.403
	1	612	509	102.903	10589
	0	662	519	142.916	20424.9
				SSQ	38249.9
				MSE	7649.98
0.2	4	499	499	0	0
	3	583	499	83.7531	7014.59
	2	522	516	6.5043	42.3058
	1	612	517	94.7149	8970.91
	0	662	536	126.075	15894.8
				SSQ	31922.6
				MSE	6384.53
0.3	4	499	499	0	0
	3	583	499	83.7531	7014.59
	2	522	524	-1.871	3.5007
	1	612	523	88.2017	7779.54
	0	662	550	112.044	12553.9
				SSQ	27351.5
				MSE	5470.3
0.4	4	499	499	0	0
	3	583	499	83.7531	7014.59
	2	522	532	-10.246	104.987
	1	612	528	83.3636	6949.5
	0	662	562	100.321	10064.3
				SSQ	24133.4
				MSE	4826.67
0.5	4	499	499	0	0
	3	583	499	83.7531	7014.59
	2	522	541	-18.622	346.766
	1	612	531	80.2006	6432.14
	0	662	572	90.4031	8172.72
				SSQ	21966.2
				MSE	4393.24

Table 4 continued.

0.6	4	499	499	0	0
	3	583	499	83.7531	7014.59
	2	522	549	-26.997	728.836
	1	612	533	78.7127	6195.68
	0	662	580	81.7879	6689.25
				SSQ	20628.4
				MSE	4125.67
0.7	4	499	499	0	0
	3	583	499	83.7531	7014.59
	2	522	558	-35.372	1251.2
	1	612	533	78.8998	6225.17
	0	662	588	73.9727	5471.96
				SSQ	19962.9
				MSE	3992.58
0.8	4	499	499	0	0
	3	583	499	83.7531	7014.59
	2	522	566	-43.748	1913.85
	1	612	531	80.7619	6522.49
	0	662	596	66.4552	4416.29
				SSQ	19867.2
				MSE	**3973.44**
0.9	4	499	499	0	0
	3	583	499	83.7531	7014.59
	2	522	574	-52.123	2716.8
	1	612	527	84.2992	7106.35
	0	662	603	58.7327	3449.53
				SSQ	20287.3
				MSE	4057.45

Table 5

The fitted values for all 52 cases of the previous k periods, $V_{SES}^{c-4}, V_{SES}^{c-3}, V_{SES}^{c-2}$ and V_{SES}^{c-1} and the current period, V_{SES}^{c}

Coding of Cases	α	k	V^{c-k}	V_{SES}^{c-k}	e $\left(V^{c-k} - V_{SES}^{c-k}\right)$	e^2
$C_{1,1}$	0.8	4	499	499	0	0
		3	583	499	83.7531	7014.59
		2	522	566	-43.748	1913.85
		1	612	531	80.7619	6522.49
		0	662	596	66.4552	4416.29
					SSQ	19867.2
					MSE	3973.44
$C_{1,2}$	0.9	4	213	213	0	0
		3	230	213	16.8727	284.688
		2	224	229	-4.2601	18.1485
		1	243	225	17.7877	316.404
		0	252	241	11.2046	125.542
					SSQ	744.783
					MSE	148.957
$C_{2,2}$	0.8	4	679	679	0	0
		3	844	679	165.435	27368.7
		2	718	811	-92.463	8549.41
		1	903	737	165.597	27422.3
		0	1015	869	145.55	21184.9
					SSQ	84525.3
					MSE	16905.1
$C_{2,3}$	0.9	4	101	101	0	0
		3	107	101	5.17239	26.7536
		2	108	106	1.43587	2.06172
		1	114	107	6.29892	39.6764
		0	117	113	3.95369	15.6317
					SSQ	84.1234
					MSE	16.8247
$C_{3,3}$	0.9	4	212	212	0	0
		3	228	212	16.6569	277.454
		2	223	227	-4.1269	17.031
		1	241	223	17.5771	308.954
		0	250	239	11.0665	122.468
					SSQ	725.906
					MSE	145.181

Table 5 continued.

