IMPROVEMENT OF VECTOR AUTOREGRESSION (VAR) ESTIMATION USING COMBINE WHITE NOISE (CWN) TECHNIQUE

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Abstrak

Kajian lepas mendedahkan bahawa Autoregresi Eksponen Teritlak Bersyaratkan Heteroskedastik (EGARCH) mengatasi Autoregresi Vektor (VAR) apabila data menunjukkan heteroskedastisiti. Walau bagaimanapun, penganggaran EGARCH tidak cekap apabila data mempunyai kesan keumpilan. Oleh itu, dalam kajian ini, kelemahan VAR dan EGARCH dimodel menggunakan Gabungan Hingar Putih (CWN). Model CWN dibangunkan dengan mengintegrasikan hingar putih VAR dengan EGARCH menggunakan Model Pemurataan Bayesian (BMA) untuk meningkatkan anggaran VAR. Pertama, reja piawai bagi ralat EGARCH (varians heteroskedastik) telah diuraikan menjadi varians sama dan ditakrifkan sebagai siri hingar putih. Kemudian, siri tersebut diubah menjadi model CWN melalui BMA. CWN disahkan menggunakan kajian perbandingan berdasarkan simulasi dan data sebenar Keluaran Dalam Negara Kasar (GDP) bagi empat buah negara. Data disimulasi dengan menggabungkan tiga saiz sampel dengan nilai keumpilan dan kepencongan rendah, sederhana, dan tinggi. Model CWN dibandingkan dengan tiga model sedia ada (VAR, EGARCH dan Purata Bergerak (MA)). Ralat piawai, log-kebolehjadian, kriteria maklumat dan ukuran ralat telahan digunakan untuk menilai prestasi kesemua model tersebut. Dapatan simulasi menunjukkan bahawa CWN mengatasi tiga model yang lain apabila menggunakan saiz sampel 200 dengan keumpilan tinggi dan kepencongan sederhana. Keputusan yang sama diperolehi bagi data sebenar di mana CWN mengatasi tiga model yang lain dengan keumpilan tinggi dan kepencongan sederhana menggunakan GDP Perancis. CWN juga mengatasi tiga model yang lain apabila menggunakan data GDP dari tiga negara lain. CWN merupakan model yang paling tepat dengan anggaran 70 peratus berbanding dengan model VAR, EGARCH dan MA. Dapatan simulasi dan data sebenar ini menunjukkan bahawa CWN adalah lebih tepat dan menyediakan alternatif yang lebih baik untuk memodelkan data heterokedastik dengan kesan keumpilan.

Kata kunci: Autoregresi Eksponen Teritlak Bersyaratkan Heteroskedastik, Autoregresi Vektor, Kesan Keumpilan, Model Pemurataan Bayesian, Gabungan Hingar Putih.

Abstract

Previous studies revealed that Exponential Generalized Autoregressive Conditional Heteroscedastic (EGARCH) outperformed Vector Autoregression (VAR) when data exhibit heteroscedasticity. However, EGARCH estimation is not efficient when the data have leverage effect. Therefore, in this study the weaknesses of VAR and EGARCH were modelled using Combine White Noise (CWN). The CWN model was developed by integrating the white noise of VAR with EGARCH using Bayesian Model Averaging (BMA) for the improvement of VAR estimation. First, the standardized residuals of EGARCH errors (heteroscedastic variance) were decomposed into equal variances and defined as white noise series. Next, this series was transformed into CWN model through BMA. The CWN was validated using comparison study based on simulation and four countries real data sets of Gross Domestic Product (GDP). The data were simulated by incorporating three sample sizes with low, moderate and high values of leverages and skewness. The CWN model was compared with three existing models (VAR, EGARCH and Moving Average (MA)). Standard error, log-likelihood, information criteria and forecast error measures were used to evaluate the performance of the models. The simulation findings showed that CWN outperformed the three models when using sample size of 200 with high leverage and moderate skewness. Similar results were obtained for the real data sets where CWN outperformed the three models with high leverage and moderate skewness using France GDP. The CWN also outperformed the three models when using the other three countries GDP data sets. The CWN was the most accurate model of about 70 percent as compared with VAR, EGARCH and MA models. These simulated and real data findings indicate that CWN are more accurate and provide better alternative to model heteroscedastic data with leverage effect.

Keywords: Exponential Generalized Autoregressive Conditional Heteroscedastic, Vector Autoregression, Leverage Effect, Bayesian Model Averaging, Combine White Noise.

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List of Publications

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Table of Contents

List of Tables

List of Figures

List of Abbreviations

AC	autocorrelation
ACF	autocorrelation function
ADF	Augmented Dickey-Fuller
AIC	Akaike information criterion
AR	autoregression
ARCH	autoregressive conditional heteroscedastic
ARIMA	autoregressive integrated moving average
ARMA	autoregressive moving average
ARMAX	autoregressive moving average including predetermined variables
AU	Australia
BAMSE	Bartlett’s m specification error test
BHHH	Berndt, Hall, Hall and Hausman
BIC	Bayesian information criterion
BMA	Bayesian model averaging
CARMA	controlled autoregressive moving average
CWN	combine white noise
DSGE	dynamic stochastic general equilibrium
EGARCH	exponential generalized autoregressive conditional heteroscedastic
EIV	error in variables
FAVAR	factor-augmented vector autoregression
GARCH	generalized autoregressive conditional heteroscedastic
GDP	gross domestic product
GNP	gross national product
GRMSE	geometric root mean square error
HC2	heteroscedasticity consistent 2
HC3	heteroscedasticity consistent 3
HCCM	heteroscedasticity consistent covariance matrix
HCCME	heteroscedasticity consistent covariance matrix estimator
HVI	heteroscedasticity variance index

IGARCH	integrated generalized autoregressive conditional heteroscedastic
ICC	intraclass correlation coefficient
LIML	limited information maximum likelihood
LOO	leave-one-out
LR	likelihood ratio
MA	moving average
MAE	mean absolute error
MAPE	mean absolute percentage error
MAR	moving average representation
MEM-GARCH	multiplicative error model generalized autoregressive conditional heteroscedastic
MLE	maximum likelihood estimation
OLS	ordinary least square
PAC	partial autocorrelation function
PMB	posterior model probability
PMW	posterior model weight
QMLE	quasi-maximum likelihood estimator
RBC	real business cycle
RCCNMA	random coefficient complex non-linear moving average
RCMA	random coefficient moving average
RESET	regression specification error test
RLA	robust latent autoregression
RMSE	root mean square error
SARIMA	seasonal autoregressive integrated moving average
SSE	sum of square error
STBL	survey of terms of business lending
SVAR	structural vector autoregression
TGARCH	threshold generalized autoregressive conditional heteroscedastic
UK	United Kingdom
US	United States
VAR	vector autoregression

VARX	vector autoregression including predetermined variables
VMA	vector moving average
WMD	weighted minimum distance
WMDF	weighted minimum distance fuller-modified version

CHAPTER ONE
INTRODUCTION

1.1 Background of the Study

Sims (1980) introduced Vector Autoregression (VAR) models that provide macro econometric system a better execution of the models as choices to simultaneous equations with error term called white noise (Harvey). A univariate autoregression is defined as a single-variable model in which the current estimation of a variable is clarified by its lagged values. A VAR is a *k*-equation, *k*-variable direct model in which every variable is regressed by its personal lagged values, in addition to present and past estimations of the left over *k-1* variable. VAR accompany the certification of giving a consistent and dependable methodology to information, interpretation, forecasting, structural inference and policy examination. The tools that accompany VAR are not difficult to use and interpret, to capture the rich dynamics in various time series.

VAR consist of three forms; reduced, recursive and structural (Stock & Watson, 2001). The reduced form VAR passes on that each variable in the model serve as a direct capacity of its own past qualities together with all different variables past values that are measured and a serially uncorrelated error term called white noise. Regression of ordinary least squares is utilized for the estimation of every model. The surprise activities in the variables are the error terms in the regression model following the consideration of its previous values. The reduced structure model that contains error terms shall be connected crosswise over equations, when diverse variables are joined with one another (Stock & Watson, 2001). The reduced form coefficients which are not linear combinations of the structural coefficients are the restricted, reduced form that refers to as restricted VAR. The mutually needy variables as functions of the predetermined variables is only being expressed as the set of linear equations without restriction of the coefficient values in the equations is called unrestricted reduced form known as unrestricted VAR (Charemza & Deadman, 1992; 1997).

The recursive VAR strives to characterize the structure of the model by the development of the error term in individual error to be random with the error in the past mathematical equations. This is carefully considering a percentage of the mathematical equations that are contemporaneous estimations of different variables as regressors in evaluating the VAR equations.

The computation of the Choleski factorization of the reduced form VAR covariance matrix is equipped when the recursive VAR which accounts for the reduced structure estimates of VAR (Lutkepohl, 2006). Clearly, the variable arrangement changes the results of the VAR models, coefficients and residual, having n factorial recursive VAR signifying the likely arrangements entirely (Stock & Watson, 2001).

Structural VAR reveals the contemporaneous relations among the variables using economic theory (Bernanke, 1986; Blanchard & Quah, 1989; Sims, 1986). Setting up causal relations among variables needs the “identifying assumptions” of structural VAR(Stock & Watson, 2001, p. 2).The structural VAR model is rewritten in unrestricted VAR to overcome the parameter identification problem, because using ordinary least squares (OLS) estimation will yield inconsistent parameter estimation of structural VAR. The reduced form VAR which is unrestricted VAR has an easy application for forecasting the variables (Stock & Watson, 2001).

1.2 Problem Statement

Vector Autoregression (VAR) is incorporating white noise error (ε_t) in the model, which assumed zero mean, zero autocovariances at non-zero lags and constant variance (Harvey, 1993). The violation of these assumptions contributes to the inefficient VAR estimation.

First, when the mean is not zero, the Ordinary Least Square (OLS) estimation will be biased (Kennedy, 2008). This can easily be resolved by removing the non-zero mean form the error term and incorporate it in the intercept term in the estimated equation (Kennedy, 2008). Taking the expectation of the error term will make mean equal to zero (Harvey, 1993).

Second, when the autocovariances of non-zero lags are not zero (autocorrelation), forecasting reliability will be less as the forecasting error terms are liable to increase or decrease in size over time (Kelejian & Oates, 1981; Kennedy, 2008; Lazim, 2013). An essential assumption in econometric estimation is when the series of error terms at different points in time are not related (uncorrelated) to each other, which are violated by autocorrelation. Moving Average (MA) with the autocovariances of lags greater than specified lags q are zero (uncorrelated) is employed to resolve the autocorrelation problem. The random series are estimated directly from the observation, when the parameters are precisely known. The effect of autocorrelation is minimized in the errors, if suitable model is used (Box & Pierce, 1970; Newbold & Ganger, 1974). Moving Average model cannot handle the cases of unequal variances (heteroscedasticity) but MA can only handle equal variances (white noise) efficiently (White, 1980).

Third, when the variance is not constant and this is also known as heteroscedasticity. The existence of heteroscedasticity (unequal variances) leads to inefficient parameters and inconsistent covariance matrix estimates in VAR estimation (White, 1980). In 1982, Engle introduced Autoregressive Conditional Heteroscedasticity (ARCH) model to overcome the unequal variances. The ARCH process errors have some properties such as; mean zero, serially uncorrelated processes with non-constant variances conditional on the past, and constant unconditional variances (Engle, 1982) to resolve the heteroscedasticity. ARCH is necessary in order to have good result from the estimation of a model, to achieve more reasonable forecast variances and proper information for policy makers (Engle, 1982). Bollerslev (1986) also suggested Generalized Autoregressive Conditional Heteroscedasticity (GARCH) to capture the volatility persistent, which is flexible to uplift the weakness of fixed lag structure in ARCH models. There are excess kurtosis and volatility persistence in GARCH (Vivian & Wohar, 2012; Ewing & Malik, 2013).

However, the family of GARCH includes integrated GARCH (IGARCH), threshold GARCH (TGARCH) and exponential GARCH (EGARCH) solved the effect of the excess kurtosis and volatility persistence by capturing the asymmetry of the model. The GARCH family models; EGARCH, quadratic GARCH (QGARCH), TGARCH, Glosten, Jagannathan and Runkle GARCH (GJR-GARCH) and asymmetric power ARCH (APARCH) use the statistical properties for asymmetric volatility to model the leverage effect when restriction is made to satisfy the positivity, stationary and restriction of finite fourth order model, but GARCH family cannot handle leverage effect(Rodríguez &Ruiz, 2012).

Previous studies revealed that GARCH family models impose positivity restriction to model the leverage effect but EGARCH outperform the other GARCH family models with less restriction(most flexible) (Rodríguez &Ruiz, 2012).Modelling the leverage effect using EGARCH require stationary and invertibility conditions to hold (Hentschel, 1995; McAleer, 2014; McAleer & Hafner, 2014; Martinet & McAleer, 2016). The general condition of stationary of random coefficient moving average (RCMA) time series models are not easy to investigate as the models are non-linear. Hence, the derivation of EGRACH from RCMA is not possible. Linear MA (p) process invertibility conditions are easily established, but the situation in the RCMA case is more complicated. The models that are not invertible are not used for forecasting, because the white noise terms are to be estimated, which reveal the significance of invertibility (Marek, 2005).

Furthermore, McAleer and Hafner (2014) introduced a random coefficient complex non-linear moving average (RCCNMA) process. The lack of an invertibility condition for the returns shocks underlying the EGARCH model results in the non-availability of statistical properties for the quasi-maximum likelihood estimator (QMLE) of the EGARCH parameters. The derivation of EGARCH from RCCNMA process reveals the lack of statistical properties of the QMLE of EGARCH because the stationary and invertibility conditions for the RCCNMA process cannot hold. The class of random coefficient linear moving average models is not RCCNMA process. This reveals that the EGARCH parameters cannot permit the derivation of statistical properties (stationarity and invertibility) from RCCNMA process (Marek, 2005; McAleer and Hafner, 2014; Martinet & McAleer, 2016).

The unavailability of statistical properties for modelling the EGARCH leverage effect of the heteroscedastic data can be improved by decomposing the EGARCH standardized residuals into series of models and using Bayesian Model Averaging (BMA) to select the best models. Bayesian Information Criterion (BIC) value will also be used in determining the weight in BMA for the combination of the models (Hoeting et al., 1999; Shao & Gift, 2014; Hooten & Hobbs, 2015).

Therefore, there is a crucial need to develop a new model that can solve the challenges of the heteroscedastic data with leverage effect. The purpose of this study is to develop a new model for the improvement of VAR estimation using EGARCH and BMA because heteroscedastic data with leverage effect are not easy to model using EGARCH.

1.3 Objective of the Study

Based on the purpose of the study which is to develop a new model to improve the VAR estimation, the following are the objectives:

i. to divide the EGARCH standardized residuals into series of models.

ii. to use BMA to select the best models from the series of models.

iii. to develop a new model for the heteroscedastic data with leverage effect.

iv. to validate the performance of the new model using comparison study based on simulated and real data.

1.4 Significance of the Study

The new model can provide better alternative to model heteroscedastic data with leverage effect to overcome VAR, EGARCH and MA weaknesses by comparison study based on simulated and real data. The new model can improve VAR estimation using real data which can benefit the econometricians, economists and statistical modelling end users.

1.5 Thesis Outline

The thesis is divided into six chapters:

Chapter one is the introduction which includes the background of the study, problem statement, objective of the study, significance of the study and thesis outline.

Chapter two is the review of related literature on the VAR and its weaknesses. The ARCH and GARCH family with its weaknesses of errors in the models literature are reviewed, while the problem of heteroscedasticity and correlation are enumerated.MA process, linear regression model and Bayesian model averaging are discussed.

Chapter three outline the methodology that describes the main contribution of this study. There are ten steps in the development of the new model for the heteroscedastic data with leverage effect.

Chapter four validates the performances of combine white noise (CWN) conditions which are based on different sample sizes, leverages and skewness using simulation. The CWN is compared with the three models (VAR, EGARCH and MA) using standard error, log-likelihood, information criteria (AIC and BIC)and error measures.

Chapter five enumerates the performances of CWN using Gross Domestic Product (GDP) data of four countries (United States, United Kingdom, Australia and France). The CWN is compared with the three models (VAR, EGARCH and MA) using standard error, log-likelihood, information criteria (AIC and BIC) and error measures.

Chapter six summarizes the findings, limitations and suggestion of future research.

CHAPTER TWO
LITERATURE REVIEW

2.1 Introduction

VAR can only produce efficient estimation when the error terms are white noise errors which are not heteroscedastic in nature (Sims, 1980; White, 1980; Qin & Gilbert, 2001). The error terms of VAR are white noise processes which are serially uncorrelated random variables with zero mean, constant variance and zero autocovariances at non-zero lags (Harvey, 1993).

The econometrician beliefs that the theoretical models excesses are complemented by implication, the error terms of the estimated models, of which the theory provides unfinished explanations of economic systems. Accordingly, the econometrics extensive tradition has seen the dynamic evolution of the economy as a driving force having relationship directly with the theory (Qin & Gilbert, 2001).

The conviction that errors exclusively signify that in the generation of business cycles, random shocks are responsible for the failure to recognize that the regression residual properties are resolved by the empirical model, data samples and process of estimation. Alternatively, in economic theory association, "innovation residuals" model planned principles are the outcome in errors that cannot be interpreted (Qin & Gilbert, 2001, p. 426). The following sections explain the error term in VAR model, ARCH, GARCH/GARCH family models, MA, linear regression model and BMA.

2.2 Vector Autoregression (VAR) White Noise

VAR is when current variable is a function of its lagged variable with all different lagged variables in the study and serially uncorrelated error term called white noise (Harvey, 1993). VAR have assurance of providing a reasonable and convincing approach to data description, forecasting, structural inference and policy estimation (Stock & Watson, 2001).

The Cowles Commission researchers have been using VAR-type models in econometrics. There is no proposition that the VAR representations are non-structural. The Cowles Commission discussed the issues of estimation and identification context of the simultaneous equation models, the reduced form, taking an open VAR as the most general form (Qin & Gilbert, 2001). Liu (1960) first argued that the simultaneous equation model which is a particular form of reduced form as a set of a prior restriction which is truly the one that can be obtained as data that are not tampered with, which has not losing its originality.

Sims (1980) VAR methodology wholly integrated the opinion of Liu in his 1960 paper. Sargent and Sims (1977) introduced a fundamental VAR experiment to examine factor estimation in the frequency domain when there is small consistent in a prior business cycle theory that produces the cyclical dynamics which reflected in their estimated VAR. However, Sims in his well-known 1980 paper shifted back consideration to the time domain for the recognition of alternative to conventional econometrics with the employment of VAR procedures. He used unrestricted VAR to propose variables modelling and stated as follows:

$$A(L)x_t = e_t \qquad (2.1)$$

Here the matrix polynomial $A(L)$ of order n in L, x_t is the variable in time t and $E(e_1 \mid x_{t-1}, x_{t-2}, ...) = E(e_t \mid x_{t-1}) = 0$ is an innovation process with the magnitude of model-derived, the residuals are serially uncorrelated. Sims (1980) expressed ℓ_t as innovation shocks to definite associated modelled variables and named "money innovation" as the error term in a money-demand equation (Qin & Gilbert, 2001, p. 439).

The errors are interpreted as shocks which are understandable as the unrestricted VAR provided the matrix polynomial is invertible and then transformed to the moving average representation (MAR);

$$x_t = A(L)^{-1} e_t \qquad (2.2)$$

where the x_t is the variable in time t and the error series ℓ_t are interpreted as shocks that employ extensively VAR modellers for policy estimation. Through the conduction method of matrix polynomial the main generator of business cycles are the effects of the shocks.

2.2.1 VAR-Real Business Cycles (VAR-RBC) White Noise

The Real Business Cycles (RBC) models have been widely implementing impulse response estimation, with the view that VAR models are more empirical in data estimation while RBC modellers believe in theoretical aspect and as a result, preferred to calibrate instead of estimation of the unknown parameters. RBC models regard when the parameters are estimated as employing restricted VAR models (Qin & Gilbert, 2001). The specification of the error terms in RBC models arise as autoregression (AR) processes with random shocks, and measurement errors, or errors of observation, which are initiated out of the need of estimation process and are generally developed for the exogenous variables shocks because the total number of equations to be estimated are more than the number of exogenous shock terms (Qin & Gilbert, 2001).

The model misspecifications that occurred in omitting variables are not recognized by this specification. RBC modellers have a tendency to clarify the discrepancy in the models as occurring from insignificant or unexciting parts of the economy, when there is a clear difference in the values obtained in simulation and the values obtained in the actual data of the model (Kydland & Prescott, 1991). The econometricians have great doubt in RBC models that bring this type of clarifications because RBC believed in theory instead of estimation (Quah, 1995; Gregory & Smith, 1995). Linkage of seemingly contrary procedures with exogenous shocks in impulse response estimation indicates the fascinating aspect of the connection of the study for the error terms explanation. The shocks in impulse response can be discussed.

2.2.2 Shocks in Impulse Response

Impulse response traces out responses of current and future values of each variable to a unit increase in the current value of one of the VAR errors. Assumed basic VAR model as in equation (2.1) offers a valid economical description of the statistical process of which x variables is tracked as the errors in impulse response estimation interpreted as shocks. Whichever model is valid is a subset of unrestricted VAR model in equation (2.1). Still, controlling the decision on variables that will be in the vector **x** is the truth. The economist should try to be open as inadequate quantity of sample size demand for a very little quantity of variables of choice in practice. Sims (1980) that criticized the theory as generating “incredible” restrictions suggested these choices (Qin & Gilbert, 2001, p. 440). In addition to genuine “stimuli” the control errors of the e_t embraced the innovation part by implication (Qin & Gilbert, 2001, p.440). The suggestion of Sims choices now reveal the identification by sign restrictions.

2.2.3 Identification by Sign Restrictions

Identification is to examine whether the coefficients of the estimated reduced form equation can produce the parameters of the numerical structural equation estimates. If the coefficients of the estimated reduced form equation cannot produce the parameters of the numerical structural equation estimates, then, it is known as identification problem and is regarded as errors in equation (Qin & Gilbert, 2001).Identification is to be sure that the equation fits into the data is the exact required equation not any other equation or a mixture of other equations together with the required equation (Christ, 1994). Structural equations can only be estimated if these equations are identified (Qin & Gilbert, 2001).

Kilian and Murphy (2012) observed that the doubtful conventional identifying assumptions bring an alternative class of structural VAR models in which structural shocks have been identified by restricting the sign of the reactions of chosen model variables to structural shocks. VAR models identified based on sign restrictions have no point estimate of the structural impulse response functions. Unlike traditional structural VAR models based on short-run restrictions, sign-identified VAR models are only set identified. A unique solution are not implied, however, a set of solutions that all are equally consistent with the identifying assumptions.

Faust (1998), Canova and De Nicolo (2002), Uhlig (2005) established this procedure for monetary policy using VAR models. For example, Uhlig (2005) proposed that when there is no price raise and no increase in non-borrowed reserves for a while because of monetary policy shock, a sudden monetary policy reduction is related with a raise in the federal funds rate. He indicated that the results from the sign-identified models and conventional structural VAR models are different. Sign-identified VAR models are becoming more fashionable in many areas and are now useful in empirical macroeconomics. VAR model is employed to study fiscal shocks (Canova & Pappa, 2007; Mountford & Uhlig, 2009; Pappa, 2009), technology shocks (Dedola & Neri, 2007b), and several shocks in open economies (Canova & De Nicolo, 2002; Scholl & Uhlig, 2008), in oil markets (Kilian & Murphy, 2012, 2014), and in labour markets (Fujita, 2011), as instance.

When every identified shock is connected with an exceptional sign pattern, then it needs identification in sign-identified models. If sign restrictions are not dynamic that is structural shocks are not identified by restricting the sign of the reactions of chosen model variables to structural shocks, simply restrict the sign of the coefficients in the corresponding structural vector moving average (VMA) representation. Different from conventional exclusion restrictions, the economic theory straight away motivates sign restrictions. In addition, though the theoretical justification of the restrictions are regularly weak when restricting the sign reactions at longer horizons.

As the set of sign restrictions are given, considering the reduced form VAR model, the vector white noise reduced form innovations with variance-covariance matrix and the corresponding structural VAR model innovations. Then, the construction of structural impulse response functions with all models fit the data appropriately.

Various researchers suggested interpreting accordingly a set of acceptable structural impulse response functions of the VAR models based on sign restrictions. There are two procedures. First procedure is to make the set of acceptable models one using a penalty function (Uhlig, 2005). Francis, Owyang, Roush, and DiCecio (2014) recognized a technology shock as a shock that maximized the forecast-error variance distribution in labour productivity at a finite horizon and suits sign restrictions. Faust (1998) appealed to the effects of monetary policy shocks on real result concerning the comparable argument. Penalty functions help in providing evidence that some outcome were the best result, based on the set of acceptable models, to assess worst case (or best case) circumstances.

Second procedure is to enforce additional restrictions so as to bring low the set of acceptable reactions. Comparable impulse responses are obtained when the decrease in the set of acceptable models have been reduced to a small number of acceptable models that are very simple to interpret. Canova and De Nicolo (2002), and Canova and Paustian (2011) suggested enforcing extra structure in the form of sign restrictions on dynamic cross-correlations, to decrease the number of acceptable results.

These restrictions based on properties of dynamic stochastic general equilibrium (DSGE) models; encourage obtaining from data simulated by the DSGE models, the DSGE model reactions. In similar work, Kilian and Murphy (2012, 2014) have suggested extra identifying restrictions on a structural oil market, VAR model based on bounds on price elasticity's impact. This has been a special case of enforcing a prior distribution on the values of this price elasticity.

The differentiation between alternative data generating processes and develops sign-identified VAR capability are the enforcement of extra restrictions that has been revealed. It is important that the employment of all information to identify structural shocks from sign-identified models is not just an alternative. But the possibilities of deducing the true structural reactions from sign-identified VAR models can only increase on every small number of sign restrictions because of an opinion in the midst of a number of applied users that remain doubtful. One absolutely supposed that every satisfactory model was possibly a prior to building the posterior distribution of the structural reactions in which the opinion is incorrect (Kilian & Murphy, 2012). For example, Kilian and Murphy (2012, 2014) expressed that except these reactions can be cancelled just by enforcing a bound on the short-run price elasticity of oil supply, oil market VAR models identified by sign restrictions may only involve great reactions of the real price of oil to oil supply shocks. As such, it has been confirmed with reliable judgment in the literature and improper empirical proof that this elasticity is close to zero. They indicated further that neglecting to enforce this extra identifying information, may lead researchers to give much weight to oil supply shocks simply because of the empirical data estimation.

Inoue and Kilian (2013) have argued that the usual approach to sign-identified impulse response functions required comprehensive economic interpretation and fall short of expressing the uncertainty about the structural response functions. Thus, they proposed models that allow both the exactly identified and the sign-identified VAR model in the estimation. The VAR white noise explanation above leads to the VAR white noise application.

2.2.4 VAR White Noise Application

Buch, Eickmeier, and Prieto (2014) have discovered that the risk taking of banks may affect monetary policy decisions (Rajan, 2006).Particularly on low risk investments, as a decrease in the policy rate lowers returns. The bank managers maintained the average return on assets, stable; they have reasons to change into high risk credit market sections. Banks "search for yield" can weaken financial strength which may be encouraged by the expansion of monetary policy (Rajan, 2006, p. 501). VAR is employed to carry out the empirical data of United States (US) banks in response to financial policy disturbances. The empirical data for the model are gross domestic product (GDP) growth, GDP deflator inflation, the monetary policy interest rate, and banking factors. Summary of the monetary lending story presented in the federal reserve's survey of terms of business lending (STBL) is that of banking factors. The information obtained by the bank about the credit of the borrower determines the new loan risk, cash flow, credit rating, access to different supplies of funding, management quality, collateral, and quality of the guarantor using the STBL questionnaire to request for the information. There are organized loans into various risk categories based on the reports about the borrowers. Changes across risk categories involve changing bank risk taking. Investigation revealed the differences among local bank, big bank and foreign banks (Buch *et al*., 2014).

The exploitation differences among various banks and loan market sections, revealed the effects the financial policy shocks of risk-taking. The discrimination of reactions to obtain new loans and loan disbursement through various kinds of banks with series of risk categories loan are revealed. The findings revealed that following the expansion of financial policy reactions, with the average of sample period, local banks notably raise new loans to high risk of the borrowers. The masterpiece of loan supply of local banks changes towards giving loans with high risk. Although bulks of the loan portfolio of big banks do not shift considerably as more new high risk loans is given out (Buch *et al*., 2014).

VAR model has great number of pieces of information on banks which permits to model the direct relationship between the banking sector and the macro economy. Past studies employing panel studies (longitudinal studies) are more restrictive in modelling the macroeconomic shocks, but permitted modelling, bank heterogeneity which are differences in the levels of bank regulation and competition (Altunbas, Gambacorta, & Marques-Ibanez, 2010).

VAR model deals with the connections between macroeconomic factors and the banking system by observing identified effect, mutually orthogonal and macroeconomic shocks. Panel studies usually degenerate risk procedures on interest rates monetary policy with supplementary explanatory variables. There is no response from banks to the macro economy, while the permission of interest rates and other macroeconomic factors have effect on banks, according to the studies.

Swamy (2014) have argued that in VAR approach estimation, the satisfaction of economic logic by the established interdependence and co-movement of the banking stability covariates, in the banking dominated emerging economy. With this, the continued stability of the banking system is demonstrated in India when compared other countries' economies. Keeping up economic growth is a reliable and functional banking system which is important.

A reasonable number of literatures trying to reveal the effects of monetary policy employing restricted multivariate time series models. The earliest effort came from Friedman and Schwartz (1982). They accepted that there is a very good relationship between the result and prices in monetary aggregates. They suggested that the relationships cannot signify inactive reactions of monetary aggregates to the developments in the private sector. However, majorly, there is variation in monetary policy effects on the private sector. The declaration is supported with an indication that the relationships continue, as the variations in monetary aggregates that can forecast the current or the immediately previous expansion in the private sector. This is an indication of nonresponsive in the monetary aggregates (Sims & Zha, 2006).

Sims and Zha (2006) argued that the error terms (innovations) display a better performance in short-term interest rate policy changes than the error terms in money stock in some parts, though this can be called “price puzzle” (p. 234)by interpretation, the failing of monetary reduction clearly created a decline in prices. Sims (1986), with various other studies, like (Christiano, Eichenbaum, & Evan, 1996; Gordon & Leeper, 1994; Bernanke & Mihov, 1998) estimated the reactions of monetary policy changes in interest rates decline, development of the money stock, and increasing prices for the expansion of monetary policy shock, with informal arguments to justify their thrive in using restricted VAR time-series models for US data. Cushman and Zha (1997) enlarged the study for modelling open economies using VAR. The VAR white noise applications of Subsection 2.2.4 revealed the VAR white noise weakness in Subsection 2.2.5.

2.2.5 VAR White Noise Weaknesses

Cooley and LeRoy (1985) assumed that if the interpretations of VAR models are non-structural and are equivalent versions of the same model, the observationally equivalent versions of a given model have different causal interpretations. The important applications of VAR models have this invariance property.

A theoretical (not based on theory) macro econometrics has been credited for its use in analysing causal orderings and policy interventions. The criticism depends on whether VAR models are interpreted as structural or non-structural. If the models are structural in nature and interpreted as non-structural, the conclusions are not supported. Excluding prior identifying restrictions, when the adopted a theoretical macro econometrics is not arbitrary renormalized with restrictions on error distributions, the models are interpreted as structural. The conclusions are not supported, if the requirement for theory justification failed.

Blanchard and Quah (1989) assumed that the unemployment and output dynamics provide two types of shocks, the effects of first type of shock on output is permanent, the second shock effects is temporary and the two shocks being interpreted as supply and demand shocks. In graphical form the vertical axes denoted simultaneously the log of output and the rate of unemployment; the horizontal axis denotes time in quarters. The demand shocks have a hump-shaped effect on output and unemployment. They concluded that demand shocks with considerable contribution to the fluctuations of result at short and medium term horizons, and which after about two or three years the unemployment vanishes. The supply shocks have an effect on the level of output which cumulated steadily over time. In the base case, the peak response is about eight times the initial effect and takes place after eight quarters. The effect decreases to stabilize eventually, the long-run impact is roughly estimated for good statistical reasons. The effect of supply shocks on result adds up over time to attain a level after five years. They identified the dynamic effects of supply and demand shocks on real GNP with procedure based on estimation of a bivariate VAR system. Blanchard and Quah concluded that demand shocks majorly drive the result fluctuations as resulted from their estimation and identification. The study of Blanchard (1989) concluded that the particular identification restrictions imposed on the model result on demand shocks is robust and also based on an arbitrary supposition about the moving average representation are the results derived from VAR estimated. Lippi and Reichlin (1993,

p. 644) argued that Blanchard and Quah's econometric work may be on the "wrong" side of the unit circle which leads to a moving average representation equal zero. An alternative moving average representation which is equivalent to a given estimated VAR is advocated. Lippi and Reichlin (1993) argued that the estimated VAR empirical results on nonstandard moving average representations that present economically reasonable alternatives to imaginative representations being compared with Blanchard and Quah's results.

Paruolo and Rehbek (1999) revealed that vector autoregressive model approach is weak in finding the shock of monetary policy to inflation and economic movement. In their results, exchange rate has a significant response on inflation and bank lending having significant impact on result, but the interest rate is not significant. No reaction to money supply of inflation and result in the model estimation. Paruolo and Rehbek (1999) showed that the inconsistent estimation in VAR integration of order two is the weak exogeneity wrong assumption of consistency and efficiency of the conditional system estimator. The inclusion of drift terms in VAR model does not affect the main conclusion, that is, the inclusion or exclusion of drift terms in VAR give the same result. In the same way, Atabaev and Ganiyev (2013) have argued that there is weakness in the shock of monetary policy to inflation and economic activity. Employing VAR estimation and money supply is not responding to inflation and result in the estimation. This brings low competition among the banks and the external financial have power over capital inflow in the economy of Kyrgyzstan.

Gunnemann, Gunnemann, and Faloutsos (2014) have presented Robust Latent Autoregression (RLA) model to discover the users' base rating behaviour and anomalies in rating distributions. Gunnemann *et al.,* (2014) argued that the RLA results indicated that the highest error is shown in non-robust VAR and Kalman Filtering. Since the unknown structure of the data cannot be identified, for their error increases rapidly for a high number of anomalies. RLA does better than the robust VAR method and RLA is more robust to the anomalies. RLA has less error compared to robust VAR while the non-robust VAR has the highest error. When predicting the future rating distribution, any method with a high number of anomalies is more challenging. Since the non-robust VAR is having the highest error, this indicated the weakness in VAR error terms.

Gordon and Leeper (1994), Christiano *et al.* (1996), and Strongin (1995) estimation of the big impacts of monetary policy shocks on real result, demanding the history of considerable part of variance result. Bernanke and Mihov (1998) in their study argued that the majority of its specifications, demand for brief historical post war business cycle instabilities, and find out very weak effects of policy innovations.

Though, Gordon and Leeper (1994), Christiano *et al.* (1996), and Strongin (1995) employed numerous variables and several released suppositions, using a general device to acquire identification and claimed that sector changes of the economy interrupted the reactions to monetary policy. Bernanke and Blinder (1992), Gordon and Leeper (1994), and Christiano *et al.* (1996) stressed the need for a list of variables that reasonably influence policy, with the variables in the inertial-sector block that penetrated the policy response function. Bernanke and Blinder (1992), Gordon and Leeper (1994), and Christiano *et al.* (1996) argued that contemporaneous consequence on policy are not presently given in the variables, a contemporaneous reaction to policy is also deprived. The studies revealed inconclusive opinions on the economic possibility of nonexistence of contemporaneous reactions to policy compared with the occurrence of contemporaneous impacts on policy for the variables.

The Bernanke and Blinder (1992) employed a further difficult identification scheme. It is an unreasonable supposition that the public sale of market prices like commodity prices is shunned. Gordon and Leeper (1994) examined that the long interest rate has no simultaneous reaction to monetary policy. The inaction suppositions raise during the literature make sense, yet the argument is that the traditional cost adjustment cost and sticky-price models cannot create a stochastic performance that agreed with VAR literature suppositions. For correct identification, the permission for a number of channels of instant reaction of the private sector to policy shock may be important, also even is the indication of the existence of inertia in the private sector. A structural stochastic equilibrium model is presented so as VAR identification scheme generate correct results. The restrictions that validated the other identification schemes emerged unfair, from the viewpoint of the model used in (Bernanke & Blinder, 1992) study.

The implication that monetary policy shocks are of less influence in production decline in the United States over the used period of sample, though, can be the biggest estimated effects. The specifications have the same result, which monetary policy reacts to inflationary shocks initiating in the private sector by constricting the money stock. It means, monetary policy powerfully opposed inflationary and deflationary demands than it supposed under a rule fixing the amount of money or its growth rate. The calculation of the reaction of the economy to inflationary disturbances under the supposition that policy reacted to all disturbances not as much as it has historically, and concluded that real policy may now react to the price level instabilities reductions (Bernanke & Blinder, 1992; Gordon & Leeper, 1994).

The monetary policy disturbances have very strong effects on prices, very weak effects on result. The experimental connection involving high interest rates and succeeding low result is in these interpretations because of the principal source of inflationary demands, not to contractionary monetary policy itself (Bernanke & Blinder, 1992; Christiano *et al.*, 1996).

The discovery of weak effects and a small historical function for monetary policy in producing business cycle instabilities related to monetary policy disturbances; to irregular disparity in monetary policy. The outcome appeared that much of the practical disparity in monetary policy variables is analytically reacting to the economy stand; this is an anticipation of any effective monetary policy. The results are reliable, but bad monetary policy, unlike historical, can generate a high degree of unstable inflation and simultaneously likely, it also generated a high degree of instability in result which attributed to VAR weakness (Bernanke & Blinder, 1992).

VAR models cannot implement greatly bank disorganised reports (Angeloni, Faia, &Lo Duca, 2011; Eickmeier & Hofmann, 2013; Lang & Nakamura, 1995). The univariate regressions (De Nicolo, Dell'Ariccia, Laeven, & Valencia, 2010) cannot evaluate heterogeneity (diverse or dissimilar). White (1980) discovered heteroscedastic behaviour of error term in the data which cannot be modelled by VAR. The heteroscedastic error is enumerated as follows.

2.3 Autoregressive Conditional Heteroscedastic (ARCH) and Generalized Autoregressive Conditional Heteroscedastic (GARCH) Models

Forecasting models have serious challenges in terms of heteroscedastic errors (White, 1980; Engle, 1982; Engle, 1983). The Autoregressive Conditional Heteroscedastic (ARCH) models overcome these challenges.

2.3.1 ARCH Model

Engle (1982) proposed Autoregressive Conditional Heteroscedastic (ARCH) model because of time varying volatility. The equations are on normal distribution, comparing with change in stock market distribution and fat tail measuring effect, and this effect was named ARCH. ARCH models were able to grip group errors and can withstand any changes made by economic forecaster. But ARCH cannot handle the abnormalities like crashes, mergers, news effect or threshold effects in the financial and economic sector. Bollerslev (1986) introduced generalized ARCH (GARCH) tocapture the volatility persistent, which was flexible to the uplift of the weakness of ARCH model.

2.3.2 GARCH Model

When the series is heteroscedastic with variance varying over time, which was the major application of GARCH, and GARCH permitted large lag structure with extended memory. An investigation revealed that there are excess kurtosis and volatility persistence in GARCH (Vivian & Wohar, 2012; Ewing & Malik, 2013). Hassan, Hossny, Nahavandi, and Creighton (2012) discussed these tests on heteroscedastic have not given deviations of the homoscedasticity of the checked time series data. In order to support their argument, they proposed Heteroscedasticity Variance Index (HVI) that gave more information about the time series behaviour. They used linear filtering to obtain local variances and the variance of the local variances was as the estimated quantified heteroscedasticity with criticism that there is a quadratic boundless function. Hassan, Hossny, Nahavandi, and Creighton (2013) modified their 2012 proposition by testing the distance of series of heteroscedastic from homoscedasticity using quantifying method. The proposed index of the heteroscedasticity is quantified by calculating the average tangent angle of local variance function as following;

$$\mu_\theta(\sigma_y^2) = \frac{1}{n}\int_{t=1}^{N}[\tan^{-1}(\frac{d}{dt}\sigma_y^2(t \mid \omega))]dt \qquad (2.3)$$

Where $\theta(\sigma_y^2)$ is the local tangent angles function of $\sigma_y^2(t \mid \omega)$, the length of time series is n and the average local tangent angles of the same function which correlates theoretically with the change of local variances is $\mu_\theta(\sigma_y^2)$, hence quantifies heteroscedasticity. The proposed measure has a lower bound of 0^0 for a completely homoscedastic dataset and an upper bound asymptote of 90^0 for an ultimate heteroskedastic dataset. The proposed index and the popular heteroscedasticity results indicated consistency. In their estimation, with the employment of local variance approach, they failed to determine the current drawback of heteroscedasticity test with volatile mean.

2.3.3 Family of GARCH Model

ARCH and GARCH models focus on the variances of the error terms that are not constant, being known as heteroscedasticity which VAR cannot model efficiently but it can only model white noise error term efficiently. ARCH and GARCH models correct this heteroscedasticity challenge by modelling the variance (Engle, 2001).

Engle (1982) and Bollerslev (1986) introduced linear ARCH and GARCH model specifications for variance and focus on the magnitude of returns; disregarded the information on the direction of returns, and volatility affects the direction of return (Nelson, 1991; Hentschel, 1995; Berument, Metin-Ozcan, & Neyapti, 2001). This is the adventure of GARCH family. Volatility has to be a shock, which is a reaction to the news. The news timing can provide a rise to an expected volatility component, like economic announcements, which may not be a shock (Engle, 2001).

The integrated generalized autoregressive conditional heteroscedastic (IGARCH) model shows a similarity with ARIMA (0, 1, 1) model as the definition of an ACF of squared sample size, if the data (samples) are stationary in first difference, then the model is known as IGARCH (Harvey, 1993). Threshold GARCH (TGARCH) and

EGARCH capture the asymmetric effects of positive and negative shocks of the same dimension on conditional volatility in various ways. Leverage is a particular case of asymmetry.

2.3.4 EGARCH Model

Exponential GARCH (EGARCH) uplifted the weaknesses of GARCH which are excess kurtosis and volatility persistence. EGARCH is a non-linear model in which the conditional variance is able to respond to the asymmetric volatility behaviour (Harvey, 1993). EGARCH overcame the problem of measuring whether the shocks to conditional variance are persistent (Harvey, 1993). Mutunga*et al.* (2015) emphasized that the EGARCH model has the minimum mean square error and mean absolute error when compared with Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model; this reveals that EGARCH forecast has been more precise.

Quadratic GARCH (QGARCH), TGARCH, GJR-GARCH, EGARCH and asymmetric power ARCH (APARCH) models guarantee positivity of conditional variances, stationarity, and existence of fourth-order moments, when the models are restricted. APARCH estimates have a very small percentage of the series satisfy by the finite kurtosis restriction, while GJR-GARCH and EGARCH estimates have larger percentage of the series at the finite kurtosis condition (Nelson, 1991; Hentschel, 1995; Rodríguez & Ruiz, 2012; McAleer, 2014).

QGARCH model guarantee positivity of the conditional variances with severe restrictions and the asymmetry of QGARCH model have very limited representation in practice. The TGARCH asymmetry parameter promised stationary and finite kurtosis with restrictions and these restrictions are not tough on the leverage effect provided the persistence is small. GJR-GARCH estimates satisfied the finite kurtosis condition when restricted. TGARCH model imposition of restrictions on leverage effect is very comparable with EGARCH, but EGARCH has been more flexible in the asymmetric response of volatility. The EGARCH models imposed less restriction among the GARCH family, which allowing it to be most flexible model (Rodríguez &Ruiz, 2012).Positivity restriction on the parameters of the model made EGARCH to capture the asymmetry, but cannot model the leverage efficiently (Nelson, 1991; Hentschel, 1995; McAleer, 2014; McAleer & Hafner, 2014; Martinet & McAleer, 2016).

It has been a known fact, that positive shocks may have less impact on the volatility than the negative shocks of the same magnitudes. As both the positive and negative shocks are assigned an equal degree of importance in the simple GARCH model which cannot remove leverage effect (Nelson, 1991; Hentschel, 1995; McAleer, 2014; McAleer & Hafner, 2014; Martinet & McAleer, 2016). Although, Nelson (1991) proposed the EGARCH to overcome the leverage effect but it can only capture the asymmetric volatility. While a negative shock will add more volatility, as the coefficient of the conditional variance will be negative. The positivity restriction positioned on each conditional variance follows the simple GARCH specification and the conditional variance without restriction necessitated the conditional volatility to be negative. Therefore, EGARCH modelling leverage effect is not possible, even; the general statistical properties (stationarity and invertibility) to estimate the EGARCH parameters to model the leverage effect are lacking (McAleer, 2014; McAleer & Hafner, 2014; Martinet & McAleer, 2016).

The general condition of stationary of random coefficient moving average (RCMA) time series models are not easy to investigate. Linear MA (p) process invertibility conditions are easily established, but the situation in the RCMA case is more complicated, because of the non-linear model. The models that are not invertible are not used for forecasting because the white noise terms are to be estimated, which has revealed the significance of invertibility (Marek, 2005).

McAleer and Hafner (2014) have introduced a random coefficient complex nonlinear moving average (RCCNMA) process. The lack of an invertibility condition for the returns shocks underlying the EGARCH model results in the non-availability of statistical properties for the quasi-maximum likelihood estimator (QMLE) of the EGARCH parameters. The derivation of EGARCH from RCCNMA process revealed the lack of statistical properties of the QMLE of EGARCH because the stationary and invertibility conditions for the RCCNMA process are not known. The class of random coefficient linear moving average models is not RCCNMA process. This reveals that the EGARCH parameters cannot permit the derivation of statistical properties (stationarity and invertibility) from RCCNMA process (Marek, 2005; McAleer and Hafner, 2014; Martinet & McAleer 2016).The error term major challenges are heteroscedastic and autocorrelation errors (White, 1980; Kennedy, 2008; Lazim, 2013).

2.3.5 The Effect of Heteroscedastic Errors

In econometric modelling, the assumption is that error terms have the same(constant) variance which is generally called homoscedasticity. When this assumption is violated and the error terms are not having the same variance which indicates variances vary over time is known as heteroscedasticity (Lazim, 2013).

In matrix form, the error terms of off-diagonal elements of variance-covariance matrix are assumed to be zero; however the diagonal elements are varying in size over time with an independent variable. As large as an independent variable, so also the error variance will be large (Kennedy, 2008).

The detection of the heteroscedasticity presence with the use of a modification of Bartlett's M specification error test (BAMSET) is considered for simple heteroscedasticity (Ramsey, 1969). Any model that exhibits heteroscedasticity can be detected by a heteroscedasticity discrete outcome model with greater heterogeneous flexibility of choice models (Williams, 2009; Savolainen, Mannering, Lord, & Quddus, 2011). White (1980) used ordinary least squares (OLS) with additional condition to have a consistent estimator of OLS parameter covariance matrix which permits to test directly for heteroscedasticity. White emphasized that correct inferences and confidence interval are achieved, which permits heteroscedasticity consistent covariance matrix, even, when the heteroscedasticity is not totally removed in the estimation process.

Antoine and Lavergne (2014) proposed a Weighted Minimum Distance (WMD) estimator that is consistent and asymptotically normal. WMD estimator does not depend on instrumental variables. They argued that without prior knowledge of the weakness pattern of identification, Wald testing is considered for estimation and heteroscedasticity presence produces robust inference. They recommended that when heteroscedasticity is present, WWD or Fuller-Modified version (WMDF) can be used for robust inference, that is, insensitive to deviations from the assumptions under which it was derived.

Cribari-Neto and Galva (2003) stated that the unbiased and consistent of parameters when using OLS estimation on the vector of regression which display some form of heteroscedasticity is still valid, but for inference, the estimated covariance matrix has to be consistent. They proposed improved estimators in which the numerical results favour modification of HC2 (heteroscedasticity consistent 2) and Heteroscedasticity Consistent Covariance Matrix (HCCM) estimator.

Ahmed, Aslam, and Pasha (2011) revealed that despite the fact that the conventional HCCM estimators are obtained from the OLS estimators, the conventional HCCM estimators describe more correct inferences in terms of fewer size distortion. The available literature advocated that having heteroscedastic regression models, the use of the HCCM estimators with many adaptive estimators [e.g., heteroscedasticity consistent (HC3)] results in an efficient estimation only. The adaptive estimators performed better than OLS estimators, but the tests did not perform admirably well with these estimators. However, some weighted versions of HCCMEs are computed, similar to HCCMEs obtained from the OLS residuals, and these are based on the residuals of adaptive estimators. The weighted version of HCCM performs well, when the original model is transformed in an attempt to remove heteroscedasticity. Correct inferences are drawn when the heteroscedasticity consistent covariance matrix estimators are used and error terms display heteroscedasticity.

Uchôa, Cribari-Neto, and Menezes (2014) constructed the heteroscedasticity consistent covariance matrix estimators using both unrestricted and restricted residuals for inference test in fixed effects regression models under an unknown form of heteroscedasticity. They proposed that the test statistic of quasi-t tests used Arellano estimator, and consider with and without high leverage data points of the regression structures. Their results indicated that the unrestricted residuals and restricted residuals tests produce the most accurate asymptotic approximations. But the numerical evidence, when the sample size is small, quasi-t test inference is unreliable.

Various tests are developed to study the existence of heteroscedastic behaviour; Brensch and Pagan (1979), White (1980), Engle (1982), Dovonon and Renault (2013), and Chao, Hausman, Newey, Swanson, and Woutersen (2014). These tests clearly indicate whether the time series tested has heteroscedastic behaviour or not. The null hypothesis of heteroscedasticity is assumed. The existence of heteroscedasticity in a series is to accept the null hypothesis, while heteroscedastic series' failure is to reject the null hypothesis. The effect of autocorrelation errors in error term can be discussed.

2.3.6 The Effect of Autocorrelation

When the series of error terms in different periods of time are not correlated with each other and assume that the present error term is independent of past error terms and future error terms. If this assumption is violated, then the error terms are autocorrelated or serial correlation exists (Kennedy, 2008; Lazim, 2013). The omitted important factors of regression cause the correlation amidst of those that are included in the regression which are also important factors; the autocorrelation across the periods may be because of omitted important factors should have been in the regression model (Greene, 2008).

In matrix form, the error terms are autocorrelated when the variance-covariance matrix with off-diagonal elements of the error term is zero. Three main reasons for autocorrelation existence:

i. the effects of random shocks persisted over one period of time.
ii. there is likely an influence of positive shock in a previous period activity to current periods.
iii. with closed ties, the effect of random shock in one region may cause changes in the next region.

The autocorrelation in omitting the relevant explanatory variable produce autocorrelation shock (Kennedy, 2008).

First order autocorrelation is taking as the specific type and has been the most commonly used among the order of autocorrelations. This is because the variance minimization corresponds to first order autocorrelation of zero and to know if the corrections of autocorrelation are suboptimal that directs the avoidable large variability (Van Beers, Van der Meer, & Veerman, 2013). First order autocorrelation is when error in present period is a function of the error in the past period, that is, the present period error is correlated with the past period error. This first order autocorrelation occurs when the present period error is equal to the past period error plus spherical error (shock) which is written mathematically as:

$$e_t = \rho e_{t-1} + u_t \tag{2.4}$$

where ρ is a parameter less than one and is called the autocorrelation coefficient and u_t is the spherical error. When ρ is positive, errors tend to have the same sign as the error in the past period (Kelejian & Oates, 1981; Kennedy, 2008).

The autocorrelation occurrence mark size distortion which suffers with the commonly applied approach for testing directional forecasts, but Blaskowitz and Herwartz (2014) proposed a bootstrap approach test that reveals the size distortions in small samples which are minimized compared with traditional approaches, and bootstrap approach holds appealing power. The effects of autocorrelation errors revealed it detection.

Detecting Autocorrelation

The error terms have positive autocorrelation when the positive error term in a time period is likely to produce another positive error term in subsequent periods and the negative error term in a time period is likely to produce another negative error term in subsequent periods, then positive autocorrelation can produce cyclical pattern over time (Lazim, 2013).

The error terms have negative autocorrelation when the positive error term in a period of time is likely to produce the negative error term in subsequent periods and the negative error term in a period of time is likely to produce the positive error term in subsequent periods. Then the negative autocorrelation in error term can produce an alternating pattern over time (Kelejian & Oates, 1981; Bowerman, O'Connell, & Koehler, 2005; Kennedy, 2008; Lazim, 2013).

When the error terms have no positive or negative autocorrelation, then the error terms appear in a random pattern over time which signified the error terms are statistically independent. There may be less reliable in forecasting as forecasting error terms are likely to increase or decrease in size over time, when autocorrelation is positive (Kelejian & Oates, 1981; Bowerman, O'Connell, & Koehler, 2005;Kennedy, 2008; Lazim, 2013). Moving average is used to model the autocorrelation error detected.

2.4 Moving Average (MA) Process

In time series, moving average process is regarded as linear regression, which is a function of the present value of the progression with the present and past white noise error terms or random shocks. The progressions are correlated for all lags within the specified lags of the progressions, but uncorrelated for all lags greater than the specified lags of the progressions (Box & Pierce, 1970; Godfrey, 1978). Durbin (1959) proposed a broad approach for theory of testing autocorrelation when the lagged dependent variables of the regressors of a regression equation are incorporated (Godfrey, 1978). This test is asymptotically corresponding to the suitable Likelihood Ratio (LR) test, and testing for the null hypothesis of serially uncorrelated series against the alternative that a steady first order Autoregressive process produced the errors of the regression model. In empirical estimation this test is generally employed. It has been obvious that the moving average error model of order n is a more reasonable alternative hypothesis than the autoregressive of order one scheme because the null hypothesis is that the moving average error are independent and normally distributed with zero mean and constant variance (Godfrey, 1978). However, fourth order autocorrelation cannot be discovered by this test (Godfrey, 1978).

Godfrey (1978) suggested large sample tests of the serially uncorrelated supposition suitable for the broad alternative hypotheses of autoregressive of order n and moving average error model of order n errors. These tests contain the properties that are asymptotically identical to the corresponding Likelihood Ratio (LR) tests and these tests are based on Silvey Lagrange multiplier procedure. It needs no iterative computations. The multipliers in these tests have equal asymptotic variance-covariance matrix under the null hypothesis H_O, and the test statistic for the MA alternative is precisely equivalent to the test statistic for the AR alternative. The Durbin testing method is not equivalent as these tests (Godfrey, 1978).

Zhang, Jia, and Ding (2012) offered a hierarchical least squares iterative estimation for multivariable Box–Jenkins. Ding and Chen (2005) proposed the least squares based iterative algorithms for Hammerstein nonlinear autoregressive moving average including predetermined variables (ARMAX) systems. A two-stage recursive least squares parameter estimation is proposed for result error models and a two-stage least squares based iterative identification algorithm is proposed for controlled autoregressive moving average (CARMA) systems. Hu and Ding (2013) have suggested the multistage identification approach for feedback nonlinear systems employing the hierarchical identification technique, and this approach produced more precise parameter estimates following some iterations. When MA minimized the effect of autocorrelation errors, the standardized residuals of EGARCH series are modelled using linear regression model.

2.5 Linear Regression Model

The term regression was first initiated by a British biologist; Francis Galton in 1908,when he was studying heredity. Linear regression involves the model to be linear in regression parameters. Regression estimation is the technique to determine the link connecting one or more response variables (equally known as dependent variables, explained variables, predicted variables, or regressands, usually denoted by y) and the predictors (equally known as independent variables, explanatory variables, control variables, or regressors, generally represented by $x_1, x_2, ..., x_p$).

The simple linear regression is considered in this study for modelling the linear connection between two variables. One is the dependent variable y and the other is the independent variable x. The simple regression model is in the form of the dependent variable is a function of constant term, the product of the independent variable with its coefficients which is the slope of the regression regarded as middle term plus error term. The assumption is that error term is normally distributed with mean zero and a constant variance.

Simple linear regression is to examine the linear connection between one dependent variable and one independent variable. Applications of regression estimation can be applied scientifically in various areas like medicine, biology, agriculture, economics, engineering, sociology, geology, etc. The principles of regression estimation are:

i. institute a causal connection between response variable y and regressor x.

ii. predict y based on the value of x.

The most essential step is to recognize the real life situation that fall into a specific scientific area (Yan & Su, 2009). Bayesian model averaging is used to select the best model from the several linear regression models.

2.6 Bayesian Model Averaging (BMA)

Bayesian model averaging (BMA) is an approach for variable selection which computes the multiple models value so that the suitable model can be selected for a given variable outcome (dependent variables). The best model has the lowest BIC and highest posterior probability in the BMA output (Raftery, 1995; Raftery, Painter, & Volinsky, 2005).

The characteristics of each model are when a group of predictors (independent variables) of the outcome variable (dependent variables) are the application of all predictors and given an outcome variable of interest. This produced a posterior distribution of the outcome variable which has been a weighted average of the posterior distributions of the outcome for every likely model (Raftery, Painter, & Volinsky, 2005)

Bayesian model averaging (BMA) is basically used to produce the most relevant models from the numerous models that have been discussed to achieve the aim of this study (this thesis). BMA is used to select the best white noise models in this study. Asatryan and Feld (2015) argued that BMA produces a logical method to deal with both model and parameter uncertainty in a situation of weak theoretical direction. Hoeting, Madigan, Raftery, and Volinsky (1999) investigated the performance of four different models: linear regression models, generalized linear models, survival estimation and graphical models.

Theoretically, BMA offers superior average predictive performance when compared with any single model selected and this theoretical result connecting different model and the types of data is in support of the performance in a range of applications. BMA-based confidence intervals are superior when calibrated compare with single-model based confidence intervals. The posterior effect probabilities are easy to understand, and BMA estimation took into account the parameters of interest of model uncertainty. Numerous competing models are permitted to be included in the estimation process. Bayesian model averaging offers better estimation of variance than the estimation that ignored model uncertainty (Hoeting *et al.*, 1999; Shao &Gift, 2014; Hooten & Hobbs, 2015). Bayesian information criterion (BIC) value is used in determining the weight in BMA for the combination.

Bayesian Model Averaging (BMA) Weighting

Selection of model weights needs application of more flexibility and accounting for uncertainty parameter. The priors for parameters and priors for models required Bayesian multimodel inference for clear specification. Using Akaike information criteria (AIC) weighting might favour complex models more heavily than desired, but a computational simple method is to use Bayesian information criteria (BIC) weights with prior model weights (Link & Barker, 2006).The approximation computation of posterior model weights is by using BIC. A set of models and prior probabilities are the starting of Bayesian model weighting, provided that the truth model is in the model set. In the model set, model weight is regarded as the probability of the truth model. This is to say that models are selected and weighted according to high probabilities (Link & Barker, 2006).

The conversion of prior model probabilities to posterior model probabilities are through Bayes factors procedure. Model selection and model averaging used posterior model probabilities. Bayesian model inference linked logically with the model selection and model averaging to obtain good model (Link & Barker, 2006).

In literature, attribute weighting process for naive Bayes performance is better than standard naive Bayes and weighting procedures based on each of them result with the same given input data sets (Hall, 2007).

Jin, Chai, and Si (2004) described collaborative filtering predicts as a particular user utility items which is based on various users that are given equal numbers of items for rating information. Some years ago, various collaborative filtering algorithms were just beginning (Breese, Heckerman, & Kadie, 1998; Herlocker, Konstan, Borchers, & Riedl, 1999; Soboroff & Nicholas, 2000; Resnick, Iacovou, Suchak, Bergstrom, & Riedl, 1994; Hofmann & Puzicha, 1999; Fisher *et al.*, 2000; Pennock, Horvitz, Lawrence, & Giles, 2000). Normally, they are of two classified classes: algorithms model-based and memory-based algorithms (Breese *et al.*, 1998). The training database users were first identify by memory-based algorithms and combine the users that were equivalent in terms of rating patterns, this is the obtainable particular user forecast (i.e., a test user). The group incorporated Pearson-correlation procedure (Resnick *et al.*, 1994), the vector comparison based procedure (Breese *et al.*, 1998), with the generalised vector space model expansion (Soboroff & Nicholas, 2000).

Model-based procedures assembly collectively dissimilar users in the training database into a small number of classes based on their rating sample. These procedures first group the test user into one of the predefined user classes, so as to predict the rating on a particular item from the test user and the targeted item predicted by the rating of the predicted class. Algorithms within this group contain Bayesian network procedures (Breese *et al.*, 1998) and the model part (Hofmann & Puzicha, 1999). The model-based procedures have the benefit that only the profiles of models required to be kept when judge with the memory-based procedures. While the memory-based procedures have regularly easy model-based procedures and involved small calculation offline, though, model-based procedures regularly have to go through series of circular calculation on generating model profiles.

Moreover, the model-based procedures are presuming that a little quantity of classes of user for modelling the rating samples of various users is enough which as a result, the variety of users are lost. In the end, when the quantity of training users of the memory-based procedures is less, tend to perform better than model-based procedures (Si &Jin, 2003). As a result of generating suitable bunches of users, ratings by only a little quantity of users are adequate. Combination capability of both procedures, hybrid procedures such as 'Personality Diagnosis' procedures (Pennock *et al.*, 2000) is advanced, which performed better than the procedures of various model-based and memory-based. In numerous real world applications because of the ease and robustness, the memory-based procedures have been extensively utilized. To recognize the users in the training database, of which many are identical to the test user that answers the memory-based procedures. The resemblance of two dissimilar users is regularly calculated for harmonizing the ratings of equal items grouped, given by the two users. Items are used with the same significance, for various memory-based procedures. Actually, this was not welcome since inconsistencies in various items were accounted for. Rating of several items is in another way considered by various users while others may be highly privileged by most users

Logically, in determining the user-similarity with dissimilar ratings, items with same ratings will have less impact. As it is, items with a little ratings variance have less superior items with big variation ratings (Pennock *et al.*, 2000).

Though, it may not be automatically accurate, for the complexity in rating an item can be from a large variation in the ratings of specified items with various users. In the description of Herlocker *et al.* (1999), no weighting items directed a little better results than employing rating variance for weighting items. For more information, to variance, other weights like inverse user frequency (Breese *et al.*, 1998), entropy, and mutual information (Yu, Wen, Xu, & Ester, 2001). The results in (Yu *et al.*, 2001) indicated that improvement in the performance of collaborative filtering is through few weighting schemes for items. One among the reasons is that, a large number of present weighting schemes are typically calculated by well-defined functions. There is no certainty in the objectives of what the worldwide is trying to accomplish from these weighting schemes.

Jin *et al.* (2004) introduced the latest leave-one-out (LOO) procedure weighting scheme to tackle the challenges. They stressed that the routing behaviour of some part of other users must be analogous to the rating behaviour of a personage user. Thus, items for a superior weighting scheme convey users of analogous interests nearer and temporarily divided users of dissimilar interests further apart. The user distribution is to be examined over the item space different perspective. The spanned vector space with various items of every user having a place in the space, of which, the projection on every axis is indicating the rating of an equivalent item. This item space directed a crowded together distribution of user points, it means, numerous user points are being bonded closely by every user point. The shape the original user distribution has a high-quality weighting method for items (i.e., a user distribution that is not employing whichever weights for items) to a type of crowded together distribution (Jin *et al.*, 2004).

This idea is presented to maximize the likelihood for every user which is found to be appropriate weights of the items to be alike to minimize one of the other users, with a probabilistic optimization difficulty. The training users offered the observed ratings being employed. This procedure by design calculate the fitting weights for various items that are not similar to mainly the early work on weighting schemes which are resolute by foreknowledge functions on item weights. The crowding together assumption fixed in most models-based procedures are similar to the supposition of crowding together for user locations in the item space of the distribution (Jin *et al.*, 2004).

One significant characteristic of this procedure is that, the crowded together distribution of user rating behaviours gives a low precise supposition. The algorithm only required that every user will have a minimum of one user similar to other user, not like various model-based procedures that spate users into various disjoint classes. As a report, most model-based procedures have to indicate the accurate number of crowds together, as this algorithm did not indicate the accurate number of crowds together. Dissimilarity is, a discriminative model is a new procedure with the aim of giving details of how a number of training users are related and others are not, while generative models are mainly model-based procedure with the aim of describing the observed ratings of various training users (Ng & Jordan, 2002; Jin *et al.*, 2004). With this report, the creation of various users' ratings makes most model procedure seek for the seeds of crowds together that can be of use. This algorithm examined the weights of making every user to be near to the related users from different items and different users are separated**.** The explanation of generative model is that every item of observed ratings is included in the distinguishing user's interests that are useless. The discriminative model assigned a lot of higher weight to important items. Numerous studies discovered that the performance of a discriminative model outweighs a generative model performance (Ng & Jordan, 2002).

2.7 Summary

VAR cannot model efficiently the data that is heteroscedastic in nature. ARCH model with controlled numbers of lag structure, and GARCH models with large numbers of lag structure resolved the heteroscedastic data challenges. When there are excess kurtosis and persistence volatility in GARCH, the estimation are not efficient and it cannot capture the asymmetric effect of non-linear models.

The EGARCH/GARCH family uplifted the weaknesses of GARCH, but cannot handle the leverage effect which is the major challenge. There are effects of autocorrelation errors in the error terms of GARCH family also. When the autocorrelation errors are detected, moving average (MA) process minimized the effect of autocorrelation within limited lags of MA.

When these challenges are overcome, the EGARCH/GARCH family cannot handle the leverage effect in the heteroscedastic data efficiently which is the major challenge. Therefore, there is a need to develop a new approach to resolve the heteroscedasticity with leverage effect in Chapter Three.

Table 2.1

Summary of literature review

No.	Author, year	Field of work	Method used/ proposed	Conclusion/remark
1	Sims 1980	Macroeconomics and reality	Vector Autoregression (VAR)	VAR is easy to use and interpret for forecasting and policy making.
2	White 1980	The covariance matrix estimator and heteroscedasticity test	Heteroscedasticity test	Correct inferences
3	Qin and Gilbert 2001	The error term in the history of time series econometrics	Model relationships between macroeconomic time series are inexact. VAR	Interpret errors as shocks
4	Pappa 2009	The effects of fiscal shocks on employment and the real wage.	Keynesian models.	Identify fiscal shocks/disturbances.
5	Kilian and Murphy 2012	Understanding the dynamics of oil market VAR models	Improved identification of VAR models based on sign restrictions	The resulting model estimates are broadly consistent
6	Dedola and Neri 2007	The effects of technology shocks in VAR models	VAR Model based on sign restrictions	Stochastic technology improvements persistently increase real wages, consumption, investment and output in the data

Table 2.1 continued

No.	Author, year	Field of work	Method used/ proposed	Conclusion/remark
7	Fuita 2011	Dynamics of worker flows and vacancies: Evidence from the sign restriction approach	VAR sign restriction approach	The dynamic features of the US labour market.
8	Scholl and Uhlig (2008)	New evidence on the puzzles: Results from agnostic identification on monetary policy and exchange rates	Provide an efficient algorithm to implement sign restrictions in Markov-switching SVARs.	The forward discount puzzle is robust even without delayed overshooting.
9	Canova and De Nicolo (2002)	Monetary disturbances matter for business fluctuations in the G-7	Unrestricted VAR	Identified monetary shocks have reasonable properties; that they significantly contribute to output and inflation cycles in all G-7 countries
10	Francis, Owyang, Roush, and DiCecio 2014	A flexible finite-horizon alternative to long-run restrictions with an application to technology shocks	Identifying shocks in VARs	The robust result that hours worked responds negatively to positive technology shocks.

Table 2.1 continued

No.	Author, year	Field of work	Method used/ proposed	Conclusion/remark
11	Faust (1998)	The robustness of identified VAR conclusions about money	Robustness of identified VAR	The technique reveals only weak support for the claim that monetary policy shocks contribute a small portion of the forecast error variance of post-war U.S.
12	Canova and Paustian (2011)	Business cycle measurement with some theory	The approach employs the flexibility of SVAR techniques against model misspecificati on,	The model does not require the probabilistic structure to be fully specified to be operative; it shields researchers against omitted variable biases and representation problems and requires limited computer time.
13	Inoue and Kilian (2013)	Inference on impulse response functions in structural VAR models	The use of Bayesian methods facilitates the interpretation of sign-identified VAR models	This approach has the advantage of allowing for a unified treatment of estimation and inference in both the exactly identified and the sign-identified VAR model.
14	Buch, Eickmeier, & Prieto. (2014).	Survey-based evidence on bank risk taking	Factor-augmented vector autoregressive model (FAVAR)	Based on results, an expansionary monetary policy shock, small domestic banks increase their exposure to risk. Large domestic banks do not change their risk exposure. Foreign banks take on more risk only, when interest rates are ‘too low for too long

Table 2.1 continued

No.	Author, year	Field of work	Method used/ proposed	Conclusion/remark
15	Rajan (2006)	Has finance made the world riskier?	VAR	He suggests market-friendly policies that would reduce the incentive of intermediary managers to take excessive risk .
16	Altunbas, Gambacorta & Marques-Ibanez, (2010)	Bank risk and monetary policy	Growth rate model (VAR)	They find that banks characterized by lower expected default frequency are able to offer a larger amount of credit and to better insulate their loan supply from monetary policy changes.
17	Swamy 2014	The interrelatedness of banking stability measures	vector auto regression (VAR) model	The model is able to capture the dynamics of banking stability with greater and appreciable accuracy.
18	Sims and Zha (2006)	Does monetary policy generate recessions**?**	Identified VAR	Identifying assumptions for VAR models can be discussed in the context of explicit DSGE models DSGE models that fit the data by the stiff standards of careful time series econometrics are possible.

Table 2.1 continued

No.	Author, year	Field of work	Method used/ proposed	Conclusion/remark
19	Cooley and LeROY (1985)	A theoretical macro econometrics	VAR versus SVAR	They conclude that if the models are structural in nature and interpreted as non-structural, the conclusions are not supported.
20	Blanchard and Quah (1989)	The dynamic effects of aggregate demand and supply disturbances	VAR versus Model distributed lags of two disturbances	They conclude that unemployment and output provide two shocks; permanent and temporary shocks.
21	Paruolo and Rahbek (1999)	Weak exogeneity in I(2) VAR systems	VAR	VAR is weak in finding the shock of monetary policy to inflation and economic movement.
22	Engle (1982)	Autoregressive Conditional Heteroscedasticity (ARCH) with estimates of the variance of United Kingdom inflation	ARCH	ARCH model the conditional variance. This model is used to estimate the means and variances of inflation in the U.K
23	Bollerslev (1986)	Generalized Autoregressive Conditional Heteroscedasticity (GARCH)	GARCH	GARCH lag structure is flexible with long memory. It models the uncertainty inflation rate efficiently

Table 2.1 continued

No.	Author, year	Field of work	Method used/ proposed	Conclusion/remark
24	Ewing and Malik (2013)	Volatility transmission between gold and oil futures under structural breaks	GARCH	They investigated that there are excess kurtosis and volatility persistence in GARCH
25	Hassan, Hossny, Nahavandi, and Creighton (2013)	Quantifying heteroscedasticity using slope of local variances index	Modified Heteroscedasticity Variance Index (HVI)	Heteroscedasticity results indicate consistency. They failed to determine the current drawback of heteroscedasticity test with volatile mean.
26	Nelson (1991)	Conditional heteroscedasticity in asset returns: A new approach	EGARCH	EGARCH estimated coefficients and that may unduly restrict the dynamics of the conditional variance process. Interpreting whether shocks to conditional variance "persist"
27	Mutunga, Islam and Orawo (2015)	Implementation of the estimating functions approach in asset returns volatility forecasting using first order asymmetric GARCH models	EGARCH	EGARCH forecast is more precise.

Table 2.1 continued

No.	Author, year	Field of work	Method used/ proposed	Conclusion/remark
28	Rodrʹiguez and Ruiz (2012)	Revisiting several popular GARCH models with leverage effect: Differences and similarities	Comparison of GARCH family models.	They show that when the parameters satisfy the positivity, stationarity, and finite kurtosis conditions, the dynamics that the GJR and GQARCH models can represent are heavily limited while those of the TGARCH and EGARCH models are less restricted. EGARCH is the most flexible.
29	McAleer (2014)	Asymmetry and leverage in conditional volatility models	GARCH, GJR GARCH and EGARCH	He shows that the parameters satisfy the positivity, stationarity, and finite kurtosis conditions of the asymmetry. None of the GARCH family can model leverage effect.
30	Marek (2005)	On invertibility of a random coefficient moving average model	Random Coefficient Moving Average (RCMA) model	Generally, to find an invertibility condition of RCMA(l) model is very difficult. Non-invertible models cannot be used for Forecasting

Table 2.1 continued

No.	Author, year	Field of work	Method used/ proposed	Conclusion/remark
31	McAleer and Hafner (2014)	A one line derivation of EGARCH	Random Coefficient Complex Nonlinear Moving Average (RCCNMA) process.	The EGARCH model can be derived from RCCNMA process; and the lack of statistical properties of the estimators of EGARCH under general conditions is that the stationarity and invertibility conditions for the RCCNMA process are not known.
32	Martinetand McAleer(2016)	On the invertibility of EGARCH(p, q)	EGARCH	The statistical properties of the (Quasi-)Maximum Likelihood Estimator (QMLE) of the EGARCH(p, q) parameters are not available
33	Lazim (2013)	Heteroscedasticity	Forecasting technique	Heteroscedasticity makes forecasting less reliable. As large as an independent variable, so also the error variance will be large and the less the predictable.

Table 2.1 continued

No.	Author, year	Field of work	Method used/ proposed	Conclusion/remark
34	Hoeting, Madigan, Raftery, and Volinsky (1999)	Bayesian Model Averaging (BMA): A tutorial.	BMA	BMA offers superior average predictive performance when compare with any single model selected. BMA estimation takes into account the parameters of interest of model uncertainty
35	Link and Barker (2006)	Model weights and the foundations of multimodel inference	BMA weighing	BIC weights with prior model weights. A set of models and prior probabilities are the starting of Bayesian model weighting, provided that the truth model is in the model set. models are selected and weighted according to high probabilities

CHAPTER THREE

METHODOLOGY

3.1 Introduction

This chapter explains the outline of the methodology used to achieve the objective of the study. The first objective is to develop a new model for the heteroscedastic data with leverage effect. The second objective is to validate the performance of the new model using comparison study based on simulated and real data. The validation of the suitability and appropriateness of the new model using simulated data with different sample sizes along with low, moderate and high values of leverages and skewness. The validations of the new models using four real data sets were examined.

The methodology framework in Figure 3.1 summarized the new model development which consists of ten steps and the explanation is in Section 3.2. The new model derivation is detailed in Subsection 3.2.1. Figure 3.2 which summarized the new validation process by using simulated and real data as explained in Section3.3. The performance of the new model was compared with the three models (VAR, EGARCH and MA) using standard error, log-likelihood, information criteria (AIC and BIC) and forecast error measures (Section 3.4).

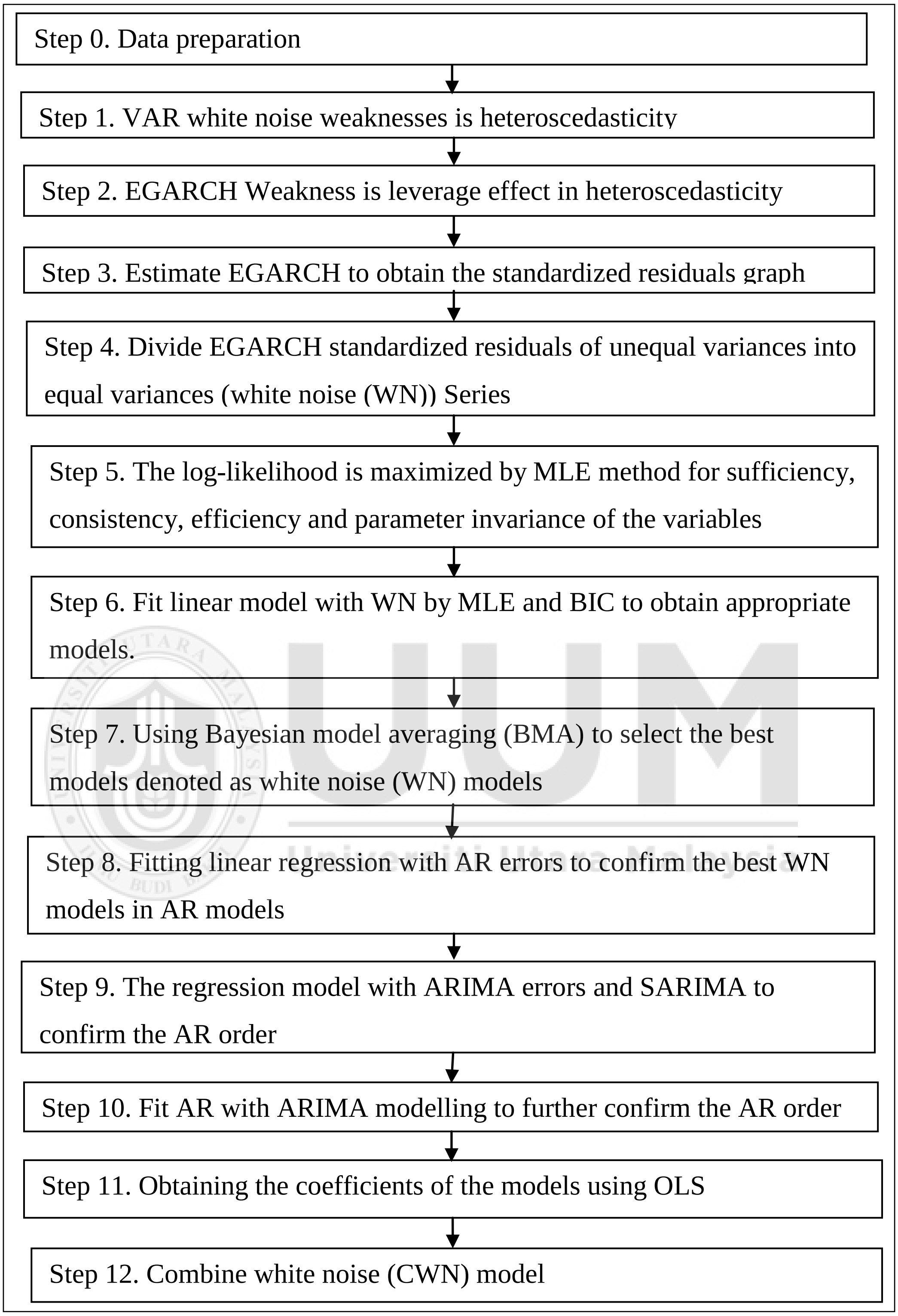


Figure 3.1. Methodology Framework of Combine White Noise (CWN) Model Development

3.2 Model Development

Figure 3.1 is the outline of the twelve (12) steps in the new model development. The data that exhibited heteroscedasticity were considered in the development of the model to improve the estimation of the VAR model.

Step 0: Data preparation was the preliminary stage of making the data ready as required for the implementation of step one to step twelve. The simulated data were based on EGARCH properties using *betategarch* package in R software. Some tests (mainly Jarque-Bera and ARCH ML tests) and estimation were made on collections of real data that were heteroscedastic in nature to obtain the EGARCH among the GARCH family models.

Step 1: VAR white noise estimation is efficient but weak in modelling heteroscedasticity.

Step 2: EGARCH estimation can model heteroscedasticity without leverage effect efficiently but cannot model heteroscedasticity with leverage effect.

Step 3: The EGARCH estimation using heteroscedastic data to obtain the standardized residuals in graphical form for further computation in step 4.

Step 4: The EGARCH estimation of standardized residuals which contained heteroscedastic errors (unequal variances) were decomposed into equal variances (white noise (WN) series) by regrouping using graphical approach. Then the log-likelihood was employed to obtain the optimal results of the WN series.

Step 5: The log-likelihood was maximized by the maximum likelihood estimation (MLE) method for the parameters optimizations. The MLE has the optimal properties of the parameter of interest in the MLE estimator was for full information, this was called sufficiency; the data asymptotically improved by generation from the parameter true value was consistent; when the parameter estimates were attained asymptotically was of minimum variance which was called efficiency; and the parameterization applied were invariant. When the log-likelihood was obtained, each group of equal variance (white noise (WN)) was fitted into regression model to obtain a model which is white noise (WN) model.

Step 6: Fitting of linear models were good model building which requires a grab of regression techniques (Stapleton, 2009; Yan & Su, 2009). Therefore, the linear model was fitted into the series using MLE and BIC to obtain the fitted WN models. In fitting these linear regression models, each WN model has mean zero and constant variance. Therefore, log-likelihood was used to compute the posterior model probability and Bayesian model averaging (BMA) to obtain the best two WN models from each standardized residuals graph of unequal variances.

Step 7: The Bayesian model averaging (BMA). The posterior model probabilities were the weights of the posterior distributions in every WN model in which Bayesian inference was based as revealed in Raftery (1995) findings. BIC is the weight. The posterior model probability BIC values was calculated as

$$BIC_k = -2L_k + p\log(n) \tag{3.1}$$

Where L is the log-likelihood, which is being maximized by the MLE, p is the number of parameters in the considered model (k^{th} model) and n is the sample size. BIC is approximately equal to marginal (integrated) likelihood (Stanford & Raftery, 2002; Shao & Gift, 2014). The BIC values are used to calculate the posterior model weight (PMW) which is also called posterior model probability (PMP).

$$\text{PMP} = \frac{\exp(-0.5BIC_k)}{\sum_{ii=1}^{K} \exp(-0.5BIC_k)} \tag{3.2}$$

When the PMPs are obtained, then Bayesian model averaging (BMA) is used to select the best WN models with lowest BIC and highest posterior probability values. There were 2^K certainty and uncertainty WN models to account for, and it summarized the best models from which the best two models were selected and considered as the overall best two WN models. Fit the linear regression model to confirm the best two WN models result of BMA selection.

Step 8: Fitting linear regression with autoregressive errors to confirm the best two WN models, with zero mean and constant variance (Higgins & Bera 1992).Regress the models obtained and run the autocorrelation function (ACF) of the model to have the order of autoregression (AR). Use regression model with ARIMA errors to obtain the order of the two WN models.

Step 9: The regression model with ARIMA errors and SARIMA. Firstly regress the models in step 8, and then run the following ACF of the models. The ACF spike of the first lag signified autoregressive (AR) of order one which was statistically significant, while the other lags were close to zero. The SARIMA (1, 0, 0) which indicated AR (1) converge with short iteration. Thus, fit AR with ARIMA modelling of time series to obtain the AR of each model for attaining the previous values of the regression model and good for dynamic forecast.

Step 10: Fit AR with ARIMA modelling of time series to obtain the AR of each WN model for proper accountability of the past values of the regression models. Use lowest AIC value to obtain and confirm the right order of AR model. Therefore, OLS was employed to obtain the coefficients of the WN models.

Step 11: Using OLS to obtain the coefficients of the WN models with maximum order and without considering the AIC value. OLS has good finite-sample properties when compared with Yule-Walker estimator, even, after the bias was corrected. OLS has the smallest mean square error for stationary models when compared with bias formula and bootstrap procedure (Engsted & Pedersen, 2014). Obtain the CWN model in step 12 below.

Step 12: The linear combination of the two WN models results in combine white noise (CWN) model. The WN series obtained from the decomposition of the graphical standardized residuals of unequal variances in step 4 through step 11 produced two WN models which the linear combination was CWN in step 12. CWN and VAR error terms are white noise. Therefore CWN can improve the VAR white noise structure.

Based on all of these steps, the following Subsection discussed the model derivation.

Model Derivation

The developments of the model require the heteroscedastic data that use EGARCH information for computation process in step 0to step 2 with respect to the equation (3.3).

The EGARCH model permits the conditional mean of financial returns to be:

$$y_t = E(y_t / I_{t-1}) + \varepsilon_t \tag{3.3}$$

$y_t = \Delta \log P_t$ stands for the log difference in prices (p_t), $I_{t-1}$is the information set at time $t-1$ and the conditional heteroscedasticity is ε_t.

Step 3: This discloses the EGARCH model for the estimation to obtain the standardized residuals graph.

The EGARCH specification is

$$\log h_t = \alpha + \beta | z_{t-1} | + \delta z_{t-1} + \gamma \log h_{t-1}, \qquad |\gamma| < 1 \tag{3.4}$$

Where $z_t = \varepsilon_t \big/ \sqrt{h_t}$ is the standardized shocks, $z_t \sim iid\ (0, A)$. $|\gamma| < 1$is when there is stability. The impact is asymmetric if $\delta \neq 0$, although, there is existence of leverage if $\delta < 0$ and $\beta < -\delta$. Since both β and δ must be positive which are the variances of two stochastic processes, then, modelling leverage effect is not possible (McAleer, 2014; McAleer & Hafner, 2014; Martinet & McAleer, 2016). Therefore, EGARCH errors which have been exhibiting unequal variances (heteroscedastic errors) behaviours in the process of estimation are obtained in graphical form for further computation in step 4.

Step 4: The graph of the standardized residuals of EGARCH which are unequal variances are rearranged and grouped into equal variances series to deal with the leverage effect of heteroscedastic data. Then, the log-likelihood is employed to obtain the optimal results of these equal variances series in step 5.

Step 5: The log-likelihood is maximized by the maximum likelihood estimation (MLE) method. Suppose that X_i are independent Bernoulli random variables probability distribution that relies on unknown parameter θ, which can make this dependence explicit by writing $f(x_i)$ as $f(x_i;\theta)$ for which the probability density function of each X_i is:

$$f(x_i;\theta) = \theta^{x_i}(1-\theta)^{1-x_i} \tag{3.5}$$

for $x_i = 0$ or 1 and $0 < \theta < 1$. The likelihood function $L(\theta)$ is:

$$L(\theta) = \prod_{i=1}^{n} f(x_i;\theta) = \theta^{x_1}(1-\theta)^{1-x_1} \times \theta^{x_2}(1-\theta)^{1-x_2} \times \ldots \times \theta^{x_n}(1-\theta)^{1-x_n} \tag{3.6}$$

For $0 < \theta < 1$. The exponents is sum up as:

$$L(\theta) = \theta^{\Sigma X_i}(1-\theta)^{n-\Sigma x_n}$$

The value of θ that maximizes the natural logarithm of the likelihood function is:

$$\log L(\theta) = (\Sigma x_i)\log(\theta) + (n - \Sigma x_i)\log(1-\theta) \tag{3.7}$$

Differentiate the log-likelihood and set to zero:

$$\frac{\partial \log L(\theta)}{\partial \theta} = \frac{\Sigma x_i}{\theta} - \frac{(n - \Sigma x_i)}{1-\theta} \equiv 0 \tag{3.8}$$

Multiplying through by $\theta(1-\theta)$:

$$(\Sigma x_i)(1-\theta) - (n - \Sigma x_i)\theta = 0$$

Simplify:

$$\Sigma x_i - \theta\Sigma x_i - n\theta + \theta\Sigma x_i = 0$$

Hence

$$\Sigma x_i - n\theta = 0$$

Therefore the parameter θ estimate is:

$$\hat{\theta} = \frac{\sum_{i=1}^{n} x_i}{n} \tag{3.9}$$

The log-likelihood is maximized by MLE method and it has the optimal properties of the parameters on equal variances series results for sufficiency, consistency, efficiency and constant parameters. Next step reveals the models of these equal variances series called white noise (WN) series (Myung, 2003).

Step 6: Moving average (MA) model is considered as fitted linear model that transformed white noise series to white noise (WN) model. Therefore, MA model is adapted for the computation of each WN model, whose sum is WN models.

$$Y_1 = \varepsilon_{1t} + \theta_{11}\varepsilon_{1,t-1} + \theta_{12}\varepsilon_{1,t-2} + \ldots\theta_{jq}\varepsilon_{j,t-q}$$

$$Y_2 = \varepsilon_{2t} + \Phi_{21}\varepsilon_{2,t-1} + \Phi_{22}\varepsilon_{2,t-2} + \ldots\Phi_{jq}\varepsilon_{j,t-q}$$

$\vdots$

$$Y_j = \varepsilon_{jt} + \phi_{j1}\varepsilon_{j,t-1} + \phi_{j2}\varepsilon_{j,t-2} + \ldots\phi_{jq}\varepsilon_{j,t-q}$$

$$Y_{jt} = \sum_{j=1}^{q}\theta_j\varepsilon_{j,t-q} + \sum_{j=1}^{q}\Phi_j\varepsilon_{j,t-q} + \ldots \tag{3.10}$$

$$= A(L)\varepsilon_t + B(L)\varepsilon_t + \ldots$$

$$= \varepsilon_t[A(L) + B(L) + \ldots] \tag{3.11}$$

$$AY_t = Q\varepsilon_t \tag{3.12}$$

Then, the invertibility condition is met

$$Y_t = A^{-1}Q\varepsilon_t \quad for\,|A^{-1}| < 1$$

$Y_t = V_t,\quad V_t \sim N(0, \sigma_c^2),\ \sigma_c^2$ is the sum of equal variances (3.13)

$Y_t = V_t$ is the sum of white noise models

Step 7: Consider the sum of white noise:

$$Y_t = V_t \tag{3.14}$$

Where $Y_t = (Y_{1t} + Y_{2t} \ldots + Y_{jt})$, and $V_t = (V_{1t} + V_{2t} \ldots + V_{jt})$ are the white noise. Considering, the best two white noise, V_1 and V_2 in the overall best models produced by the Bayesian model averaging result (Hoeting, Madigan, Raftery, & Volinsky, 1999; Asatryan & Feld, 2013; Shao & Gift, 2014; Kaplan &Chen, 2014).

Step 8 to step 12: Explain further progressions of obtaining combine white noise model.

Considering WN (1) model from equation (3.12) for the recursive processes.

$$Y_t = \mu + V_t - M_1 V_{t-1}, \tag{3.15}$$

where $Y_t = (Y_{1t}, ..., Y_{kt})', V_t$ is zero mean white noise having a non-singular covariance matrix $\sum_V$, $\mu = (\mu_1, ..., \mu_{kt})'$ is the mean vector of Y_t, $E(Y_t) = \mu$ for all t with the assumption that $\mu = 0$ that is, Y_t is a zero mean process. *K* is the number of variables.

Consider:

$$Y_t = V_t - M_1 V_{t-1}, \quad t = 0, 1, 2, ...$$

$$V_t = Y_t + M_1 V_{t-1} \tag{3.16}$$

By recursive substitution, there are:

$$V_t = Y_t + M_1(Y_{t-1} + M_1 V_{t-2}) = Y_t + M_1 Y_{t-1} + M_1^2 V_{t-2}$$

$$\vdots$$

$$= Y_t + M_1 Y_{t-1} + ... + (M_1)^n V_{t-n} + (M_1)^{n+1} V_{t-n-1}$$

$$= Y_t + \sum_{i=1}^{\infty} (M_1)^i Y_{t-i} \tag{3.17}$$

If $M_1^i \to 0$ as $i \to \infty$

Y_t become the subject of the formula,

$$Y_t = -\sum_{i=1}^{\infty} (M_1)^i Y_{t-i} + V_t, \tag{3.18}$$

Equation (3.18) is the infinite order VAR representation of the process. Since M_1^i can be zero for i greater than some finite number p, the process may be a finite order VAR (*p*).

Therefore:

$$Y_t = \sum_{i=1}^{p} \phi_i Y_{t-i} + V_t, \quad V_t \sim N\left(0, \sigma_c^2\right) \tag{3.19}$$

$E(V_t \mid Y_{t-1} + Y_{t-2} + \ldots) = E(V_t \mid Y_{t-1}) = 0,$ the combine white noise given the series (combine) variables equals zero.

Since in an original VAR model, $E(\ell_t \mid x_{t-1}, x_{t-2}, \ldots) = E(\ell_t \mid X_{t-1}) = 0,$ is in sequence, therefore, the combine white noise given the series of the variables in lags equal zero. Where $Y_t = (Y_{1t} + \ldots + Y_{kt}),\ V_t$ has zero mean of the combine white noise with a non-singular covariance matrix $\sum_V$. V_t which is the error term of combine white noise model which are encompassed in VAR representation. Therefore, combine white noise can be used to improve the VAR estimation. The derived model can be used for validation in Section 3.3.

Combination of Two Variances of the Combine White Noise Model

The combination of equal variances is σ_c^2 from equation (3.13) in Section 3.2. The combine variance of the combine white noise is:

$$\sigma_c^2 = \sigma_1^2 + \sigma_2^2 + \ldots \tag{3.20}$$

Considering the best two variances in the overall best models produced by the Bayesian model averaging result (Hoeting, Madigan, Raftery, & Volinsky, 1999; Asatryan & Feld, 2013; Shao & Gift, 2014; Kaplan &Chen, 2014). The two combine variance follows:

$$\sigma_c^2 = \sigma_1^2 + \sigma_2^2 \tag{3.21}$$

The variance of errors, σ_c^2 in the combine white noise can be written:

$$\sigma_c^2 = W^2\sigma_1^2 + (1-W)^2\sigma_2^2 + 2W\rho\sigma_1(1-W)\sigma_2 \tag{3.22}$$

where the balanced weight specified for the model is W and ρ is the intra-class correlation coefficient. The least of σ_c^2 appearing, when the equation is differentiated with respect to W and equate to zero, obtaining the models as follows:

$$\sigma_c^2 = W^2\sigma_1^2 + \sigma_2^2 - 2W\sigma_2^2 + W^2\sigma_2^2 + 2W\rho\sigma_1\sigma_2 - 2W^2\rho\sigma_1\sigma_2$$

$$\frac{\sigma_c^2}{dw} = 2W\sigma_1^2 - 2\sigma_2^2 + 2W\sigma_2^2 + 2\rho\sigma_1\sigma_2 - 4W\rho\sigma_1\sigma_2$$

since $\frac{\sigma_c^2}{dw}$ is zero (turning point)

$$2W\sigma_1^2 - 2\sigma_2^2 + 2W\sigma_2^2 + 2\rho\sigma_1\sigma_2 - 4W\rho\sigma_1\sigma_2 = 0$$

$$2W\sigma_1^2 + 2W\sigma_2^2 - 4W\rho\sigma_1\sigma_2 = 2\sigma_2^2 - 2\rho\sigma_1\sigma_2$$

$$2W(\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2) = 2\sigma_2^2 - 2\rho\sigma_1\sigma_2$$

Subsequently, the optimum value of W is:

$$W = \frac{\sigma_2^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2} \tag{3.23}$$

Where ρ is the correlation; intra-class correlation coefficient which is used for a reliable measurement (Bates & Granger, 1969; McGraw & Wong, 1996; Rodr´ıguez & Elo, 2003; Lu, & Shara, 2007; Wallis, 2011; Li, Zeng, Lin, Cazzell & Liu, 2015).

The intra-class correlation coefficient (ICC) expressed how powerfully the units in the identical group resemble one another. In measuring the same quantity, ICC was employed to evaluate the consistency or conformity of measurements made by multiple observers. ICC was used for the principal measurement of reliability in favour of quantitative measures (Rodr´ıguez & Elo, 2003; Lu, & Shara, 2007; Wallis, 2011; Li, Zeng, Lin, Cazzell & Liu, 2015).

ICC employs a pooled mean and standard deviation with the data centred and scaled, while every variable was centred and scaled by its own mean and standard deviation in Pearson correlation. All measurements were of the same quantity for the pooled scaling (Bates & Granger, 1969; McGraw & Wong, 1996; Rodr´ıguez & Elo, 2003; Lu, & Shara, 2007; Wallis, 2011; Li, Zeng, Lin, Cazzell & Liu, 2015).

3.3 Model Validation

Figure 3.2 outline the combine white noise (CWN) model validation process. The CWN model was validated using simulated and real data by comparing CWN with VAR, EGARCH and MA. In the validation process, CWN uses the VAR properties and procedures in the software because of common properties and their error terms are white noise. The Subsections 3.3.1and 3.3.2 discussed the simulated and real data processes of the estimation respectively.