$C_{3,4}$	0.9	4	27	27	0	0
		3	28	27	0.91135	0.83055
		2	29	28	1.01595	1.03216
		1	31	29	2.00881	4.03533
		0	32	30	1.55268	2.41082
					SSQ	8.30886
					MSE	1.66177
$C_{4,4}$	0.9	4	73	73	0	0
		3	76	73	3.22613	10.4079
		2	77	76	1.68316	2.83303
		1	82	77	4.35634	18.9777
		0	84	81	2.82223	7.96501
					SSQ	40.1837
					MSE	8.03674
$C_{4,5}$	0.9	4	9	9	0	0
		3	9	9	0.26826	0.07196
		2	10	9	0.39084	0.15275
		1	11	10	1.34187	1.80063
		0	12	11	1.23199	1.51779
					SSQ	3.54314
					MSE	0.70863
$C_{5,5}$	0.9	4	25	25	0	0
		3	26	25	0.84084	0.70701
		2	27	26	0.96297	0.92732
		1	29	27	1.93618	3.74877
		0	30	28	1.51642	2.29952
					SSQ	7.68262
					MSE	1.53652
$C_{5,6}$	0.9	4	2	2	0	0
		3	2	2	0.04966	0.00247
		2	2	2	0.0805	0.00648
		1	2	2	0.12121	0.01469
		0	4	2	2.02172	4.08736
					SSQ	4.111
					MSE	0.8222

Table 5 continued.

$C_{6,6}$	0.9	4	19	19	0	0
		3	19	19	0.6049	0.3659
		2	20	19	0.76098	0.57909
		1	22	20	1.69233	2.86398
		0	23	22	1.39683	1.95114
					SSQ	5.76012
					MSE	1.15202
$C_{6,7}$	0.9	4	3	3	0	0
		3	3	3	0.07577	0.00574
		2	3	3	0.12121	0.01469
		1	3	3	0.14896	0.02219
		0	5	3	2.0335	4.1351
					SSQ	4.17773
					MSE	0.83555
$C_{7,7}$	0.9	4	14	14	0	0
		3	15	14	0.44671	0.19955
		2	15	15	0.60078	0.36094
		1	17	15	1.52806	2.33498
		0	18	17	1.31841	1.7382
					SSQ	4.63367
					MSE	0.92673
$C_{7,8}$	0.9	4	2	2	0	0
		3	2	2	0.04966	0.00247
		2	2	2	0.0805	0.00648
		1	2	2	0.12121	0.01469
		0	4	2	2.02172	4.08736
					SSQ	4.111
					MSE	0.8222
$C_{8,8}$	0.9	4	16	16	0	0
		3	17	16	0.50895	0.25903
		2	17	17	0.66646	0.44417
		1	19	17	1.59278	2.53694
		0	20	19	1.34908	1.82001
					SSQ	5.06016
					MSE	1.01203

Table 5 continued.

$C_{8,9}$	0.9	4	3	3	0	0
		3	3	3	0.07577	0.00574
		2	3	3	0.12121	0.01469
		1	4	3	1.13976	1.29904
		0	5	4	1.14178	1.30365
					SSQ	2.62313
					MSE	0.52463
$C_{9,9}$	0.9	4	36	36	0	0
		3	37	36	1.28454	1.65004
		2	38	37	1.24968	1.5617
		1	40	38	2.39173	5.72037
		0	42	40	1.74797	3.05541
					SSQ	11.9875
					MSE	2.3975
$C_{10,10}$	0.8	4	718	718	0	0
		3	907	718	189.281	35827.2
		2	763	869	-105.87	11207.6
		1	977	785	192.705	37135.4
		0	1111	939	172.178	29645.4
					SSQ	113816
					MSE	22763.1
$C_{10,11}$	0.9	4	254	254	0	0
		3	276	254	22.7867	519.232
		2	266	274	-8.0898	65.4444
		1	290	267	23.5583	554.991
		0	303	288	15.0354	226.064
					SSQ	1365.73
					MSE	273.146
$C_{11,11}$	0.8	4	536	536	0	0
		3	633	536	97.4418	9494.9
		2	562	614	-52.217	2726.66
		1	666	572	94.2371	8880.63
		0	726	647	78.5646	6172.4
					SSQ	27274.6
					MSE	5454.92

Table 5 continued.