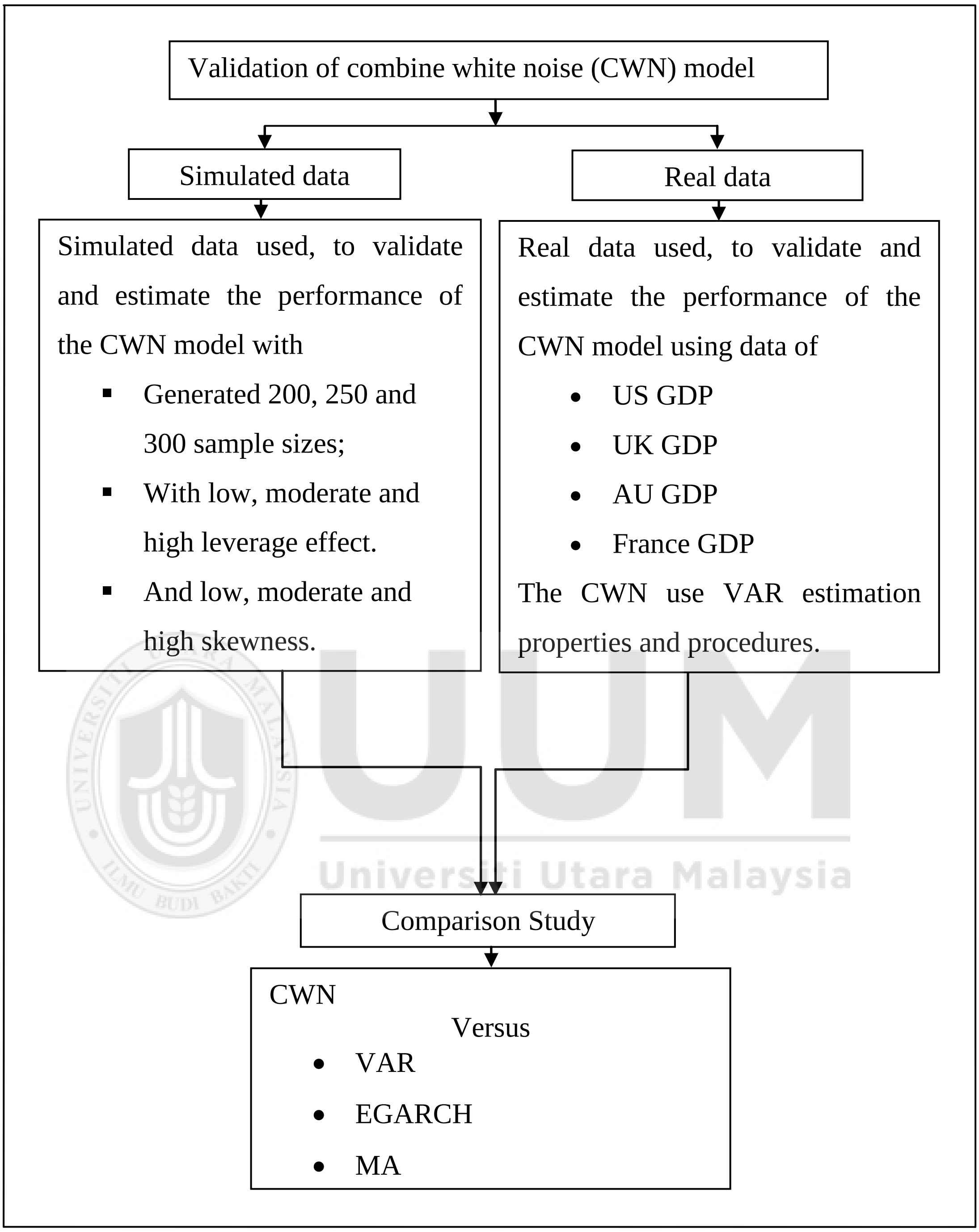


Figure 3.2. Methodology Framework of CWN Model Validation

3.3.1 Data Simulation

The data were simulated to evaluate the performance of the model with three different types of sample size. Ng and Lam (2006) evaluated the MEM-GARCH model to obtain the sample sizes by using correlation approach to calculate the effectiveness of model estimation (conditional variances). It was discovered that 200 sample size have correlation value of 0.4983 and 300 sample size have correlation value of 0.9203; the higher the sample size the higher the degree of correlation. In relation to the degree of correlation, this study considered 200 sample size as low, 250 sample size as moderate and 300 sample size as high values as reported in Table 3.1. The simulated three different sample sizes data of EGARCH with different values of leverages and skewness offered twenty seven different models to estimate each of EGARH, MA, VAR and CWN. These results produced one hundred and eight models different estimation.

Each of these sample size was used for low, moderate and high skewness. Bulmer (1979) revealed that the distribution with skewness within zero (0) and half (0.5) was fairly symmetrical, the distribution with skewness within 0.5 and one (1) was moderate and highly skewness was absolute value greater than one (1) (Piovesana & Senior, 2016). Therefore, 0.5 was considered as low skewness, 0.7 as moderate skewness and 1.2 as high skewness for this study. Sucarrat (2013) considered moderate leverage as 0.02 and strong skewness as 0.8 in simulations of generated data as reported in Table 3.1. The detailed analysis was carried out in Chapter Four.

Each of these sample size of 200, 250 and 300 simulated data used low, moderate and high leverage. Sucarrat (2013) considered 0.02 as moderate leverage for simulation of 2000 simulated sample size using the *betategarch* package in R software as reported in Table 3.1. Sucarrat and Sucarrat (2013) considered 0.05 as leverage for simulation of 500 simulated sample size, but did not specify whether 0.05 was low, moderate nor high leverage.

Therefore, in this study, 0.01, 0.05 and 0.09 were used as low, moderate and high leverage effect values respectively for the simulation of the data that exhibited heteroscedasticity with leverage effect.

Table 3.1

Conditions for Data Generation

				Conditions	
No	**Authors**	**Criteria**	**Low**	**Moderate**	**High**
1	Ng and Lam (2006)	Sample Size	200	.	300
2	Sucarrat (2013)	Leverage	.	0.02	.
3	Sucarrat (2013)	Skewness	.	0.8	.
4	Bulmer (1979)	Skewness	-0.5 to +0.5	-1.0 to -0.5 +0.5 to +1.0	< -1.0 or >+1.0
5	Pioresama and Senior (2016)	Skewness	-0.5 to +0.5	-1.0 to -0.5 +0.5 to +1.0	< -1.0 or >+1.0

The validation using simulated data, the estimation of the best models among the models including the new model revealed the right sample size with the appropriate values of leverage and skewness. The validation of simulated data is in Chapter Four.

3.3.2 Real Data

Real data that exhibited heteroscedastic errors were employed to validate the combine white noise model with the parameters being estimated. Four sets of data were employed; United States gross domestic product (US GDP), United Kingdom (UK) GDP Australia (AU) GDP and France GDP. These data were retrieved from DataStream of Universiti Utara Malaysia Library. The data that have heteroscedastic errors terms have unequal variances in estimation process. Modelling the asymmetric effect of heteroscedastic errors which was non-linear can be with or without leverage effect depends on the nature and size of the data. The data distributions have skewness since the model was asymmetric. The estimation of EGARCH models with sample sizes revealed the skewness and asymmetry with or without leverage effect. The validation of real data sets is in Chapter Five.

3.3.3 Estimation Procedure

Considering maximum likelihood estimation, when a stationary moving average of order one is assumed:

$$y_t = \varepsilon_t + \theta\varepsilon_t, \quad t = 1,...,n \tag{3.24}$$

where ε_t is independent normal random variable of a series with zero mean and constant variance, σ^2. Where the absolute of θ is less than one. $\sigma^2 v_t$ is the variance matrix of $y_1,...,y_n$ where

$$\boldsymbol{V}_n = \begin{pmatrix} 1+\theta^2 & \theta & 0 & \cdots & 0 \\ \theta & 1+\theta^2 & \theta & & \vdots \\ 0 & \theta & 1+\theta^2 & & \vdots \\ \vdots & & & \ddots & \theta \\ 0 & \vdots & \vdots & \theta & 1+\theta^2 \end{pmatrix}$$

The determinant of this is $|\boldsymbol{V}_n| = (1-\theta^{2n+2})/(1-\theta^2)$ which tends to $1/(1-\theta^2)$ for large n. Approximately, the inverse of $\boldsymbol{V}_n$ is:

$$\frac{1}{1-\theta^2}\begin{pmatrix} 1 & -\theta & \theta^2 & \cdots & (-\theta)^{n-1} \\ -\theta & 1 & -\theta & & \vdots \\ \theta^2 & -\theta & 1 & & \vdots \\ \vdots & & & & -\theta \\ (-\theta)^{n-1} & \cdots & \cdots & -\theta & 1 \end{pmatrix}$$

Hence, the approximate log-likelihood is given by

$\log L = A - \frac{1}{2}\log(1-\theta^2) - \frac{1}{2\sigma^2(1-\theta^2)}[\Sigma y_t^2 - 2\theta\Sigma y_t y_{t+1} + 2\theta^2 \Sigma y_{t+2} - ...]$. Neglecting the term $\frac{1}{2}\log(1-\theta^2)$ because of the small order in n when compared with $\log L$ present the maximum likelihood equation approximation:

$$\frac{\partial}{\partial\theta}\left[\frac{1}{(1-\theta^2)}\{\Sigma y_t^2 - 2\theta\Sigma y_t y_{t+1} + 2\theta^2\Sigma y_{t+2} - ...\}\right] = 0 \tag{3.25}$$

This results in unmanageable estimating equation. Thereby, a simple and efficient estimation based on autoregression representation is suggested (Durbin, 1959; Harvey & Philip, 1979; Myung, 2003).

Estimation based on the autoregression representation model (3.24) has the infinite autoregressive as follows:

$$y_t + \phi_1^{'} y_{t-1} + \phi_2^{'} y_{t-2} + ... = \varepsilon_t \tag{3.26}$$

where $\phi_i^{'} = (-\theta)^i$ and what is left after $k+1$ the terms of the series $y_t + \phi_1^{'} y_{t-1} + ...$ is $(-\theta)^{k+1}(y_{t-k-1} - \theta y_{t-k-2} ...) = (-\theta)^{k+1}\varepsilon_{t-k-1}$, with variance $\theta^{2k+2}\sigma^2$. This tends to zero as k tends to infinite since the absolute value of Θ is less than one. Consequentially the finite representation is:

$$y_t + \phi_1 y_{t-1} + \phi_2 y_{t-2} + ... = \varepsilon_t \tag{3.27}$$

Taking k adequately large for accuracy, is important, is always in asymptotic arguments in respect of its smallness when compared with n.

Let $a_1,..., a_k$ be the least squares estimators of $\phi_1,...,\phi_k$ which are estimators acquired by minimizing $\sum_{t=k+1}^{n}(y_t - \phi_1 y_t - ... - \phi_k y_{t-k})^2$. $a_1,..., a_k$ are asymptotically normal with means $\phi_1,...,\phi_k$ and variance matrix v_k^{-1}/n. $\sigma^2 v_t$ is the variance matrix of $y_{t-1},..., y_{t-k}$. Following, $a_1,...,a_k$ have the asymptotic distribution:

$$dP = \frac{n^{\frac{1}{2}}|V|^{\frac{1}{2}}}{(2\pi)^{\frac{1}{2}k}} \exp\left[-\frac{n}{2}\left\{(1+\theta^2)\sum_{i-1}^{k}(a_i - \phi_i)^2 + 2\theta\sum_{i-1}^{k-1}(a_i - \phi_i)(a_{i+1} - \phi_{i+1})\right\}\right] da_1 ... da_k. \tag{3.28}$$

The following relations are satisfied by autoregressive coefficients $\phi_1,...,\phi_k$

$\phi_1 c_0 + \phi_2 c_1 + ... + \phi_k c_1 \phi_k c_{k-1} = -c_1,$

$\phi_1 c_1 + \phi_2 c_0 + ... + \phi_k c_1 \phi_k c_{k-2} = -c_2,$

⋮

$\phi_1 c_{k-1} + ... + \phi_k c_0 = -c_k,$

this $\sigma^2 c_r = E(x_r x_{t+r})$. Setting $c_0 = (1+\theta^2)\sigma^2$, $c_1 = \theta\sigma^2$ and $c_r = 0 \quad (r > 1)$ to get

$$(1+\theta^2)\phi_1 + \theta\phi_2 = -\theta,$$

$$\theta\phi_{r-1} + (1+\theta^2)\phi_r + \theta\phi_{r+1} = 0 \qquad (r = 2,\ldots,k-1),$$

$$\theta\phi_{k-1} + (1+\theta^2)\phi_k = 0$$

These equations are multiplied by $-2\phi_i + \phi_i \quad (i = 1,\ldots,k)$ each and adding to obtain quadratic expression, Q, in the exponent of (3.28) (Durbin, 1959).

$$Q = (1+\theta^2)\sum_{i=1}^{k}(a_i - \phi_i)^2 + 2\theta\sum_{i=1}^{k-1}(a_i - \phi_i)(a_{i+1} - \phi_{i+1})$$

$$= (1+\theta^2)\sum_{i=1}^{k}\phi_i^2 + 2\theta\sum_{i=1}^{k-1}a_i a_{i+1} + 2\theta\phi_1 - \theta\phi_1.$$

As k is large, ϕ_1 is almost equal to $-\theta$ that results, on setting $\phi_0 = 0,$

$$Q = (1+\theta^2)\sum_{i=1}^{k}\phi_i^2 + 2\theta\sum_{i=1}^{k-1}a_i a_{i+1} - 1, \qquad (3.29)$$

to a high order of accuracy.

Estimating θ by maximizing the likelihood that was got from the distribution of $a_1,\ldots,a_k$ as $|V_k| = (1-\theta^{2k+2})/(1-\theta^2)$ of which k is adequately large for it to be approximately equal to $1/(1-\theta^2)$. The first approximation of maximizing the likelihood is proportional to minimizing the quadratic form Q. In differentiating Q with respect to $\theta,$ and equating to zero to obtain the estimator of θ:

$$b = -\frac{\sum_{i=0}^{k-1} a_i a_{i+1}}{\sum_{i=0}^{k} a_i^2} \tag{3.30}$$

The maximum likelihood estimator that is obtained from equation (3.25) is not as easy as this estimator (Durbin, 1959; Myung, 2003; Chaudhuri, Kakade, Netrapalli, Sanghavi, 2015).

Efficiency of the Estimator

$(1-\theta^2)/n$ is the minimum asymptotic variance of consistent estimators of θ. Taking the asymptotic distribution of $a_1,...,a_k$, when k is large, to be:

$$dP = \frac{n^{\frac{1}{2}|V|^{\frac{1}{2}}}}{(2\pi)^{\frac{1}{2}k}} \exp(-\frac{1}{2} nQ) da_1 ... da_k, \tag{3.31}$$

equation (3.21) gives Q this can be written in this form:

$$dP = (1-\theta^2)^{-\frac{1}{2}} f(Q) da,$$

Taking the integral with respect to $a_1,...,a_k$ as:

$$\int (1-\theta^2)^{-\frac{1}{2}} f(Q) da. = 1.$$

$$\int f(Q) da = (1-\theta^2)^{-\frac{1}{2}}. \tag{3.32}$$

Differentiate equation (3.32) and divide across with *n:*

$$\int \frac{\partial Q}{\partial \theta} f(Q) da = 0\left(\frac{1}{n}\right) \qquad i.e. \quad E\left[\frac{\partial Q}{\partial \theta}\right] = 0\left(\frac{1}{n}\right) \tag{3.33}$$

and second derivative gives

$$V(b) = \frac{E(\partial Q / \partial \theta)^2}{E(\partial^2 Q / \partial \theta^2)^2}, \tag{3.34}$$

to first order in n. Seeing that $E(\partial^2 Q / \partial \theta^2)^2 = [E(\partial^2 Q / \partial \theta^2)]^2$ to degree one, and employing equation (3.33) to obtain the asymptotic result:

$$V(b) = \frac{2}{nE(\partial^2 Q / \partial \theta^2)}.$$

To first order in n, $E\left(\frac{\partial^2 Q}{\partial \theta^2}\right) = 2\sum_{i=0}^{k} \phi_i^2$, of which large k tends to $2\sum_{i=0}^{\infty} (-\theta)^{2i} = 2/(1-\theta^2)$.

Therefore, when k is adequately large, the asymptotic variance of b is:

$$V(b) = \frac{1-\theta^2}{n} \tag{3.35}$$

as nearly as desire (Durbin,1959; Chaudhuri, Kakade, Netrapalli, & Sanghavi, 2015). The estimation procedures were used to analyze the heteroscedastic data to make appropriate comparison of the CWN with the three models.

3.4 Comparison Study

Data simulation and real data were used to compare CWN with the three models which were VAR, EGARCH and MA. The following are the outline of the three models.

VAR Model

VAR are effectively used to model the multivariate time series data that has white noise errors. VAR model can be written as:

$$y_t = \theta + \phi_1 y_{t-1} + \ldots + \phi_p y_{t-p} + \varepsilon_t \tag{3.36}$$

Let y_t be a vector of length *k*. θ is the constant, $\phi' s$ are the coefficients and ε_t is the white noise with zero mean and constant variance.

EGARCH Model

EGARCH is one of the GARCH family models that have been modelling the heteroscedastic error. It can be written as:

$$\log h_t = \alpha + \beta | z_{t-1} | + \delta z_{t-1} + \gamma \log h_{t-1}, \qquad | \gamma | < 1 \tag{3.37}$$

where $z_t = \varepsilon_t \big/ \sqrt{h_t}$ is the standardized shocks, $z_t \sim iid\,(0, A)$. $|\gamma| < 1$ is when there is stability. The impact is asymmetric if $\delta \neq 0$, although, there is existence of leverage if $\delta < 0$ and $\beta < -\delta$. Since both β and δ must be positive which are the variances of two stochastic processes, then, modelling leverage effect is not possible (McAleer, 2014; McAleer & Hafner, 2014; Martinet & McAleer, 2016).

MA Process

When a moving average process is finite, it was constantly stationary and the errors are white noise. The process can be written as:

$$Y_t = \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \ldots + \theta_q \varepsilon_{t-q} \tag{3.38}$$

θ is the coefficient of the past error terms and ε_t is the current error term which is white noise.

The performances of CWN, VAR, EGARCH and MA models were validated using standard error, log-likelihood, information criteria and forecast error measures as follow:

Standard Error

The estimation of standard error of a regression or model is the measurement of the error and the smaller the value of standard error, the appropriate the model is. It can be as written:

$$SE = \sqrt{\frac{\sum (y_i - \hat{y}_i)^2}{n-1}} \tag{3.39}$$

where y is the observation, n is the number of sample size and $\hat{y}$ was the predicted value.

Log-Likelihood

Log-likelihood is data summarizing implement to obtain evidence about unknown parameters, which is computational convenience. The highest the value of log-likelihood considered the best among the models. Mostly, log-likelihood is derived from a sample. If an independent sample $x_1, x_2, ..., x_n$ from a distribution $f(x \mid \theta)$, then the log-likelihood is:

$$l(\theta \mid x) = \log \prod_{i=1}^{n} f(x_i \mid \theta)$$

$$= \sum_{i=1}^{n} l(\theta \mid x_i) \tag{3.40}$$

Where Π is the product, Σ is the sum total and θ is the parameter (Myung, 2003; Park, Simar & Zelenyuk, 2015).

Information Criteria

Information criteria are used to select the best among some models with minimum values of Akaike information criteria (AIC) and Bayesian information criteria (BIC) which are the two information criteria considered in this study.

Akaike Information Criteria (AIC)

Snipes and Taylor (2014) revealed that the selection of the best model is resolved by the minimum AIC value among the AIC values of the models. AIC can be written as:

$$AIC = 2K - 2\log L \tag{3.41}$$

where L is the likelihood value and K is the number of parameters in the model.

Bayesian Information Criteria (BIC)

BIC selects the best model with minimum BIC value among the BIC values of the models. The parameters are panellized in BIC than AIC (Spiegelhalter, Best, Carlin, & Linde, 2014).

$$BIC = K\ln(n) - 2\ln L \tag{3.42}$$

where K is the number of free parameters to be estimated, or the number of regressors, including the intercept in the estimated model. n is the number of observations (sample size). L is the likelihood value.

Forecast Error Measures

Error measure is a criterion used to express the dissimilarities between poor forecast model and good forecast model. The criterion is to have the minimum values of error measures (Armstrong, & Collopy, 1992; Lazim, 2013).

There is no particular error measure that is good enough for good forecast model. Forecasters used more than one error measures to achieve the accuracy, reliable and consistency of the forecast evaluation results. When a particular model gives minimum values in the number of error measures considered, then the model is good for forecasting (Armstrong, & Collopy, 1992; Lazim, 2013). Four forecast error measures were used in this study; root mean square error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE), geometric root mean square error (GRMSE).

Root Mean Square Error

The root mean square error (RMSE) is the square root of the mean of the square of all of the error. RMSE gives equal weight to all the errors at any period of time. RMSE formula is:

$$RMSE = \sqrt{\frac{\sum_{t}^{n} e_t^2}{n}} \tag{3.43}$$

where $e_t = y_t - \hat{y}_t$, the actual observed value in time t is y_t and $\hat{y}_t$ is the fitted value in time t. RMSE is considered relevant in decision making (Lazim, 2013).

Mean Absolute Error

Mean absolute error (MAE) determines the closeness of the forecasts or predictions of the final outcomes in measurement. In time series analysis, the common measure of forecast error measure is mean absolute error. The MAE is:

$$MAE = \frac{1}{n}\sum_{i=1}^{n} | f_i - y_i | = \frac{1}{n}\sum_{i=1}^{n} | e_t | \tag{3.44}$$

The average absolute errors is the mean absolute error which is $| e_t | = | f_i - y_i |$ where the prediction and the true value are f_i and y_i respectively. The mean absolute error is a scale-dependent accuracy measure because the mean absolute error is on same scale of data being measured (Chai & Draxler, 2014).

Mean Absolute Percentage Error

The mean absolute percentage error (MAPE) measures the prediction exactness of forecasting method, even in trend estimation. MAPE measured in series is given as:

$$MAPE = \sum_{t=1}^{n} \frac{\left|(e_t \mid y_t)\right| * 100}{n} \tag{3.45}$$

where $\left|(e_t \mid y_t)\right| * 100$ is the absolute percentage error computed on the fitted values for a forecasting method n is the number of fitted points (Lazim, 2013).

Geometric Root Mean Square Error

The geometric root mean square error (GRMSE) overcomes the challenges of data having outliers to have a good forecast evaluation. When there are large forecast error measure terms because of the forecast that are not good, the GRMSE is employed to uplift the challenges. The GRMSE is:

$$GRMSE = (\prod_{t}^{n} e_{itp}^{2})^{\frac{1}{2n}} \tag{3.46}$$

where the number of effective data point is n, e_{itp}^{2} is the actual observation at time t for i series using method p (Fildes, 1992; Fildes, Wei, & Ismail, 2011).

3.5 Model Accuracy

Model accuracy is calculated by dividing the logarithm of predicted value with the logarithm of actual value which is equal to log (Q) or ln (Q). The result is multiplied by one hundred to obtain the percentage of ln (Q) which is also the percentage of model accuracy, is dimensionless and used in comparisons across data sets. Logarithm is used to take care of the asymmetry challenges. $\sum(\ln Q)^2$ is used mostly when the data is heteroscedastic to obtain the model accuracy.

The models accuracy:

$$\ln(Q) = \ln\left(\frac{predicted\ value}{actual\ value}\right) \tag{3.47}$$

$$= \ln(predicted\ value) - \ln(actual\ value)$$

$$\sum \ln(Q)^2 = \sum \ln\left(\frac{predicted\ values}{actual\ values}\right)^2 \tag{3.48}$$

$$= \sum [\ln(predicted\ values) - \ln(actual\ values)]^2$$

$\sum$ is the summation of the items. The models satisfied the set of desirable properties of most of the model accuracy measures (Törnqvist, P. Vartia, & Y. O. Vartia, 1985; Tofallis, 2014).

3.6 Summary

The new model called CWN has been derived based on the twelve steps using EGARCH and BMA successfully. The developments of CWN for the upliftment of the challenges of leverage effect in the heteroscedastic data were discussed. The validations of the CWN model through simulated and real data were explained.

The three different sample sizes of 200, 250 and 300 data were simulated; 200 sample size was considered as low, 250 sample size as moderate and 300 sample size as high(Ng & Lam, 2006).Each of these sample sizes was used with different values of low, moderate and high skewness. The low, moderate and high leverage for each sample size were equally examined. The estimation of the simulated data disclosed the characteristics of the heteroscedasticity with the new model. Data simulations were used to compare CWN with VAR, EGARCH and MA. Chapter Four revealed the outcome.

The validation of the model using real data was explained. Four real datasets were used to investigate the performance of the model. Real data were used to compare

CWN with VAR, EGARCH and MA. Chapter Five presented the description of the validation of the model using real data sets.

The developments and validations of CWN can overcome the leverage effect in the heteroscedastic data.

CHAPTER FOUR
VALIDATION OF COMBINE WHITE NOISE USING SIMULATED DATA

4.1 Introduction

This chapter detailed the performances of the combine white noise with simulated data of three different types of sample sizes. Each of these sample size was used for low, moderate and high values of leverages and skewness as explained with the twelve steps in Chapter Three, Section 3.2 and Subsection 3.2.1.Section 4.2 described the data simulation. The twelve steps were employed in Section 4.3for the description of model development process. Section 4.4 described the performance of the validated models by comparison with results in Subsections 4.4.1 to 4.4.3. Section 4.5 summarized the findings of the simulation study in Section 4.4.

4.2 Data Simulation

The data were simulated based on the condition that the intercept (ω) is the long-term log-volatility,GARCH parameters (α) was less than one which indicated stability, ARCH parameters (β) was less than one indicating stationary, degree of freedom(df) with different values of leverages (δ) and skewness (γ) as reported in Table 4.1.The data were simulated according to the Beta-Skew-t-EGARCH models, that is, the EGARCH models with leverages and skewness values using *betategarch* package in R software (Sucarrat, 2013). The estimated parameters for the simulated 200 sample size of data were close to the postulated model as were reported in Table 4.1. Postulated model is the model assumed as a basis of an argument. Similar results were obtained when 250 and 300 sample sizes were conducted.

Table 4.1

The Estimated Parameters of the Simulated Data for Postulated Model with different values of Leverages and Skewness of 200 Sample Size for EGARCH Model

Low leverage and low skewness						
Parameters	ω	α	β	δ	df	γ
Postulate	0.01	0.5	0.1	0.01	10	0.5
Estimates	-0.06	0.51	0.08	0.01	7.50	0.46
Low leverage and moderate skewness						
Parameters	ω	α	β	δ	df	γ
Postulate	0.01	0.5	0.1	0.01	10	0.7
Estimates	-0.03	0.69	0.04	-0.03	11.14	0.56
Low leverage and high skewness						
Parameters	ω	α	β	δ	df	γ
Postulate	0.01	0.5	0.1	0.01	10	1.2
Estimates	0.06	0.51	0.05	0.04	7.57	1.21
Moderate leverage and low skewness						
Parameters	ω	α	β	δ	df	γ
Postulate	0.01	0.5	0.1	0.05	10	0.5
Estimates	-0.09	0.06	0.05	0.07	9.99	0.41
Moderate leverage and moderate skewness						
Parameters	ω	α	β	δ	df	γ
Postulate	0.01	0.5	0.1	0.05	10	0.7
Estimates	0.01	0.63	0.06	0.01	9.99	0.60
Moderate leverage and high skewness						
Parameters	ω	α	β	δ	df	γ
Postulate	0.01	0.5	0.1	0.05	10	1.2
Estimates	0.07	0.51	0.05	0.09	8.04	1.21
High leverage and low skewness						
Parameters	ω	α	β	δ	df	γ
Postulate	0.01	0.5	0.1	0.09	10	0.5
Estimates	-0.13	0.21	0.05	0.11	9.98	0.40
High leverage and moderate skewness						
Parameters	ω	α	β	δ	df	γ
Postulate	0.01	0.5	0.1	0.09	10	0.7
Estimates	0.01	0.46	0.07	0.05	9.38	0.60
High leverage and high skewness						
Parameters	ω	α	β	δ	df	γ
Postulate	0.01	0.5	0.1	0.09	10	1.2
Estimates	0.08	0.52	0.05	0.13	8.78	1.21

4.3 Model Development

Twelve steps were employed for the development of the model.

Step 1: VAR white noise estimation was efficient but weak in modelling heteroscedasticity, the weakness were as reported in Table 4.8 to Table 4.16.

Step 2: EGARCH estimation modelled heteroscedasticity without leverage effect efficiently but weak in modelling the leverage effect in the heteroscedasticity, the weakness were as reported in Table 4.8 to Table 4.16.Therefore, the data that exhibited heteroscedasticity were simulated, estimated and the graphs of the estimated standardized residuals with unequal variances and zero mean were considered in this study to resolve the leverage effect challenges.

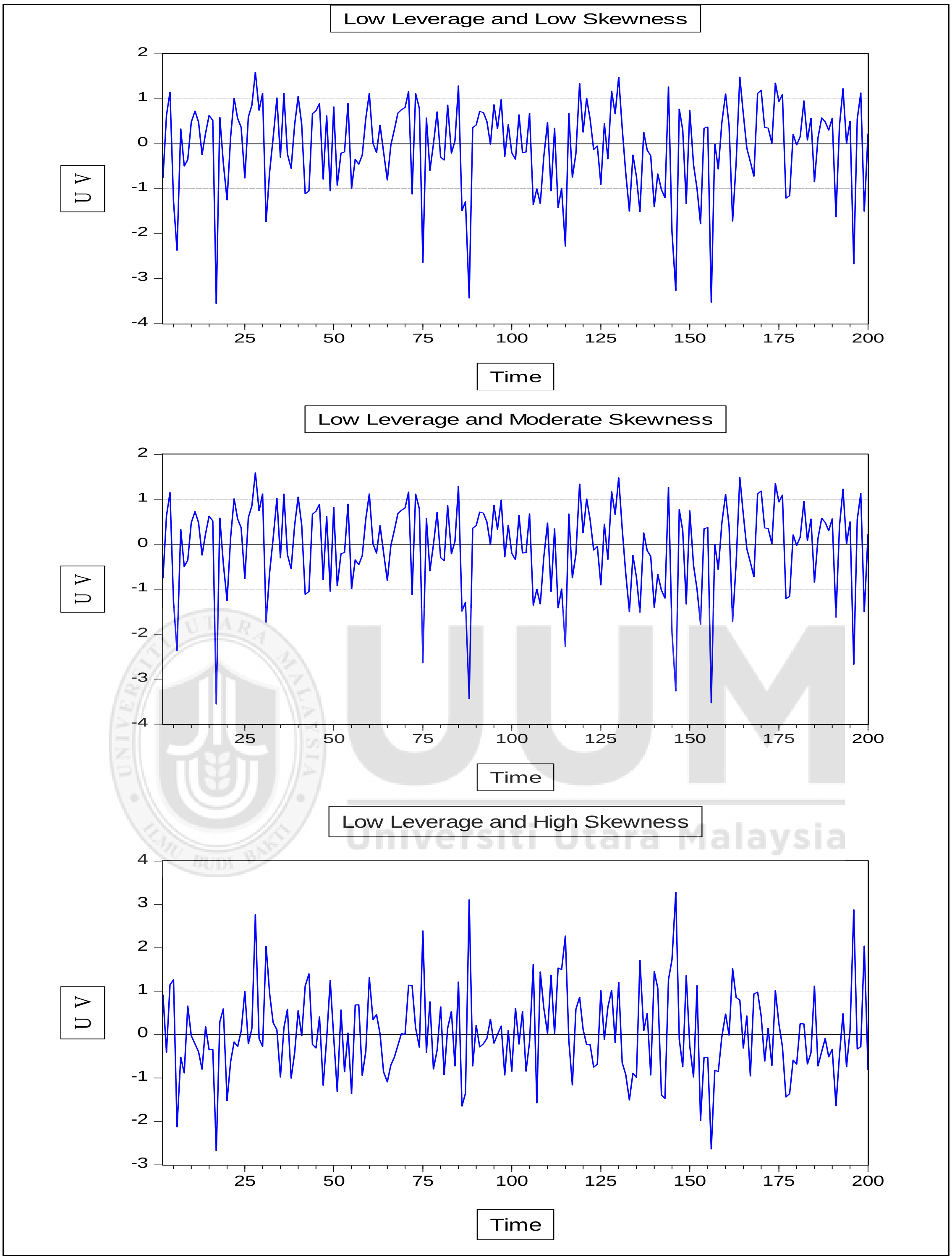
Step 3: The simulated data of 200 sample size were estimated to obtain the standardized residuals in graphical form. The graphs of standardized residuals displayed the error terms of these models for the purpose of this study. The error terms have the characteristics of heteroscedasticity with leverage effect(unequal variances), which made up the conditional variance challenges in the estimation for 200 sample size different values of leverages and different values of skewness as reported in Figure 4.1 to Figure4.3. Similar results were obtained when standardized residuals graphs of 250 and 300 sample sizes were conducted.

Step 4: The standardized residuals graphs of unequal variances were decomposed (rearranged and grouped) manually into equal variances (white noise) series to overcome the leverage effect which were examined by displaying in graphical form. The decomposition for low leverage and low skewness have forty equal variances, the low leverage and moderate skewness have forty four equal variances and the low leverage and high skewness have forty three equal variances. The decomposition for moderate leverage and low skewness have forty equal variances, the moderate leverage and moderate skewness have forty five equal variances and the moderate leverage and high skewness have forty one equal variances. The decomposition for high leverage and low skewness have forty one equal variances, the high leverage and moderate skewness have forty four equal variances and the high leverage and high skewness have forty three equal variances. Maximum likelihood estimation method was applied on each equal variance to obtain the log-likelihood.

Step 5: The Log-Likelihood

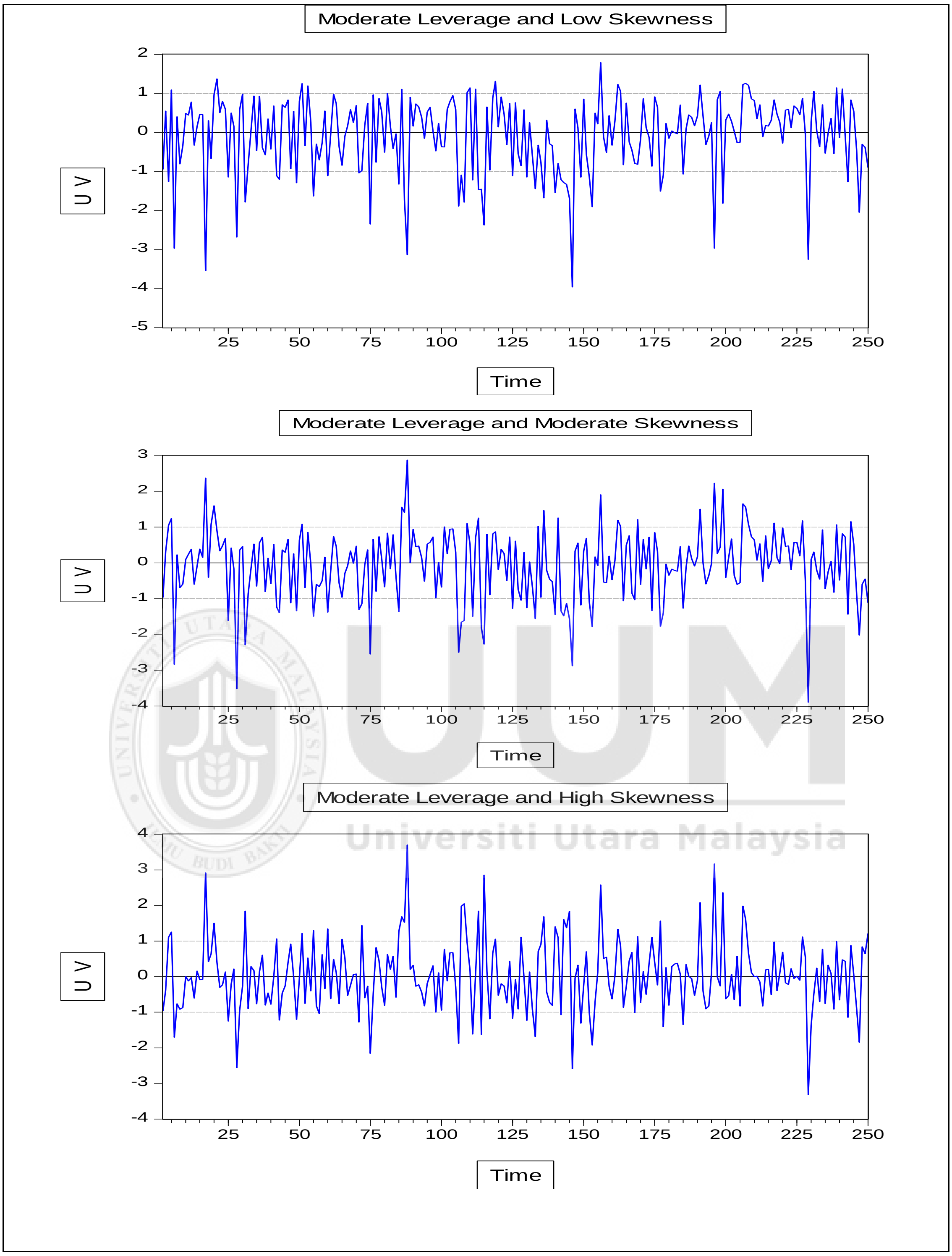
The log-likelihood was maximized by the maximum likelihood estimation method for the number of equal variances in each standardized residual. The estimation of maximum likelihood was employed to optimize the parameters for sufficiency, consistency, efficiency and invariance parameterization of the equal variances (white noise) series. The log-likelihood values were reported in Appendix C.

Based on the log-likelihood obtained, the number of equal variances (white noise) from each standardized residuals were fitted into linear model by MLE and BIC for modelling each equal variance. This revealed the equal variance model called white noise (WN) model.



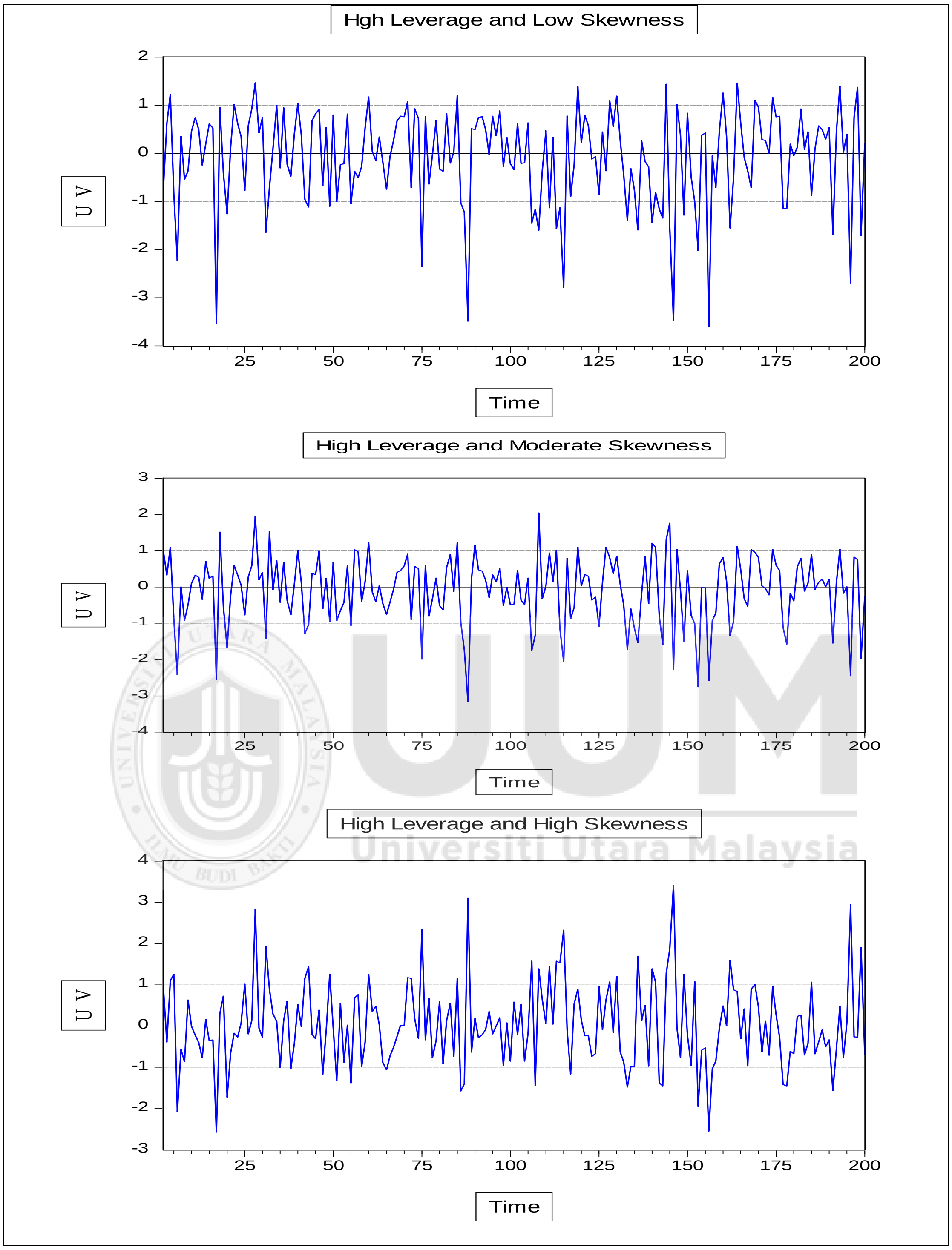
U V represented Unequal Variances

*Figure 4.1.*Graphs of Standardized Residuals for Low Leverage and Different Values of Skewness



U V represented Unequal Variances

*Figure 4.2.*Graphs of Standardized Residuals for Moderate Leverage and Different Values of Skewness



U V represented Unequal Variances

Figure 4.3. Graphs of Standardized Residuals for High Leverage and Different Values of Skewness

Step 6: Fitting Linear Model and BIC

The linear model was fitted into the series of equal variances (WN) by MLE and BIC to obtain the fitted WN models. In fitting these linear models, each WN model has mean zero and variance one (constant) and each model was significant. White noise assumed zero mean and constant variance. Therefore, WN models with zero mean and constant variance confirmed the WN. The standardized residual graphs have zero mean. The Bayesian model averaging was used for model selection.

Step 7: Bayesian Model Averaging (BMA)

There were 2^K certainty and uncertainty models to account for, and for this study K were the numbers of equal variances in step 5 that were transformed to models in step 6by fitting the linear model. Some models were selected out of 2^K uncertainty and certainty models. The best models were determined by the lowest BIC and highest posterior probability (the correct model) in BMA output. The computer outputs were in Appendix D.

The Appendix D detail were: The column “p!=0” indicated the probability that the coefficient for a given predictor is not zero. This indicated that at least one of the best models considered in the row directly under the column “p!=0”. The column “EV” displayed the BMA posterior distribution mean for each coefficient and the column “SD” displayed the BMA posterior distribution standard deviation for each coefficient. The posterior probabiliy of quantity of interest was determined by each of the models considered when the posterior propability was correct, given that one of the considered models was correct.The best five models (discribed as model 1, model 2, model 3, model 4 and model 5)were displayed. The predictors (independent variables)to be included in a regression model were determined by BMA. Two best predictors were displayed in Appendix D (number 10, number 11 and number 12).

Appendix D (number 1 to number 9) displayed the numbers of predictors for 200 sample size. Similar results were computed for 250 and 300 sample sizes. Appendix D (number 10) summarized the BMA for 200 sample size. The low leverage and low skewness revealed that predictor *A* has the best model which was in the third model discribedas model 3 with minium BICand highest posterior probability values. Predictor *B* has the best model in model 4 which was the best model. The low leverage and moderate skewness revealed that predictor *C*has the best model inmodel 4 with minium BIC and highest posterior probability values. Predictor *D* has the best model in model 3 which was the best model. The low leverage and high skewness revealed the best model was in model 3 with minium BIC and highest posterior probability values as predictor *E*. Predictor *F* has the best model in model 2 which was the best model.

The moderate leverage and low skewness revealed the best model was in model 3 with minium BIC and highest posterior probability values as predictor *G*. Predictor *H* has the best model in model 2 which was the best model. The moderate leverage and moderate skewness revealed the best model was in model 2 with minium BIC and highest posterior probability values as predictor *I*. Predictor *J* has the best model in model 3 which was the best model. The moderate leverage and high skewness revealed the best model was in model 2 with minium BIC and highest posterior probability values as predictor *K*. Predictor *L* has the best model in model 3 which was the best model.

The high leverage and low skewness revealed the best model was in model 3 with minium BIC and highest posterior probability values as predictor *M*. Predictor *N* has the best model in model 2 which was the best model. The high leverage and moderate skewness revealed the best model was in model 2 with minium BIC and highest posterior probability values as predictor *P*. Predictor *Q* has the best model in model 3 which was the best model. The high leverage and high skewness revealed the best model was in model 2 with minium BIC and highest posterior probability values as predictor *R*. Predictor *S* has the best model in model 3 which was the best model.

Appendix D (number 11) summarized the BMA for 250 sample size. The low leverage and low skewness revealed the best model was in model 3 with minium BIC and highest posterior probability values as predictor A_1. Predictor B_1has the best model in model 2 which was the best model. The low leverage and moderate skewness revealed the best model was in model 3 with minium BIC and highest posterior probability values as predictor C_1. Predictor D_1 has the best model in model 4 which was the best model. The low leverage and high skewness revealed the best model was in model 2 with minium BIC and highest posterior probability values as predictor E_1. Predictor F_1 has the best model in model 3 which was the best model.

The moderate leverage and low skewness revealed the best model was in model 1 with minium BIC and highest posterior probability values as predictor G_1. Predictor H_1 has the best model in model 2 which was the best model. The moderate leverage and moderate skewness revealed the best model was in model 4 with minium BIC and highest posterior probability values as predictor I_1. Predictor J_1 has the best model in model 3 which was the best model. The moderate leverage and high skewness revealed the best model was in model 2 with minium BIC and highest posterior probability values as predictor K_1. Predictor L_1 has the best model in model 3 which was the best model.

The high leverage and low skewness revealed the best model was in model 3 with minium BIC and highest posterior probability values as predictor M_1. Predictor N_1 has the best model in model 2 which was the best model. The high leverage and moderate skewness revealed the best model was in model 3 with minium BIC and highest posterior probability values as predictor P_1. Predictor Q_1 has the best model in model 2 which was the best model. The high leverage and high skewness revealed the best model was in model 2 with minium BIC and highest posterior probability values as predictor R_1. Predictor S_1 has the best model in model 3 which was the best model.

Appendix D (number 12) summarized the BMA for 200 sample size. The low leverage and low skewness revealed the best model was in model 3 with minium BIC and highest posterior probability values as predictor A_2. Predictor B_2has the best model in model 2 which was the best model. The low leverage and moderate skewness revealed the best model was in model 3 with minium BIC and highest posterior probability values as predictor C_2. Predictor D_2 has the best model in model 3 which was the best model. The low leverage and high skewness revealed the best model was in model 1 with minium BIC and highest posterior probability values as predictor E_2. Predictor F_2 has the best model in model 2 which was the best model.

The moderate leverage and low skewness revealed the best model was in model 4 with minium BIC and highest posterior probability values as predictor G_2. Predictor H_2 has the best model in model 3 which was the best model. The moderate leverage and moderate skewness revealed the best model was in model 3 with minium BIC and highest posterior probability values as predictor I_2. Predictor J_2 has the best model in model 2 which was the best model. The moderate leverage and high skewness revealed the best model was in model 2 with minium BIC and highest posterior probability values as predictor K_2. Predictor L_2 has the best model in model 3 which was the best model.

The high leverage and low skewness revealed the best model was in model 2 with minium BIC and highest posterior probability values as predictor M_2. Predictor N_2 has the best model in model 3 which was the best model. The high leverage and moderate skewness revealed the best model was in model 2 with minium BIC and highest posterior probability values as predictor P_2. Predictor Q_2 has the best model in model 3 which was the best model. The high leverage and high skewness revealed the best model was in model 3 with minium BIC and highest posterior probabilityvalues as predictor R_2. Predictor S_2 has the best model in model 2 which was the best model.

Step 8: Fitting Linear Regression with Autoregressive Errors

Fitting linear regression with autoregressive errors of which 200 were the numbers of sample size, with zero mean and variance one for each model to confirm that the white noise models were invertible to AR models (Higgins & Bera 1992). The estimated values of the fitted linear regression with autoregressive errors, based on significant code asterisk showed the best models for different values of leverages and skewness as reported in Appendix E (number 1 to number 9) for 200 sample size. Similar results were computed for 250 and 300 sample sizes. The best two models were summarized in Appendix E (number 10, number 11 and number 12).These confirmed the best selected models by BMA.

P-values revealed the significant values of each best two models in different values of leverages and skewness. The more significant of each of the two models indicated the dependent variable for the combine white noise (CWN) in step 12. When the two models were having equal significant values as models G and H in Appendix E (number 10), models R_1 and S_1 in Appendix E (number 11), models A_2 and B_2 in Appendix E (number 12) of the three different sample sizes respectively, the best which has the minium BIC and highest posterior probability values in step 7 were considered as dependent variable.

Appendix E (number 10) displayed the fitted linear regression with autoregressive errors for 200 sample size.The low leverage and low skewness shown that Model *A*was the dependent variable because its value was more significant than model *B*. The low leverage and moderate skewness shown that Model *D* was considered as dependent variable because its value was more significant than model *C*. The low leverage and high skewness shown that Model *F* was the dependent variable because its value was more significant than model *E*.

The moderate leverage and low skewness shown that Model *G* and model *H* were having the same value of significant figures, model *H*was considered as dependent variable in step 12 because predictor *H*value was in model 2 step 7 as reported in Appedix D (number 10) with minium BIC and high posterior probability values.The moderate leverage and moderate skewness shown that Model *I* was considered as dependent variable because its value was more significant than model *J*.

The high leverage and low skewness shown that Model *M* was considered as dependent variable because its value was more significant than model *N*. The high leverage and moderate skewness shown that Model *P* was considered as dependent variable because its value was more significant than model *Q*. The high leverage and high skewness shown that Model *S* was considered as dependent variable because its value was more significant than model *R*.

Appendix E (number 11) displayed the fitted linear regression with autoregressive errors for 200 sample size. The low leverage and low skewness shown that Model A_1 was considered as dependent variable because its value was more significant than model B_1. The low leverage and moderate skewness shown that Model C_1 was considered as dependent variable because its value was more significant than model D_1. The low leverage and high skewness shown that Model E_1 was considered as dependent variable because its value was more significant than model F_1.

The moderate leverage and low skewness shown that Model G_1 was considered as dependent variable because its value was more significant than model H_1. The moderate leverage and moderate skewness shown that Model J_1 was considered as dependent variable because its value was more significant than model I_1. The moderate leverage and high skewness shown that Model K_1 was considered as dependent variable because its value was more significant than model L_1.

The high leverage and low skewness shown that Model N_1 was considered as dependent variable because its value was more significant than model M_1. The high leverage and moderate skewness shown that model Q_1 was considered as dependent variable because its value was more significant than model P_1. The high leverage and high skewness shown that model R_1 and model S_1 were having the same value of significant figures, model $R_1$was considered as dependent variable in step 12 because predictor R_1 value was in model 2 in step 7 with minium BIC and high posterior probability values.

Appendix E (number 12) displayed the fitted linear regression with autoregressive errors for 200 sample size. The low leverage and low skewness shown that model A_2 and model B_2 were having the same value of significant figures, model B_2 was considered as dependent variable because predictor B_2 value was in model 2 in step 7 with minium BIC and high posterior probability values. The low leverage and moderate skewness shown that Model C_2 was considered as dependent variablebecause its value was more significant than model D_2. The low leverage and high skewness shown that Model F_2 was considered as dependent variable because its value was more significant than model E_2.

The moderate leverage and low skewness shown that Model H_2 was considered as dependent variable because its value was more significant than model G_2. The moderate leverage and moderate skewness shown that Model I_2 was considered as dependent variable because its value was more significant than model J_2. The moderate leverage and high skewness shown that Model K_2 was considered as dependent variable because its value was more significant than model L_2.

The high leverage and low skewness shown that Model M_2 was considered as dependent variable because its value was more significant than model N_2. The high leverage and moderate skewness shown that Model $P_2$was considered as dependent variable because its value was more significant than model Q_2. The high leverage and high skewness shown that Model R_2 was considered as dependent variablebecause its value was more significant than model S_2.

The SARIMA models were used for the lag selection of autoregressive order of the models with *Y* as dependent variable of a model.

Step 9: The Regression Model with ARIMA Errors

Firstly, regress with the models obtained in step 8, and then run the following ACF of the models. The ACF spike of the first lag signified autoregressive (AR) of order one which was significant, while the rest lags were close to zero which signified that the orders were zero. SARIMA (1, 0, 0) indicated AR (1) converge with short iteration. Therefore, SARIMA (1, 0, 0) were considered as the best.

The confirmation of two models from the result of BMA in step 7by fitting the linear regression with autoregressive errors in step 8 revealed that the first columns for the first model with 200 sample size of leverages and skewness values as shown in Figure

4.4 to Figure 4.6which displayed AR order one for the first columns. The second columns displayed AR order one for the second models with 200 sample sizes values of leverages and skewness. These revealed that all the ARs were of order one as displayed in Figure 4.4 to Figure 4.6. This was to confirm the right order, and all the models were of order one.

With these reports, autoregressive model of order one was considered for 200 sample size as displayed in Figure 4.4 to Figure 4.6. Therefore, this confirmed the autoregressive model of order one [AR (1)] in the following computation**.** Similar results were obtained when 250 and 300 sample sizes were conducted and ARs were of order one.

Step 10: Fit AR with ARIMA Modelling of Time Series

This was to obtain the autoregressive model (AR) of each model. Use lowest AIC value to obtain and confirm the right order of AR model. Only models of lowest AIC values were reported which were AR of order one as reported in Table 4.2 to Table 4.4. The computer outputs were reported in Appendix G for 200 sample size. Similar results were computed for 250 and 300 sample sizes.

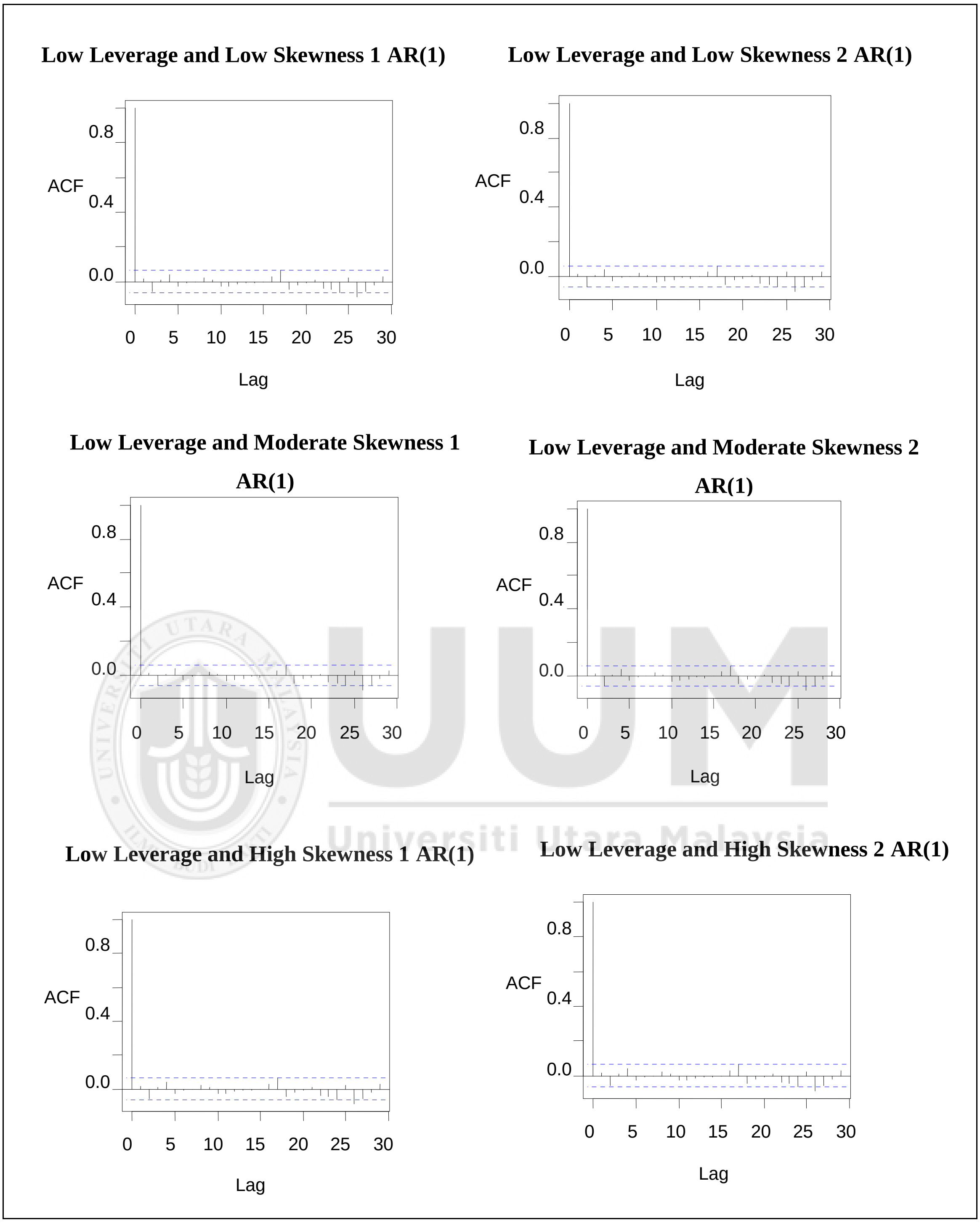


*Figure 4.4.*The ACF of Low Leverage and different Values of Skewness

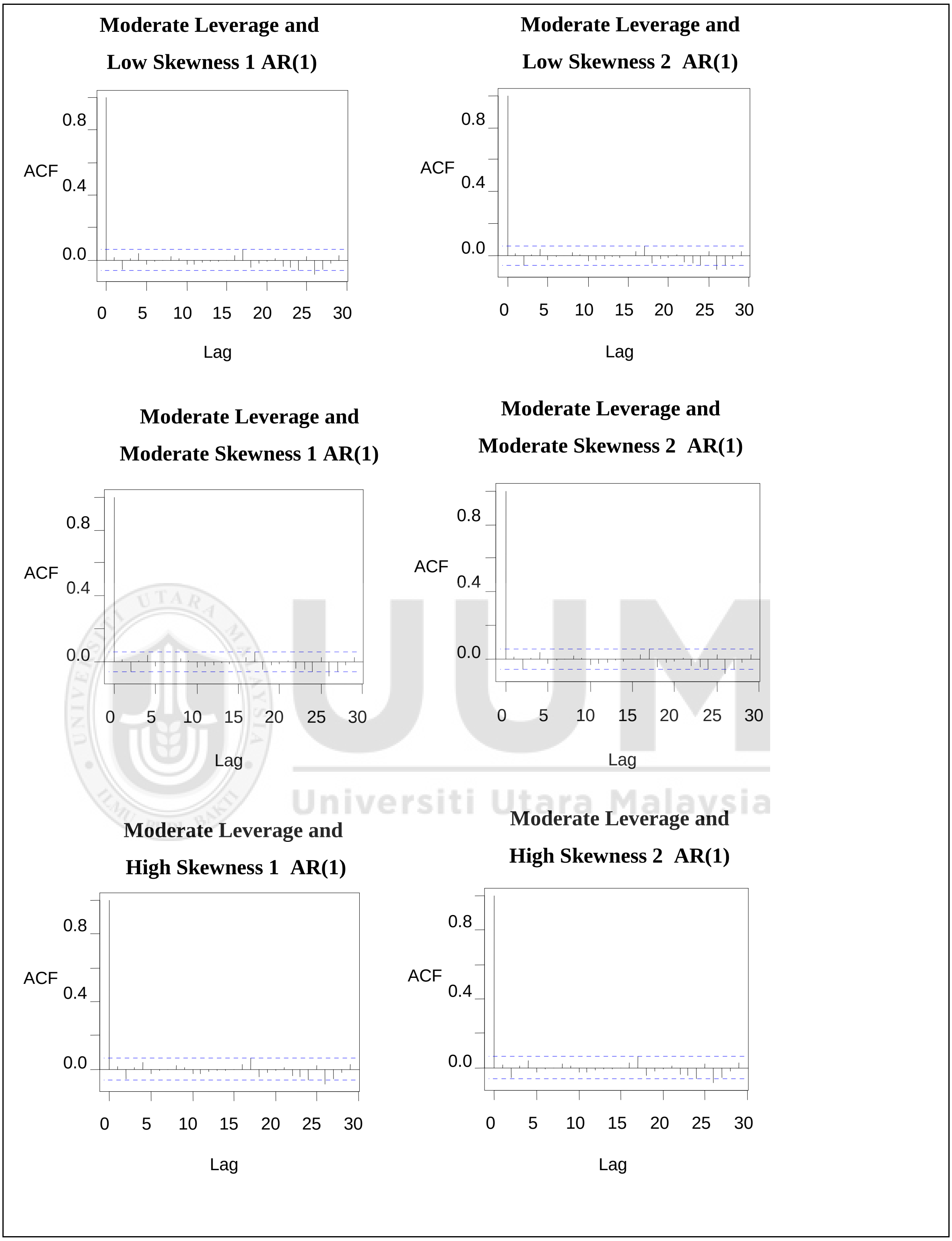


Figure 4.5. The ACF of Moderate Leverage and different Values of Skewness

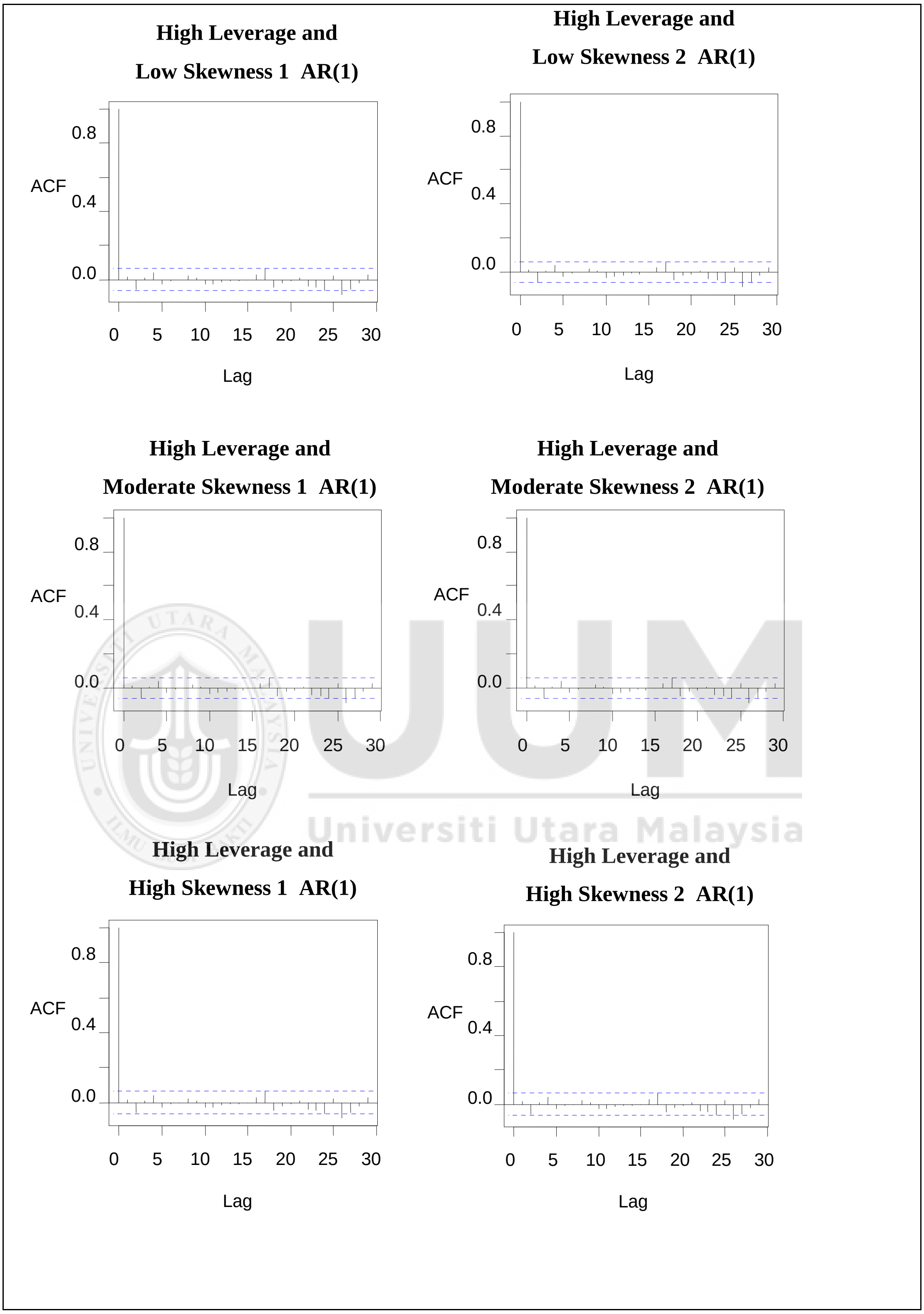


*Figure 4.6.*The ACF of High Leverage and different Values of Skewness

All the models were having ARIMA (1, 0, 0): AR (1) with the smallest AIC values. Y was the dependent variable of the AR (1). Ordinary least square (OLS) method was used to obtain the coefficients of the models.

Table 4.2

Obtaining the AR Order of Each Model for 200 Sample Size

	Each Model is Order One					
ARIMA	$x=A$	$x=B$	$x=Y$	$x=C$	$x=D$	$x=Y$
Order	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)
AIC	602.47	594.21	675.97	602.47	646.95	675.97
ARIMA	$x=E$	$x=F$	$x=Y$	$x=G$	$x=H$	$x=Y$
Order	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)
AIC	594.21	646.95	675.97	571.8	623.13	675.97
ARIMA	$x=I$	$x=J$	$x=Y$	$x=K$	$x=L$	$x=Y$
Order	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)
AIC	655.72	610.03	675.97	627.53	594.21	675.97
ARIMA	$x=M$	$x=N$	$x=Y$	$x=P$	$x=Q$	$x=Y$
Order	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)
AIC	623.13	655.72	675.97	623.13	655.72	675.97
ARIMA	$x=R$	$x=S$	$x=Y$			
Order	(1,0,0)	(1,0,0)	(1,0,0)			
AIC	2727.17	2727.17	675.97			

Table 4.3

Obtaining the AR Order of Each Model for 250 Sample Size

Each Model is Order One						
ARIMA	$x = A_1$	$x = B_1$	$x = Y_1$	$x = C_1$	$x = D_1$	$x = Y_1$
Order	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)
AIC	2727.17	2744.34	675.97	2720.1	2749.49	675.97
ARIMA	$x = E_1$	$x = F_1$	$x = Y_1$	$x = G_1$	$x = H_1$	$x = Y_1$
Order	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)
AIC	2778.11	2749.49	675.97	2721.1	633.09	675.97
ARIMA	$x = I_1$	$x = J_1$	$x = Y_1$	$x = K_1$	$x = L_1$	$x = Y_1$
Order	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)
AIC	2773.99	2761.73	675.97	2845.17	2758.76	675.97
ARIMA	$x = M_1$	$x = N_1$	$x = Y_1$	$x = P_1$	$x = Q_1$	$x = Y_1$
Order	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)
AIC	2720.1	633.09	675.97	2847.7	2806.7	675.97
ARIMA	$x = R_1$	$x = S_1$	$x = Y_1$			
Order	(1,0,0)	(1,0,0)	(1,0,0)			
AIC	2735.71	2768.72	675.97			

Table 4.4

Obtaining the AR Order of Each Model for 300 Sample Size

Each Model is Order One						
ARIMA	$x = A_2$	$x = B_2$	$x = Y_2$	$x = C_2$	$x = D_2$	$x = Y_2$
Order	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)
AIC	2720.1	2735.57	675.97	2735.71	2735.57	675.97
ARIMA	$x = P_2$	$x = Q_2$	$x = Y_2$	$x = W_2$	$x = Z_2$	$x = Y_2$
Order	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)
AIC	2720.1	2735.57	675.97	2727.17	2777.82	675.97
ARIMA	$x = A_2$	$x = B_2$	$x = Y_2$	$x = C_2$	$x = D_2$	$x = Y_2$
Order	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)
AIC	2735.57	2795.3	675.97	2733.88	2721.1	675.97
ARIMA	$x = P_2$	$x = Q_2$	$x = Y_2$	$x = W_2$	$x = Z_2$	$x = Y_2$
Order	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)
AIC	2823.76	2720.1	675.97	2847.7	2823.76	675.97
ARIMA	$x = A_2$	$x = B_2$	$x = Y_2$			
Order	(1,0,0)	(1,0,0)	(1,0,0)			
AIC	2720.51	2845.17	675.97			

Step 11: To obtain the Coefficients of the Model using OLS

Using OLS to obtain the coefficients of the AR, maximum order of one andAIC value was not applied. OLS has good finite-sample properties when compared with Yule-Walker estimator, even after the bias was corrected. OLS has the smallest mean square error for stationary models when compared with bias formula and bootstrap procedure (Engsted & Pedersen, 2014) as reported in Table 4.5 to Table 4.7.

Table 4.5

Using OLS to obtain the Coefficients of the Models for200 Sample Size

Model	Coefficients	Maximum order	Sigma^2 estimated	Intercept
A	0.0484	1	1.0330	0.0005
B	0.0160	1	0.9557	-0.0009
Y	0.1326	1	1.6770	-0.0003
C	-0.0309	1	0.9608	7.981e-05
D	-0.0379	1	0.9604	0.0005
Y	0.1326	1	1.6770	-0.0003
E	0.0078	1	0.9449	-0.0008
F	-0.0379	1	0.9604	0.0005
Y	0.1326	1	1.6770	-0.0003
G	0.0139	1	0.9517	0.0008
H	0.0039	1	1.0030	0.0006
Y	0.1326	1	1.6770	-0.0003
I	-0.0379	1	0.9604	0.0005
J	-0.0269	1	1.0200	0.0011
Y	0.1326	1	1.6770	-0.0003
K	-0.0056	1	0.9589	0.0003
L	-0.0030	1	0.9453	-0.0011
Y	0.1326	1	1.6770	-0.0003
M	0.0291	1	1.0510	0.0001
N	0.0078	1	0.9449	-0.0008
Y	0.1326	1	1.6770	-0.0003
P	0.0014	1	1.0770	-0.0007
Q	0.0291	1	1.0510	0.0001
Y	0.1326	1	1.6770	-0.0003
R	0.0102	1	0.9450	-0.0010
S	-0.0307	1	1.0750	-0.0002
Y	0.1326	1	1.6770	-0.0003

Table 4.6

Using OLS to obtain the Coefficients of the Models for250 Sample Size

Model	Coefficients	Maximum order	Sigma^2 estimated	Intercept
A_1	0.0139	1	0.9517	0.0008
B_1	-0.0718	1	0.9663	0.0018
Y_1	0.1326	1	1.6770	-0.0003
C_1	0.0078	1	0.9449	-0.0008
D_1	7e-04	1	0.9741	-0.0006
Y_1	0.1326	1	1.6770	-0.0003
E_1	0.0318	1	1.0030	0.0004
F_1	7e-04	1	0.9741	-0.0006
Y_1	0.1326	1	1.6770	-0.0003
G_1	-0.0030	1	0.9453	-0.0011
H_1	-0.0072	1	1.0170	-0.0010
Y_1	0.1326	1	1.6770	-0.0003
I_1	-0.0080	1	0.9913	0.0028
J_1	0.0014	1	0.9865	0.0005
Y_1	0.1326	1	1.6770	-0.0003
K_1	-0.0307	1	1.0750	-0.0002
L_1	-0.0200	1	0.9836	0.0003
Y_1	0.1326	1	1.6770	-0.0003
M_1	0.0078	1	0.9449	-0.0008
N_1	-0.0072	1	1.0170	-0.0072
Y_1	0.1326	1	1.6770	-0.0003
P_1	0.0014	1	1.0770	-0.0007
Q_1	0.0484	1	1.0330	0.0005
Y_1	0.1326	1	1.6770	-0.0003
R_1	-0.0309	1	0.9608	7.98e-05
S_1	0.0452	1	0.9934	0.0006
Y_1	0.1326	1	1.6770	-0.0003

Table 4.7

Using OLS to obtain the Coefficients of the Models for 300 Sample Size

Model	Coefficients	Maximum order	Sigma^2 estimated	Intercept
A_2	0.0078	1	0.9449	-0.0008
B_2	-0.0379	1	0.9604	0.0005
Y_2	0.1326	1	1.6770	-0.0003
C_2	-0.0309	1	0.9608	7.98e-05
D_2	-0.0379	1	0.9604	0.0005
Y_2	0.1326	1	1.6770	-0.0003
E_2	0.0078	1	0.9449	-0.0008
F_2	-0.0379	1	0.9604	0.0006
Y_2	0.1326	1	1.6770	-0.0003
G_2	0.0139	1	0.9517	0.0008
H_2	0.0039	1	1.0030	0.0006
Y_2	0.1326	1	1.6770	-0.0003
I_2	-0.0379	1	0.9604	0.0005
J_2	-0.0269	1	1.0200	0.0010
Y_2	0.1326	1	1.6770	-0.0003
K_2	-0.0056	1	0.9589	0.0003
L_2	-0.0030	1	0.9453	-0.0011
Y_2	0.1326	1	1.6770	-0.0003
M_2	0.0291	1	1.0510	0.0001
N_2	0.0078	1	0.9449	-0.0008
Y_2	0.1326	1	1.6770	-0.0003
P_2	0.0014	1	1.0770	-0.0007
Q_2	0.0291	1	1.0510	0.0001
Y_2	0.1326	1	1.6770	-0.0003
R_2	0.0102	1	0.9450	-0.0010
S_2	-0.0307	1	1.0750	-0.0002
Y_2	0.1326	1	1.6770	-0.0003

Step 12: The CWN Model

A linear combination is one in which each variable is multiplied by a coefficient and the products are summed (Bates, Maechler, Bolker & Walker, 2014). The combination of two WN models revealed the CWN model. The models combination considered the coefficients of the models in Table 4.5 to Table 4.7. The dependent variables were revealed in step 8. The predictors in step 7 went through step 8 to step 10 processes of transformation and step 11 derived the coefficients of the models.

The models linear combinations of CWN were:

$$A_t = 0.016B_{t-1} + 0.048A_{t-1} + \varepsilon_t \tag{4.1}$$

$$D_t = -0.039C_{t-1} - 0.0379D_{t-1} + \varepsilon_t \tag{4.2}$$

$$F_t = -0.0379F_{t-1} + 0.0078E_{t-1} + \varepsilon_t \tag{4.3}$$

$$H_t = 0.0039H_{t-1} + 0.00139G_{t-1} + \varepsilon_t \tag{4.4}$$

$$I_t = -0.0269J_{t-1} - 0.0379I_{t-1} + \varepsilon_t \tag{4.5}$$

$$K_t = -0.003L_{t-1} - 0.0056K_{t-1} + \varepsilon_t \tag{4.6}$$

$$M_t = 0.0078N_{t-1} + 0.0291M_{t-1} + \varepsilon_t \tag{4.7}$$

$$P_t = 0.0291Q_{t-1} + 0.0014P_{t-1} + \varepsilon_t \tag{4.8}$$

$$S_t = -0.0307S_{t-1} + 0.0102R_{t-1} + \varepsilon_t \tag{4.9}$$

Equations (4.1) to Equation (4.9) were the CWN models for the different values of leverages and skewness of 200 sample size data simulated.

Models for 250 Sample Size

$$B_{1,t} = -0.0718B_{1,t-1} + 0.0139A_{1,t-1} + \varepsilon_{1,t} \quad (4.10)$$

$$C_{1,t} = 0.0078C_{1,t-1} + 0.0007D_{1,t-1} + \varepsilon_{1,t} \quad (4.11)$$

$$E_{1,t} = 0.0007F_{1,t-1} + 0.0138E_{1,t-1} + \varepsilon_{1,t} \quad (4.12)$$

$$G_{1,t} = -0.0072H_{1,t-1} - 0.003G_{1,t-1} + \varepsilon_{1,t} \quad (4.13)$$

$$J_{1,t} = 0.0014J_{1,t-1} - 0.008I_{1,t-1} + \varepsilon_{1,t} \quad (4.14)$$

$$K_{1,t} = -0.02L_{1,t-1} - 0.0307K_{1,t-1} + \varepsilon_{1,t} \quad (4.15)$$

$$N_{1,t} = -0.0072N_{1,t-1} + 0.0078M_{1,t-1} + \varepsilon_{1,t} \quad (4.16)$$

$$Q_{1,t} = 0.0484Q_{1,t-1} + 0.0014P_{1,t-1} + \varepsilon_{1,t} \quad (4.17)$$

$$R_{1,t} = 0.0452S_{1,t-1} - 0.0309R_{1,t-1} + \varepsilon_{1,t} \quad (4.18)$$

Equations (4.10) to Equation (4.18) were the CWN models for the different values of leverages and skewness of 250 sample size data simulated.

Models for 300 Sample Size

$$B_{2,t} = -0.0379B_{2,t-1} + 0.078A_{2,t-1} + \varepsilon_{2,t} \quad (4.19)$$

$$C_{2,t} = -0.0309C_{2,t-1} - 0.0379D_{2,t-1} + \varepsilon_{2,t} \quad (4.20)$$

$$F_{2,t} = -0.0379F_{2,t-1} + 0.0078E_{2,t-1} + \varepsilon_{2,t} \quad (4.21)$$

$$H_{2,t} = 0.0039H_{2,t-1} + 0.0139G_{2,t-1} + \varepsilon_{2,t} \quad (4.22)$$

$$I_{2,t} = -0.0269\,J_{2,t-1} - 0.0379\,I_{2,t-1} + \varepsilon_{2,t} \quad (4.23)$$

$$K_{2,t} = -0.003L_{2,t-1} - 0.0056K_{2,t-1} + \varepsilon_{2,t} \quad (4.24)$$

$$M_{2,t} = 0.0078N_{2,t-1} + 0.0291M_{2,t-1} + \varepsilon_{2,t} \quad (4.25)$$

$$P_{2,t} = 0.0291Q_{2,t-1} + 0.0014P_{2,t-1} + \varepsilon_{2,t} \quad (4.26)$$

$$S_{2,t} = -0.0307S_{2,t-1} + 0.0102R_{2,t-1} + \varepsilon_{2,t} \quad (4.27)$$

Equations (4.19) to Equation (4.27) were the CWN models for the different values of leverages and skewness of 300 sample size data simulated. The parameters of simulated data and models can be estimated, to obtain its fitness and perform the forecast evaluation by comparison.

4.4 Models Comparison

The validation of combine white noise (CWN) model was compared with VAR, EGARCH and MA models using simulated data for 200, 250 and 300 sample sizes.

4.4.1 Results for 200 sample size

The simulation of 200 sample size with different values of leverages and skewness were used for the estimation of CWN, VAR, EGARCH and MA as reported in Table 4.8 to Table 4.10. The computer output for VAR, EGARCH and MA were in Appendix H. Similar results were obtained when computer output for VAR, EGARCH and MA were conducted for 250 and 300 sample sizes.

Table 4.8 to Table 4.10 presented that CWN have the least values of standard error of regression, indicating the reliability of the model. The estimated log-likelihood parameter of CWN indicated highest value among the models, revealing a good model distribution fit. The Akaike information criteria (AIC) and Bayesian information criteria (BIC) with minimum values indicated the best fit of CWN.

CWN has the least standard error values, highest log-likelihood values and minimum information criteria (AIC and BIC) values in 200 sample size with low leverage and high skewness among the CWN in Table 4.8. This indicated the best model fit.

The root mean square error (RMSE) values, mean absolute error (MAE) values, mean absolute percentage error (MAPE) values and geometric root mean square error (GRMSE) values were forecast error measures that determined the forecast accuracy with minimum values when it was compared with the three models for forecasting.

CWN presented minimum values of forecast error measure among the models; the best was in low leverage and high skewness as reported in Table 4.8.Theseshowed that CWN was the best among the models as reported in Table 4.8.

Table 4.9 disclosed that CWN has the best model fit in 200 sample size with moderate leverage and moderate skewness with RMSE, MAE and GRMSE minimum forecast error measure while MAPE value was high among the CWN. On average, 200 sample size with moderate leverage and high skewness were considered as the best forecast as reported in Table 4.9.

Table 4.10 presented that CWN has the best model fit and best forecast evaluation values in 200 sample size with high leverage and moderate skewness, except that standard error has the minimum value in 200 sample size with high leverage and low skewness.

Table 4.8

Sample Size of 200 with Low Leverage and different Values of Skewness

Estimation	**CWN**	**VAR**	**EGARCH**	**MA**
Low leverage and low skewness				
Standard Error	0.1330	1.0070	1.0051	0.9999
Log-likelihood	138.31	137.63	-269.76	-282.76
AIC	-1.5735	-1.2626	2.7815	2.8476
BIC	-1.4742	-1.0640	2.8973	2.8806
RMSE	0.1630	1.0063	1.0610	1.0078
MAE	0.1058	1.7655	1.7208	1.7596
MAPE	10.670	110.81	275.94	110.97
GRMSE	0.0024	0.8736	0.4321	0.7448
Low leverage and moderate skewness				
Standard Error	0.1242	1.0374	1.0051	1.0012
Log-likelihood	26.718	-74.666	-269.76	-293.76
AIC	-3.2412	0.8710	2.7815	2.9647
BIC	-3.1419	1.0700	2.8973	2.9380
RMSE	0.1247	0.7804	0.8179	0.8145
MAE	0.0623	0.5872	0.6434	0.6430
MAPE	6.2157	279.42	241.29	228.98
GRMSE	0.0011	0.4145	0.3466	0.2649
Low leverage and high skewness				
Standard Error	0.4051	0.0308	0.0307	0.0307
Log-likelihood	426.08	379.24	420.11	413.85
AIC	-3.8688	-3.6908	4.1518	-4.1185
BIC	-3.7695	-3.4922	4.0360	-4.0855
RMSE	0.6527	0.0354	0.0356	0.0366
MAE	0.2588	0.0270	0.0276	0.0284
MAPE	3.9112	114.15	108.79	112.50
GRMSE	1.34E-05	0.4597	0.2631	0.5511

Table 4.9

Sample Size of 200 with Moderate Leverage and different Values of Skewness

Estimation	**CWN**	**VAR**	**EGARCHMA**	
Moderate leverage and low skewness				
Standard Error	0.1282	1.4310	1.4279	1.4233
Log-Likelihood	259.24	-91.145	-334.48	-353.37
AIC	-3.6822	1.0366	3.4320	3.5537
BIC	-3.5829	1.2352	3.5478	3.5867
RMSE	0.1504	1.2115	1.2254	1.2118
MAE	0.0898	0.8751	0.9074	0.8958
MAPE	9.1120	92.302	97.754	96.246
GRMSE	0.0021	0.7366	0.4862	0.9097
Moderate leverage and moderate skewness				
Standard Error	0.0239	1.0359	1.0331	1.0301
Log-Likelihood	417.94	132.52	-279.36	-288.72
AIC	-7.8471	-1.2111	2.8780	2.9072
BIC	-7.7478	-1.0125	2.9938	2.9402
RMSE	0.0240	1.0248	0.8180	0.8146
MAE	0.0116	0.7780	0.6434	0.6430
MAPE	5.7104	96.180	241.29	228.99
GRMSE	3.06E-05	0.3772	0.4179	0.2823
Moderate leverage and high skewness				
Standard Error	0.2502	1.4310	1.2975	1.2951
Log-Likelihood	253.76	-351.65	-317.24	-339.77
AIC	-0.2413	3.5745	3.2918	3.4450
BIC	-0.1420	3.6407	3.4085	3.4946
RMSE	0.4098	0.7916	1.3108	1.3012
MAE	0.3509	0.5987	0.9567	0.9487
MAPE	1.3992	287.54	118.49	103.47
GRMSE	0.0002	0.0990	0.3898	0.5566

Table 4.10

Sample Size of 200 with High Leverage and different Values of Skewness

Estimation	**CWN**	**VAR**	**EGARCH**	**MA**
High leverage and low skewness				
Standard Error	0.0118	0.0280	0.0279	0.0279
Log-Likelihood	500.07	431.48	439.85	433.38
AIC	-4.8349	-0.7956	-4.3503	4.3138
BIC	-4.7356	-0.5972	-4.2344	4.2808
RMSE	0.0124	0.0322	0.0319	0.0317
MAE	0.0121	0.0247	0.0245	0.0243
MAPE	1.9950	142.34	119.67	116.90
GRMSE	3.00E-05	0.4718	0.4064	0.8929
High leverage and moderate skewness				
Standard Error	0.0959	1.0247	1.0331	1.0301
Log-Likelihood	651.11	71.815	-279.36	-288.72
AIC	-5.0664	-0.6012	2.8780	2.9072
BIC	-4.9671	-0.4026	2.9938	2.9402
RMSE	0.0417	0.8154	0.8179	0.8145
MAE	0.0087	0.6352	0.6434	0.6430
MAPE	1.0861	229.99	241.29	228.98
GRMSE	1.63E-05	0.5238	0.5238	0.3380
High leverage and high skewness				
Standard Error	0.2120	1.4991	1.4977	1.4913
Log-Likelihood	97.841	-118.55	-353.26	-362.72
AIC	-0.8731	1.3120	3.6207	3.6472
BIC	-0.7738	1.5106	3.7337	3.6801
RMSE	0.1617	1.7888	1.7956	1.7989
MAE	0.0349	1.2955	1.2660	1.3016
MAPE	1.1621	281.98	381.12	272.79
GRMSE	0.0002	0.8282	0.3553	0.5331

CWN estimation outperformed VAR, EGARCH and MA using three sample sizes. CWN has the best fit in 200 sample size with moderate leverage and moderate skewness, while the best forecast was in high leverage and moderate skewness.

4.4.2 Results for 250 sample size

The simulation of 250 sample size with leverage and skewness were used for the estimation of CWN, VAR, EGARCH and MA as reported in Table 4.11to Table 4.13.

Table 4.11 to Table 4.13revealed that CWN have the minimum standard error values when compared with the three models estimated in this study. The estimated log-likelihood parameter of CWN indicated highest value among the models, revealing a good distribution fit. The information criteria with minimum values of AIC and BIC indicated the best fit of CWN among the models.

The root mean square error (RMSE) values, mean absolute error (MAE) values, mean absolute percentage error (MAPE) values and geometric root mean square error (GRMSE) values were considered as forecast error measure in this study. CWN revealed the minimum values of forecast error measures among the models. These revealed that CWN was the best among the models estimated as reported in Table 4.11 to Table 4.13.

The CWN showed that the 250 sample size with low leverage and low skewness has the best model fit among the CWN in Table 4.11. CWN in Table 4.11 presented that the 250 sample size with low leverage and high skewness has the minimum forecast error measure values. This offered the least forecast error measure evaluated among the CWN in Table 4.11.

CWN has the best model fit in 250 sample size with moderate leverage and high skewness with RMSE, MAE and GRMSE having minimum forecast error measure values while MAPE value was high among the CWN in this Table 4.12. On average, 250 sample size with moderate leverage and moderate skewness were considered having the minimum forecast error measure value as reported in Table 4.12.

Table 4.11

Sample Size of 250 with Low Leverage and different Values of Skewness

Estimation	**CWN**	**VAR**	**EGARCH**	**MA**
Low leverage and low skewness				
Standard Error	0.0152	1.71127	1.7131	1.7039
Log-Likelihood	14.010	-110.05	-474.92	-486.97
AIC	-4.9064	0.9804	3.8708	3.9117
BIC	-4.8216	1.1499	3.9697	3.9399
RMSE	0.1478	1.5260	1.5586	1.5368
MAE	0.0723	0.9976	1.0523	1.0552
MAPE	6.0500	112.12	166.95	133.65
GRMSE	0.0011	0.4660	0.2860	0.4270
Low leverage and moderate skewness				
Standard Error	0.0190	0.0346	1.0345	0.0345
Log-Likelihood	602.99	601.73	488.67	488.33
AIC	-4.8594	-4.7368	-3.8689	-3.8906
BIC	-4.7746	-4.5673	-3.7700	-3.8625
RMSE	0.0172	0.0373	0.0377	0.0371
MAE	0.0150	0.0281	0.0287	0.0283
MAPE	4.6671	132.14	100.83	138.78
GRMSE	0.0015	0.2283	0.2712	0.2516
Low leverage and high skewness				
Standard Error	0.2257	1.1185	1.1117	1.1087
Log-Likelihood	-42.450	-279.22	-373.49	-379.53
AIC	-0.2751	2.3581	3.0561	3.0522
BIC	-0.1903	2.5286	3.1550	3.0804
RMSE	0.1518	1.3036	1.3044	1.2982
MAE	0.0288	0.9805	0.9786	0.9820
MAPE	0.9009	113.55	101.26	113.72
GRMSE	0.0002	0.9813	0.1462	0.1703

Table 4.12

Sample Size of 250 with Moderate Leverage and different Values of Skewness

Estimation	**CWN**	**VAR**	**EGARCH**	**MA**
Moderate leverage and low skewness				
Standard Error	0.0153	0.0314	0.0312	0.0311
Log-Likelihood	564.39	453.23	514.06	513.81
AIC	-4.6738	-3.1947	-4.0728	-4.0945
BIC	-4.5890	-3.3642	-3.9739	-4.0663
RMSE	0.0135	0.0335	0.0345	0.0336
MAE	0.0132	0.0251	0.0265	0.0257
MAPE	1.5127	106.62	100.19	113.44
GRMSE	0.0002	0.3716	0.2736	0.3809
Moderate leverage and moderate skewness				
Standard Error	0.2245	1.3033	1.3046	1.2849
Log-Likelihood	-393.30	-410.94	-407.85	-414.23
AIC	0.1657	3.3971	3.3321	3.3513
BIC	0.2505	3.5666	3.4310	3.3936
RMSE	0.0244	0.0252	0.0254	0.0257
MAE	0.0176	0.0207	0.0210	0.0213
MAPE	0.9522	225.84	203.24	184.02
GRMSE	0.0005	0.2658	0.2221	0.2010
Moderate leverage and high skewness				
Standard Error	0.0456	1.2388	1.2372	1.2110
Log-Likelihood	710.91	619.09	-395.46	-399.49
AIC	-8.1678	-4.8762	3.2326	3.2329
BIC	-8.0830	-4.7067	3.3315	3.2752
RMSE	0.0356	1.5521	1.5613	1.5563
MAE	0.0128	1.2408	1.2414	1.2491
MAPE	3.1603	108.03	99.940	109.63
GRMSE	3.64E-05	0.1632	0.1605	0.1587

In Table 4.13, CWN has the best model fit and best forecast error measure values in250 sample size with high leverage and high skewness.

CWN estimation outperformed VAR, EGARCH and MA using three sample sizes. CWN has the best fit in 250 sample size with moderate leverage and high skewness, while the best forecast was in low leverage and high skewness.

Table 4.13

Sample Size of 250 with High Leverage and different Values of Skewness

Estimation	CWN	VAR	EGARCH	MA
High leverage and low skewness				
Standard Error	0.1598	1.7085	1.7097	1.6885
Log-Likelihood	220.50	-138.80	-69.90	481.68
AIC	-2.6686	1.2113	3.8305	3.8930
BIC	-2.5838	1.3808	3.9294	3.9354
RMSE	0.0162	0.0267	0.0283	0.0285
MAE	0.0105	0.0219	0.0232	0.0235
MAPE	7.3638	433.23	294.35	266.58
GRMSE	0.0019	0.3506	0.2807	0.3854
High leverage and moderate skewness				
Standard Error	0.0278	0.0323	0.0325	0.0328
Log-Likelihood	542.73	492.80	503.52	500.66
AIC	-4.2950	-3.8980	-3.3056	-3.9892
BIC	-4.2103	-3.7285	-3.4045	-3.9611
RMSE	0.0109	0.0280	0.0283	0.0283
MAE	0.0160	0.0226	0.0232	0.0232
MAPE	7.4827	230.94	294.35	302.56
GRMSE	0.0008	0.2083	0.2000	0.2285
High leverage and high skewness				
Standard Error	0.0447	1.3845	1.3817	1.3804
Log-Likelihood	702.57	-49.99	-410.21	-434.07
AIC	-7.9857	0.4819	3.2342	3.2617
BIC	-7.9009	0.6514	3.3331	3.3040
RMSE	0.0431	1.1663	1.1804	1.1779
MAE	0.0186	0.8773	0.9013	0.9011
MAPE	4.6617	105.54	102.06	105.57
GRMSE	5.97E-05	0.1490	0.0884	0.3612

4.4.3 Results for 300 sample size

The simulation of 300 sample size data with different values of leverages and skewness were used for the estimation of CWN, VAR, EGARCH and MA as reported in Table 4.14 to Table4.16.

Table 4.14 to Table 4.16 reported that the regression standard errors values of CWN have minimum error when compared with the three models estimated in this study. The estimated log-likelihood parameter of CWN indicated highest values among the models. The information criteria with minimum values of AIC and BIC indicated the best fit of CWN among the models.

The root mean square error (RMSE) values, mean absolute error (MAE) values, mean absolute percentage error (MAPE) values and geometric root mean square error (GRMSE) values were considered as forecast error measures in this study. CWN provided the minimum values of forecast error measure. These showed that CWN models were the best among the models estimated as reported in Table4.14 to Table 4.16.

CWN has the best model fit in 300 sample size with low leverage and low skewness as reported in Table 4.14.MAE, MAPE and GRMSE values have minimum forecast error measures in low leverage and high skewness among CWN as reported in Table 4.14.

Table 4.14

Sample Size of 300 with Low Leverage and different Values of Skewness

Estimation	CWN	VAR	EGARCH	MA
Low leverage and low skewness				
Standard Error	0.1249	1.6713	1.6881	1.6691
Log-Likelihood	903.61	333.91	-555.95	-575.94
AIC	-6.0041	-2.1532	3.7655	3.9096
BIC	-5.9298	-2.0047	3.8522	3.8873
RMSE	0.1253	1.4725	1.5642	1.5639
MAE	0.0633	1.1327	1.1356	1.1352
MAPE	6.2777	112.60	99.750	98.940
GRMSE	0.0011	1.0105	0.5472	0.5515
Low leverage and moderate skewness				
Standard Error	0.1191	1.3004	1.3061	1.2909
Log-Likelihood	426.74	-69.060	-488.75	-499.09
AIC	-2.8143	0.5422	3.3161	3.3956
BIC	-2.7401	0.6907	3.4027	3.3734
RMSE	0.1363	1.2940	1.2972	1.2941
MAE	0.0743	0.9416	0.9514	0.9525
MAPE	7.4340	102.46	101.42	104.77
GRMSE	0.0018	0.3622	0.4123	0.3765
Low leverage and high skewness				
Standard Error	0.0911	1.2563	1.2575	1.2563
Log-Likelihood	434.35	-48.380	-479.92	-490.98
AIC	-2.8652	0.4039	3.2570	3.3413
BIC	-2.7909	0.5524	3.3436	3.3191
RMSE	0.1356	1.1774	1.2001	1.2000
MAE	0.0615	0.8983	0.9277	0.9278
MAPE	5.1043	107.55	103.50	101.14
GRMSE	0.0010	0.3545	0.3537	0.3514

Table 4.15showed that CWN has the best model fit in moderate leverage and low skewness, but standard error has the highest value. The best forecast evaluation values were in 300 sample size with moderate leverage and moderate skewness.