$C_{11,12}$	0.9	4	54	54	0	0
		3	56	54	2.1339	4.55352
		2	57	55	1.56441	2.44737
		1	60	57	3.25577	10.6
		0	62	60	2.20838	4.87693
					SSQ	22.4779
					MSE	4.49557
$C_{12,12}$	0.9	4	129	129	0	0
		3	136	129	7.40862	54.8877
		2	136	135	0.73595	0.54162
		1	145	136	8.51265	72.4653
		0	149	144	5.29187	28.0038
					SSQ	155.898
					MSE	31.1797
$C_{12,13}$	0.9	4	16	16	0	0
		3	17	16	0.50895	0.25903
		2	17	17	0.66646	0.44417
		1	19	17	1.59278	2.53694
		0	20	19	1.34908	1.82001
					SSQ	5.06016
					MSE	1.01203
$C_{13,13}$	0.9	4	56	56	0	0
		3	58	56	2.27312	5.16706
		2	60	58	1.59442	2.54219
		1	63	60	3.39662	11.537
		0	65	63	2.28546	5.22334
					SSQ	24.4696
					MSE	4.89392
$C_{13,14}$	0.9	4	7	7	0	0
		3	7	7	0.21154	0.04475
		2	8	7	0.31653	0.10019
		1	9	8	1.28246	1.64472
		0	10	9	1.20505	1.45214
					SSQ	3.24179
					MSE	0.64836

Table 5 continued.

$C_{14,14}$	0.9	4	24	24	0	0
		3	25	24	0.8061	0.6498
		2	26	25	0.9357	0.87553
		1	28	26	1.90035	3.61133
		0	29	28	1.49864	2.24591
					SSQ	7.38257
					MSE	1.47651
$C_{14,15}$	0.1	4	6	6	0	0
		3	6	6	0.1837	0.03374
		2	7	6	0.42551	0.18106
		1	4	6	-2.3528	5.53584
		0	9	6	2.91025	8.46954
					SSQ	14.2202
					MSE	2.84404
$C_{15,15}$	0.9	4	17	17	0	0
		3	18	17	0.54059	0.29224
		2	18	18	0.6985	0.4879
		1	20	18	1.62563	2.64268
		0	21	20	1.36476	1.86258
					SSQ	5.2854
					MSE	1.05708
$C_{15,16}$	0.9	4	0	0	0	0
		3	0	0	-0.0015	2.4E-06
		2	0	0	-0.0026	6.6E-06
		1	1	0	1.05812	1.11963
		0	2	1	1.10661	1.22459
					SSQ	2.34423
					MSE	0.46885
$C_{16,16}$	0.9	4	15	15	0	0
		3	16	15	0.47766	0.22816
		2	16	16	0.63389	0.40182
		1	18	16	1.56026	2.4344
		0	19	18	1.33363	1.77856
					SSQ	4.84293
					MSE	0.96859

Table 5 continued.

$C_{16,17}$	0.9	4	1	1	0	0
		3	1	1	0.02388	0.00057
		2	1	1	0.03924	0.00154
		1	1	1	0.0938	0.0088
		0	3	1	2.01018	4.04082
					SSQ	4.05173
					MSE	0.81035
$C_{17,17}$	0.9	4	6	6	0	0
		3	6	6	0.1837	0.03374
		2	7	6	0.27856	0.07759
		1	8	7	1.25326	1.57066
		0	9	8	1.19193	1.42069
					SSQ	3.10268
					MSE	0.62054
$C_{17,18}$	0.9	4	2	2	0	0
		3	2	2	0.04966	0.00247
		2	2	2	0.0805	0.00648
		1	2	2	0.12121	0.01469
		0	4	2	2.02172	4.08736
					SSQ	4.111
					MSE	0.8222
$C_{18,18}$	0.9	4	12	12	0	0
		3	12	12	0.35593	0.12669
		2	12	12	0.49824	0.24824
		1	14	12	1.43348	2.05486
		0	15	14	1.27415	1.62345
					SSQ	4.05324
					MSE	0.81065
$C_{1,19}$	0.9	4	46	46	0	0
		3	37	46	-8.4962	72.1851
		2	32	38	-5.8101	33.7572
		1	28	33	-4.8977	23.9872
		0	24	29	-4.582	20.995
					SSQ	150.925
					MSE	30.1849

Table 5 continued.