CWN has the best model fit in 300 sample size with high leverage and moderate skewness. While the best forecast was in 300 sample size with high leverage and high skewness on average as reported in Table 4.16.

CWN has the best forecast in 300 sample size with high leverage and high skewness on average as the best described in Table 4.14 to Table 4.16.

Table 4.15

Sample Size of 300 with Moderate Leverage and different Values of Skewness

Estimation	**CWN**	**VAR**	**EGARCH**	**MA**
Moderate leverage and low skewness				
Standard Error	0.1694	1.6470	1.6549	1.6439
Log-Likelihood	807.35	244.63	-550.26	-571.38
AIC	-5.3602	-1.5560	3.7275	3.8420
BIC	-5.2859	-1.4075	3.8141	3.8791
RMSE	0.1634	1.5276	1.5529	1.5398
MAE	0.0890	1.1025	1.1133	1.1138
MAPE	7.4251	104.61	114.95	97.940
GRMSE	0.0026	0.5706	0.5033	0.5672
Moderate leverage and moderate skewness				
Standard Error	0.0978	1.2910	1.2898	1.2786
Log-Likelihood	493.96	-1.1150	-485.20	-496.23
AIC	-3.2639	0.0878	3.2923	3.3765
BIC	-3.1897	0.2363	3.3789	3.3542
RMSE	0.0789	1.2895	1.2911	1.2901
MAE	0.0309	0.9400	0.9486	0.9501
MAPE	3.8897	121.06	98.628	103.75
GRMSE	3.14E-05	1.0603	0.3730	0.3551
Moderate leverage and high skewness				
Standard Error	0.1254	1.2753	1.2805	1.2563
Log-Likelihood	563.63	72.890	-477.14	-492.59
AIC	-3.7300	-0.4100	3.2603	3.3373
BIC	-3.6557	-0.2608	3.3473	3.3746
RMSE	0.1130	1.2533	1.2574	1.2971
MAE	0.0510	0.9349	0.9515	0.9829
MAPE	5.1075	106.46	101.41	111.67
GRMSE	0.0007	0.3539	0.4094	0.3424

Table 4.16

Sample Size of 300 with High Leverage and different Values of Skewness

Estimation	**CWN**	**VAR**	**EGARCH**	**MA**
High leverage and low skewness				
Standard Error	0.0249	1.6432	41.653	1.6425
Log-Likelihood	827.11	268.08	-544.49	-571.13
AIC	-5.4924	-1.7130	3.6889	3.8428
BIC	-5.4181	-1.5644	3.7755	3.8799
RMSE	0.0221	1.5142	1.5505	1.5391
MAE	0.0098	1.0737	1.1026	1.1044
MAPE	4.8911	104.61	113.34	97.180
GRMSE	2.54E-05	0.5160	0.4781	0.5359
High leverage and moderate skewness				
Standard Error	0.0266	1.2942	1.2961	1.2846
Log-Likelihood	867.44	371.86	-481.41	-497.64
AIC	-5.7621	-2.4071	3.2670	3.3488
BIC	-5.6879	-2.2586	3.3536	3.3859
RMSE	0.0197	1.3090	1.3076	1.2959
MAE	0.0077	0.9424	0.9543	0.9539
MAPE	3.8896	98.960	99.840	116.05
GRMSE	0.0017	0.3185	0.3694	0.3734
High leverage and high skewness				
Standard Error	0.0492	1.3103	1.3160	1.3113
Log-Likelihood	759.88	260.76	-478.54	-503.79
AIC	-5.0426	-1.6639	3.2478	3.3899
BIC	-4.9684	-1.5154	3.3344	3.4270
RMSE	0.0722	1.3542	1.3481	1.3304
MAE	0.0306	0.9926	0.9846	0.9659
MAPE	3.3643	108.96	102.56	122.20
GRMSE	0.0002	0.4078	0.3781	0.2552

CWN estimation outperformed VAR, EGARCH and MA using three sample sizes. CWN has the best fit in 300 sample size with low leverage and low skewness, while the best forecast was in high leverage and high skewness.

4.5 Summary

In 200 sample size of simulated data with moderate leverage and moderate skewness results, CWN outperformed the VAR,EGARCH and MA with the values of least standard error, log-likelihood highest and minimum information criteria, AIC and BIC. This made the model to be the best fit among the low, moderate and high values of leverages and skewness of the 200 data simulated sample size. The best forecast for CWN results were in 200 data simulated sample size with high leverage and moderate skewness values of RMSE, MAE, MAPE and GRMSE.

In 250 sample size of simulated data with moderate leverage and high skewness results, CWN outperformed the EGARCH, VAR and MA. The minimum information criteria values of AIC, BIC, standard error and log-likelihood highest value revealed the best result. This made the model to be the best fit among the low, moderate and high values of leverages and skewness of the 250 simulated sample size. The best forecast for CWN results were in 250 data simulated sample size with low leverage and high skewness minimum values of RMSE, MAE, MAPE and GRMSE.

In 300 sample size of simulated data with low leverage and low skewness results, CWN outperformed the EGARCH, VAR and MA. The minimum information criteria values of AIC, BIC and log-likelihood highest value displayed the best result, but the lowest standard error value was in low leverage and high skewness. This made the model to be the best fit among the low, moderate and high values of leverages and skewness of the 300 data simulated sample size. The minimum forecast error measure values of RMSE, MAE, MAPE and GRMSE revealed the best forecast for 300 data simulated sample size with high leverage and high skewness values.

The CWN outperformed the VAR, EGARCH and MA estimation results. The CWN have the best result among the models estimated with different values of leverages and skewness using the three sample sizes as reported in Table 4.8 to Table 4.16

The overall best forecast model for CWN result was in 200 data simulated sample size with high leverage and moderate skewness which has minimum values of RMSE, MAE, MAPE and GRMSE.

The CWN outperformed the VAR estimation results as CWN were having the best results among the VAR models estimated with different values of leverages and skewness using the three sample sizes as reported in Table 4.8 to Table 4.16. CWN and VAR error terms are white noise. Therefore, CWN can be used to improve VAR using the three sample sizes with different values of leverages and skewness.

CHAPTER FIVE
VALIDATION OF COMBINE WHITE NOISE (CWN) MODEL USING REAL DATA

5.1 Introduction

This chapter explained the development and estimation of combine white (CWN) model using real data as described in Chapter Three, Sections 3.2 to 3.3 and Subsection 3.3.2. Real data that exhibit heteroscedastic errors were used to validate the performance of CWN model as compared to VAR, EGARCH and MA. The four sets of data that were used for the validations were United States gross domestic product (US GDP), United Kingdom gross domestic product (UK GDP), Australia gross domestic product (AU GDP) and France gross domestic product (GDP). These data sets were retrieved from DataStream of Universiti Utara Malaysia Library.

Section 5.2 described the type of real data. The twelve steps were employed in Section 5.3for the description of model development process. Followed by, Section 5.4 that described the performance of the validated models by comparison; the results were in Subsection 5.4.1. Subsection 5.4.2explained the reliability of the measurements of degree of relationship between the data distribution and using Levene's test of equal variances to solve the challenges of non-normality in the data distribution. Subsection 5.4.3 explained the combination of two variances. Then, Section 5.5 explained the different values of leverages and skewness. Section 5.6 summarized the findings based on the four sets of the real data in Sections 5.4 and 5.5.

5.2 Real Data

Figure 5.1 displayed the quarterly data sets of US GDP, UK GDP, AU GDP and France GDP which consist of 220 data point each. US GDP and UK GDP data sets started from quarter one 1960 to quarter four 2014, while AU GDP and France GDP data sets started from quarter three 1960 to quarter two 2015. The time plot of four countries GDP data in level indicated trend behaviour. The slope of the time plot of each data varies according to the behaviour of the data sets. The four countries GDP data sets have similar characteristics which showed that the data sets were heteroscedastic in nature, given the assurance for further tests. Statistics and normality tests were conducted to confirm the heteroscedastic nature of the data sets.

Table 5.1 summarized the statistics and normality tests for the four countries GDP which showed that the Jarque-Bera test values were significant. Jarque-Bera test revealed the type of data distribution and showed whether the data sets were heteroscedastic in nature or not. It indicated non-normal distribution for the four countries GDP with kurtosis and skewness which revealed the data sets were heteroscedastic in nature. Standard deviation in each distribution was greater than one which was an indication of non-normal distribution. These were characteristics of heteroscedastic data sets as reported in Table 5.1.

The behaviours of the level sets of data signified the presence of heteroscedasticity and the level data were transformed (from level data series to return series) for confirmation.

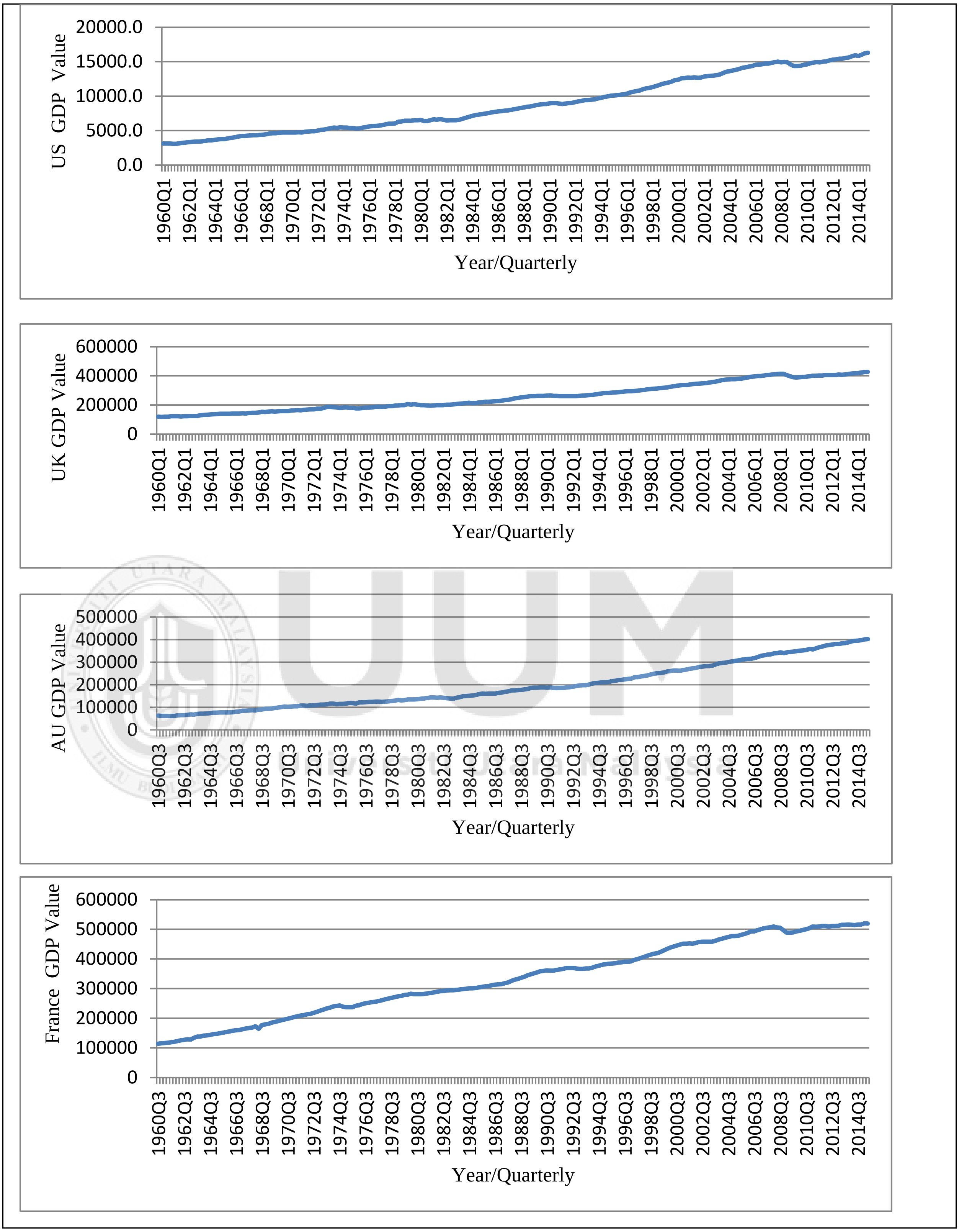


*Figure 5.1.*The Time Plot of Four GDP Quarterly Data

Table 5.1

Statistical Summary and Normality Tests for the Four Countries Real Data Sets

	Standard deviation	**Skewness**	**Kurtosis**	**Jarque-Bera**
US GDP	4026.92	0.3257	1.7447	18.34 (0.0001)***
UK GDP	95359.32	0.3341	1.7809	17.72 (0.0001)***
AU GDP	98689.88	0.5403	2.0839	18.40 (0.0001)***
France GDP	124321.80	-0.0667	1.7959	13.45 (0.0012)***

P-values ( ) *** significant at 1%, ** significant at 5%, * significant at 10%

Data Preparation

The transformation of level data series to return series was through differentiating the log of the level data multiply by one hundred (ST=100*dlog (y)) which displayed more stationary behaviour empirically (McAleer, 2014; McAleer & Hafner, 2014). This was also to observe whether the data exhibited volatility clustering, skewness and kurtosis. These were the characteristics of the presence of heteroscedasticity.

The transformed data sets for three countries GDP revealed that the standard deviation values were approaching one, while AU GDP standard deviation value was greater than one as showed in Table 5.2. Jarque-Bera test values were highly significant. It indicated non-normal distribution for all the data sets. The four countries GDP data sets showed that there were excess kurtosis and skewness in the distributions. France GDP distribution has the highest values of kurtosis and skewness which could be the attribute of leverage effect in the heteroscedastic data.

Autoregressive processes were used for the transformed data series (return series) computation to obtain ARCH effect and performed ARCH LM tests to know the effect of heteroscedasticity. *F*-Statistic and Obs*R-squared were significant which were indications of ARCH presence in the data. The ARCH presence in the data was a justification of using GARCH model as GARCH is the generalization of ARCH.

Table 5.2 displayed the specification of ARCH and GARCH models in which ARCH LM tests were significant in three countries GDP as revealed by *F*-Statistics and Observation*R-squared but highly significant in France GDP.

Table 5.2

Statistical Summary, Normality and ARCH Tests for the Four Countries Real Data Set

	Standard deviation	**Skewness**	**Kurtosis**	**Jarque-Bera**	***F*-Statistic**	**Obs*R-squared**
US GDP	0.8405	-0.3204	4.5160	24.720 (0.0000)***	1.3727 (0.0406)**	1.3767 (0.0414)**
UK GDP	0.9669	0.3755	7.0150	152.24 (0.0000)***	0.0602 (0.0064)***	0.0607 (0.0053)***
AU GDP	1.0556	0.3647	3.9497	13.090 (0.0014)***	4.9084 (0.0003)***	22.580 (0.0004)***
France GDP	0.9205	0.7639	22.98	3663.37 (0.0000)***	21.0033 (0.0000)***	71.690 (0.0000)***

P-values ( ) *** significant at 1%, **significant at 5%, *significant at 10%

Specification of ARCH and GARCH Family Models Using Real Data

ARCH-Normal distribution specification: ARCH is normally distributed when the mean of the variable is zero and autocovariances are zero. The variances were positive for all values of alpha which were the coefficients of ARCH in all the four countries GDP. The coefficients were highly significant in three countries, while it was not significant in AU GDP. France GDP has the minimum information criteria (AIC and

BIC) and highest log-likelihood values among the four countries GDP as displayed in Table 5.3.

GARCH-Normal distribution specification: GARCH is normally distributed when the mean of the variable is zero and autocovariances are zero. The coefficients of mean equations were highly significant but not significant in AU GDP model estimation. The coefficients of variance equations were highly significant. The sum of the coefficients of mean and variance equations was less than one, it means stationary (Bollerslev, 1987) and it was a mean reverting variance process with slowly mean reverting (Engle, 2001) in AU GDP and France GDP. While USGDP and UK GDP were not stationary since the sum of the coefficients of mean and variance equations of each was greater than one and was unstable (Bollerslev, 1986). Volatility persistence took place when the addition of the ARCH and GARCH coefficients were close to one. France GDP estimation process has the minimum information criteria (AIC and BIC) and highest value of log-likelihood.

TGARCH-Normal distribution specification: The TGARCH is normally distributed when the mean of the variable is zero and autocovariances are zero. The coefficients of mean equations were highly significant except AU GDP that was not significant. The coefficients of variance equations were significant. None of the thresholds of the asymmetries of the variance equations were significant. USGDP and UK GDP were not stationary, while AU GDP and France GDP were stationary. France GDP has the minimum information criteria (AIC and BIC) (AIC and BIC) and highest value of log-likelihood.

Table 5.3

Specification of ARCH, GARCH and TGARCH Models Using Real Data

	α	β	δ	**AIC**	**BIC**	**LL**
US GDP						
ARCH	0.3613 (0.0000)	.	.	2.3813	2.4434	-255.57
GARCH	0.3664 (0.0000)	0.8055 (0.0000)	.	2.2972	2.3748	-245.40
TGARCH Normal	0.3783 (0.0000)	0.7717 (0.0000)	0.1155 (0.2367)	2.3017	2.3948	-244.88
TGARCH Student's *t*	0.3351 (0.0000)	0.7670 (0.0000)	0.0500 (0.6747)	2.2663	2.3750	-240.02
UK GDP						
ARCH	0.3349 (0.0003)	.	.	2.6844	2.7465	-288.59
GARCH	0.4105 (0.0000)	0.8962 (0.0000)	.	2.5260	2.6037	-270.34
TGARCH Normal	0.4231 (0.0000)	0.9113 (0.0000)	-0.0722 (0.0549)	2.5244	2.6175	-269.15
TGARCH Student's *t*	0.2838 (0.0000)	0.9027 (0.0000)	-0.1328 (0.1592)	2.3398	2.4485	-248.04
AU GDP						
ARCH	0.0135 (0.8451)	.	.	2.9073	2.9694	-312.10
GARCH	-0.0765 (0.1573)	1.0167 (0.0000)	.	2.6317	2.7094	-281.86
TGARCH Normal	-0.0423 (0.4567)	0.9960 (0.0000)	-0.0328 (0.0724)	2.6547	2.7479	-283.36
TGARCH Student's *t*	-0.0515 (0.2422)	1.0061 (1.0061)	-0.0260 (0.2591)	2.6532	2.7619	-282.20
France GDP						
ARCH	0.4745 (0.0000)	.	.	1.9446	2.0067	-207.96
GARCH	0.5123 (0.0000)	0.3120 (0.0000)	.	1.9229	2.0005	-204.59
TGARCH Normal	0.4568 (0.0000)	0.3758 (0.0000)	-0.2956 (0.0966)	1.9274	2.0205	-204.08
TGARCH Student's *t*	0.5856 (0.0000)	0.2221 (0.2878)	0.0687 (0.8136)	1.7002	1.8089	-178.09

P-values ( ), represented the values of ARCH, GARCH, TGARCH Normal and Student's *t*

TGARCH-Student's *t* distribution specification: The TGARCH is Student's *t* distributed when an additional parameter, called degrees of freedom, which changes its shape from standard normal distribution. The coefficients of mean equations were highly significant except AU GDP that was not significant. The coefficients of variance equations were highly significant in US GDP and UK GDP, while AU GDP and France GDP were not significant. None of the thresholds of the asymmetries of the variance equations were significant. France has the minimum information criteria (AIC and BIC) and highest value of log-likelihood.

EGARCH-Normal distribution specification: EGARCH is normally distributed when the mean of the variable is zero and autocovariances are zero. The coefficients of mean equations were highly significant except AU GDP that was not significant. The coefficients of variance equations were highly significant in three countries, while AU GDP was not significant. There were stabilities because the coefficient of the past log term was less than one, but AU GDP was not stable. France GDP has the minimum information criteria (AIC and BIC) and highest value of log-likelihood as described in Table 5.4.

EGARCH-Student's *t* distribution specification: The EGARCH is Student's *t* distributed when an additional parameter, called degrees of freedom, which changes its shape from standard normal distribution (shape).The coefficients of mean equations were significant except in AU GDP. Almost all the coefficients of variance equations were significant. The excess kurtosis can relaxed the assumption that the conditional returns were normally distributed with the assumption that the returns followed a

Student's t distribution of fat tails (Bollerslev, 1987; Harvey & Sucarrat, 2014). The stabilities conditions were met as the past log term value for each of the three countries model was less than one. The stability was not met in AU GDP because the coefficient of the past log term value was greater than one. There were asymmetries effects in UKGDP and AU GDP model estimation. There were existence of leverage effects in USGDP and France GDP. France GDP has the minimum information criteria (AIC and BIC) and highest value of log-likelihood in the estimation.

EGARCH-Generalized error distribution specification: EGARCH is the generalized error distributed when the symmetrical unimodal member of the exponential family, locates the mode of the distribution, and defines the dispersion of the distribution which controls the skewness. The coefficients of mean equations were significant except AU GDP. Most of the coefficients of variance equations were significant. The stabilities conditions were met except AU GDP which the coefficient of the past log term was greater than one. There was asymmetry effect in UK GDP. There were existence of leverage effects in US GDP, AU GDP and France GDP. France GDP has the minimum information criteria (AIC and BIC) and highest value of log-likelihood.

Table 5.4

Specification of EGARCH Models Using Real Data

	α	β	δ	γ	**AIC**	**BIC**	**LL**
UK GDP							
EGARCH Normal	0.3627	0.2991	.	0.9364	2.2949	2.3725	-245.14
	(0.0000)	(0.0002)		(0.0000)			
EGARCH Student's t	0.3277	0.3206	-0.0656	0.8915	2.2678	2.3764	-240.19
	(0.0000)	(0.0163)	(0.3964)	(0.0000)			
EGARCH GED	0.3044	0.3244	-0.0883	0.8916	2.2645	2.3732	-239.83
	(0.0000)	(0.0223)	(0.2909)	(0.0000)			
UK GDP							
EGARCH Normal	0.4036	0.2448	.	0.9857	2.5169	2.5955	-269.34
	(0.0000)	(0.0000)		(0.0000)			
EGARCH Student's *t*	0.2913	0.2182	0.0933	0.9900	2.3515	2.4601	-249.31
	(0.0000)	(0.0106)	(0.1228)	(0.0000)			
EGARCH GED	0.3188	0.1896	0.0619	0.9876	2.3790	2.4877	-252.31
	(0.0000)	(0.017)	(0.2853)	(0.0000)	.	.	.
AU GDP							
EGARCH Normal	-0.0526	-0.0107	.	1.0118	2.6400	2.7178	-282.75
	(0.3690)	(0.8116)		(0.0000)			
EGARCH Student's *t*	-0.0462	-0.0157	0.0203	1.0106	2.6532	2.7619	-282.20
	(0.4481)	(0.8111)	(0.4224)	(0.0000)			
EGARCH GED	-0.0042	0.1177	-0.0201	1.0276	2.8115	2.9202	-299.45
	(0.9402)	(0.0000)	(0.3909)	(0.0000)			
France GDP							
EGARCH Normal	0.5748	0.9482	.	0.7299	1.8575	1.9507	-196.47
	(0.0000)	(0.0000)		(0.0000)			
EGARCH Student's *t*	0.5839	0.4966	-0.1223	0.6544	1.7018	1.8104	-178.49
	(0.0000)	(0.0018)	(0.1715)	(0.0000)			
EGARCH GED	0.5378	0.7243	-0.0113	0.6875	1.7230	1.8317	-180.81
	(0.0000)	(0.0000)	(0.8843)	(0.0000)			

The ARCH and GARCH family models were considered for the estimation in this study. EGARCH model with generalized error distribution for USGDP, and EGARCH model with Student's *t* distribution were selected for UK GDP, AU GDP and France GDP with values of minimum information criteria (AIC and BIC) and highest log-likelihood (Almeida & Hotta, 2014) as reported in Table 5.4.

Therefore, EGARCH which showed the standardized residuals of unequal variances which was heteroscedastic in nature was used for the development of the model.

5.3 Model Development

Twelve steps were employed for the development of the models.

Step 1: VAR white noise estimation was efficient but weak in modelling heteroscedasticity, the weakness were as reported in Table 5.10.

Step 2: EGARCH estimation modelled heteroscedasticity without leverage effect efficiently but weak in modelling the leverage effect in the heteroscedasticity, the weakness were as reported in Table 5.10.

Therefore, the data sets that exhibit heteroscedasticity were simulated, estimated and the graphs of the estimated standardized residuals with unequal variances and zero mean were considered in this study to resolve the leverage effect challenges.

Step 3:The estimation of EGARCH model with generalized error distribution for USGDP and EGARCH model with Student's *t* distribution for UK GDP, AU GDP and France GDP were selected to obtain the standardized residuals in graphical form.

The graphs of standardized residuals displayed the error terms of EGARCH models for the purpose of this study. The error terms have the characteristics of heteroscedasticity with leverage effect (unequal variances), which made up the conditional variance challenges in the estimation processes as reported in Figure 5.2. The irregular movement in the standardized residuals graphs revealed the unequal variances of the heteroscedastic behaviours.

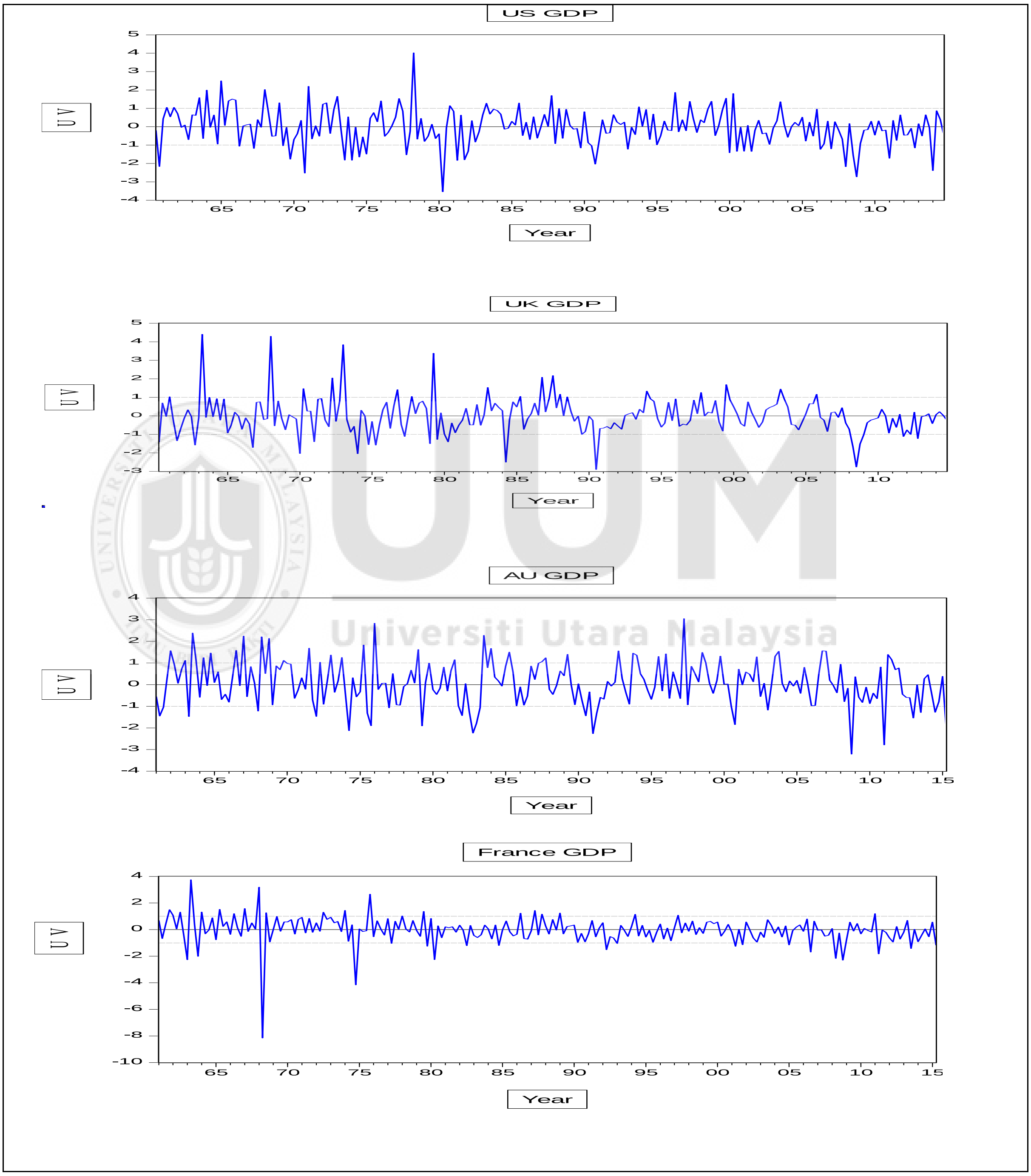
Step 4: Graphs the standardized residuals with unequal variances and zero mean.

The standardized residuals graphs of unequal variances were decomposed into equal variances by rearrangement.

The standardized residuals of unequal variances for US GDP were decomposed (rearranged and grouped) manually into equal variances to overcome the leverage effect. Forty six equal variances were obtained. UK GDP standardized residuals were rearranged and grouped manually into unequal variances and there were forty two equal variances. The standardized residuals of AU GDP were rearranged and grouped manually into equal variances. There were forty three equal variances obtained from the decomposition of the standardized residuals. Forty one equal variances were obtained from the rearranged and grouped manually of standardized residuals of France GDP. Then, maximum likelihood estimation method was applied on each equal variance to obtain the log-likelihood.

Step 5: The Log-Likelihood

The log-likelihood was maximized by the maximum likelihood estimation method for the number of equal variances in each data. The estimation of maximum likelihood was employed for sufficiency, consistency, efficiency and invariance parameterization of the variables that is, the equal variances series. The log-likelihood values were reported in Table 5.5.



U V represented Unequal Variances

*Figure 5.2.*Graphs of Standardized Residuals for the Four Countries GDP

The log-likelihood were obtained, therefore, the number of equal variances for each data were fitted into linear model for modelling each equal variance. This revealed the equal variance model which is known as white noise (WN) model.

Table 5.5

The Log-Likelihood Values for Real Data

The log-likelihood values of 46 equal variances for US GDP					
-336.96	-316.13	-323.23	-311.15	-303.51	301.02
-305.21	-304.45	-312.37	-315.65	-300.62	306.49
-307.55	-325.56	-306.47	-319.86	307.51	324.03
-332.68	-314.61	-324.95	-314.67	312.07	311.71
-319.87	-302.40	-303.62	-325.78	322.77	320.04
-280.92	-312.75	-315.61	-310.27	-319.22	307.02
-299.20	-305.31	-326.84	-307.77	-317.05	339.26
-315.89	-311.74	-292.93	-330.01		
The log-likelihood values of 42 equal variances for UK GDP					
-304.49	-310.65	-317.68	-327.68	-318.53	-311.53
-309.68	-315.40	-308.23	-319.72	-325.96	-284.02
-302.04	-303.99	-297.92	-314.74	-314.44	-305.62
-311.45	-319.65	-310.42	-319.29	-288.65	-321.13
-312.28	-289.96	-311.51	-328.83	-320.06	-323.16
-321.01	-319.78	-293.71	-299.77	-313.98	-314.26
-324.36	-314.25	-304.68	-302.43	-316.50	-314.73
The log-likelihood values of forty-three equal variances for AU GDP					
-304.94	-322.88	-287.19	-299.12	-299.77	-296.52
-332.98	-309.33	-320.99	-294.04	-307.10	-319.86
-313.99	-303.70	-285.96	-300.80	-319.56	-322.76
-311.41	-311.48	-313.37	-315.79	-293.04	-317.11
-312.26	-324.01	-321.22	-323.92	-321.94	-315.80
-314.01	-297.40	-297.14	-310.77	-299.30	-310.91
-306.87	-305.57	-313.36	-311.22	-321.54	-324.28
-317.40					
The log-likelihood values of forty-one equal variances for France GDP					
-299.63	-309.29	-318.16	-321.73	-309.49	-304.24
-318.88	-319.36	-297.45	-299.58	-311.90	-320.82
-322.99	-285.08	-294.42	-302.66	-303.02	-309.51
-312.25	-315.83	-315.38	-333.21	-308.63	-279.96
-327.88	-299.00	-316.30	-325.28	-303.63	-283.97
-331.54	-288.75	-288.72	-323.35	-312.66	-314.07
-330.14	-332.29	-312.19	-322.88	-315.49	

Step 6: Fitting Linear Model into the white noise by MLE and BIC

The linear model was fitted into the series of equal variances (WN) by MLE and BIC to obtain the fitted WN models. In fitting these linear models, each WN model has mean zero and variance one (constant), and each model was significant. White noise assumed zero mean and constant variance. Therefore, WN models with zero mean and constant variance confirmed the graphical equal variances with zero mean.

The equal variances models were the white noise (WN) models with significant coefficients, significant constant terms, and BIC values, zero mean and constant variance. Therefore, use the log-likelihood to compute the Bayesian model averaging.

Step 7: Bayesian Model Averaging (BMA)

There were 2^K certainty and uncertainty models to account for, and for this study, K is the number of equal variances models (WN models) obtained in step 6, which were forty-six, forty two, forty three and forty one for US GDP, UK, AU GDP and France GDP countries data sets respectively (Hoeting *et al.*, 1999; Shao & Gift, 2014; Hooten & Hobbs, 2015). Some models were selected out of 2^{46}, 2^{42}, 2^{43} and 2^{41} uncertainty and certainty models for each country. Summary of the best models were shown and the best models for US GDP, UK GDP, AU GDP and France GDP computations were as reported in Table 5.6.

The Table 5.6 summarized BMA results details were: The column "p!=0" indicated the probability that the coefficient for a given predictor is not zero. This indicated that at least one of the best models considered in the row directly under the column "p!=0".

The column "EV" displayed the BMA posterior distribution mean for each coefficient and the column "SD" displayed the BMA posterior distribution standard deviation for each coefficient. The posterior probabiliy of quantity of interest was determined by each of the models considered in the study when the posterior propability was correct, given that one of the considered models was correct.The best five models (discribed as model 1, model 2, model 3, model 4 and model 5)were displayed. The predictors (independent variables) to be included in a regression model were determined by BMA. Two best predictors were displayed in Table 5.6.

Table 5.6 summarized the BMA for US GDP, UK GDP, AU GDP and France GDP. US GDP revealed the best model was the first model discribed as model 1 with minium BIC and highest posterior probability values as predictor *A*. Predictor *B* has the best model in model 1 which was the best model. UK GDP revealed the best model was in model 2 with minium BIC and highest posterior probability values as predictor *C*. Predictor *D* has the best model in model 3 which was the best model. The AU GDP revealed the best model was in model 1 with minium BIC and highest posterior probability values as predictor *E*. Predictor *F* has the best model in model 2 which was the best model. The France GDP revealed the best model was in model 1 with minium BIC and highest posterior probability values as predictor *E*. Predictor *F* has the best model in model 2 which was the best model.

Step 8: Fitting Linear Regression with Autoregressive Errors

Fitting linear regression with autoregressive errors which were 220 number of sample sizes, with zero mean and variance one (Higgins & Bera 1992). The estimated values of the fitted linear regression with autoregressive errors, based on significant code asterisk showed the selected best models. The best models for each country GDP were reported in Table 5.7.

P-values in Table 5.7 displayed the significant values of each best two models in the four countries model estimation. The more significant out of the two models indicated the dependent variable for the combine white noise in step 12. Where the two models were having equal significant values as in models A and B in Table 5.7, the overall best model for US GDP which was *A* model in step 7 was considered as the dependent variables in step 10 for the combine white noise model.

Table 5.6

BMA Summary for Real Data

Predictor	**p!=0**	**EV**	**SD**	**model 1**	**Model 2**	**model 3**	**model 4**	**model 5**
US GDP								
Intercept	100	1.0070	0.0621	1.0105	1.0026	1.0032	1.0105	1.0185
A	61	9.75e-02	0.0926	0.1632	.	.	0.1489	0.1661
B	66	1.04e-01	0.0903	0.1616	0.1481	.	.	0.1608
UK GDP								
Intercept	100	1.0817	0.1491	1.0707	1.0892	1.0827	1.0998	1.0687
C	40	0.1304	0.1834	.	0.3259	.	0.3111	.
D	34	0.1066	0.1726	.	.	0.3233	0.3077	.
AU GDP								
Intercept	100	1.17E-01	0.0621	0.1204	0.1164	0.1173	0.1263	0.1224
E	87	1.49E-01	0.0813	0.1671	0.1741	0.1718	0.1719	0.1796
F	36	5.10E-02	0.0794	.	0.1451	.	0.1519	.
France GDP								
Intercept	100	1.63E-02	0.0114	0.0160	0.0172	0.0170	0.0170	0.0169
G	69	1.20e-01	0.0996	0.1807	0.1686	.	0.1739	.
H	38	5.83e-02	0.0867	.	0.1455	.	.	0.1601
nVar				3	2	1	2	4
r^2				0.8180	0.8130	0.8080	0.8120	0.8210
BIC				-358.8	-357.7	-357.6	-357.4	-357.2
post prob				0.087	0.052	0.048	0.045	0.040

nvar, r^2, BIC and post prob values were reported for US GDP in Table 5.6.

Table 5.7

Confirmation of the Fitted Linear Regression with Autoregressive Errors Using Real Data

Model	**Estimate**	**Std. Error**	***t* value**	**Pr(>|*t*|)**
US GDP				
(Intercept)	1.01e+00	2.37e-17	4.28e+16	<2e-16 ***
A	1.48e-01	2.40e-17	6.17e+15	<2e-16 ***
B	2.03e+00	2.52e-17	8.07e+16	<2e-16 ***
UK GDP				
(Intercept)	1.0998	0.1471	7.474	1.89e-12 ***
C	0.3077	0.1473	2.089	0.0378 *
D	0.3111	0.1446	2.151	0.0326 *
AU GDP				
(Intercept)	0.1263	0.0617	2.048	0.0418 *
P	0.1719	0.0615	2.793	0.0057 **
Q	-0.1114	0.0581	-1.917	0.0566.
France GDP				
(Intercept)	0.0399	0.0649	0.615	0.5391
W	0.1686	0.0690	2.446	0.0153 *
X	-0.1114	0.0581	-1.917	0.0566.

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1 Residual standard error: 0.9630 on 173 degrees of freedom Multiple R-squared: 0.8457, Adjusted R-squared: 0.8047 *F*-statistic: 20.62 on 46 and 173 DF, *p*-value: <2.2e-16.

The footnotes were reported for US GDP in Table 5.7.

The SARIMA models were used for the lag selection of the autoregressive order of the models. *Y* was dependent variable of a model.

Step 9: The Regression Model with ARIMA Errors

Firstly, regress the models in step 8, and then run the following ACF of the models. The ACF spike of the first lag signified autoregressive (AR) of order one for all the data sets which were statistically significant, while the rest lag were close to zero. The SARIMA (1, 0, 0) which indicated AR (1) converged with short iteration. Therefore SARIMA (1, 0, 0) were considered as the best. The computer outputs of SARIMA were in Appendix F.

The confirmation of two models from the result of BMA in step 7 by fitting the linear regression with autoregressive errors in step 8showed that the first columns for the first model with US Figure 5.3 displayed AR order one. While the second columns for the second model with US presented AR order one. These revealed that all the ARs were of order one as displayed in Figure 5.3.

With these reports, autoregressive model of order one was considered as shown in Figure 5.3.The other three countries GDP estimation process displayed similar figures of order one. Therefore, this confirmed the autoregressive model of order one [AR (1)] in the following computation**.**

Step 10: Fit AR using ARIMA Modelling of Time Series

This was to obtain the AR of each model. Use lowest AIC value to obtain and confirm the right order of AR model. Only models of lowest AIC values were reported which were AR model of order one as reported in Table 5.8.

Table 5.8 showed $Y_1, Y_2, Y_3, Y_4, A, B, C, D, P, Q, W$ and Z were having ARIMA (1, 0, 0): AR (1) with the smallest AIC values indicating model of order one. The dependent variable of the model was *Y*. Then, ordinary least square method was used to obtain the coefficient of the models.

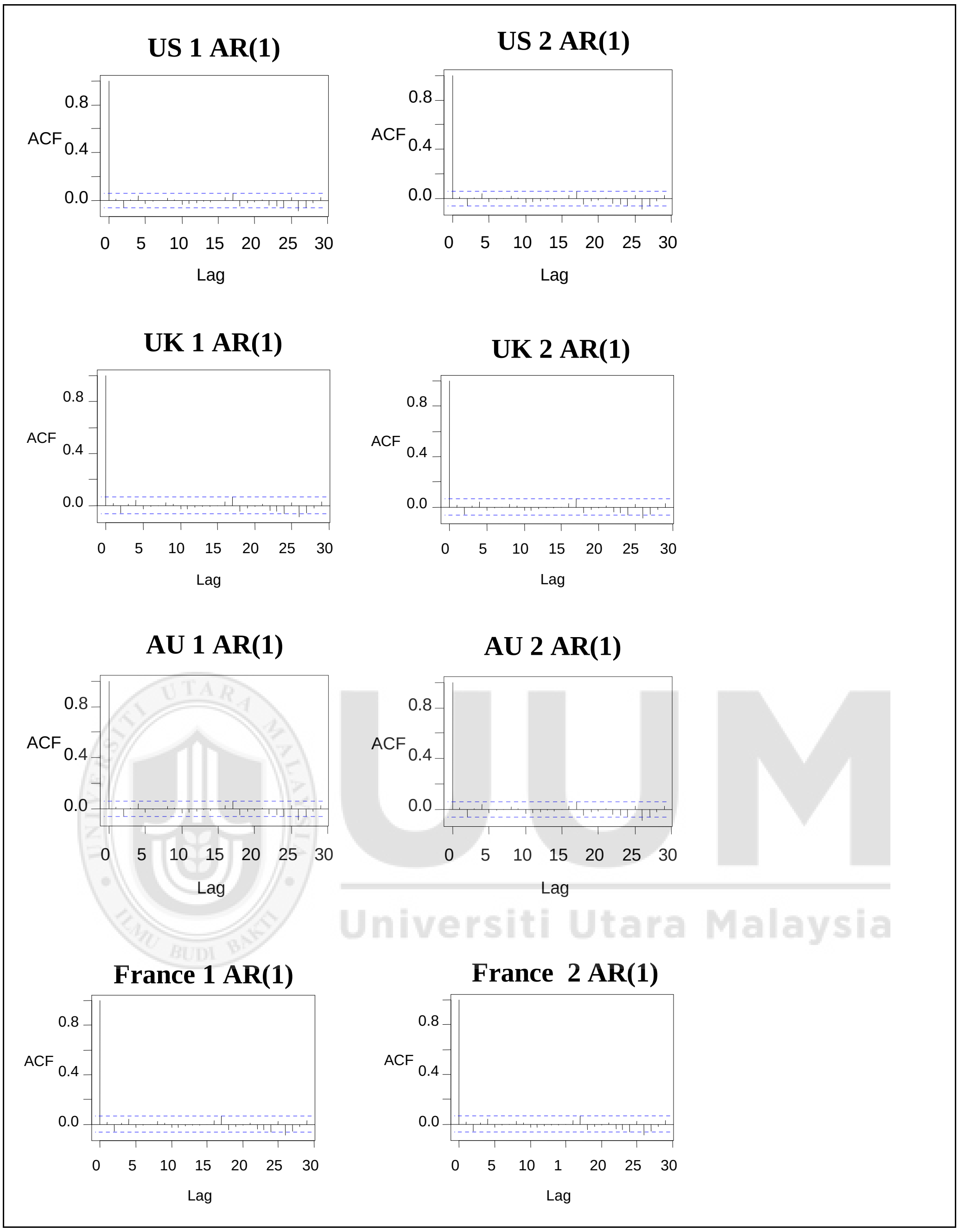


*Figure 5.3.*The ACF of Real Data Sets

Step 11: To obtain the Coefficients of the Model using OLS

Using OLS to obtain the coefficients of the AR, maximum order of one and AIC value was not considered. OLS has good finite-sample properties when compared with Yule-Walker estimator, even, after the bias was corrected. OLS has the smallest mean square error for stationary models when compared with bias formula and bootstrap procedure (Engsted & Pedersen, 2014). These revealed the coefficients of the models as reported in Table 5.9.

Table 5.8

Obtaining the AR Order of Each Model

Each Model is Order One						
ARIMA	$x = A$	$x = B$	$x = Y_1$	$x = C$	$x = D$	$x = Y_2$
Order	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)
AIC	635.19	666.02	921.55	626.89	637.54	591.61
ARIMA	$x = P$	$x = Q$	$x = Y_3$	$x = W$	$x = Z$	$x = Y_4$
Order	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)	(1,0,0)
AIC	628.94	651.32	591.61	602.47	594.21	675.97

Table 5.9

Using OLS to obtain the Coefficients of the Models

Model	**Coefficients**	**Maximum order**	**Sigma^2 estimated**	**Intercept**
A	-0.0694	1	0.9767	-0.0020
B	0.0482	1	0.8901	-0.0039
Y_1	0.0396	1	3.7610	-0.0083
C	-0.1078	1	0.9690	-0.0087
D	0.0078	1	1.0380	-0.0007
Y_2	0.1459	1	0.8370	0.0044
P	-0.0059	1	0.9973	0.0020
Q	-0.1118	1	1.1020	0.0025
Y_3	0.1459	1	0.8368	0.0044
W	0.0484	1	1.0330	0.0005
Z	0.0160	1	0.9557	-0.0009
Y_4	0.1326	1	1.6770	-0.0003

Step 12: The Model

A linear combination is one in which each variable is multiplied by a coefficient and the products are summed (Bates, Maechler, Bolker & Walker, 2014). The combination of two WN models revealed the CWN model. The models combination considered the coefficients of the models in Table 5.9. The dependent variables were revealed in step 8. The predictors in step 7 went through step 8 to step 10 processes of transformation and step 11 derived the coefficients of the models.

The models linear combinations of CWN were:

$$A_t = 0.0482B_{t-1} - 0.0694A_{t-1} + \varepsilon_t \tag{5.5}$$

$$D_t = -0.1078C_{t-1} + 0.0078D_{t-1} + \varepsilon_t \tag{5.6}$$

$$P_t = -0.0059P_{t-1} - 0.1118Q_{t-1} + \varepsilon_t \tag{5.7}$$

$$W_t = 0.0484W_{t-1} + 0.016\,Z_{t-1} + \varepsilon_t \tag{5.8}$$

The Equation (5.5) to Equation (5.8) were the CWN models derived for each of the four countries. The models can now be estimated, to obtain its fitness and perform the forecast evaluation by comparison.

5.4 Models Comparison

The estimation of combine white noise (CWN) model was compared with VAR, EGARCH and MA models using four countries GDP.

5.4.1 Results of the Real Data

Table 5.10 summarized the real data tests and estimation for CWN having the least standard error of regression values. CWN have the highest log-likelihood values and indicated good distribution fit. AIC value and BIC value revealed the minimum information criteria (AIC and BIC) and best fit of CWN among the models. Considering CWN estimation; least standard error, minimum information criteria (AIC and BIC) values of AIC, BIC and highest value of log-likelihood were revealed

Using Australia GDP. The Jarque-Bera of residual normality tests were significant and indicated non-normality of the data distribution. Then, Levene's test for equal variances was conducted in Section 5.4.2 to justify the equal variances of CWN.

The dynamic forecast evaluation revealed that CWN has the minimum forecast error measures values of root mean square error (RMSE), mean absolute error (MAE), mean absolute percentage error (MAPE) and geometric root of mean square error (GRMSE) among the models. These were forecast error measures that determined the forecast accuracy when it was compared with other models for forecasting. The model that has the minimum forecast error measure values revealed the best forecast accuracy as reported in Table5.10.Considering the estimation of CWN; the minimum forecast error measure values of RMSE and MAE were revealed for Australia GDP, the minimum forecast error measure value of MAPE was for United States GDP and the minimum value of GRMSE was for France GDP. The CWN estimated results outperformed the VAR estimated as reported in Table 5.10.

Table 5.10

Summary of the Four Countries GDP Tests and Estimation

Estimation	**CWN**	**VAR**	**EGARCH**	**MA**
US Summary				
Standard Error	0.3926	68.999	0.7904	65.791
Log-Likelihood	63.320	-1167.9	-239.83	-1226.1
AIC	-0.5235	10.775	2.2645	11.224
BIC	-0.4306	10.961	2.3732	11.271
Normality Tests	Not Normal	Not Normal	Not Normal	Not Normal
RMSE	0.4828	312.08	0.6628	305.84
MAE	0.1140	244.14	0.4691	237.82
MAPE	1.3871	1.7039	147.66	1.6582
GRMSE	0.0588	24.837	0.2287	0.2569
Normality Tests	Not Normal	Not Normal	Normal	Not Normal
UK Summary				
Standard Error	0.1955	2209.8	0.9685	1.2047
Log-Likelihood	383.16	-1606.9	-249.31	-1969.8
AIC	-3.4444	14.785	2.3515	18.099
BIC	-3.3515	14.971	2.4601	18.146
Normality Tests	Not Normal	Not Normal	Not Normal	Not Normal
RMSE	0.1673	35951	0.6534	2465.6
MAE	0.0400	30655	0.4088	1584.5
MAPE	1.4280	8.4933	169.70	137.81
GRMSE	0.0197	1.1262	0.2042	693.77
Normality Tests	Not Normal	Not Normal	Normal	Not Normal
AU Summary				
Standard Error	0.0451	1519.9	1.0597	0.0105
Log-Likelihood	699.81	-1211.9	-282.20	-1914.4
AIC	-6.3362	11.178	2.6532	17.510
BIC	-6.2433	11.364	2.7619	17.557
Normality Tests	Not Normal	Not Normal	Normal	Normal
RMSE	0.0403	53254	0.4899	2328.7
MAE	0.0109	46227	0.3665	1915.1
MAPE	1.8160	15.617	107.61	0.6225
GRMSE	0.0050	675.92	0.2133	0.0021
Normality Tests	Not Normal	Not Normal	Normal	Not Normal

Table 5.10 continued

France Summary				
Standard Error	0.0601	2121.9	1.0579	2086.2
Log-Likelihood	515.95	-1464.0	-178.49	-1983.1
AIC	-4.6571	13.479	1.7018	18.138
BIC	-4.5642	13.665	1.8104	18.184
Normality Tests	Not Normal	Not Normal	Not Normal	Not Normal
RMSE	0.0532	2401.7	1.3941	1892.5
MAE	0.0145	1689.6	0.6684	1068.5
MAPE	1.8169	0.9997	100.07	0.6656
GRMSE	0.0021	152.52	0.3192	620.93
Normality Tests	Not Normal	Not Normal	Not Normal	Not Normal

5.4.2 Intra-class Correlation Coefficient and Levene's Test

CWN was not normally distributed as reported in Table 5.10.The Intra-class correlation coefficients (ICC) was used to test the reliability of the measurements of degree of relationship between the data distribution(Caceres, Hall, Zelaya, F. Williams, & Mehta, 2009; Li, Zeng, Lin, Cazzell & Liu, 2015) and the relationships were poor as reported in Appendix B. This can be as a result of the data sets were not normally distributed, but passed the Levene's test.

An independent samples test was conducted to test whether the CWN data sets have equal variances or not. The test revealed that the variability in the distribution of the two data sets was no significantly different with the value which was greater than the p-value of 0.05when the two data sets were having equal variances. US GDP, AU GDP and France GDP were having equal variances as the p-values were greater than significant value of 0.05, while UK GDP has unequal variances because the *p*-value was less than significant value of 0.05 (Lim & Loh, 1996; Boos & Brownie, 2004; Bast *et al.*, 2015)as reported in Appendix B. Therefore, combine variance which revealed less value than each of the variances in the combine white noise estimation that were employed.

5.4.3 Combination of Two Variances of the Combine White Noise Model

In US GDP estimation process, the standard errors of dependent variables A and B were used to calculate the variances of each: variance of A was 0.0550, and B variance was 0.0004. Obtaining combine variance, σ_c^2 of the combine white noise, where K is the balanced weight and ρ is the correlation, but used intra-class correlation for reliability measurements. The explanations were in Chapter Three, Section 3.2 and Subsection 3.2.1 with equation (3.19) to equation (3.22) (Bates & Grangers, 1969; Caceres, Hall, Zelaya, F. Williams, & Mehta, 2009).

Then;

$\rho = 0.01, \quad \sigma_1 = 0.2346, \quad \sigma_2 = 0.0195$

$$K = \frac{\sigma_2^2 - \rho\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2}$$

$$= 0.0061$$

$$\sigma_c^2 = k^2\sigma_1^2 + (1-k)^2\sigma_2^2 + 2\rho k\sigma_1(1-k)\sigma_2$$

$$= 0.0004$$

This was the combine variance which was less than each of the variances, indicated that combine variance is more appropriate. Following the estimation computational procedure above; the combine variance of UK GDP, AU GDP and France GDP values were 0.0036, 0.0022 and 0.0026 respectively. The processes of estimation computations have shown that US GDP has the least combine variance among the countries GDP.

The values of combine variances were the smallest variances. Therefore, the combine white noise (equal variance) error term was encompassed in the vector auto regression (VAR) model for estimation. The inversion of MA (1) process to AR infinite, was in accordance with the multiple series encompassed. The results of CWN showed that the data distributions were not normal, but passed the Levene's test of equal variances. The different values of leverages and skewness were discussed.

5.5 Leverage and Skewness for the Four Countries GDP

The outperformed CWN among the models were used for the four countries transformed data sets which displayed that there were low leverage and low skewness for US GDP. There were high leverage and moderate skewness for France data distribution and estimation. The leverages range of values was determined by the numbers of data that exhibited leverage effect in the estimation. Three countries were having low skewness while France has moderate skewness in the distribution. AU GDP and UK GDP were asymmetric in this study as reported in Table 5.11.

Table 5.11

Leverage and Skewness for the Four Countries GDP

	Asymmetry	Leverage	Skewness
Transformed data			
US GDP	.	-0.0883	-0.3204
UKGDP	0.0933	.	0.3755
AU GDP	0.0203	.	0.3647
France GDP	.	-0.1223	0.7639

5.6 Model Accuracy

CWN outperformed the three models (VAR, EGARCH and MA) using the four countries data sets for the model accuracy in percentage form when the data sets were heteroscedastic in nature. Percentage of accuracy of VAR was the least, while MA and EGARCH percentages were low because of low leverage and low skewness displayed using US GDP. VAR and MA were having the least percentages of accuracy with higher percentage of accuracy for EGARCH because of the high leverage and moderate skewness using France GDP as reported in Table 5.11 to Table 5.12. The percentages of model accuracy for VAR and MA were high in UK GDP and AU GDP as compared with that of US GDP and France GDP; this was because UK GDP and AU GDP have no leverage effect (asymmetry).UK GDP and AU GDP were asymmetric with low skewness which revealed the high percentages of accuracy for MA model as compared with EGARCH model. CWN have the highest percentages of accuracy and were the most accurate models as reported in Table 5.12.

Table 5.12

Model Accuracy in percentages for the Four Countries GDP

Four Countries GDP	CWN	VAR	EGARCH	MA
US GDP	63%	1%	37%	25%
UK GDP	70%	31%	19%	47%
AU GDP	69%	2%	47%	49%
France GDP	69%	1%	64%	1%

5.7 Summary

Among the models estimated for the four countries GDP, CWN presented the least standard error, the minimum information criteria (AIC and BIC)of AIC, BIC and log-likelihood highest values using Australia GDP. RMSE and MAE minimum forecast error measure values were for Australia GDP, the minimum MAPE values was for United Kingdom GDP and GRMSE was having the minimum value using France GDP. The results of CWN showed that none of the data distributions were normal, but United States GDP, Australia GDP and France GDP passed the Levene's test of equal variances.

CWN outperformed VAR, EGARCH and MA in all the four countries GDP discussed in this study. CWN outperformed the VAR in the four different countries GDP in both model fit and forecasting. CWN and VAR error terms were white noise. This was an assurance that CWN can be used to improve the VAR estimation as reported in Table5.10.

CWN was the most accurate model when compared with VAR, EGARCH and MA models as reported in Table 5.12. CWN outperformed the three models using other indicators of model performance which were standard error, log-likelihood, information criteria (AIC and BIC) and forecast error measures.

The estimation of CWN outperformed the EGARCH whether the heteroscedastic data contains leverage effect or not. There were leverage effect in United States GDP and France GDP. There were asymmetric in the United Kingdom GDP and Australia GDP data distributions. Therefore, the countries real data sets that have leverage effects showed that CWN outperformed the three models with high leverage and moderate skewness using France GDP.

CHAPTER SIX
CONCLUSION

6.1 Introduction

This Chapter summarizes the development of the model, validation of CWN using simulated and real data in Section 6.2. Section 6.3 reveals the limitation and future research.

6.2 Summary

The estimation of VAR and GARCH family models are inefficient when the heteroscedastic data have leverage effect which has motivated this study in creating new model by improving vector auto regression (VAR) estimation through combining the white noise. Thus, this new model is named combine white noise (CWN) model. The derivation of CWN involves twelve steps.

The first step to third step are for the collections of heteroscedastic data which are the weaknesses of VAR and EGARCH, then, use EGARCH estimation to obtain standardized residuals graph of unequal variances. The fourth step sort out by decomposition(rearrangement and grouping) manually the standardized residuals graph of unequal variances into equal variances series. The fifth step is the application of log-likelihood which is maximized by maximum likelihood estimation (MLE) procedure to obtain optimal results of sufficiency, consistency, efficiency and parameter invariant of the unequal variances series which are also called white noise (WN) series. The sixth step describes the transformation of WN series into WN model using linear model. The seventh step Bayesian moving average (BMA) is used to obtain the best two WN models for each group as explained in fifth and sixth steps. The eighth step confirms the best two WN models for each group in seventh step using linear regression with auto regression errors. The ninth step regress the models in the eighth step and obtain the models order using regression model with ARIMA error. The tenth step fit the auto regression (AR) using ARIMA modelling of time series to obtain the AR of each model. The eleventh step finds the coefficients of the models using ordinary least square (OLS). The twelfth step obtains combine white noise (CWN) model using linear combination approach. It is named CWN because it is derived from the white noise (equal variances) obtained in the EGARCH standardized residuals of unequal variances. Thus, the validations processes are examined using simulated and real data.

The validation of the performance of combine white noise model with simulation was carried out with three different sample sizes in connection with the low, moderate and high values of leverages and skewness in ordered form. Combine white noise performed well in validation process.

The simulated data of 200 sample size with high leverage and moderate skewness has the best forecast model among the different values of leverages and skewness, while the simulated data fit the best model with moderate leverage and moderate skewness. The 250 sample size of simulated data with moderate leverage and high skewness fit the best model among the different values of leverages and skewness, while the simulated data for the best forecast model was in low leverage and high skewness. The

300 sample size of simulated data with low leverage and low skewness fit the best model among the different values of leverages and skewness, while the simulated data for the best forecast model was in high leverage and high skewness.

The validation of the performance of the combine white noise (CWN) model using real data was implemented. The model estimation for the four countries GDP disclosed that CWN have least standard error, the minimum information criteria (AIC and BIC)of AIC, BIC and log-likelihood highest values using Australia GDP. The four GDP were not normally distributed. RMSE and MAE minimum forecast error measure values were disclosed using Australia GDP for CWN estimation, the minimum MAPE values was for United Kingdom GDP and GRMSE was having the minimum value for France GDP. The four countries used the equal number of sample size. Heteroscedastic data with leverage effects were discovered in United States GDP and France GDP. While Australia GDP and United Kingdom GDP revealed that the heteroscedastic data did not contain leverage effects. The behaviours of the heteroscedastic data presented the outcomes of the estimation; France GDP has the highest values of kurtosis and skewness in the transformed data distribution. The results of CWN showed that France GDP with high leverage and moderate skewness outperformed the US GDP with low leverage and low skewness.

The CWN outperformed the VAR estimated values in the four different countries. Equally, CWN outperformed VAR in simulation processes. This provided the assurance that CWN can be used to improve the VAR estimation. The results of simulated and real data application revealed that CWN model is suitable in modelling heteroscedastic data when compared with the three models.

CWN result presented the overall best forecast model with high leverage and moderate skewness of 200 sample size using simulated data. CWN reported the overall best forecast model with high leverage and moderate skewness using real data that have leverage effects.

CWN reported the most accurate model with about 70 percent as compared with VAR, EGARCH and MA models. CWN outperformed the three models using other indicators of model performance which were standard error, log-likelihood, information criteria (AIC and BIC) and forecast error measures.

Therefore, CWN model was developed for modelling the heteroscedastic data with leverage effect efficiently by decomposing (dividing) EGARCH standardized residuals into series of models and using BMA to select the best models from the series of models. The validation of the performance of CWN with the three models using comparison study was revealed based on simulated and real data. CWN can improve VAR estimation using real data which can benefit the econometricians, economists and statistical modelling end users.

6.3 Limitations and Future Research

The combine white noise (CWN) model has successfully outperformed the three models (VAR, EGARCH and MA) estimated based on simulated and real data studies. The main challenge is the process of obtaining white noise series from the standardized residuals of the EGARCH which is time consuming. Therefore, further study will be conducted to simplify the process in ensuring the future innovation in automating this new model to be embedded in software.

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Appendix A

GDP Real Data

1. Quarterly United States Gross Domestic Product (US GDP) Data for fifty five years

Year/ Quarterly	GDP	Year/ Quarterly	GDP	Year/ Quarterly	GDP	Year/ Quarterly	GDP
1960Q1	3123.2	1968Q3	4599.3	1977Q1	5799.2	1985Q3	7655.2
1960Q2	3111.3	1968Q4	4619.8	1977Q2	5913.0	1985Q4	7712.6
1960Q3	3119.1	1969Q1	4691.6	1977Q3	6017.6	1986Q1	7784.1
1960Q4	3081.3	1969Q2	4706.7	1977Q4	6018.2	1986Q2	7819.8
1961Q1	3102.3	1969Q3	4736.1	1978Q1	6039.2	1986Q3	7898.6
1961Q2	3159.9	1969Q4	4715.5	1978Q2	6274.0	1986Q4	7939.5
1961Q3	3212.6	1970Q1	4707.1	1978Q3	6335.3	1987Q1	7995.0
1961Q4	3277.7	1970Q2	4715.4	1978Q4	6420.3	1987Q2	8084.7
1962Q1	3336.8	1970Q3	4757.2	1979Q1	6433.0	1987Q3	8158.0
1962Q2	3372.7	1970Q4	4708.3	1979Q2	6440.8	1987Q4	8292.7
1962Q3	3404.8	1971Q1	4834.3	1979Q3	6487.1	1988Q1	8339.3
1962Q4	3418.0	1971Q2	4861.9	1979Q4	6503.9	1988Q2	8449.5
1963Q1	3456.1	1971Q3	4900.0	1980Q1	6524.9	1988Q3	8498.3
1963Q2	3501.1	1971Q4	4914.3	1980Q2	6392.6	1988Q4	8610.9
1963Q3	3569.5	1972Q1	5002.4	1980Q3	6382.9	1989Q1	8697.7
1963Q4	3595.0	1972Q2	5118.3	1980Q4	6501.2	1989Q2	8766.1
1964Q1	3672.7	1972Q3	5165.4	1981Q1	6635.7	1989Q3	8831.5
1964Q2	3716.4	1972Q4	5251.2	1981Q2	6587.3	1989Q4	8850.2
1964Q3	3766.9	1973Q1	5380.5	1981Q3	6662.9	1990Q1	8947.1
1964Q4	3780.2	1973Q2	5441.5	1981Q4	6585.1	1990Q2	8981.7
1965Q1	3873.5	1973Q3	5411.9	1982Q1	6475.0	1990Q3	8983.9
1965Q2	3926.4	1973Q4	5462.4	1982Q2	6510.2	1990Q4	8907.4
1965Q3	4006.2	1974Q1	5417.0	1982Q3	6486.8	1991Q1	8865.6
1965Q4	4100.6	1974Q2	5431.3	1982Q4	6493.1	1991Q2	8934.4
1966Q1	4201.9	1974Q3	5378.7	1983Q1	6578.2	1991Q3	8977.3
1966Q2	4219.1	1974Q4	5357.2	1983Q2	6728.3	1991Q4	9016.4
1966Q3	4249.2	1975Q1	5292.4	1983Q3	6860.0	1992Q1	9123.0
1966Q4	4285.6	1975Q2	5333.2	1983Q4	7001.5	1992Q2	9223.5
1967Q1	4324.9	1975Q3	5421.4	1984Q1	7140.6	1992Q3	9313.2
1967Q2	4328.7	1975Q4	5494.4	1984Q2	7266.0	1992Q4	9406.5
1967Q3	4366.1	1976Q1	5618.5	1984Q3	7337.5	1993Q1	9424.1
1967Q4	4401.2	1976Q2	5661.0	1984Q4	7396.0	1993Q2	9480.1
1968Q1	4490.6	1976Q3	5689.8	1985Q1	7469.5	1993Q3	9526.3
1968Q2	4566.4	1976Q4	5732.5	1985Q2	7537.9	1993Q4	9653.5

Quarterly United States Gross Domestic Product (US GDP) Data for fifty five years continued

Year/ Quarterly	GDP	Year/ Quarterly	GDP	Year/ Quarterly	GDP	Year/ Quarterly	GDP
1994Q1	9748.2	1999Q2	11962.5	2004Q3	13830.8	2009Q4	14541.9
1994Q2	9881.4	1999Q3	12113.1	2004Q4	13950.4	2010Q1	14604.8
1994Q3	9939.7	1999Q4	12323.3	2005Q1	14099.1	2010Q2	14745.9
1994Q4	10052.5	2000Q1	12359.1	2005Q2	14172.7	2010Q3	14845.5
1995Q1	10086.9	2000Q2	12592.5	2005Q3	14291.8	2010Q4	14939.0
1995Q2	10122.1	2000Q3	12607.7	2005Q4	14373.4	2011Q1	14881.3
1995Q3	10208.8	2000Q4	12679.3	2006Q1	14546.1	2011Q2	14989.6
1995Q4	10281.2	2001Q1	12643.3	2006Q2	14589.6	2011Q3	15021.1
1996Q1	10348.7	2001Q2	12710.3	2006Q3	14602.6	2011Q4	15190.3
1996Q2	10529.4	2001Q3	12670.1	2006Q4	14716.9	2012Q1	15275.0
1996Q3	10626.8	2001Q4	12705.3	2007Q1	14726.0	2012Q2	15336.7
1996Q4	10739.1	2002Q1	12822.3	2007Q2	14838.7	2012Q3	15431.3
1997Q1	10820.9	2002Q2	12893.0	2007Q3	14938.5	2012Q4	15433.7
1997Q2	10984.2	2002Q3	12955.8	2007Q4	14991.8	2013Q1	15538.4
1997Q3	11124.0	2002Q4	12964.0	2008Q1	14889.5	2013Q2	15606.6
1997Q4	11210.3	2003Q1	13031.2	2008Q2	14963.4	2013Q3	15779.9
1998Q1	11321.2	2003Q2	13152.1	2008Q3	14891.6	2013Q4	15916.2
1998Q2	11431.0	2003Q3	13372.4	2008Q4	14577.0	2014Q1	15831.7
1998Q3	11580.6	2003Q4	13528.7	2009Q1	14375.0	2014Q2	16010.4
1998Q4	11770.7	2004Q1	13606.5	2009Q2	14355.6	2014Q3	16205.6
1999Q1	11864.7	2004Q2	13706.2	2009Q3	14402.5	2014Q4	16293.7

2. Quarterly United Kingdom Gross Domestic Product (UK GDP) Data for fifty five years

Year/ Quarterly	GDP	Year/ Quarterly	GDP	Year/ Quarterly	GDP	Year/ Quarterly	GDP
1960Q1	119158	1968Q3	158159	1977Q1	163732	1985Q3	223107
1960Q2	118220	1968Q4	158979	1977Q2	166606	1985Q4	224158
1960Q3	120089	1969Q1	157884	1977Q3	169572	1986Q1	225834
1960Q4	120819	1969Q2	161652	1977Q4	170207	1986Q2	228391
1961Q1	122782	1969Q3	163263	1978Q1	170314	1986Q3	229928
1961Q2	123267	1969Q4	164771	1978Q2	174840	1986Q4	234262
1961Q3	122633	1970Q1	163732	1978Q3	175260	1987Q1	236229
1961Q4	122412	1970Q2	166606	1978Q4	178032	1987Q2	239505
1962Q1	123001	1970Q3	169572	1979Q1	186968	1987Q3	245364
1962Q2	124166	1970Q4	170207	1979Q2	187241	1987Q4	248254
1962Q3	124919	1971Q1	170314	1979Q3	185345	1988Q1	252941
1962Q4	124416	1971Q2	174840	1979Q4	184558	1988Q2	254603
1963Q1	125097	1971Q3	175260	1980Q1	179528	1988Q3	258558
1963Q2	130461	1971Q4	178032	1980Q2	182105	1988Q4	260772
1963Q3	131075	1972Q1	186968	1980Q3	183246	1989Q1	261846
1963Q4	134096	1972Q2	187241	1980Q4	180483	1989Q2	263514
1964Q1	134864	1972Q3	185345	1981Q1	180603	1989Q3	263651
1964Q2	137228	1972Q4	184558	1981Q2	177509	1989Q4	263719
1964Q3	137740	1973Q1	179528	1981Q3	176931	1990Q1	265371
1964Q4	139872	1973Q2	182105	1981Q4	179080	1990Q2	266644
1965Q1	139483	1973Q3	183246	1982Q1	182043	1990Q3	263704
1965Q2	139602	1973Q4	180483	1982Q2	181669	1990Q4	262665
1965Q3	140784	1974Q1	180603	1982Q3	184000	1991Q1	261838
1965Q4	141663	1974Q2	177509	1982Q4	188037	1991Q2	261442
1966Q1	141872	1974Q3	176931	1983Q1	188138	1991Q3	260779
1966Q2	142667	1974Q4	179080	1983Q2	186977	1991Q4	261240
1966Q3	143183	1975Q1	182043	1983Q3	188264	1992Q1	261346
1966Q4	142577	1975Q2	181669	1983Q4	191472	1992Q2	261067
1967Q1	144536	1975Q3	184000	1984Q1	192949	1992Q3	262816
1967Q2	146529	1975Q4	188037	1984Q2	195341	1992Q4	264742
1967Q3	147194	1976Q1	158159	1984Q3	197898	1993Q1	266762
1967Q4	147960	1976Q2	158979	1984Q4	199843	1993Q2	268180
1968Q1	153354	1976Q3	157884	1985Q1	198861	1993Q3	270418
1968Q2	152761	1976Q4	161652	1985Q2	207589	1993Q4	272389

Quarterly United Kingdom Gross Domestic Product (UK GDP) Data for fifty five years continued

Year/ Quarterly	GDP	Year/ Quarterly	GDP	Year/ Quarterly	GDP	Year/ Quarterly	GDP
1994Q1	275836	1999Q2	319560	2004Q3	376942	2009Q4	391685
1994Q2	279116	1999Q3	324767	2004Q4	378470	2010Q1	393678
1994Q3	282336	1999Q4	329111	2005Q1	381142	2010Q2	397525
1994Q4	283840	2000Q1	332555	2005Q2	385058	2010Q3	400096
1995Q1	284637	2000Q2	334960	2005Q3	389023	2010Q4	400195
1995Q2	285751	2000Q3	336221	2005Q4	394268	2011Q1	402341
1995Q3	288862	2000Q4	337211	2006Q1	396566	2011Q2	403260
1995Q4	290247	2001Q1	341026	2006Q2	398553	2011Q3	406068
1996Q1	293666	2001Q2	343637	2006Q3	399251	2011Q4	406008
1996Q2	294490	2001Q3	345468	2006Q4	402258	2012Q1	406283
1996Q3	295521	2001Q4	346546	2007Q1	405329	2012Q2	405560
1996Q4	296474	2002Q1	348115	2007Q2	407767	2012Q3	408938
1997Q1	297909	2002Q2	350978	2007Q3	411205	2012Q4	407557
1997Q2	301318	2002Q3	354058	2007Q4	413131	2013Q1	409985
1997Q3	303490	2002Q4	357286	2008Q1	414424	2013Q2	412620
1997Q4	307560	2003Q1	360733	2008Q2	413465	2013Q3	415577
1998Q1	309517	2003Q2	365803	2008Q3	406584	2013Q4	417265
1998Q2	311857	2003Q3	370428	2008Q4	397522	2014Q1	420091
1998Q3	314098	2003Q4	374127	2009Q1	390406	2014Q2	423249
1998Q4	317295	2004Q1	375324	2009Q2	389388	2014Q3	426022
1999Q1	318806	2004Q2	376455	2009Q3	390167	2014Q4	428347

3. Quarterly Australia Gross Domestic Product (AU GDP) Data for fifty five years

Year/ Quarterly	GDP	Year/ Quarterly	GDP	Year/ Quarterly	GDP	Year/ Quarterly	GDP
1960Q3	62699	1969Q1	93056	1977Q3	124632	1986Q1	160962
1960Q4	62391	1969Q2	94927	1977Q4	124221	1986Q2	160681
1961Q1	62420	1969Q3	96497	1978Q1	125099	1986Q3	161140
1961Q2	61661	1969Q4	98681	1978Q2	126118	1986Q4	163738
1961Q3	61314	1970Q1	100712	1978Q3	128077	1987Q1	165301
1961Q4	62081	1970Q2	102754	1978Q4	129122	1987Q2	168137
1962Q1	63864	1970Q3	102569	1979Q1	132635	1987Q3	171077
1962Q2	65094	1970Q4	103033	1979Q2	130506	1987Q4	174366
1962Q3	65610	1971Q1	104275	1979Q3	131781	1988Q1	175224
1962Q4	66768	1971Q2	104745	1979Q4	134307	1988Q2	175811
1963Q1	68278	1971Q3	108033	1980Q1	134898	1988Q3	177124
1963Q2	67373	1971Q4	107666	1980Q2	135231	1988Q4	179544
1963Q3	70159	1972Q1	106369	1980Q3	136029	1989Q1	181572
1963Q4	71638	1972Q2	108774	1980Q4	138348	1989Q2	185374
1964Q1	71569	1972Q3	108196	1981Q1	138871	1989Q3	186661
1964Q2	73362	1972Q4	109307	1981Q2	140977	1989Q4	186430
1964Q3	73820	1973Q1	112153	1981Q3	143892	1990Q1	187981
1964Q4	75876	1973Q2	112380	1981Q4	143269	1990Q2	188081
1965Q1	76488	1973Q3	113533	1982Q1	142109	1990Q3	187067
1965Q2	77689	1973Q4	116324	1982Q2	143358	1990Q4	188029
1965Q3	77511	1974Q1	116330	1982Q3	142456	1991Q1	185693
1965Q4	77662	1974Q2	113955	1982Q4	140178	1991Q2	185172
1966Q1	77415	1974Q3	115442	1983Q1	138772	1991Q3	185757
1966Q2	78474	1974Q4	115443	1983Q2	138449	1991Q4	186175
1966Q3	80749	1975Q1	115866	1983Q3	142754	1992Q1	187862
1966Q4	81234	1975Q2	119545	1983Q4	144818	1992Q2	189180
1967Q1	84393	1975Q3	118291	1984Q1	148331	1992Q3	190794
1967Q2	84272	1975Q4	116453	1984Q2	149866	1992Q4	194658
1967Q3	85912	1976Q1	121621	1984Q3	151245	1993Q1	196472
1967Q4	86611	1976Q2	122005	1984Q4	152282	1993Q2	197362
1968Q1	85818	1976Q3	123044	1985Q1	154754	1993Q3	197472
1968Q2	89147	1976Q4	124072	1985Q2	158237	1993Q4	201303
1968Q3	90328	1977Q1	123363	1985Q3	160243	1994Q1	204882
1968Q4	93674	1977Q2	125146	1985Q4	159842	1994Q2	207148

Quarterly Australia Gross Domestic Product (AU GDP) Data for fifty five years continued

Year/ Quarterly	GDP	Year/ Quarterly	GDP	Year/ Quarterly	GDP	Year/ Quarterly	GDP
1994Q3	209087	1999Q4	258749	2005Q1	305482	2010Q2	352372
1994Q4	210231	2000Q1	260643	2005Q2	307082	2010Q3	354131
1995Q1	210744	2000Q2	262675	2005Q3	310839	2010Q4	358039
1995Q2	212214	2000Q3	262854	2005Q4	313150	2011Q1	356698
1995Q3	215944	2000Q4	261697	2006Q1	313828	2011Q2	361486
1995Q4	217011	2001Q1	265039	2006Q2	314635	2011Q3	365720
1996Q1	221027	2001Q2	266972	2006Q3	317949	2011Q4	369377
1996Q2	221541	2001Q3	270001	2006Q4	322966	2012Q1	373199
1996Q3	224250	2001Q4	272834	2007Q1	327956	2012Q2	375378
1996Q4	225878	2002Q1	275108	2007Q2	330675	2012Q3	377463
1997Q1	226534	2002Q2	279434	2007Q3	333118	2012Q4	379566
1997Q2	233386	2002Q3	280491	2007Q4	335004	2013Q1	380471
1997Q3	233340	2002Q4	282770	2008Q1	339193	2013Q2	383444
1997Q4	236606	2003Q1	282866	2008Q2	340345	2013Q3	384740
1998Q1	239213	2003Q2	285042	2008Q3	342712	2013Q4	388070
1998Q2	241212	2003Q3	289448	2008Q4	339942	2014Q1	391553
1998Q3	245592	2003Q4	294214	2009Q1	343341	2014Q2	393991
1998Q4	249099	2004Q1	296426	2009Q2	345003	2014Q3	395491
1999Q1	251023	2004Q2	298099	2009Q3	346396	2014Q4	397658
1999Q2	252217	2004Q3	300683	2009Q4	348902	2015Q1	401153
1999Q3	254503	2004Q4	302837	2010Q1	350233	2015Q2	401816

4. Quarterly France Gross Domestic Product (France GDP) Data for fifty five years

Year/ Quarter	GDP	Year/ Quarter	GDP	Year/ Quarter	GDP	Year/ Quarter	GDP
1960Q3	114272	1969Q1	181172	1977Q3	258288	1986Q1	308498
1960Q4	115484	1969Q2	184946	1977Q4	260391	1986Q2	311999
1961Q1	117087	1969Q3	187464	1978Q1	263824	1986Q3	313604
1961Q2	117833	1969Q4	190293	1978Q2	266635	1986Q4	313910
1961Q3	118947	1970Q1	193239	1978Q3	268767	1987Q1	314745
1961Q4	121035	1970Q2	196425	1978Q4	271679	1987Q2	318759
1962Q1	123534	1970Q3	198435	1979Q1	274054	1987Q3	321141
1962Q2	125407	1970Q4	201057	1979Q2	275524	1987Q4	325800
1962Q3	127832	1971Q1	204266	1979Q3	279153	1988Q1	330048
1962Q4	129202	1971Q2	206415	1979Q4	279886	1988Q2	332798
1963Q1	128484	1971Q3	209240	1980Q1	282924	1988Q3	336774
1963Q2	133793	1971Q4	211233	1980Q2	280863	1988Q4	339979
1963Q3	138086	1972Q1	213576	1980Q3	281356	1989Q1	344994
1963Q4	138138	1972Q2	215401	1980Q4	280938	1989Q2	348237
1964Q1	141087	1972Q3	218493	1981Q1	281911	1989Q3	351547
1964Q2	142709	1972Q4	221923	1981Q2	283496	1989Q4	354986
1964Q3	144026	1973Q1	225800	1981Q3	285546	1990Q1	358597
1964Q4	146064	1973Q2	229431	1981Q4	287243	1990Q2	359991
1965Q1	146929	1973Q3	233036	1982Q1	289498	1990Q3	361148
1965Q2	149422	1973Q4	235602	1982Q2	291545	1990Q4	360888
1965Q3	151584	1974Q1	239572	1982Q3	291900	1991Q1	360956
1965Q4	153851	1974Q2	241254	1982Q4	293485	1991Q2	363503
1966Q1	155281	1974Q3	243506	1983Q1	294469	1991Q3	364800
1966Q2	157638	1974Q4	239287	1983Q2	294824	1991Q4	366789
1966Q3	159611	1975Q1	237727	1983Q3	295123	1992Q1	369983
1966Q4	160759	1975Q2	237076	1983Q4	296693	1992Q2	369878
1967Q1	163358	1975Q3	237162	1984Q1	298517	1992Q3	369322
1967Q2	165188	1975Q4	242059	1984Q2	299362	1992Q4	368319
1967Q3	167210	1976Q1	244436	1984Q3	301262	1993Q1	366004
1967Q4	168891	1976Q2	247810	1984Q4	301260	1993Q2	366572
1968Q1	173102	1976Q3	250549	1985Q1	302019	1993Q3	367791
1968Q2	164395	1976Q4	252236	1985Q2	304422	1993Q4	368427
1968Q3	177107	1977Q1	255139	1985Q3	306359	1994Q1	370297
1968Q4	179393	1977Q2	255962	1985Q4	307560	1994Q2	374699

Quarterly France Gross Domestic Product (France GDP) Data for fifty five years continued

Year/ Quarterly	GDP	Year/ Quarterly	GDP	Year/ Quarterly	GDP	Year/ Quarterly	GDP
1994Q3	377229	1999Q4	433082	2005Q1	477175	2010Q2	497884
1994Q4	380473	2000Q1	438491	2005Q2	478415	2010Q3	500788
1995Q1	382227	2000Q2	441746	2005Q3	480972	2010Q4	503409
1995Q2	384168	2000Q3	444561	2005Q4	484669	2011Q1	509219
1995Q3	384439	2000Q4	448322	2006Q1	487849	2011Q2	508803
1995Q4	385122	2001Q1	451268	2006Q2	492945	2011Q3	509799
1996Q1	387505	2001Q2	451408	2006Q3	492913	2011Q4	511046
1996Q2	388449	2001Q3	452756	2006Q4	496738	2012Q1	511258
1996Q3	390330	2001Q4	451864	2007Q1	500164	2012Q2	509776
1996Q4	390729	2002Q1	454400	2007Q2	503460	2012Q3	511124
1997Q1	392332	2002Q2	457117	2007Q3	505475	2012Q4	511075
1997Q2	396680	2002Q3	458387	2007Q4	506852	2013Q1	511761
1997Q3	399759	2002Q4	457818	2008Q1	509256	2013Q2	515619
1997Q4	403777	2003Q1	458113	2008Q2	506482	2013Q3	515016
1998Q1	406967	2003Q2	457916	2008Q3	505031	2013Q4	516114
1998Q2	411283	2003Q3	461191	2008Q4	497016	2014Q1	515222
1998Q3	414149	2003Q4	465244	2009Q1	489186	2014Q2	514610
1998Q4	417290	2004Q1	468149	2009Q2	488813	2014Q3	515823
1999Q1	419674	2004Q2	471646	2009Q3	489482	2014Q4	516402
1999Q2	423406	2004Q3	473663	2009Q4	492688	2015Q1	519856
1999Q3	428117	2004Q4	476863	2010Q1	494954	2015Q2	519796

Appendix B

Intra-class correlation coefficient and Levene's Test Real Data

1. Intra-class correlation coefficient for USGDP

	Intraclass Correlation[a]	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound	F Test with True Value: Value	df1	df2	Sig
Single Measures	.01[b]	-.12	.14	1.02	22	22	.44
Average Measures	.02[c]	-.28	.25	1.02	22	22	.44

A two-way mixed effects model where people effects are random and measures effects are fixed.

A Type C intra-class correlation coefficients using a consistent definition-the between-measure variance are excluded from the denominator variance.

b. The estimator is the same, whether the interaction effect is present or not.

c. This estimate is computed assuming the interaction effect is absent, because it is not estimable otherwise.

Levene's Test for Equal Variances Independent Samples Test for US GDP

Independent samples test

	Levene's Test for Equality of Variances: F	Sig.	t-test for Equality of Means: t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference: Lower	Upper
Equal variances assumed	1.414	0.235	2.159	438	0.031	0.059	0.027	0.005	0.113
Equal variances not assumed			2.159	255.24	0.032	0.059	0.027	0.005	0.113

2. Intra-class correlation coefficient for UK GDP

	Intraclass Correlation[a]	95% Confidence Interval Lower Bound	Upper Bound	F Test with True Value Value	df1	df2	Sig
Single Measures	-.014[b]	-.146	.118	.972	219	219	.583
Average Measures	-.029	-.341	.211	.972	219	219	.583

A two-way mixed effects model where people effects are random and measures effects are fixed.

a. Type C intra-class correlation coefficients using a consistent definition-the between-measure variances are excluded from the denominator variance.
b. The estimator is the same, whether the interaction effect is present or not.
c. This estimate is computed assuming the interaction effect is absent, because it is not estimable otherwise.

Levene's test for equal variances Independent Samples Test for UK GDP

Independent samples test									
	Levene's Test for Equality of Variances		t-test for Equality of Means					95% Confidence Interval of the Difference	
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	5.504	.019	1.133	438	.258	.015	.014	-.011	.042
Equal variances not assumed			1.133	255.50	.258	.015	.014	-.011	.042

3. Intra-class correlation coefficient for AU GDP

	Intraclass Correlation[a]	95% Confidence Interval Lower Bound	Upper Bound	F Test with True Value Value	df1	df2	Sig
Single Measures	.020[b]	-.112	.152	1.042	219	219	.381
Average Measures	.040[c]	-.252	.264	1.042	219	219	.381

A two-way mixed effects model where people effects are random and measures effects are fixed.

a. Type C intra-class correlation coefficients using a consistent definition-the between-measure variance are excluded from the denominator variance.

b. The estimator is the same, whether the interaction effect is present or not.

c. This estimate is computed assuming the interaction effect is absent, because it is not estimable otherwise.

Levene's test for equal variances Independent Samples Test for AU GDP

	Independent samples test								
	Levene's Test for Equality of Variances		t-test for Equality of Means					95% Confidence Interval of the Difference	
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	.045	.833	-2.994	438	.003	-.014	.005	-.023	-.005
Equal variances not assumed			-2.994	424.76	.003	-.014	.005	-.023	-.005

4. Intra-class correlation coefficient for France GDP

	Intraclass Correlation[a]	95% Confidence Interval Lower Bound	Upper Bound	F Test with True Value Value	df1	df2	Sig
Single Measures	.016[b]	-.116	.148	1.033	219	219	.405
Average Measures	.032[c]	-.262	.258	1.033	219	219	.405

A two-way mixed effects model where people effects are random and measures effects are fixed.

a. Type C intra-class correlation coefficients using a consistent definition-the between-measure variance are excluded from the denominator variance.

b. The estimator is the same, whether the interaction effect is present or not.

c. This estimate is computed assuming the interaction effect is absent, because it is not estimable otherwise.

Levene's test for equal variances Independent Samples Test for France GDP

Independent samples test									
	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	.271	.603	2.684	438	.008	.020	.008	.005	.035
Equal variances not assumed			2.684	373.49	.008	.020	.008	.005	.035

Appendix C

The Log-likelihood

1. The Log-Likelihood Values for 200 Sample size of Low Leverage and different Values of Skewness

Low leverage and low skewness for 40 equal variances					
-1480.69	-1476.68	-1454.77	-1341.02	-1458.05	-1448.52
-1360.76	-1347.23	-1336.74	-1362.25	-1407.78	-1433.82
-1374.86	-1318.28	-1383.83	-1437.41	-1397.73	-1430.27
-1396.86	-1361.00	-1362.71	-1327.10	-1394.40	-1440.67
-1381.62	-1334.76	-1432.11	-1429.17	-1392.74	-1413.88
-1368.69	-1278.90	-1389.05	-315.431	-1433.04	-1403.91
-1300.23	-1417.90	-1374.60	-1323.74		

Low leverage and moderate skewness for 44 equal variances					
-1408.88	-1412.84	-1417.85	-1305.38	-1361.89	-1402.64
-1400.01	-1325.60	-1378.95	-1442.18	-1279.72	-1433.54
-1406.93	-1429.50	-1348.56	-1353.01	-1323.97	-1331.18
-1304.29	-1386.27	-1367.69	-1381.06	-1331.04	-1371.04
-1352.89	-1404.61	-1392.78	-1411.30	-1367.00	-1336.88
-1321.96	-1361.73	-1357.76	-326.07	-1334.46	-1278.75
-1429.75	-1335.85	-1210.69	-1393.98	-1378.41	-1404.55
-1462.58	-1394.88				

Low leverage and high skewness for 43 equal variances					
-1408.88	-1408.88	-1417.85	-1305.38	-1361.89	-1402.64
-1400.01	-1325.60	-1378.95	-1442.18	-1279.72	-1433.54
-1406.93	-1429.50	-1348.56	-1353.01	-1323.97	-1331.18
-1304.29	-1386.27	-1367.69	-1381.06	-1331.04	-1371.04
-1411.21	-1366.90	-1336.03	-1322.11	-1359.97	-1357.66
-1443.71	-1333.36	-1278.70	-317.01	-1335.13	-1208.27
-1394.36	-1379.15	-1404.40	-1462.98	-1395.24	-1439.32
-1402.98					

2. The Log-Likelihood Values for 200 Sample size of Moderate Leverage and different Values of Skewness

Moderate leverage and low skewness for 40 equal variances					
-1321.80	-1328.98	-1433.48	-1345.96	-1356.46	-2172.37
-1422.38	-1396.74	-1428.70	-1343.75	-1389.54	-1286.90
-1329.06	-1384.08	-1368.80	-1405.64	-1360.08	-1456.30
-1401.54	-1275.89	-1294.41	-1426.36	-1383.56	-1421.46
-1313.38	-1323.85	-1423.28	-1376.72	-1455.41	-1384.32
-1335.79	-1383.15	-1365.69	-303.93	-1408.91	-1352.43
-1307.73	-1375.32	-1361.76	-1496.46		
Moderate leverage and moderate skewness for 45 equal variances					
-1430.34	-1631.54	2817.20	-1408.94	-1402.86	-1365.49
-1437.46	-1395.26	-1403.64	-1442.39	-1323.14	-1432.56
-1387.93	-1366.33	-1368.59	-1395.98	-1416.47	-1273.47
-1394.08	-1333.64	-1394.20	-1384.32	-1366.24	-1412.68
-1336.02	-1339.85	-1370.12	-1332.26	-1332.99	-1350.02
-1337.44	-1391.55	-1393.67	-307.17	-1326.93	-1282.96
-1338.00	-1392.69	-1283.39	-1336.35	-1416.69	-1359.39
-1380.94	-1342.92	-1361.48			
Moderate leverage and high skewness for 41 equal variances					
-1350.24	-1353.32	-1358.54	-1368.06	-1293.26	-1339.24
-1324.88	-1437.11	-1410.32	-1365.82	-1354.34	-1412.71
-1337.87	-1429.30	-1363.61	-1375.96	-1370.92	-1372.37
-1358.24	-1393.90	-1304.10	-1393.50	-1401.12	-1409.85
-1396.00	-1458.76	-1350.42	-1450.58	-1421.99	-1375.44
-1401.42	-1290.98	-1405.77	-304.96	-1274.79	-1329.78
-1324.23	-1390.19	-1448.74	-1355.07	-1393.68	

3. The Log-Likelihood Values for200 Sample size of High Leverage and different Values of Skewness

High leverage and low skewness for 41 equal variances					
-1408.88	-1412.48	-1417.85	-1305.38	-1361.89	-1402.64
-1400.01	-1325.60	-1378.95	-1442.18	-1279.72	-1433.54
-1406.93	-1429.50	-1348.56	-1353.01	-1323.97	-1331.18
-1304.29	-1386.27	-1367.69	-1381.06	-1331.04	-1371.04
-1352.89	-1404.61	-1392.78	-1411.30	-1367.00	-1336.88
-1321.96	-1361.73	-1357.76	-326.07	-1334.46	-1278.75
-1429.75	-1335.85	-1210.69	-1393.98	-1378.41	
High leverage and moderate skewness for 44 equal variances					
-1408.876	-1402.668	-1396.69	-1403.38	-1399.27	-1325.87
-1377.588	-1441.646	-1279.49	-1434.55	-1406.82	-1428.66
-1349.202	-1353.458	-1323.84	-1330.25	-1303.92	-1386.74
-1368.038	-1381.240	-1331.04	-1371.68	-1352.92	-1405.82
-1393.210	-1411.105	-1366.66	-1336.61	-1322.01	-1361.76
-1357.083	-1444.048	-1334.38	-286.13	-1429.24	-1335.51
-1211.513	-1393.551	-1379.15	-1404.97	-1462.81	-1394.86
-1439.359	-1402.866				
High leverage and high skewness for 43 equal variances					
-1385.99	-1387.98	-1396.69	-1403.38	-1399.27	-1325.87
-1377.59	-1441.65	-1279.49	-1434.55	-1406.82	-1428.66
-1349.20	-1353.46	-1323.84	-1330.25	-1303.92	-1386.74
-1368.04	-1381.24	-1331.04	-1371.68	-1352.92	-1405.82
-1393.21	-1411.11	-1366.66	-1336.61	-1322.01	-1361.76
-1357.08	-1444.05	-286.13	-1334.38	-1429.24	-1335.51
-1211.51	-1393.55	-1379.15	-1404.97	-1462.81	-1394.86
-1439.36					

4. The Log-Likelihood Values for 250 Sample size of Low Leverage and different Values of Skewness

Low leverage and low skewness for 45 equal variances					
-1476.42	-1462.35	1449.84	-1342.84	-1425.49	-1404.73
-1370.48	-1444.24	-1347.29	-1405.56	-1375.90	-1445.99
-1369.85	-1347.22	-1443.71	-1393.35	-1364.73	-1415.22
-1324.51	-1407.95	-1330.58	-1346.45	-1372.82	-1386.91
-1473.02	-1386.46	-1390.39	-1345.02	-1382.86	-1397.24
-1308.67	-1376.17	-1417.94	-1417.94	-1405.89	-1325.59
-1431.57	-1391.72	-1422.62	-1369.12	-1436.26	-1344.94
-1398.69	-1371.16	-1357.20			
Low leverage and moderate skewness for 48 equal variances					
-1407.61	-1407.61	-1335.67	-1337.59	-1378.93	-1378.93
-1351.28	-1366.68	-1375.19	-1453.98	-1390.95	-1331.27
-1405.29	-1347.73	-1405.82	-1343.84	-1402.93	-1320.52
-1370.74	-1300.55	-1294.20	-1354.28	-1361.77	-1398.92
-1390.28	-1402.72	-1297.32	-1370.12	-1343.01	-1409.22
-1409.10	-1388.73	-1360.61	-1380.25	-1386.09	-1372.22
-1367.02	-1451.00	-1407.49	-1411.59	-1444.01	-1453.08
-1430.76	-1447.09	-1331.38	-1377.92	-1388.95	-1383.34
Low leverage and high skewness for 46 equal variances					
-1344.60	-1341.42	-1357.34	-1370.40	-1383.75	-1415.46
-1456.02	-1467.32	-1365.12	-1439.89	-1369.30	-1389.48
-1368.49	-1410.85	-1358.19	-1390.97	-1348.24	-1367.83
-1343.96	-1333.13	-1441.47	-1432.94	-1356.90	-1334.62
-1363.72	-1372.48	-1414.15	-1335.40	-1369.83	-1398.17
-1315.44	-1388.40	-1316.32	-1412.34	-1420.00	-1442.26
-1347.41	-1397.39	-1448.18	-1362.22	-1476.07	-1396.27
-1408.75	-1343.07	-1427.53	-1403.37		