$C_{2,19}$	0.9	4	62	62	0	0
		3	76	62	13.5554	183.749
		2	87	74	12.423	154.332
		1	96	85	10.8975	118.756
		0	105	95	9.79015	95.847
					SSQ	552.684
					MSE	110.537
$C_{3,19}$	0.9	4	63	63	0	0
		3	75	63	11.7835	138.852
		2	83	73	9.76363	95.3285
		1	90	82	7.94082	63.0566
		0	96	89	6.74402	45.4818
					SSQ	342.718
					MSE	68.5437
$C_{4,19}$	0.9	4	65	65	0	0
		3	69	65	4.24711	18.0379
		2	71	69	2.09705	4.39761
		1	72	71	0.9344	0.8731
		0	72	72	0.33323	0.11104
					SSQ	23.4197
					MSE	4.68393
$C_{5,19}$	0.9	4	65	65	0	0
		3	69	65	3.42257	11.714
		2	70	68	1.4556	2.11878
		1	70	70	0.43224	0.18683
		0	70	70	-0.0862	0.00743
					SSQ	14.027
					MSE	2.80541
$C_{6,19}$	0.9	4	66	66	0	0
		3	62	66	-3.8958	15.1773
		2	59	62	-3.2908	10.8296
		1	56	59	-2.9467	8.68291
		0	54	57	-2.7869	7.76702
					SSQ	42.4568
					MSE	8.49136

Table 5 continued.

$C_{7,19}$	0.9	4	59	59	0	0
		3	48	59	-10.809	116.84
		2	42	49	-6.596	43.5076
		1	38	43	-5.0519	25.5213
		0	34	38	-4.4743	20.0195
					SSQ	205.889
					MSE	41.1778
$C_{8,19}$	0.9	4	35	35	0	0
		3	31	35	-4	16
		2	27	31	-4.4	19.36
		1	23	27	-4.44	19.7136
		0	19	23	-4.444	19.7491
					SSQ	74.8227
					MSE	14.9645
$C_{9,19}$	0.9	4	66	66	0	0
		3	61	66	-4.3663	19.0645
		2	58	62	-3.5488	12.5938
		1	56	59	-3.1145	9.69993
		0	53	56	-2.9165	8.50614
					SSQ	49.8644
					MSE	9.97287
$C_{10,19}$	0.9	4	66	66	0	0
		3	64	66	-2	4.00002
		2	62	64	-2.2	4.83997
		1	60	62	-2.22	4.9284
		0	58	60	-2.222	4.93729
					SSQ	18.7057
					MSE	3.74114
$C_{11,19}$	0.9	4	61	61	0	0
		3	77	61	15.9992	255.974
		2	93	75	17.5992	309.733
		1	109	91	17.759	315.383
		0	125	107	17.778	316.057
					SSQ	1197.15
					MSE	239.43

Table 5 continued.

$C_{12,19}$	0.9	4	61	61	0	0
		3	77	61	15.9153	253.296
		2	93	75	17.3667	301.601
		1	108	91	17.4118	303.17
		0	124	107	17.3392	300.648
					SSQ	1158.71
					MSE	231.743
$C_{13,19}$	0.9	4	66	66	0	0
		3	65	66	-0.5878	0.34551
		2	64	65	-1.3301	1.76921
		1	63	64	-1.6199	2.6242
		0	61	63	-1.7497	3.06153
					SSQ	7.80046
					MSE	1.56009
$C_{14,19}$	0.9	4	66	66	0	0
		3	61	66	-4.8333	23.3612
		2	58	61	-3.8	14.44
		1	55	58	-3.2765	10.7353
		0	52	55	-3.0414	9.25035
					SSQ	57.7869
					MSE	11.5574
$C_{15,19}$	0.9	4	59	59	0	0
		3	48	59	-10.809	116.84
		2	42	49	-6.596	43.5076
		1	38	43	-5.0519	25.5213
		0	34	38	-4.4743	20.0195
					SSQ	205.889
					MSE	41.1778
$C_{16,19}$	0.9	4	49	49	0	0
		3	40	49	-9.5564	91.324
		2	34	41	-6.162	37.9706
		1	30	35	-5.0058	25.0581
		0	26	31	-4.601	21.1695
					SSQ	175.522
					MSE	35.1044

Table 5 continued.

$C_{17,19}$	0.9	4	35	35	0	0
		3	31	35	-4	16
		2	27	31	-4.4	19.36
		1	23	27	-4.44	19.7136
		0	19	23	-4.444	19.7491
					SSQ	74.8227
					MSE	14.9645
$C_{18,19}$	0.9	4	52	52	0	0
		3	42	52	-10.274	105.563
		2	37	43	-6.4079	41.0606
		1	32	37	-5.0737	25.7425
		0	28	33	-4.5996	21.1563
					SSQ	193.522
					MSE	38.7045

Table 6

V_{SES}^{c+1} The Projected Values of All Cases (52 Cases) for the Following Future Period (2014) with the best value of α

Coding of Cases	α	V_{SES}^{c+1}
$C_{1,1}$	0.8	649
$C_{1,2}$	0.9	251
$C_{2,2}$	0.8	986
$C_{2,3}$	0.9	117
$C_{3,3}$	0.9	249
$C_{3,4}$	0.9	32
$C_{4,4}$	0.9	84
$C_{4,5}$	0.9	12
$C_{5,5}$	0.9	30
$C_{5,6}$	0.9	4
$C_{6,6}$	0.9	23
$C_{6,7}$	0.9	5
$C_{7,7}$	0.9	18
$C_{7,8}$	0.9	4
$C_{8,8}$	0.9	20
$C_{8,9}$	0.9	5
$C_{9,9}$	0.9	42
$C_{10,10}$	0.8	1077
$C_{10,11}$	0.9	301
$C_{11,11}$	0.8	710
$C_{11,12}$	0.9	62
$C_{12,12}$	0.9	148
$C_{12,13}$	0.9	20
$C_{13,13}$	0.9	65
$C_{13,14}$	0.9	10
$C_{14,14}$	0.9	29
$C_{14,15}$	0.1	6
$C_{15,15}$	0.9	21
$C_{15,16}$	0.9	2
$C_{16,16}$	0.9	19
$C_{16,17}$	0.9	3

Table 6 continued.

$C_{17,17}$	0.9	9
$C_{17,18}$	0.9	4
$C_{18,18}$	0.9	15
$C_{1,19}$	0.9	24
$C_{2,19}$	0.9	104
$C_{3,19}$	0.9	95
$C_{4,19}$	0.9	72
$C_{5,19}$	0.9	70
$C_{6,19}$	0.9	54
$C_{7,19}$	0.9	34
$C_{8,19}$	0.9	19
$C_{9,19}$	0.9	53
$C_{10,19}$	0.9	58
$C_{11,19}$	0.9	123
$C_{12,19}$	0.9	122
$C_{13,19}$	0.9	61
$C_{14,19}$	0.9	52
$C_{15,19}$	0.9	34
$C_{16,19}$	0.9	26
$C_{17,19}$	0.9	19
$C_{18,19}$	0.9	28

Appendix B

Making Projection

Table 1

Actual Number of Muslims in a Marital System for the Year 2014 for 18 Transient States

Transient States	Actual Number of Muslims in the Marital System for the Year 2014
M: Under 26	755
M: 26-30	1058
M: 31-35	309
M: 36-40	99
M: 41-45	39
M: 46-50	33
M: 51-55	34
M: 56-60	17
M: Above 60	47
F: Under 26	1281
F: 26-30	857
F: 31-35	169
F: 36-40	62
F: 41-45	40
F: 46-50	24
F: 51-55	26
F: 56-60	13
F: Above 60	15

Table 2

Projected Number of Muslims in the Marital System for the Following Future Year (2014) for Each of the Transient States Using MC Model

Transient States	Projected Number of Muslims in the Marital System for the Year 2014
M: Under 26	927
M: 26-30	802
M: 31-35	133
M: 36-40	26
M: 41-45	3
M: 46-50	3
M: 51-55	4
M: 56-60	9
M: Above 60	8
F: Under 26	1059
F: 26-30	514
F: 31-35	49
F: 36-40	20
F: 41-45	6
F: 46-50	4
F: 51-55	4
F: 56-60	2
F: Above 60	2

Table 3

Projected Number of Muslims in the Marital System for the Following Future Year (2014) for Each of the Transient States Using MC_{SMA} Model

Transient States	Projected Number of Muslims in the Marital System for the Year 2014
M: Under 26	624
M: 26-30	1171
M: 31-35	311
M: 36-40	71
M: 41-45	15
M: 46-50	8
M: 51-55	7
M: 56-60	10
M: Above 60	18
F: Under 26	1025
F: 26-30	953
F: 31-35	153
F: 36-40	46
F: 41-45	16
F: 46-50	9
F: 51-55	7
F: 56-60	3
F: Above 60	5