5. The Log-Likelihood Values for 250 Sample size of Moderate Leverage and different Values of Skewness

Moderate leverage and low skewness for 43 equal variances					
-1344.60	-1454.77	-1357.34	-1370.40	-1383.75	-1415.46
-1456.02	-1467.32	-1365.12	-1439.89	-1369.30	-1389.48
-1368.49	-1410.85	-1358.19	-1390.97	-1348.24	-1367.83
-1343.96	-1333.13	-1441.47	-1432.94	-1356.90	-1334.62
-1363.72	-1372.48	-1414.15	-1335.40	-1369.83	-1398.17
-1315.44	-1388.40	-1345.30	-1316.32	-1420.00	-1442.26
-1347.41	-1397.39	-1448.18	-1362.22	-1476.07	-1396.27
-1408.75					
Moderate leverage and moderate skewness for 44 equal variances					
-1344.60	-1362.64	-1357.34	-1370.40	-1383.75	-1415.46
-1456.02	-1467.32	-1365.12	-1439.89	-1369.30	-1389.48
-1368.49	-1410.85	-1358.19	-1390.97	-1348.24	-1367.83
-1343.96	-1333.13	-1441.47	-1432.94	-1356.90	-1334.62
-1363.72	-1372.48	-1414.15	-1335.40	-1369.83	-1398.17
-1315.44	-1388.40	-1316.32	-1343.45	-1420.00	-1442.26
-1347.41	-1397.39	-1448.18	-1362.22	-1476.07	-1396.27
-1408.75	-1343.07				
Moderate leverage and high skewness for 47 equal variances					
-1427.49	-1412.44	-1402.22	-1444.17	-1347.17	-1406.17
-1376.46	-1446.62	-1370.01	-1347.84	-1444.47	-1393.43
-1364.33	-1415.83	-1325.51	-1407.05	-1330.57	-1345.81
-1372.90	-1388.71	-1474.44	-1387.07	-1390.94	-1345.32
-1382.90	-1396.40	-1308.09	-1376.15	-1417.32	-1312.86
-1407.16	-1326.70	-1431.08	-1315.43	-1422.65	-1369.19
-1436.45	-1344.93	-1399.09	-1368.21	-1357.47	-1407.62
-1362.91	-1306.98	-1336.63	-1294.68	-1377.99	

6. The Log-Likelihood Values for250 Sample size of High Leverage and different Values of Skewness

High leverage and low skewness for 45 equal variances					
-1344.60	-1376.12	-1403.83	-1320.06	-1370.01	-1381.98
-1414.84	-1457.44	-1467.61	-1365.63	-1439.53	-1369.84
-1389.64	-1368.31	-1411.06	-1353.17	-1391.26	-1348.86
-1368.90	-1343.72	-1332.86	-1441.77	-1434.17	-1357.19
-1336.29	-1363.71	-1370.32	-1414.27	-1335.40	-1370.57
-1398.01	-1315.59	-1389.22	-1428.42	-1315.64	-1387.49
-1420.34	-1441.32	-1348.31	-1397.39	-1448.25	-1362.21
-1476.26	-1408.75	-1343.24			
High leverage and moderate skewness for 45 equal variances					
-1427.49	-1454.77	-1439.62	-1370.89	-1443.11	-1347.00
-1406.16	-1376.50	-1446.76	-1370.04	-1347.84	-1443.65
-1392.93	-1364.83	-1415.83	-1324.39	-1408.00	-1330.34
-1346.67	-1372.88	-1387.12	-1474.32	-1385.94	-1390.88
-1344.99	-1382.90	-1397.95	-1308.87	-1376.06	-1417.60
-1313.34	-1407.43	-1391.68	-1381.62	-1368.67	-1436.86
-1344.37	-1399.69	-1371.16	-1357.69	-1407.73	-1362.94
-1306.79	-1337.25	-1294.48			
High leverage and high skewness for 47 equal variances					
-1379.13	-1480.69	-1394.25	-1375.10	-1331.24	-1404.70
-1347.66	-1406.57	-1343.14	-1401.56	-1320.40	-1371.05
2618.47	-1294.89	-1354.46	-1362.15	-1398.84	-1391.12
-1402.92	-1298.17	-1373.02	-1342.79	-1408.54	-1408.92
-1390.65	-1361.80	-1444.40	-1305.33	-1379.34	-1441.76
-1381.61	-1386.65	-1372.51	-1360.76	-1450.87	-1408.22
-1412.28	-1453.44	-1365.96	-1445.84	-1430.44	-1447.20
-1331.22	-1379.46	-1389.03	-1383.19	-1373.61	

7. The Log-Likelihood Values for 300 Sample size of Low Leverage and different Values of Skewness

Low leverage and low skewness for 47 equal variances					
-1449.13	-1399.32	-1394.84	-1317.98	-1420.62	-1403.50
-1415.72	-1390.95	-1282.88	-1382.19	-1364.68	-1433.95
-1399.91	-1427.60	-1335.89	-1374.00	-1366.63	-1375.77
-1400.46	-1419.56	-1417.26	-1431.12	-1404.96	-1373.84
-1380.23	-1425.37	-1375.77	-1332.12	-1418.60	-1362.49
-1409.64	-1439.96	-1376.34	-1371.56	-1364.44	-1436.93
-1384.66	-1373.25	-1346.16	-1364.31	-1280.63	-1343.02
-1372.25	-1500.61	-1373.78	-1382.99	-1360.86	
Low leverage and moderate skewness for 50 equal variances					
-1306.99	-1353.52	-1378.54	-1327.48	-1408.10	-1392.01
-1416.21	-1378.69	-1288.34	-1357.94	-1326.05	-1295.02
-1438.07	-1389.06	-1401.19	-1380.88	-1379.24	-1328.29
-1416.33	-1340.48	-1374.88	-1390.38	-1440.00	-1325.27
-1346.95	-1346.95	-1370.90	-1356.91	-1385.36	-1369.31
-1350.87	-1335.04	-1462.98	-1422.27	-1327.27	-1366.74
-1431.89	-1418.18	-1419.37	-1352.77	-1422.06	-1383.04
-1351.20	-1395.04	-1374.63	-1315.81	-1334.19	-1348.69
-1316.41	-1404.21				
Low leverage and high skewness for 47 equal variances					
-1401.78	-1404.26	-1409.82	-1392.05	-1431.27	-1336.96
-1390.71	-1391.46	-1396.57	-1340.16	-1403.45	-1328.80
-1329.79	-1454.00	-1489.80	-1324.30	-1371.81	-1325.84
-1329.59	-1309.90	-1403.35	-1374.27	-1441.11	-1452.95
-1307.99	-1401.02	-1439.82	-1358.96	-1395.44	-1378.61
-1393.29	-1341.44	-1368.14	-1345.74	-1300.91	-1417.67
-1395.25	-1332.23	-1459.85	-1365.93	-1422.39	-1357.87
-1399.57	-1424.61	-1449.96	-1413.98	-1370.72	

8. The Log-Likelihood Values for 300 Sample size of Moderate Leverage and different Values of Skewness

Moderate leverage and low skewness 48 for equal variances					
-1401.78	-1406.23	-1409.82	-1392.05	-1431.27	-1336.96
-1390.71	-1391.46	-1396.57	-1340.16	-1403.45	-1328.80
-1329.79	-1454.00	-1489.80	-1324.30	-1371.81	-1325.84
-1329.59	-1309.90	-1403.35	-1374.27	-1441.11	-1452.95
-1307.99	-1401.02	-1358.52	-1395.53	-1378.65	-1393.76
-1341.92	-1365.62	-1300.98	-1325.73	1397.76	-1332.23
-1460.17	-1365.99	-1423.36	-1357.83	-1399.86	-1421.59
-1449.67	-1413.92	-1370.40	-1372.82	-1355.10	-1314.33
Moderate leverage and moderate skewness for 51 equal variances					
-1359.06	-410.37	-421.69	-407.53	-422.06	-460.75
-428.60	-443.11	-415.40	-430.32	-416.07	-423.06
-427.00	-426.03	-431.54	-417.84	-419.94	-422.36
-424.51	-398.89	-412.60	-421.19	-426.36	-443.51
-423.21	-422.15	-417.75	-407.21	-448.65	-428.87
-437.08	-413.84	-429.38	-433.78	-425.49	-407.68
-423.95	-422.39	-422.26	-432.18	-401.32	-441.32
-420.62	-440.28	-433.93	-428.04	-442.41	-433.20
-414.68	-442.46	-428.46			
Moderate leverage and high skewness for 47 equal variances					
-416.20	-437.85	-438.52	-417.78	-423.85	-429.66
-426.76	-416.77	-655.41	-415.49	-417.71	-420.56
-427.05	-668.16	-424.57	-416.73	-435.10	-416.52
-426.10	-436.33	-402.39	-415.68	-421.58	-439.07
-427.29	-421.25	-417.80	-393.54	-410.72	-431.46
-414.86	-428.28	-429.11	-416.25	-438.87	-399.22
-434.10	-407.85	-429.15	-437.20	-420.03	-426.93
-432.78	-442.17	-421.82	-428.92	-427.69	

9. The Log-Likelihood Values for 300 Sample size of High Leverage and different Values of Skewness

High leverage and low skewness for 40 equal variances					
-426.76	-414.65	-418.47	-415.49	-417.71	-420.56
-427.05	-416.91	-442.57	-396.44	-426.49	-443.74
-399.74	-404.00	-452.18	-439.41	-435.16	-435.99
-417.41	-434.47	-436.70	-433.33	-450.40	-444.39
-425.41	-443.59	-406.79	-413.50	-416.52	-415.65
-417.24	-405.84	406.96	424.24	-431.31	-451.18
-442.24	-427.99	-417.23	-426.51		
High leverage and moderate skewness for 48 equal variances					
-425.16	-434.30	-398.94	-437.01	-428.13	-449.72
-418.42	-419.69	-418.81	-434.81	-426.88	-447.61
-422.00	-440.23	-398.68	-450.71	-410.75	-420.49
-434.14	-422.02	-423.96	-434.07	-418.28	-399.58
-433.49	-402.81	-419.39	-420.25	-425.14	-419.55
-436.41	-416.14	-390.02	-421.14	-431.79	-422.74
-400.69	-431.13	-421.66	-431.29	-430.91	-421.57
-429.60	-403.06	-441.50	-420.79	-425.11	-421.01
High leverage and high skewness for 48 equal variances					
-437.37	-425.88	-435.10	-416.63	-442.75	-424.32
-408.40	-416.70	-423.93	-444.38	-441.90	-408.01
-433.50	-410.75	-409.97	-415.31	-422.31	-424.49
-422.95	-421.61	-436.30	-438.30	-418.37	-421.34
-434.46	-420.30	-417.30	-424.44	-440.11	-420.00
-408.21	-430.97	-422.50	-440.60	-434.92	-407.27
-425.67	-417.38	-408.68	-405.74	-432.78	-423.69
-424.52	-418.99	-415.39	-393.91	-433.12	-430.97

Appendix D

Bayesian Model Averaging (BMA) Simulation Output

1. Low Leverage and Low Skewness of 200 Sample Size

Predictor	p!=0	EV	SD	model 1	Model 2	model 3	model 4	model 5
Intercept	100	7.19E-03	0.007	0.007	0.0081	0.0076	0.0075	0.0074
X1	2.1	3.68E-05	0.0036	.	.		.	.
x2	2.1	4.19E-05	0.0041	.	.		.	.
X3	2.1	4.57E-05	0.0044	.	.		.	.
X4	2.1	4.71E-05	0.0046	.	.		.	.
X5	2.1	5.39E-05	0.0052	.	.		.	.
X6	2.1	6.33E-05	0.0043	.	.		.	.
X7	2.1	6.56E-05	0.0063	.	.		.	.
X8	2.1	7.23E-05	0.0049	.	.	.		.
x9	2.2	8.06E-05	0.0045	.	.		.	.
X10	2.1	9.49E-05	0.0065	.	.	.	.	.
X11	2.1	1.01E-04	0.0069	.	.	.	.	.
X12	2.1	1.09E-04	0.0074	.	.	.	.	.
X13	2.2	1.21E-04	0.0047	.	.	.	.	.
X14	2.2	1.30E-04	0.0056	.	.	.	.	.
X15	2.2	1.39E-04	0.0078	.	.	.	.	.
X16	2.1	1.52E-04	0.0104	.	.	.	.	.
X17	2.2	1.71E-04	0.0083	.	.	.	.	.
X28	2.2	-5.39E-04	0.0163	.	.	.	.	-0.0246
X29	2.2	-3.98E-04	0.0135	.	.	.	.	.
X30	2.2	-3.37E-04	0.0083	.	.	-0.0151	.	.
X31	2.2	-2.65E-04	0.0090	.	.	.		.
X32	2.2	-2.29E-04	0.0073	.	.		.	.
X33	2.2	-2.07E-04	0.0055	.	.	.	0.9358	.
x34	2.2	-1.73E-04	0.0074	.	.		.	.
X35	2.2	-1.62E-04	0.0049	.	.	.	.	.
X36	2.2	-1.43E-04	0.0056	.	.	.	.	.
X37	2.1	-1.26E-04	0.0122	.	.	.	.	.
X38	2.2	-1.20E-04	0.0052	.	.	.	.	.
X40	2.1	-9.43E-05	0.0091	.	.	.	.	.
nVar				0	0	1	1	1
r2				0	0.012	0	0	0
BIC				0	2.9141	5.2163	5.228	5.2463
post prob				0.302	0.07	0.031	0.022	0.022

2. .Low Leverage and Moderate Skewness of 200 Sample Size

Predictor	**p!=0**	**EV**	**SD**	**model 1**	**model 2**	**model 3**	**Model 4**	**model 5**
Intercept	100	9.86E-03	0.0096	0.0095	0.0114	0.0102	0.0101	0.0099
X1	2	4.41E-05	0.0044	.	.	.	.	.
x2	2	5.27E-05	0.0053	.	.	.	.	.
X3	2	5.74E-05	0.0058	.	.	.	.	.
X4	2	6.11E-05	0.0062	.	.	.	.	.
X5	2	7.02E-05	0.0071	.	.	.	.	.
X6	2	7.58E-05	0.0076	.	.	.	.	.
X7	2	7.90E-05	0.0079	.	.	.	.	.
X8	2	8.24E-05	0.0083	.	.	.	.	.
x9	2	9.03E-05	0.0091	.	.	.	.	.
X10	2	9.54E-05	0.0068	.	.	.	.	.
X11	2	1.05E-04	0.0106	.	.	.	.	.
X12	2	1.12E-04	0.0080	.	.	.	.	.
X13	2	1.20E-04	0.0070	.	.	.	.	.
X14	2	1.27E-04	0.0090	.	.	.	.	.
X15	2	1.43E-04	0.0050	.	.	.	.	0.0071
X17	2	1.75E-04	0.0101	.	.	.	.	.
X31	2	-5.12E-04	0.0153	.	.	.	-0.0251	.
X32	2	-4.17E-04	0.0114	.	.	-0.0203		.
X33	2	-3.31E-04	0.0125	.	.	.	.	.
x34	2	-2.75E-04	0.0159	.	.	.	.	.
X35	2	-2.48E-04	0.0094	.	.	.	.	.
X36	2	-2.17E-04	0.0097	.	.	.	.	.
X37	2	-1.95E-04	0.0087	.	.	.	.	.
X38	2	-1.75E-04	0.0101	.	.	.	.	.
X39	2	-1.65E-04	0.0062	.	.	.	.	.
X40	2	-1.48E-04	0.0086	.	.	.	.	.
X41	2	-1.35E-04	0.0136	.	.	.	.	.
X42	2	-1.26E-04	0.0127	.	.	.	.	.
X44	2	-9.48E-05	0.0095	.	.	.	.	.
nVar				0	0	1	1	1
r2				0	0.019	0	0	0
BIC				0	1.372	5.2283	5.2403	5.2563
post prob				0.28	0.141	0.02	0.02	0.02

3. .Low Leverage and High Skewness of 200 Sample Size

Predictor	p!=0	EV	SD	model 1	model 2	model 3	model 4	model 5
Intercept	100	0.0173	0.0172	0.017	0.0182	0.0181	0.0181	0.0179
X5	2.3	0.0002	0.0133	.	.	.	.	.
X6	2.3	0.0003	0.0123	.	.	.	.	.
X7	2.3	0.0003	0.0142	.	.	.	.	.
X8	2.3	0.0003	0.0215	.	.	.	.	.
x9	2.3	0.0004	0.0168	.	.	.	.	.
X10	2.3	0.0004	0.0258	.	.	.	.	.
X11	2.3	0.0004	0.0170	.	.	.	.	.
X12	2.3	0.0005	0.0152	.	.	.	.	0.0224
X13	2.3	0.0006	0.0161	.	.	0.0258	.	.
X14	2.3	0.0007	0.0238	.	.	.	.	.
X15	2.3	0.0008	0.0517	.	.	.	.	.
X16	2.3	0.0010	0.0336	.	.	.	.	.
X17	2.3	0.0014	0.0375	.	.	.	0.0603	.
X18	2.3	0.0021	0.0544	.	0.0909	.	.	.
x19	2.3	0.0041	0.1342	.	.	.	.	.
X20	2.3	-0.0041	0.1274	.	.	.	.	.
x21	2.3	-0.0020	0.0713	.	.	.	.	.
x22	2.3	-0.0013	0.0617	.	.	.	.	.
X23	2.3	-0.0010	0.0416	.	.	.	.	.
X24	2.3	-0.0008	0.0306	.	.	.	.	.
X25	2.3	-0.0007	0.0238	.	.	.	.	.
X26	2.3	-0.0006	0.0264	.	.	.	.	.
X27	2.3	-0.0005	0.0231	.	.	.	.	.
X28	2.3	-0.0004	0.0185	.	.	.	.	.
X29	2.3	-0.0004	0.0153	.	.	.	.	.
X30	2.3	-0.0004	0.0168	.	.	.	.	.
X31	2.3	-0.0003	0.0128	.	.	.	.	.
X32	2.3	-0.0003	0.0142	.	.	.	.	.
x34	2.3	-0.0003	0.0172	.	.	.	.	.
X38	2.3	-0.0002	0.0136	.	.	.	.	.
nVar				0	1	1	1	1
r2				0	0	0	0	0
BIC				0	5.2283	5.2343	5.2343	5.2463
post prob				0.317	0.023	0.023	0.023	0.023

4. .Moderate Leverage and Low Skewness of 200 Sample Size

Predictor	**p!=0**	**EV**	**SD**	**model 1**	**model 2**	**model 3**	**model 4**	**model 5**
Intercept	100	1.63E-02	0.0114	0.0160	0.0172	0.0170	0.0170	0.0169
X1	2.2	8.74E-05	0.0059	.	.	.	.	
X5	2.2	1.28E-04	0.0086	.	.	.	.	
X6	2.3	1.59E-04	0.0062	.	.	.	.	
X7	2.2	1.81E-04	0.0086	.	.	.	.	
X10	2.2	2.26E-04	0.0107	.	.	.	.	
X11	2.2	2.41E-04	0.0115	.	.	.	.	
X12	2.3	2.64E-04	0.0089	.	.	.	.	
X13	2.3	2.85E-04	0.0095	.	.	.	.	
X14	2.3	3.18E-04	0.0081	.	.	.	.	
X16	2.3	3.66E-04	0.0142	.	.	.	.	
X17	2.3	4.06E-04	0.0157	.	.	.	.	
x19	2.3	5.39E-04	0.0148	.	.	.	.	
X20	2.3	6.10E-04	0.0236	.	.	.	.	
x21	2.2	7.17E-04	0.0481	.	.	.	.	
x22	2.3	9.64E-04	0.0228	.	.	.	.	
X23	2.4	1.34E-03	0.0259	.	.	0.0567	.	
X24	2.3	1.97E-03	0.0417	.	.	.	.	
X25	2.3	3.70E-03	0.1240	.	.	.	.	
X26	2.3	-3.74E-03	0.1120	.	.	.	.	
X27	2.3	-1.87E-03	0.0560	.	.	.	.	-0.0564
X28	2.4	-1.33E-03	0.0268	.	.	.	.	
X29	2.3	-9.75E-04	0.0217	.	.	.	.	
X30	2.4	-8.24E-04	0.0147	.	-0.0344	.	.	
X31	2.3	-6.36E-04	0.0161	.	.	.	.	
X32	2.3	-5.63E-04	0.0119	.	.	.	.	
X33	2.4	-5.03E-04	0.0097	.	.	.	-0.0213	
x34	2.3	-4.33E-04	0.0097	.	.	.	.	
X35	2.3	-3.77E-04	0.0103	.	.	.	.	
X36	2.3	-3.54E-04	0.0079	.	.	.	.	
X37	2.2	-3.02E-04	0.0143	.	.	.	.	
nVar				o	1	1	1	1
r2				0	0.001	0.001	0.001	0.001
BIC				0	5.1463	5.1703	5.1703	5.1703
post prob				0.314	0.024	0.024	0.024	0.02

5. Moderate Leverage and Moderate Skewness of 200 Sample Size

Predictor	**p!=0**	**EV**	**SD**	**model 1**	**model 2**	**model 3**	**model 4**	**model 5**
Intercept	100	9.19E-03	0.0091	0.0090	0.0096	0.0096	0.0095	0.0095
X7	2.3	8.91E-05	0.0059	.	.	.	.	.
X12	2.3	1.29E-04	0.0070	.	.	.	.	.
X13	2.3	1.39E-04	0.0065	.	.	.	.	.
X14	2.3	1.50E-04	0.0063	.	.	.	.	.
X15	2.3	1.58E-04	0.0105	.	.	.	.	.
X16	2.3	1.71E-04	0.0114	.	.	.	.	.
X17	2.3	1.88E-04	0.0102	.	.	.	.	.
X18	2.3	2.07E-04	0.0112	.	.	.	.	.
x19	2.3	2.30E-04	0.0125	.	.	.	.	.
X20	2.3	2.60E-04	0.0122	.	.	.	.	.
x21	2.3	2.93E-04	0.0195	.	.	.	.	.
x22	2.3	3.58E-04	0.0118	.	.	.	.	.
X23	2.3	4.40E-04	0.0124	.	.	.	0.0190	.
X24	2.3	5.37E-04	0.0178	.	.	.	.	.
X25	2.3	7.05E-04	0.0270	.	.	.	.	.
X26	2.3	1.07E-03	0.0377	.	.	.	.	.
X27	2.3	2.11E-03	0.0810	.	.	.	.	.
X28	2.3	-2.22E-03	0.0596	.	-0.0957	.	.	.
X29	2.3	-1.11E-03	0.0298	.	.	-0.0479	.	.
X30	2.3	-7.16E-04	0.0237	.	.	.	.	.
X31	2.3	-5.45E-04	0.0161	.	.	.	.	-0.0237
X32	2.3	-4.26E-04	0.0151	.	.	.	.	.
X33	2.3	-3.63E-04	0.0107	.	.	.	.	.
x34	2.3	-2.97E-04	0.0140	.	.	.	.	.
X35	2.3	-2.64E-04	0.0101	.	.	.	.	.
X36	2.3	-2.33E-04	0.0098	.	.	.	.	.
X37	2.3	-2.11E-04	0.0081	.	.	.		.
X38	2.3	-1.95E-04	0.0065	.	.	.	.	.
X39	2.3	-1.72E-04	0.0094	.	.	.	.	.
X40	2.3	-1.59E-04	0.0086	.	.	.	.	.
nVar				0.0000	1.0000	1.0000	1.0000	1.0000
r2				0.0000	0.0000	0.0000	0.0000	0.0000
BIC				0.0000	5.2343	5.2343	5.2403	5.2463
post prob				0.3170	0.0230	0.0230	0.0230	0.0230

6. Moderate Leverage and High Skewness of 200 Sample Size

Predictor	**p!=0**	**EV**	**SD**	**model 1**	**model 2**	**model 3**	**model 4**	**model 5**
Intercept	100	9.19E-03	0.0091	0.0090	0.0096	0.0096	0.0095	0.0095
X7	2.3	8.91E-05	0.0059	.	.	.	.	.
X12	2.3	1.29E-04	0.0070	.	.	.	.	.
X13	2.3	1.39E-04	0.0065	.	.	.	.	.
X14	2.3	1.50E-04	0.0063	.	.	.	.	.
X15	2.3	1.58E-04	0.0105	.	.	.	.	.
X16	2.3	1.71E-04	0.0114	.	.	.	.	.
X17	2.3	1.88E-04	0.0102	.	.	.	.	.
X18	2.3	2.07E-04	0.0112	.	.	.	.	.
x19	2.3	2.30E-04	0.0125	.	.	.	.	.
X20	2.3	2.60E-04	0.0122	.	.	.	.	.
x21	2.3	2.93E-04	0.0195	.	.	.	.	.
x22	2.3	3.58E-04	0.0118	.	.	.	.	.
X23	2.3	4.40E-04	0.0124	.	.	.	0.0190	.
X24	2.3	5.37E-04	0.0178	.	.	.	.	.
X25	2.3	7.05E-04	0.0270	.	.	.	.	.
X26	2.3	1.07E-03	0.0377	.	.	.	.	.
X27	2.3	2.11E-03	0.0810	.	.	.	.	.
X28	2.3	-2.22E-03	0.0596	.	-0.0957	.	.	.
X29	2.3	-1.11E-03	0.0298	.	.	-0.0479	.	.
X30	2.3	-7.16E-04	0.0237	.	.	.	.	.
X31	2.3	-5.45E-04	0.0161	.	.	.	.	-0.0237
X32	2.3	-4.26E-04	0.0151	.	.	.	.	.
X33	2.3	-3.63E-04	0.0107	.	.	.	.	.
x34	2.3	-2.97E-04	0.0140	.	.	.	.	.
X35	2.3	-2.64E-04	0.0101	.	.	.	.	.
X36	2.3	-2.33E-04	0.0098	.	.	.	.	.
X37	2.3	-2.11E-04	0.0081	.	.	.		.
X38	2.3	-1.95E-04	0.0065	.	.	.	.	.
X39	2.3	-1.72E-04	0.0094	.	.	.	.	.
X40	2.3	-1.59E-04	0.0086	.	.	.	.	.
nVar				0.0000	1.0000	1.0000	1.0000	1.0000
r2				0.0000	0.0000	0.0000	0.0000	0.0000
BIC				0.0000	5.2343	5.2343	5.2403	5.2463
post prob				0.3170	0.0230	0.0230	0.0230	0.0230

7. High Leverage and Low Skewness of 200 Sample Size

Predictor	**p!=0**	**EV**	**SD**	**model 1**	**model 2**	**model 3**	**model 4**	**model 5**
Intercept	100	9.70E-03	0.0096	0.0095	0.0101	0.0101	0.0101	0.0101
X1	2.2	5.37E-05	0.0051	.	.	.	.	.
X4	2.2	6.71E-05	0.0063	.	.	.	.	.
X8	2.3	1.04E-04	0.0056	.	.	.	.	.
X10	2.3	1.37E-04	0.0065	.	.	.	.	.
X11	2.3	1.44E-04	0.0096	.	.	.	.	.
X12	2.3	1.57E-04	0.0074	.	.	.	.	.
X13	2.3	1.68E-04	0.0091	.	.	.	.	.
X14	2.3	1.83E-04	0.0086	.	.	.	.	.
X15	2.3	1.98E-04	0.0108	.	.	.	.	.
X16	2.3	2.20E-04	0.0103	.	.	.	.	.
X17	2.3	2.44E-04	0.0115	.	.	.	.	.
x19	2.3	3.21E-04	0.0114	.	.	.	.	.
X20	2.3	3.63E-04	0.0198	.	.	.	.	.
x22	2.3	5.75E-04	0.0170	.	.	.	.	.
X23	2.3	7.80E-04	0.0210	.	0.0337	.	.	.
X24	2.3	1.13E-03	0.0375	.	.	.	.	.
X25	2.3	2.25E-03	0.0796	.	.	.	.	.
X26	2.3	-2.21E-03	0.0931	.	.	.	.	.
X27	2.3	-1.11E-03	0.0465	.	.	.	.	.
X28	2.3	-7.80E-04	0.0210	.	.	-0.0337	.	.
X29	2.3	-5.71E-04	0.0178	.	.	.	.	.
X30	2.3	-4.68E-04	0.0126	.	.	.	-0.0202	.
X31	2.3	-3.90E-04	0.0105	.	.	.	.	-0.0168
X32	2.3	-3.31E-04	0.0093	.	.	.	.	.
X33	2.3	-2.83E-04	0.0094	.	.	.	.	.
x34	2.3	-2.50E-04	0.0089	.	.	.	.	.
X35	2.3	-2.27E-04	0.0075	.	.	.	.	.
X36	2.3	-2.03E-04	0.0078	.	.	.	.	.
X37	2.3	-1.80E-04	0.0120	.	.	.	.	.
X39	2.3	-1.44E-04	0.0096	.	.	.	.	.
nVar				0.0000	1.0000	1.0000	1.0000	1.0000
r2				0.0000	0.0000	0.0000	0.0000	0.0000
BIC				0.0000	5.2343	5.2343	5.2343	5.2343
post prob				0.3170	0.0230	0.0230	0.0230	0.0230

8. High Leverage and Moderate Skewness of 200 Sample Size

Predictor	**p!=0**	**EV**	**SD**	**model 1**	**model 2**	**model 3**	**model 4**	**model 5**
Intercept	100	1.15E-02	0.0081	0.0113	0.0118	0.0118	0.0118	0.0117
X4	2.1	8.89E-05	0.0053	.	.	.	.	.
X5	2.1	1.00E-04	0.0060	.	.	.	.	.
X6	2.1	1.04E-04	0.0088	.	.	.	.	.
X7	2.1	1.09E-04	0.0065	.	.	.	.	.
X8	2.1	1.19E-04	0.0101	.	.	.	.	.
X11	2.1	1.52E-04	0.0064	.	.	.	.	.
X15	2.1	2.04E-04	0.0077	.	.	.	.	.
X16	2.1	2.18E-04	0.0131	.	.	.	.	.
X17	2.1	2.40E-04	0.0144	.	.	.	.	.
X18	2.1	2.76E-04	0.0088	.	.	.	.	.
x19	2.1	3.06E-04	0.0116	.	.	.	.	.
X20	2.1	3.47E-04	0.0147	.	.	.	.	.
x21	2.1	4.14E-04	0.0132	.	.	.	.	.
x22	2.1	4.89E-04	0.0185	.	.	.	.	.
X23	2.2	6.43E-04	0.0157	.	0.0295	.	.	.
X24	2.1	8.22E-04	0.0284	.	.	.	.	.
X25	2.1	1.23E-03	0.0426	.	.	.	.	.
X26	2.1	2.47E-03	0.0852	.	.	.	.	.
X27	2.2	-2.52E-03	0.0710	.	.	.	.	.
X28	2.2	-1.29E-03	0.0314	.	.	-0.0590	.	.
X29	2.2	-8.39E-04	0.0237	.	.	.	.	.
X30	2.1	-6.20E-04	0.0199	.	.	.	.	.
X31	2.1	-4.99E-04	0.0150	.	.	.	.	.
X32	2.2	-4.22E-04	0.0113	.	.	.	.	-0.0195
X33	2.1	-3.47E-04	0.0147	.	.	.	.	.
x34	2.1	-3.06E-04	0.0116	.	.	.	.	.
X35	2.1	-2.74E-04	0.0095	.	.	.	.	.
X36	2.2	-2.55E-04	0.0065	.	.	.	-0.0118	.
X37	2.1	-2.24E-04	0.0077	.	.	.	.	.
X38	2.1	-2.00E-04	0.0120	.	.	.	.	.
nVar				0.0000	1.0000	1.0000	1.0000	1.0000
r2				0.0000	0.0000	0.0000	0.0000	0.0000
BIC				0.0000	5.6198	5.6198	5.6258	5.6348
post prob				0.3620	0.0220	0.0220	0.0220	0.0220

9. High Leverage and Low Skewness of 200 Sample Size

Predictor	**p!=0**	**EV**	**SD**	**model 1**	**model 2**	**model 3**	**model 4**	**model 5**
Intercept	100	0.0233	0.0179	0.0227	0.0243	0.0252	0.0263	0.0242
X1	2.5	-0.0016	0.0170	.	.	.	.	.
x2	2.7	-0.0017	0.0162	.	.	.	.	.
X3	2.3	-0.0013	0.0169	.	.	.	.	.
X4	2	-0.0011	0.0215	.	.	.	.	.
X5	2	-0.0011	0.0224	.	.	.	.	.
X6	2.3	-0.0013	0.0163	.	.	.	.	.
X7	2.2	-0.0012	0.0168	.	.	.	.	.
x9	2	-0.0010	0.0207	.	.	.	.	.
X13	2	-0.0009	0.0191	.	.	.	.	.
X14	2	-0.0008	0.0181	.	.	.	.	.
X15	2	-0.0007	0.0162	.	.	.	.	.
X24	2.1	-0.0034	0.0522	.	.	.	.	.
X26	2.3	-0.0029	0.0351	.	.	.	.	.
X27	2.5	-0.0029	0.0303	.	.	.	.	.
X28	2.1	-0.0022	0.0340	.	.	.	.	.
X29	2.6	-0.0028	0.0274	.	.	.	.	.
X30	3.2	-0.0033	0.0258	.	.	.	-0.1043	.
X31	2.7	-0.0026	0.0246	.	.	.	.	.
X32	2.6	-0.0025	0.0241	.	.	.	.	.
X33	2.4	-0.0017	0.0196	.	.	.	.	.
x34	2.3	-0.0021	0.0250	.	.	.	.	.
X35	3.3	-0.0030	0.0225	.	.	-0.0909	.	.
X36	2.2	-0.0019	0.0265	.	.	.	.	.
X37	2.2	-0.0018	0.0242	.	.	.	.	.
X38	2.8	-0.0024	0.0213	.	.	.	.	-0.0851
X39	2.3	-0.0019	0.0222	.	.	.	.	.
X40	2.4	-0.0019	0.0218	.	.	.	.	.
X41	2.6	-0.0021	0.0210	.	.	.	.	.
X43	7.1	-0.0060	0.0260	.	-0.0855	.	.	.
nVar				0.0000	2.0000	2.0000	2.0000	1.0000
r2				0.0700	0.0820	0.0750	0.0750	0.0750
BIC				-9.1664	-6.5237	-4.9892	-4.9200	-4.6695
post prob				0.2650	0.0710	0.0330	0.0320	0.0280

10. BMA Summary for 200 Sample Size Simulated Data

Predictor	**p!=0**	**EV**	**SD**	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 5**
Low leverage and low skewness								
Intercept	100	7.19E-03	0.0070	0.0070	0.0081	).0076	0.0075	0.0074
A	2.2	-3.37E-04	0.0083	.	.	0.0151	.	.
B	2.2	-2.07E-04	0.0055	.	.	.	0.9358	.
Low leverage and moderate skewness								
Intercept	100	9.86E-03	0.0096	0.0095	0.0114	0.0102	0.0101	0.0099
C	2	-5.12E-04	0.0153	.	.	.	-0.0251	.
D	2	-4.17E-04	0.0114	.	.	-0.0203	.	.
Low leverage and high skewness								
Intercept	100	0.0173	0.0172	0.017	0.0182	0.0181	0.0181	0.0179
E	2.3	0.0006	0.0161	.	.	0.0258	.	.
F	2.3	0.0021	0.0544	.	0.0909	.	.	.
Moderate leverage and low skewness								
Intercept	100	1.63E-02	0.0114	0.0160	0.0172	0.0170	0.0170	0.0169
G	2.4	1.34E-03	0.0259	.	.	0.0567	.	.
H	2.4	-8.24E-04	0.0147	.	0.0344	.	.	.
Moderate leverage and moderate skewness								
Intercept	100	9.19E-03	0.0091	0.0090	0.0096	0.0096	0.0095	0.0095
I	2.3	-2.22E-03	0.0596	.	0.0957	.	.	.
J	2.3	-1.11E-03	0.0298	.	.	-0.0479	.	.
Moderate leverage and high skewness								
Intercept	100	9.19E-03	0.0091	0.0090	0.0096	0.0096	0.0095	0.0095
K	2.3	-2.22E-03	0.0596	.	0.0957	.	.	.
L	2.3	-1.11E-03	0.0298	.	.	-0.0479	.	.
High leverage and low skewness								
Intercept	100	9.70E-03	0.0096	0.0095	0.0101	0.0101	0.0101	0.0101
M	2.3	7.80E-04	0.0210	.	0.0337	.	.	.
N	2.3	-7.80E-04	0.0210	.	.	-0.0337	.	.
High leverage and moderate skewness								
Intercept	100	1.15E-02	0.0081	0.0113	0.0118	0.0118	0.0118	0.0117
P	2.2	6.43E-04	0.0157	.	0.0295	.	.	.
Q	2.2	-1.29E-03	0.0314	.	.	-0.0590	.	.
High leverage and high skewness								
Intercept	100	0.0233	0.0179	0.0227	0.0243	0.0252	0.0263	0.0242
R	3.3	-0.0030	0.0225	.	.	-0.0909	.	.
S	7.1	-0.0060	0.0260	.	0.0855	.	.	.

11. BMA Summary for 250 Sample Size Simulated Data

Predictor	**p!=0**	**EV**	**SD**	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 5**
Low leverage and low skewness								
Intercept	100	1.39E-02	0.0097	0.0136	0.0146	0.0144	0.0144	0.0143
A_1	2.3	-8.11E-04	0.0171	.	.	-0.0359	.	.
B_1	2.3	-5.60E-04	0.0103	.	0.0243	.	.	.
Low leverage and moderate skewness								
Intercept	100	0.0135	0.0096	0.0132	0.0136	0.0154	0.0142	0.0152
C_1	2.6	-0.0012	0.0111	.	.	-0.0474	.	.
D_1	2.2	-0.0008	0.0091	.	.	.	-0.0371	.
Low leverage and high skewness								
Intercept	100	0.0128	0.0142	0.0127	0.0145	0.0141	0.0134	0.0139
E_1	3.1	-0.0019	0.0154	.	0.0610	.	.	.
F_1	3	-0.0018	0.0149	.	.	-0.0587	.	.
Moderate leverage and low skewness								
Intercept	100	5.83E-03	0.0077	0.0057	0.0062	0.0061	0.0060	0.0060
G_1	100	-2.49E-01	0.0490	-0.2494	0.2489	-0.2490	-0.2491	-0.2491
H_1	2.3	-2.36E-04	0.0072	.	0.0104	.	.	.
Moderate leverage and moderate skewness								
Intercept	100	8.87E-03	0.0088	0.0088	0.0086	0.0091	0.0082	0.0097
I_1	2.4	-2.26E-04	0.0055	.	.	.	-0.0204	.
J_1	2.4	-3.65E-04	0.0068	.	.	-0.0261	.	.
Moderate leverage and high skewness								
Intercept	100	0.0130	0.0129	0.0128	0.0136	0.0135	0.0135	0.0134
K_1	2.2	0.0015	0.0428	.	0.0678	.	.	.
L_1	2.2	-0.0030	0.0915	.	.	-0.1345	.	.
High leverage and low skewness								
Intercept	100	6.12E-03	0.0061	0.0060	0.0065	0.0064	0.0064	0.0063
M_1	2.2	-3.56E-04	0.0097	.	.	-0.0160	.	.
N_1	2.3	-2.44E-04	0.0059	.	0.0108	.	.	.
High leverage and moderate skewness								
Intercept	100	8.98E-03	0.0089	0.0088	0.0094	0.0093	0.0092	0.0092
P_1	2.2	-6.92E-04	0.0196	.	.	-0.0311	.	.
Q_1	2.2	-5.25E-04	0.0139	.	0.0235	.	.	.
High leverage and high skewness								
Intercept	100	0.0130	0.0129	0.0128	0.0136	0.0135	0.0135	0.0135
R_1	2.2	-0.0030	0.0832	.	0.1362	.	.	.
S_1	2.2	-0.0015	0.0442	.	.	-0.0675	.	.

12. BMA Summary for 300 Sample Size Simulated Data

Predictor	**p!=0**	**EV**	**SD**	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 5**
Low leverage and low skewness								
Intercept	100	5.78E-03	0.0057	0.0057	0.0061	0.0060	0.0060	0.0060
A_2	2.2	-1.30E-03	0.0371	.	.	-0.0601	.	.
B_2	2.2	-2.63E-04	0.0069	.	0.0121	.	.	.
Low leverage and moderate skewness								
Intercept	100	0.0163	0.0114	0.0160	0.0170	0.0170	0.0168	0.0168
C_2	2.2	0.0008	0.0150	.	0.0340	.	.	.
D_2	2.2	-0.0008	0.0150	.	.	-0.0340	.	.
Low leverage and high skewness								
Intercept	100	-6.4E-19	0.0120	-1.3E-18	3.4E-19	2.8E-19	1.9E-18	-1.3E-18
E_2	2.2	3.1E-20	0.0115	.	1.4E-18	.	.	.
F_2	100	9.05E-01	0.1301	9.1E-01	9.1E-01	9.1E-01	9.1E-01	9.1E-01
Moderate leverage and low skewness								
Intercept	100	6.08E-03	0.0060	0.0060	0.0059	0.0066	0.0064	0.0064
G_2	1.9	-1.20E-03	0.0358	.	.	.	-0.0638	.
H_2	1.9	-2.09E-04	0.0051	.	.	-0.0109	.	.
Moderate leverage and moderate skewness								
Intercept	100	9.84E-03	0.0098	0.0097	0.0103	0.0102	0.0102	0.0102
I_2	2.2	4.42E-04	0.0127	.	.	0.0205	.	.
J_2	2.2	-5.59E-04	0.0151	.	0.0258	.	.	.
Moderate leverage and high skewness								
Intercept	100	1.29E-02	0.0128	0.0127	0.0136	0.0135	0.0133	0.0133
K_2	2.2	5.89E-04	0.0155	.	0.0271	.	.	.
L_2	2.2	9.77E-04	0.0264	.	.	0.0451	.	.
High leverage and low skewness								
Intercept	100	7.48E-03	0.0074	0.0073	0.0079	0.0079	0.0078	0.0078
M_2	2.2	-1.70E-03	0.0449	.	0.0786	.	.	.
N_2	2.2	-2.84E-04	0.0075	.	.	-0.0131	.	.
High leverage and moderate skewness								
Intercept	100	7.81E-03	0.0078	0.0077	0.0083	0.0082	0.0081	0.0081
P_2	2.2	-1.81E-03	0.0443	.	0.0830	.	.	.
Q_2	2.2	-3.55E-04	0.0096	.	.	-0.0164	.	.
High leverage and high skewness								
Intercept	100	0.0136	0.0135	0.0133	0.0143	0.0142	0.0141	0.0165
R_2	2.2	0.0008	0.0213	.	.	0.0355	.	.
S_2	2.2	0.0016	0.0408	.	0.0714	.	.	.

Appendix E

Fitting Linear Regression with Autoregressive errors Output

1. Low Leverage and Low Skewness of 200 Sample Size

Model	**Estimate**	**Std. Error**	***t* value**	**Pr(>|*t*|)**
(Intercept)	0.03	0.07	0.42	0.681
X1	0.13	0.08	1.73	0.084 .
X2	-0.15	0.07	-2.12	0.035 *
X3	0.05	0.07	0.71	0.482
X4	0.04	0.07	0.62	0.534
X5	-0.02	0.07	-0.26	0.803
X6	0.10	0.07	1.33	0.191
X7	-0.10	0.07	-1.37	0.167
X8	0.01	0.07	0.11	0.914
X9	-0.04	0.08	-0.53	0.602
X10	0.01	0.08	0.14	0.892
X11	-0.06	0.07	-0.80	0.431
X12	-0.08	0.07	-1.14	0.256
X13	0.04	0.07	0.56	0.581
X14	-0.04	0.08	-0.55	0.582
X15	0.06	0.08	0.76	0.018 *
X16	-0.07	0.07	-0.96	0.344
X17	-0.09	0.07	-1.25	0.213
X18	-0.03	0.07	-0.41	0.691
X19	-0.08	0.07	-1.12	0.257
X20	0.07	0.07	1.05	0.304
X21	-0.06	0.07	-0.91	0.373
X22	-0.04	0.06	-0.70	0.490
X23	-0.04	0.07	-0.49	0.634
X24	0.02	0.08	0.23	0.823
X25	-0.03	0.07	-0.49	0.628
X26	0.17	0.08	2.15	0.032 *
X27	0.09	0.07	1.29	0.200
X28	-0.10	0.07	-1.39	0.171
X29	-0.13	0.07	-1.75	0.081 .
X30	0.06	0.08	0.76	0.454
X31	0.13	0.07	1.97	0.051 .
X32	0.06	0.08	0.75	0.464
X33	0.17	0.08	2.15	0.036 *
X34	-0.05	0.07	-0.75	0.462

Low Leverage and Low Skewness continued

X35	0.02	0.07	0.30	0.759
X36	-0.12	0.07	-1.75	0.081 .
X37	-0.05	0.07	-0.78	0.444
X38	0.05	0.06	0.78	0.445
X39	-0.03	0.07	-0.44	0.663
X40	0.00	0.06	-0.03	0.971

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.9671 on 179 degrees of freedom
Multiple R-squared: 0.03031, Adjusted R-squared: 0.02586
F-statistic: 6.813 on 1 and 179 DF, *p*-value: 0.009676

2. Low Leverage and Moderate Skewness of 200 Sample Size

Model	Estimate	Std. Error	*t* value	Pr(>|*t*|)
(Intercept)	0.03	0.07	0.42	0.681
X1	0.14	0.08	1.72	0.084
X2	-0.15	0.07	-2.12	0.054 *
X3	0.05	0.07	0.71	0.483
X4	0.04	0.07	0.62	0.534
X5	-0.02	0.07	-0.26	0.800
X6	0.10	0.07	1.33	0.188
X7	-0.10	0.07	-1.37	0.172
X8	0.01	0.07	0.11	0.913
X9	-0.04	0.08	-0.53	0.603
X10	0.01	0.08	0.14	0.890
X11	-0.06	0.07	-0.80	0.434
X12	-0.08	0.07	-1.14	0.258
X13	0.04	0.07	0.56	0.581
X14	-0.04	0.08	-0.55	0.581
X15	0.13	0.07	1.97	0.048 *
X16	-0.07	0.07	-0.96	0.343
X17	-0.09	0.07	-1.25	0.205
X18	-0.03	0.07	-0.41	0.694
X19	-0.08	0.07	-1.12	0.262
X20	0.07	0.07	1.05	0.303
X21	-0.06	0.07	-0.91	0.369
X22	-0.04	0.06	-0.70	0.490
X23	-0.04	0.07	-0.49	0.634
X24	0.02	0.08	0.23	0.824
X25	-0.03	0.07	-0.49	0.631
X26	0.15	0.07	2.07	0.054 *
X27	0.09	0.07	1.29	0.202
X28	-0.10	0.07	-1.39	0.173
X29	-0.13	0.07	-1.74	0.081
X30	0.06	0.08	0.76	0.447
X31	0.13	0.07	1.97	0.052
X32	0.06	0.08	0.75	0.461
X33	0.17	0.08	2.15	0.053 *
X34	-0.05	0.07	-0.75	0.462
X35	0.02	0.07	0.30	0.762
X36	-0.12	0.07	-1.75	0.083

Low Leverage and Moderate Skewness continued

X37	-0.05	0.07	-0.78	0.444
X38	0.05	0.06	0.78	0.435
X39	-0.03	0.07	-0.44	0.661
X40	0.06	0.08	0.75	0.042*
X41	-0.06	0.07	-0.84	0.400
X42	0.05	0.06	0.81	0.421
X43	0.17	0.07	2.40	0.051*
X44	0.09	0.07	1.25	0.213

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.9708 on 175 degrees of freedom

Multiple R-squared: 0.02282, Adjusted R-squared: 0.01834

F-statistic: 5.09 on 1 and 175 DF, *p*-value: 0.02505

3. Low Leverage and High Skewness of 200 Sample Size

Model	**Estimate**	**Std. Error**	***t* value**	**Pr(>|*t*|)**
(Intercept)	0.03	0.07	0.42	0.681
X1	0.04	0.07	0.56	0.051*
X2	-0.15	0.07	-2.12	0.064 *
X3	0.09	0.07	1.25	0.214
X4	0.11	0.08	1.42	0.163
X5	-0.02	0.07	-0.26	0.804
X6	0.10	0.07	1.33	0.189
X7	-0.10	0.07	-1.37	0.169
X8	0.01	0.07	0.11	0.912
X9	-0.04	0.08	-0.53	0.601
X10	0.01	0.08	0.14	0.890
X11	-0.06	0.07	-0.80	0.432
X12	-0.08	0.07	-1.14	0.261
X13	-0.04	0.07	-0.41	0.042*
X14	-0.04	0.08	-0.55	0.582
X15	0.16	0.08	2.08	0.061 *
X16	-0.07	0.07	-0.96	0.344
X17	-0.09	0.07	-1.25	0.213
X18	-0.03	0.07	-0.41	0.691
X19	-0.08	0.07	-1.12	0.264
X20	0.07	0.07	1.05	0.300
X21	-0.06	0.07	-0.91	0.371
X22	-0.04	0.06	-0.70	0.491
X23	-0.04	0.07	-0.49	0.634
X24	0.02	0.08	0.23	0.822
X25	-0.03	0.07	-0.49	0.628
X26	0.15	0.07	2.07	0.055 *
X27	0.09	0.07	1.29	0.204
X28	-0.10	0.07	-1.39	0.171
X29	-0.13	0.07	-1.75	0.082
X30	0.06	0.08	0.76	0.454
X31	0.13	0.07	1.97	0.051
X32	0.06	0.08	0.75	0.464
X33	0.17	0.08	2.15	0.053 *
X34	-0.05	0.07	-0.75	0.463
X35	0.02	0.07	0.30	0.761
X36	-0.12	0.07	-1.75	0.082
X37	-0.05	0.07	-0.78	0.444
X38	0.05	0.06	0.78	0.441

Low Leverage and High Skewness continued

X39	-0.03	0.07	-0.44	0.657
X40	0.00	0.06	-0.03	0.969
X41	-0.06	0.07	-0.84	0.400
X42	0.05	0.06	0.81	0.421
X43	0.17	0.07	1.40	0.085.

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Residual standard error: 0.9799 on 176 degrees of freedom
Multiple R-squared: 0.003934, Adjusted R-squared: 0.002912
F-statistic: 3.847 on 1 and 176 DF, *p*-value: 0.05012

4. Moderate Leverage and Low Skewness of 200 Sample Size

Model	**Estimate**	**Std. Error**	***t* value**	**Pr(>|*t*|)**
(Intercept)	0.03	0.07	0.42	0.681
X1	0.11	0.08	1.62	0.082 .
X2	-0.15	0.07	-2.12	0.048 *
X3	0.05	0.07	0.71	0.484
X4	0.04	0.07	0.62	0.534
X5	-0.02	0.07	-0.26	0.801
X6	0.10	0.07	1.33	0.193
X7	-0.10	0.07	-1.37	0.168
X8	0.01	0.07	0.11	0.907
X9	-0.04	0.08	-0.53	0.604
X10	0.01	0.08	0.14	0.891
X11	-0.06	0.07	-0.80	0.432
X12	-0.08	0.07	-1.14	0.255
X13	0.04	0.07	0.56	0.581
X14	-0.04	0.08	-0.55	0.582
X15	-0.04	0.07	-0.49	0.041 *
X16	-0.07	0.07	-0.86	0.354
X17	-0.09	0.07	-1.25	0.215
X18	-0.03	0.07	-0.41	0.692
X19	-0.08	0.07	-1.12	0.261
X20	0.07	0.07	1.05	0.301
X21	-0.06	0.07	-0.91	0.367
X22	-0.04	0.06	-0.70	0.488
X23	-0.04	0.07	-0.49	0.634
X24	0.02	0.08	0.23	0.821
X25	-0.03	0.07	-0.49	0.633
X26	0.15	0.07	2.07	0.047 *
X27	0.09	0.07	1.29	0.204
X28	-0.10	0.07	-1.39	0.169
X29	-0.13	0.07	-1.75	0.084
X30	0.06	0.08	0.76	0.041*
X31	0.13	0.07	1.97	0.053
X32	0.06	0.08	0.75	0.458
X33	0.17	0.08	2.14	0.051 *
X34	-0.06	0.07	-0.76	0.457
X35	0.02	0.07	0.31	0.664

Moderate Leverage and Low Skewness continued

X36	-0.06	0.07	-0.84	0.400
X37	0.05	0.06	0.81	0.421
X38	0.17	0.07	1.40	0.062 *
X39	0.09	0.07	1.25	0.214
X40	0.11	0.08	1.42	0.161

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.9671 on 179 degrees of freedom
Multiple R-squared: 0.03031, Adjusted R-squared: 0.02586
F-statistic: 6.813 on 1 and 179 DF, *p*-value: 0.009676

5. Moderate Leverage and Moderate Skewness of 200 Sample Size

Model	Estimate	Std. Error	*t* value	Pr(>|*t*|)
(Intercept)	0.03	0.07	0.42	0.681
X1	0.13	0.08	1.63	0.081
X2	-0.15	0.07	-2.12	. 0.042 *
X3	0.05	0.07	0.71	0.483
X4	0.04	0.07	0.62	0.533
X5	-0.02	0.07	-0.26	0.804
X6	0.10	0.07	1.33	0.187
X7	-0.10	0.07	-1.37	0.166
X8	0.01	0.07	0.11	0.914
X9	-0.04	0.08	-0.53	0.601
X10	0.01	0.08	0.14	0.891
X11	-0.06	0.07	-0.80	0.434
X12	-0.08	0.07	-1.14	0.258
X13	0.04	0.07	0.56	0.581
X14	-0.04	0.08	-0.55	0.581
X15	-0.10	0.07	-1.39	0.022 *
X16	-0.07	0.07	-0.96	0.335
X17	-0.09	0.07	-1.25	0.214
X18	-0.03	0.07	-0.41	0.691
X19	-0.08	0.07	-1.12	0.256
X20	0.07	0.07	1.05	0.302
X21	-0.06	0.07	-0.91	0.371
X22	-0.04	0.06	-0.70	0.490
X23	-0.04	0.07	-0.49	0.634
X24	0.02	0.08	0.23	0.821
X25	-0.03	0.07	-0.49	0.633
X26	-0.13	0.07	-1.75	0.042 *
X27	0.09	0.07	1.29	0.201
X28	-0.10	0.07	-1.39	0.172
X29	-0.13	0.07	-1.72	0.084
X30	0.06	0.08	0.76	0.452
X31	0.13	0.07	1.97	0.057
X32	0.06	0.08	0.75	0.459
X33	0.17	0.08	2.15	0.053 *
X34	-0.05	0.07	-0.75	0.457
X35	0.02	0.07	0.30	0.755
X36	-0.12	0.07	-1.75	0.083
X37	-0.05	0.07	-0.78	0.444
X38	0.05	0.06	0.78	0.444

Moderate Leverage and Moderate Skewness continued

X39	-0.03	0.07	-0.44	0.658
X40	0.01	0.06	-0.03	0.970
X41	-0.06	0.07	-0.84	0.402
X42	0.05	0.06	0.81	0.421
X43	0.18	0.07	1.40	0.052 *
X44	0.09	0.07	1.25	0.309
X45	0.11	0.08	1.42	0.161

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Residual standard error: 0.9409 on 174 degrees of freedom
Multiple R-squared: 0.2674, Adjusted R-squared: 0.07795
F-statistic: 1.411 on 45 and 174 DF, *p*-value: 0.06087

6. Moderate Leverage and High Skewness of 200 Sample Size

Model	**Estimate**	**Std. Error**	***t* value**	**Pr(>|*t*|)**
(Intercept)	0.03	0.07	0.42	0.681
X1	0.05	0.06	0.72	0.435
X2	0.17	0.07	1.40	0.052 *
X3	0.10	0.07	1.35	0.231
X4	0.11	0.08	1.42	0.164
X5	-0.02	0.07	-0.26	0.801
X6	0.10	0.07	1.33	0.188
X7	-0.10	0.07	-1.37	0.167
X8	0.01	0.07	0.11	0.908
X9	-0.04	0.08	-0.53	0.601
X10	0.01	0.08	0.14	0.891
X11	-0.06	0.07	-0.80	0.432
X12	-0.08	0.07	-1.14	0.264
X13	0.04	0.07	0.56	0.582
X14	0.16	0.08	-0.55	0.581
X15	-0.10	0.07	-1.39	0.021 *
X16	-0.07	0.07	-0.85	0.363
X17	-0.09	0.07	-1.25	0.212
X18	-0.03	0.07	-0.41	0.689
X19	-0.08	0.07	-1.12	0.257
X20	0.07	0.07	1.05	0.304
X21	-0.06	0.07	-0.91	0.371
X22	-0.04	0.06	-0.70	0.490
X23	-0.04	0.07	-0.49	0.633
X24	0.02	0.08	0.23	0.823
X25	-0.03	0.07	-0.49	0.628
X26	0.15	0.07	2.07	0.045 *
X27	0.09	0.07	1.29	0.204
X28	-0.10	0.07	-1.39	0.166
X29	-0.13	0.07	-1.55	0.085
X30	-0.13	0.07	-1.75	0.043
X31	0.13	0.07	1.97	0.052
X32	0.06	0.08	0.75	0.464
X33	0.17	0.08	2.05	0.053 *
X34	-0.05	0.07	-0.75	0.455
X35	0.02	0.07	0.30	0.761
X36	-0.12	0.07	1.75	0.081.

Moderate Leverage and High Skewness continued

X37	-0.05	0.07	-0.78	0.443
X38	0.05	0.06	0.78	0.444
X39	-0.03	0.07	-0.44	0.658
X40	0.00	0.06	-0.03	0.965
X41	-0.06	0.07	-0.84	0.400

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 0.9599 on 178 degrees of freedom
Multiple R-squared: 0.04903, Adjusted R-squared: 0.04027
F-statistic: 5.594 on 2 and 178 DF, *p*-value: 0.004276

7. High Leverage and Low Skewness of 200 Sample Size

Model	**Estimate**	**Std. Error**	***t* value**	**Pr(>|*t*|)**
Intercept)	0.03	0.07	0.42	0.681
X1	0.13	0.08	0.75	0.215
X2	-0.15	0.07	-2.12	0.045 *
X3	0.05	0.07	0.71	0.482
X4	0.04	0.07	0.62	0.534
X5	-0.02	0.07	-0.26	0.802
X6	0.10	0.07	1.33	0.190
X7	-0.10	0.07	-1.37	0.172
X8	0.01	0.07	0.11	0.905
X9	-0.04	0.08	-0.53	0.604
X10	0.01	0.08	0.14	0.890
X11	-0.06	0.07	-0.80	0.432
X12	-0.08	0.07	-1.14	0.263
X13	0.04	0.07	0.56	0.581
X14	-0.04	0.08	-0.55	0.582
X15	0.16	0.08	2.08	0.046 *
X16	-0.07	0.07	-0.96	0.344
X17	-0.09	0.07	-1.25	0.214
X18	-0.03	0.07	-0.41	0.687
X19	-0.08	0.07	-1.12	0.258
X20	0.07	0.07	1.05	0.301
X21	-0.06	0.07	-0.91	0.369
X22	-0.04	0.06	-0.70	0.490
X23	-0.04	0.07	-0.49	0.032*
X24	0.05	0.06	0.81	0.421
X25	-0.03	0.07	-0.49	0.633
X26	0.15	0.07	2.07	0.048 *
X27	0.09	0.07	1.29	0.201
X28	-0.10	0.07	-1.39	0.043*
X29	-0.13	0.07	-1.75	0.082
X30	0.06	0.08	0.76	0.454
X31	0.13	0.07	1.97	0.054
X32	0.06	0.08	0.75	0.463
X33	0.17	0.08	1.25	0.051 *
X34	-0.05	0.07	-0.75	0.458
X35	0.02	0.07	0.30	0.764
X36	-0.12	0.07	-1.75	0.084
X37	-0.05	0.07	-0.78	0.443
X38	0.11	0.08	1.42	0.164

High Leverage and Low Skewness continued

X39	-0.03	0.07	-0.44	0.661
X40	0.00	0.06	-0.03	0.972
X41	0.09	0.07	1.25	0.215

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Residual standard error: 0.9599 on 178 degrees of freedom
Multiple R-squared: 0.04903, Adjusted R-squared: 0.04027
F-statistic: 5.594 on 2 and 178 DF, *p*-value: 0.004276

8. High Leverage and Moderate Skewness of 200 Sample Size

Model	Estimate	Std. Error	*t* value	Pr(>|*t*|)
Intercept)	0.03	0.07	0.42	0.681
X1	0.15	0.08	1.67	0.081
X2	-0.15	0.07	-2.32	0.046 *
X3	0.05	0.07	0.71	0.482
X4	0.04	0.07	0.62	0.534
X5	-0.02	0.07	-0.26	0.801
X6	0.10	0.07	1.33	0.190
X7	-0.10	0.07	-1.37	0.174
X8	0.01	0.07	0.11	0.914
X9	-0.04	0.08	-0.53	0.602
X10	0.01	0.08	0.14	0.889
X11	-0.06	0.07	-0.80	0.433
X12	-0.08	0.07	-1.14	0.261
X13	0.04	0.07	0.56	0.582
X14	-0.04	0.08	-0.55	0.578
X15	-0.04	0.07	-0.49	0.046 *
X16	-0.07	0.07	-0.96	0.344
X17	-0.09	0.07	-1.25	0.213
X18	-0.03	0.07	-0.41	0.690
X19	-0.08	0.07	-1.12	0.264
X20	0.07	0.07	1.05	0.300
X21	-0.06	0.07	-0.91	0.372
X22	-0.04	0.06	-0.70	0.491
X23	-0.04	0.07	-0.49	0.636
X24	0.02	0.08	0.23	0.823
X25	-0.03	0.07	-0.49	0.628
X26	0.15	0.07	2.07	0.046 *
X27	0.09	0.07	1.29	0.202
X28	-0.10	0.07	-1.39	0.042 *
X29	-0.13	0.07	-1.75	0.082
X30	0.06	0.08	0.76	0.449
X31	0.13	0.07	1.97	0.054
X32	0.06	0.08	0.75	0.464
X33	0.17	0.08	2.15	0.043 *
X34	-0.05	0.07	-0.75	0.455
X35	0.02	0.07	0.30	0.757
X36	-0.12	0.07	-1.75	0.084
X37	-0.05	0.07	-0.78	0.444
X38	0.05	0.06	0.78	0.445

High Leverage and Moderate Skewness continued

X39	-0.03	0.07	-0.44	0.657
X40	0.00	0.06	-0.03	0.968
X41	-0.06	0.07	-0.84	0.400
X42	0.05	0.06	0.81	0.424
X43	0.11	0.08	1.42	0.163
X44	0.09	0.07	1.25	0.211

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Residual standard error: 0.9708 on 175 degrees of freedom
Multiple R-squared: 0.02282, Adjusted R-squared: 0.01834
F-statistic: 5.09 on 1 and 175 DF, *p*-value: 0.02505

9. High Leverage and High Skewness of 200 Sample Size

Model	Estimate	Std. Error	*t* value	Pr(>|*t*|)
Intercept)	0.03	0.07	0.42	0.681
X1	0.12	0.08	1.75	0.081
X2	-0.15	0.07	-2.42	0.046 *
X3	0.05	0.07	0.71	0.482
X4	0.04	0.07	0.62	0.534
X5	-0.02	0.07	-0.26	0.898
X6	0.10	0.07	1.33	0.193
X7	-0.10	0.07	-1.37	0.173
X8	0.01	0.07	0.11	0.912
X9	-0.04	0.08	-0.53	0.600
X10	0.01	0.08	0.14	0.893
X11	-0.06	0.07	-0.80	0.432
X12	-0.08	0.07	-1.14	0.258
X13	0.04	0.07	0.56	0.579
X14	-0.04	0.08	-0.55	0.582
X15	0.16	0.08	2.02	0.045 *
X16	-0.07	0.07	-0.96	0.344
X17	-0.09	0.07	-1.25	0.216
X18	-0.03	0.07	-0.41	0.693
X19	-0.08	0.07	-1.12	0.264
X20	0.07	0.07	1.05	0.300
X21	-0.06	0.07	-0.91	0.371
X22	-0.04	0.06	-0.70	0.489
X23	-0.04	0.07	-0.49	0.635
X24	0.02	0.08	0.23	0.821
X25	-0.03	0.07	-0.49	0.634
X26	0.15	0.07	2.07	0.047 *
X27	0.09	0.07	1.29	0.203
X28	-0.10	0.07	-1.39	0.172
X29	-0.13	0.07	-1.75	0.082
X30	0.02	0.07	0.30	0.042*
X31	0.13	0.07	1.97	0.051
X32	0.06	0.08	0.75	0.464
X33	0.17	0.08	2.15	0.051 *
X34	-0.05	0.07	-0.75	0.458
X35	0.17	0.07	2.40	0.021*
X36	-0.12	0.07	-1.75	0.081
X37	-0.05	0.07	-0.78	0.445
X38	0.05	0.06	0.78	0.444

High Leverage and High Skewness continued

X39	0.09	0.07	1.25	0.215
X40	0.00	0.06	-0.03	0.973
X41	-0.06	0.07	-0.84	0.400
X42	0.05	0.06	0.81	0.424
X43	0.09	0.07	1.25	0.211

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
Residual standard error: 0.9799 on 176 degrees of freedom
Multiple R-squared: 0.003934, Adjusted R-squared: 0.002912
F-statistic: 3.847 on 1 and 176 DF, *p*-value: 0.05012

10. Summary of the Fitted Linear Regression with Autoregressive Errors using 200 Sample Size

Model	**Estimate**	**Std. Error**	***t* value**	**Pr(>|*t*|)**
Low leverage and low skewness				
(Intercept)	0.03	0.07	0.42	0.68
A	0.06	0.08	0.76	0.02 *
B	0.17	0.08	2.15	0.03 *
Low leverage and moderate skewness				
(Intercept)	0.03	0.07	0.42	0.68
C	0.13	0.07	1.97	0.05 *
D	0.06	0.08	0.75	0.04 *
Low leverage and high skewness				
(Intercept)	0.03	0.07	0.42	0.68
E	0.04	0.07	0.56	0.05 *
F	-0.03	0.07	-0.41	0.04 *
Moderate leverage and low skewness				
(Intercept)	0.03	0.07	0.42	0.68
G	-0.04	0.07	-0.49	0.04 *
H	0.06	0.08	0.76	0.04*
Moderate leverage and moderate skewness				
(Intercept)	0.03	0.07	0.42	0.68
I	-0.10	0.07	-1.39	0.02 *
J	-0.13	0.07	-1.75	0.04 *
Moderate leverage and high skewness				
(Intercept)	0.03	0.07	0.42	0.68
K	-0.10	0.07	-1.39	0.02 *
L	-0.13	0.07	-1.75	0.04 *
High leverage and low skewness				
(Intercept)	0.03	0.07	0.42	0.68
M	-0.04	0.07	-0.49	0.03 *
N	-0.10	0.07	-1.39	0.04 *
High leverage and moderate skewness				
(Intercept)	0.03	0.07	0.42	0.68
P	-0.04	0.07	-0.49	0.03 *
Q	-0.10	0.07	-1.39	0.0.4 *
High leverage and high skewness				
Intercept)	0.03	0.07	0.42	0.68
R	0.02	0.07	0.30	0.04 *
S	0.17	0.07	2.40	0.02 *

11. Summary of the Fitted Linear Regression with Autoregressive Errors using 250 Sample Size

Model	**Estimate**	**Std. Error**	***t* value**	**Pr(>|*t*|)**
Low leverage and low skewness				
(Intercept)	0.03	0.07	0.42	0.68
A_1	0.17	0.08	2.15	0.03 *
B_1	0.02	0.07	0.30	0.02 *
Low leverage and moderate skewness				
(Intercept)	0.03	0.07	0.42	0.68
C_1	-0.12	0.07	-1.75	0.03 *
D_1	-0.06	0.07	-0.84	0.04 *
Low leverage and high skewness				
(Intercept)	0.03	0.07	0.42	0.68
E_1	-0.05	0.07	-0.78	0.03 *
F_1	-0.06	0.07	-0.84	0.04 *
Moderate leverage and low skewness				
(Intercept)	0.03	0.07	0.42	0.68
G_1	-0.03	0.07	-0.41	0.02 *
H_1	-0.05	0.07	-0.75	0.03 *
Moderate leverage and moderate skewness				
(Intercept)	0.03	0.07	0.42	0.68
I_1	0.13	0.08	1.73	0.03 *
J_1	0.09	0.07	1.25	0.02 *
Moderate leverage and high skewness				
(Intercept)	0.03	0.07	0.42	0.68
K_1	-0.08	0.07	-1.12	0.02 *
L_1	-0.06	0.07	-0.91	0.03 *
High leverage and low skewness				
(Intercept)	0.03	0.07	0.42	0.68
M_1	-0.05	0.07	-0.75	0.04 *
N_1	-0.12	0.07	-1.75	0.03 *
High leverage and moderate skewness				
(Intercept)	0.03	0.07	0.42	0.68
P_1	0.15	0.07	2.07	0.04 *
Q_1	0.09	0.07	1.29	0.02 *
High leverage and high skewness				
Intercept)	0.03	0.07	0.42	0.68
R_1	-0.04	0.06	-0.70	0.04 *
S_1	-0.04	0.07	-0.49	0.04 *

12. Summary of the Fitted Linear Regression with Autoregressive Errors using 300 Sample Size

Model	**Estimate**	**Std. Error**	***t* value**	**Pr(>|*t*|)**
Low leverage and low skewness				
(Intercept)	0.03	0.07	0.42	0.68
A_2	0.06	0.08	0.75	0.04 *
B_2	-0.12	0.07	-1.75	0.04 *
Low leverage and moderate skewness				
(Intercept)	0.03	0.07	0.42	0.68
C_2	-0.04	0.07	-0.49	0.03 *
D_2	0.06	0.08	0.75	0.04 *
Low leverage and high skewness				
(Intercept)	0.03	0.07	0.42	0.68
E_2	0.01	0.08	0.14	0.04 *
F_2	-0.03	0.07	-0.49	0.03 *
Moderate leverage and low skewness				
(Intercept)	0.03	0.07	0.42	0.68
G_2	0.06	0.08	0.76	0.04 *
H_2	0.02	0.07	0.30	0.03 *
Moderate leverage and moderate skewness				
(Intercept)	0.03	0.07	0.42	0.68
I_2	0.02	0.08	0.23	0.02 *
J_2	0.06	0.08	0.75	0.04 *
Moderate leverage and high skewness				
(Intercept)	0.03	0.07	0.42	0.68
K_2	-0.07	0.07	-0.96	0.03 *
L_2	-0.03	0.07	-0.41	0.04*
High leverage and low skewness				
Intercept)	0.03	0.07	0.42	0.68
M_2	0.13	0.07	1.97	0.03 *
N_2	-0.12	0.07	-1.75	0.04 *
High leverage and moderate skewness				
(Intercept)	0.03	0.07	0.42	0.68
P_2	0.09	0.07	1.29	0.02 *
Q_2	0.13	0.07	1.97	0.04 *
High leverage and high skewness				
Intercept)	0.03	0.07	0.42	0.68
R_2	-0.09	0.07	-1.25	0.04 *
S_2	-0.08	0.07	-1.12	0.03 *

Appendix F

The SARIMA Model Output

US GDP

FOR Y=A
adjreg1 = sarima (y, 1, 0, 0, xreg = x); this is considered for this study. AR(1)
initial value 0.663160
iter   2 value 0.662375
iter   3 value 0.662373
iter   4 value 0.662371
iter   4 value 0.662371
iter   4 value 0.662371
final value 0.662371
converged

initial value 0.661860
iter   2 value 0.661854
iter   3 value 0.661851
iter   3 value 0.661851
iter   3 value 0.661851
final value 0.661851
converged

Y=B
adjreg = sarima (y, 1, 0, 0, xreg = x); this is considered for this study. AR(1)
initial value 0.663160
iter   2 value 0.662375
iter   3 value 0.662373
iter   4 value 0.662371
iter   4 value 0.662371
iter   4 value 0.662371
final value 0.662371
converged

initial value 0.661860
iter   2 value 0.661854
iter   3 value 0.661851
iter   3 value 0.661851
iter   3 value 0.661851
final value 0.661851
converged

UK GDP
Y=C
adjreg = sarima (y, 1, 0, 0, xreg = x); this is considered for this study. AR(1)
initial value 0.791950
iter   2 value 0.780303

iter 3 value 0.780218
iter 4 value 0.780186
iter 5 value 0.780186
iter 5 value 0.780186
iter 5 value 0.780186
final value 0.780186
converged

initial value 0.778065
iter 2 value 0.778064
iter 3 value 0.778063
iter 4 value 0.778063
iter 4 value 0.778063
iter 4 value 0.778063
final value 0.778063
converged

Y=D
adjreg = sarima(y, 1, 0,0, xreg = x); this is considered for this study. AR(1)
initial value 0.789797
iter 2 value 0.780195
iter 3 value 0.779809
iter 4 value 0.779785
iter 5 value 0.779778
iter 5 value 0.779778
iter 5 value 0.779778
final value 0.779778
converged

initial value 0.779815
iter 2 value 0.779815
iter 2 value 0.779815
iter 2 value 0.779815
final value 0.779815
converged

AU GDP
Y=P
adjreg1 = sarima (y, 1, 0, 0, xreg = x); this is considered for this study. AR(1)
initial value -0.078320
iter 2 value -0.089107
iter 3 value -0.089109
iter 4 value -0.089110
iter 4 value -0.089110
iter 4 value -0.089110
final value -0.089110
converged

initial value -0.087981
iter 2 value -0.087992

iter 3 value -0.088001
iter 3 value -0.088001
iter 3 value -0.088001
final value -0.088001
converged

Y=Q
adjreg = sarima (y, 1, 0, 0, xreg = x); this is considered for this study. AR(1)
initial value 0.663160
iter 2 value 0.662375
iter 3 value 0.662373
iter 4 value 0.662371
iter 4 value 0.662371
iter 4 value 0.662371
final value 0.662371
converged

initial value 0.661860
iter 2 value 0.661854
iter 3 value 0.661851
iter 3 value 0.661851
iter 3 value 0.661851
final value 0.661851
converged

France GDP
Y=W
adjreg = sarima(y, 1, 0,0, xreg = x) ; this is considered for this study. AR(1)
initial value 0.778937
iter 2 value 0.778936
iter 3 value 0.778936
iter 4 value 0.778936
iter 4 value 0.778936
iter 4 value 0.778936
final value 0.778936
converged

initial value 0.776859
iter 2 value 0.776858
iter 2 value 0.776858
iter 3 value 0.776858
iter 3 value 0.776858
iter 3 value 0.776858
final value 0.776858
converged

```
Y=X
adjreg = sarima(y, 1, 0,0,  xreg = x) ; this is considered for this study. AR(1)
initial value 0.774611
iter  2 value 0.771018
iter  3 value 0.769513
iter  4 value 0.769370
iter  4 value 0.769370
iter  4 value 0.769370
final value 0.769370
converged

initial value 0.793377
iter   2 value 0.793297
iter   3 value 0.793296
iter   3 value 0.793296
iter   3 value 0.793296
final value 0.793296
converged
```

Appendix G

Fit AR with ARIMA Modelling Time Series

1. Low Leverage and Low Skewness of 200 Sample Size

arima(x = X26, order = c(1, 0, 0))

Coefficients:

```
        ar1 intercept
     0.0832     -0.040
s.e.  0.0670      0.069
sigma^2 estimated as 0.881:  log likelihood = -298.24,  aic = 602.47
```

arima(x = X15, order = c(1, 0, 0))

Coefficients:

```
        ar1 intercept
     -0.0546    -0.0646
s.e.   0.0688     0.0589
sigma^2 estimated as 0.8485:  log likelihood = -294.1,  aic = 594.21
```

arima(y, order = c(1,0,0))

Call:

arima(x = y, order = c(1, 0, 0))

Coefficients:

```
        ar1 intercept
     0.132    -0.0142
s.e.  0.070     0.1051
sigma^2 estimated as 1.668:  log likelihood = -334.99,  aic = 675.97
```

2. Low Leverage and Moderate Skewness of 200 Sample Size

arima(X26, order = c(1,0,0))

Call:

arima(x = X26, order = c(1, 0, 0))

Coefficients:

ar1 intercept

0.0832 -0.040

s.e. 0.0670 0.069

sigma^2 estimated as 0.881: log likelihood = -298.24, aic = 602.47

arima(X40, order = c(1,0,0))

Call:

arima(x = X40, order = c(1, 0, 0))

Coefficients:

ar1 intercept

0.1468 -0.0488

s.e. 0.0666 0.0820

sigma^2 estimated as 1.078: log likelihood = -320.47, aic = 646.95

arima(y, order = c(1,0,0))

Call:

arima(x = y, order = c(1, 0, 0))

Coefficients:

ar1 intercept

0.132 -0.0142

s.e. 0.070 0.1051

sigma^2 estimated as 1.668: log likelihood = -334.99, aic = 675.97

3. Low Leverage and High Skewness of 200 Sample Size

arima(X15, order = c(1,0,0))

Call:

arima(x = X15, order = c(1, 0, 0))

Coefficients:

ar1 intercept

-0.0546 -0.0646

s.e. 0.0688 0.0589

sigma^2 estimated as 0.8485: log likelihood = -294.1, aic = 594.21

arima(X40, order = c(1,0,0))

Call:

arima(x = X40, order = c(1, 0, 0))

Coefficients:

	ar1	intercept
	0.1468	-0.0488
s.e.	0.0666	0.0820

sigma^2 estimated as 1.078: log likelihood = -320.47, aic = 646.95

4. Moderate Leverage and Low Skewness of 200 Sample Size

arima(X30, order = c(1,0,0))

Call:

arima(x = X30, order = c(1, 0, 0))

Coefficients:

	ar1	intercept
	0.0983	-0.0901
s.e.	0.0669	0.0654

sigma^2 estimated as 0.7663: log likelihood = -282.9, aic = 571.8

arima(X23, order = c(1,0,0))

Call:

arima(x = X23, order = c(1, 0, 0))

Coefficients:

	ar1	intercept
	0.0245	0.1381
s.e.	0.0682	0.0680

sigma^2 estimated as 0.9678: log likelihood = -308.56, aic = 623.13

arima(y, order = c(1,0,0))

Call:

```
arima(x = y, order = c(1, 0, 0))
Coefficients:
        ar1  intercept
      0.132    -0.0142
s.e.  0.070     0.1051
sigma^2 estimated as 1.668:  log likelihood = -334.99,  aic = 675.97
```

5. Moderate Leverage and Moderate Skewness of 200 Sample Size

```
 arima(X28, order = c(1,0,0))
Call:
arima(x = X28, order = c(1, 0, 0))
Coefficients:
         ar1  intercept
      0.0618     0.0565
s.e.  0.0673     0.0761
sigma^2 estimated as 1.122:  log likelihood = -324.86,  aic = 655.72


 arima(X29, order = c(1,0,0))
Call:
arima(x = X29, order = c(1, 0, 0))
Coefficients:
         ar1  intercept
      0.1212    -0.0287
s.e.  0.0672     0.0732
sigma^2 estimated as 0.9118:  log likelihood = -302.02,  aic = 610.03


 arima(y, order = c(1,0,0))
Call:
arima(x = y, order = c(1, 0, 0))
Coefficients:
```

ar1 intercept

0.132   -0.0142

s.e. 0.070   0.1051

sigma^2 estimated as 1.668: log likelihood = -334.99, aic = 675.97

6. Moderate Leverage and High Skewness of 200 Sample Size

arima(X35, order = c(1,0,0))

Call:

arima(x = X35, order = c(1, 0, 0))

Coefficients:

ar1 intercept

0.1308   -0.0064

s.e. 0.0669   0.0770

sigma^2 estimated as 0.9873: log likelihood = -310.76, aic = 627.53

arima(X15, order = c(1,0,0))

Call:

arima(x = X15, order = c(1, 0, 0))

Coefficients:

ar1 intercept

-0.0546   -0.0646

s.e.   0.0688   0.0589

sigma^2 estimated as 0.8485: log likelihood = -294.1, aic = 594.21

arima(y, order = c(1,0,0))

Call:

arima(x = y, order = c(1, 0, 0))

Coefficients:

ar1 intercept

0.132   -0.0142

s.e. 0.070   0.1051

sigma^2 estimated as 1.668: log likelihood = -334.99, aic = 675.97

7. High Leverage and Low Skewness of 200 Sample Size

arima(X23, order = c(1,0,0))

Call:

arima(x = X23, order = c(1, 0, 0))

Coefficients:

ar1 intercept

0.0245 0.1381

s.e. 0.0682 0.0680

sigma^2 estimated as 0.9678: log likelihood = -308.56, aic = 623.13

arima(X28, order = c(1,0,0))

Call:

arima(x = X28, order = c(1, 0, 0))

Coefficients:

ar1 intercept

0.0618 0.0565

s.e. 0.0673 0.0761

sigma^2 estimated as 1.122: log likelihood = -324.86, aic = 655.72

arima(y, order = c(1,0,0))

Call:

arima(x = y, order = c(1, 0, 0))

Coefficients:

ar1 intercept

0.132 -0.0142

s.e. 0.070 0.1051

sigma^2 estimated as 1.668: log likelihood = -334.99, aic = 675.97

8. High Leverage and Moderate Skewness of 200 Sample Size

arima(X23, order = c(1,0,0))

Call:

arima(x = X23, order = c(1, 0, 0))

Coefficients:

```
        ar1  intercept
     0.0245     0.1381
s.e. 0.0682     0.0680
```

sigma^2 estimated as 0.9678: log likelihood = -308.56, aic = 623.13

arima(X28, order = c(1,0,0))

Call:

arima(x = X28, order = c(1, 0, 0))

Coefficients:

```
        ar1  intercept
     0.0618     0.0565
s.e. 0.0673     0.0761
```

sigma^2 estimated as 1.122: log likelihood = -324.86, aic = 655.72

arima(y, order = c(1,0,0))

Call:

arima(x = y, order = c(1, 0, 0))

Coefficients:

```
       ar1  intercept
     0.132    -0.0142
s.e. 0.070     0.1051
```

sigma^2 estimated as 1.668: log likelihood = -334.99, aic = 675.97

9. High Leverage and High Skewness of 200 Sample Size

arima(x35, order = c(1,0,0))

Call:

arima(x = x35, order = c(1, 0, 0))

Coefficients:

```
        ar1  intercept
     0.0138    -0.0425
```

```
s.e.  0.0320     0.0317
sigma^2 estimated as 0.9514:  log likelihood = -1360.58,  aic = 2727.17

  arima(x30, order = c(1,0,0))
Call:
arima(x = x30, order = c(1, 0, 0))
Coefficients:
         ar1 intercept
      0.0039    -0.0525
s.e.  0.0320     0.0322
sigma^2 estimated as 1.002:  log likelihood = -1385.91,  aic = 2777.82

arima(y, order = c(1,0,0))
Call:
arima(x = y, order = c(1, 0, 0))
Coefficients:
        ar1 intercept
      0.132    -0.0142
s.e.  0.070     0.1051
sigma^2 estimated as 1.668:  log likelihood = -334.99,  aic = 675.97>
```

Appendix H

EGARCH, VAR and MA Computer Output for 200 Simulated Data

1. EGARCH estimation N=200(0.01 low leverage and 0.5 low skewness)

Dependent Variable: Y

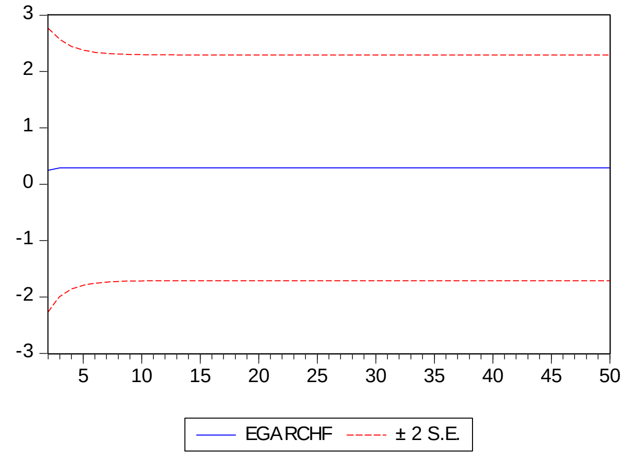
Method: ML - ARCH (Marquardt) - Student's t distribution
Date: 08/15/17 Time: 10:27
Sample (adjusted): 2 200
Included observations: 199 after adjustments
Convergence achieved after 26 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)
*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.000134	0.062959	-0.002124	0.9983
AR(1)	0.002396	0.073010	0.032815	0.9738
		Variance Equation		
C(3)	-0.651026	0.290762	-2.239035	0.0252
C(4)	0.688630	0.270382	2.546878	0.0109
C(5)	0.071728	0.125992	0.569308	0.5691
C(6)	-0.425210	0.223455	-1.902884	0.0571
T-DIST. DOF	7.354583	3.892371	1.889486	0.0588

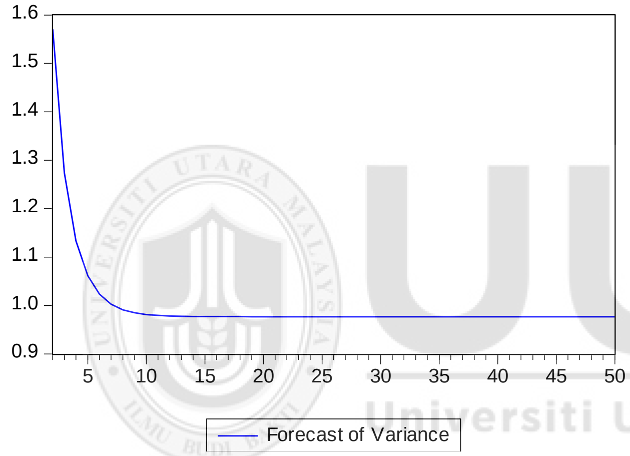
R-squared	-0.005128	Mean dependent var	-0.072121
Adjusted R-squared	-0.010230	S.D. dependent var	1.000000
S.E. of regression	1.005102	Akaike info criterion	2.781458
Sum squared resid	199.0154	Schwarz criterion	2.897303
Log likelihood	-269.7550	Hannan-Quinn criter.	2.828343
Durbin-Watson stat	1.966213		

Inverted AR Roots	.00

2. EGARCH forecast N=200(0.01 low leverage and 0.5 low skewness)



Forecast: EGARCHF	
Actual: D01	
Forecast sample: 1 50	
Adjusted sample: 2 50	
Included observations: 49	
Root Mean Squared Error	1.061001
Mean Absolute Error	0.720844
Mean Abs. Percent Error	275.9369
Theil Inequality Coefficient	0.812123
Bias Proportion	0.083719
Variance Proportion	0.906895
Covariance Proportion	0.009386



3. MA estimation N=200(0.01 low leverage and 0.5 low skewness)

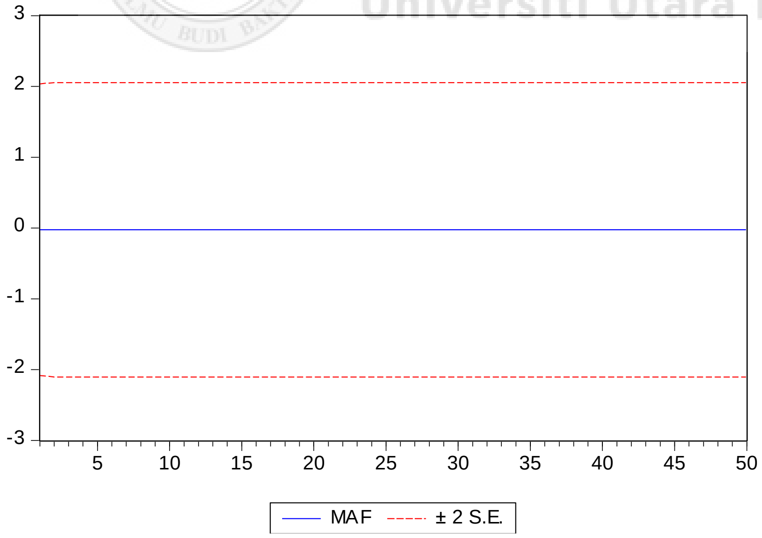
Dependent Variable: Y
Method: Least Squares
Date: 08/15/17 Time: 10:25
Sample: 1 200
Included observations: 200
Convergence achieved after 5 iterations
MA Backcast: 0

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.071917	0.071774	-1.001992	0.3176
MA(1)	0.015207	0.071082	0.213932	0.8308

R-squared	0.000192	Mean dependent var	-0.071946
Adjusted R-squared	-0.004858	S.D. dependent var	0.997487
S.E. of regression	0.999907	Akaike info criterion	2.847641
Sum squared resid	197.9632	Schwarz criterion	2.880624
Log likelihood	-282.7641	Hannan-Quinn criter.	2.860989
F-statistic	0.038025	Durbin-Watson stat	2.002003
Prob(F-statistic)	0.845594		

Inverted MA Roots	-.02

4. MA estimation N=200(0.01 low leverage and 0.5 low skewness



Forecast: MAF
Actual: D01
Forecast sample: 1 50
Included observations: 50

Root Mean Squared Error	1.007804
Mean Absolute Error	0.759568
Mean Abs. Percent Error	110.9657
Theil Inequality Coefficient	0.975212
Bias Proportion	0.000000
Variance Proportion	0.999983
Covariance Proportion	0.000017

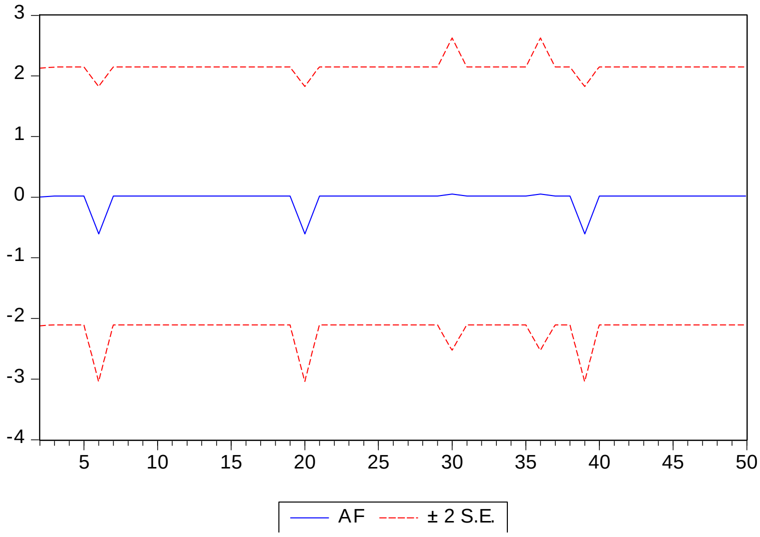
5. VAR estimation N=200(0.01 low leverage and 0.5 low skewness)

Vector Autoregression Estimates
Date: 08/15/17 Time: 10:48
Sample (adjusted): 2 200
Included observations: 199 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	Y	X15	X26
Y(-1)	0.012751	-0.002319	0.007566
	(0.07160)	(0.00787)	(0.00467)
	[0.17809]	[-0.29468]	[1.62083]
X15(-1)	-0.295836	-0.016021	0.015777
	(0.65122)	(0.07159)	(0.04245)
	[-0.45428]	[-0.22380]	[0.37162]
X26(-1)	-0.034475	0.047702	0.053849
	(1.08989)	(0.11981)	(0.07105)
	[-0.03163]	[0.39815]	[0.75788]
C	-0.074679	-0.014675	0.015034
	(0.07400)	(0.00813)	(0.00482)
	[-1.00914]	[-1.80392]	[3.11615]
R-squared	0.001227	0.001514	0.016719
Adj. R-squared	-0.014139	-0.013847	0.001591
Sum sq. resids	197.7570	2.389742	0.840483
S.E. equation	1.007045	0.110703	0.065652
F-statistic	0.079857	0.098586	1.105182
Log likelihood	-281.7453	157.6321	261.6060
Akaike AIC	2.871812	-1.544041	-2.589005
Schwarz SC	2.938010	-1.477844	-2.522808
Mean dependent	-0.072121	-0.013568	0.015075
S.D. dependent	1.000000	0.109944	0.065704

Determinant resid covariance (dof adj.)	5.35E-05
Determinant resid covariance	5.03E-05
Log likelihood	137.6262
Akaike information criterion	-1.262575
Schwarz criterion	-1.063983

6. VAR estimation N=200(0.01 low leverage and 0.5 low skewness)



Forecast: AF	
Actual: A	
Forecast sample: 1 50	
Adjusted sample: 2 50	
Included observations: 49	
Root Mean Squared Error	1.006279
Mean Absolute Error	0.765508
Mean Abs. Percent Error	110.8082
Theil Inequality Coefficient	0.861777
Bias Proportion	0.000000
Variance Proportion	0.741081
Covariance Proportion	0.258919

7. EGARCH estimation N=200(0.01 low leverage and 0.7 moderate skew)

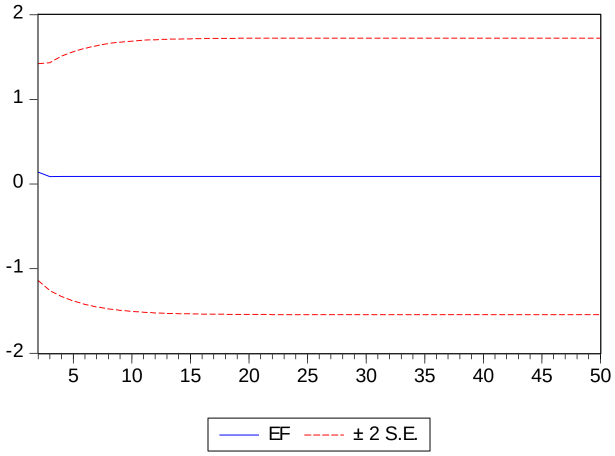
Dependent Variable: Y
Method: ML - ARCH (Marquardt) - Student's t distribution
Date: 08/15/17 Time: 12:00
Sample (adjusted): 2 200
Included observations: 199 after adjustments
Convergence achieved after 26 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)
*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	-0.000134	0.062959	-0.002124	0.9983
AR(1)	0.002396	0.073010	0.032815	0.9738
Variance Equation				
C(3)	-0.651026	0.290762	-2.239035	0.0252
C(4)	0.688630	0.270382	2.546878	0.0109
C(5)	0.071728	0.125992	0.569308	0.5691
C(6)	-0.425210	0.223455	-1.902884	0.0571
T-DIST. DOF	7.354582	3.892372	1.889486	0.0588

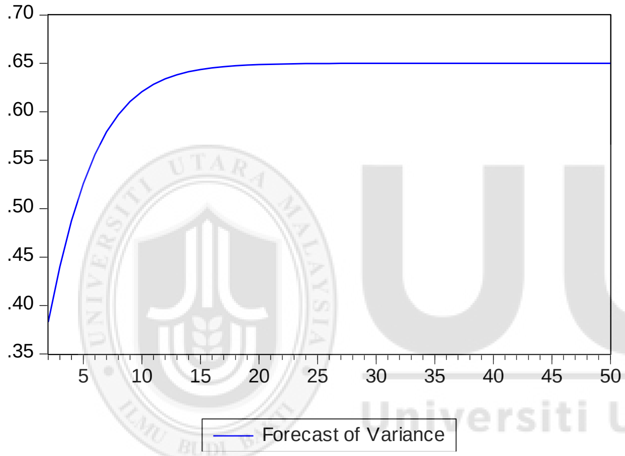
R-squared	-0.005128	Mean dependent var	-0.072121
Adjusted R-squared	-0.010230	S.D. dependent var	1.000000
S.E. of regression	1.005102	Akaike info criterion	2.781458
Sum squared resid	199.0154	Schwarz criterion	2.897303
Log likelihood	-269.7550	Hannan-Quinn criter.	2.828343
Durbin-Watson stat	1.966213		

Inverted AR Roots .00

8. EGARCH estimation N=200(0.01 low leverage and 0.7 moderate skewness)



Forecast: EF	
Actual: E	
Forecast sample: 1 50	
Adjusted sample: 2 50	
Included observations: 49	
Root Mean Squared Error	0.817929
Mean Absolute Error	0.643386
Mean Abs. Percent Error	241.2879
Theil Inequality Coefficient	0.894688
Bias Proportion	0.000022
Variance Proportion	0.983621
Covariance Proportion	0.016357



9. MA estimation N=200(0.01 low leverage and 0.7 moderate skewness)

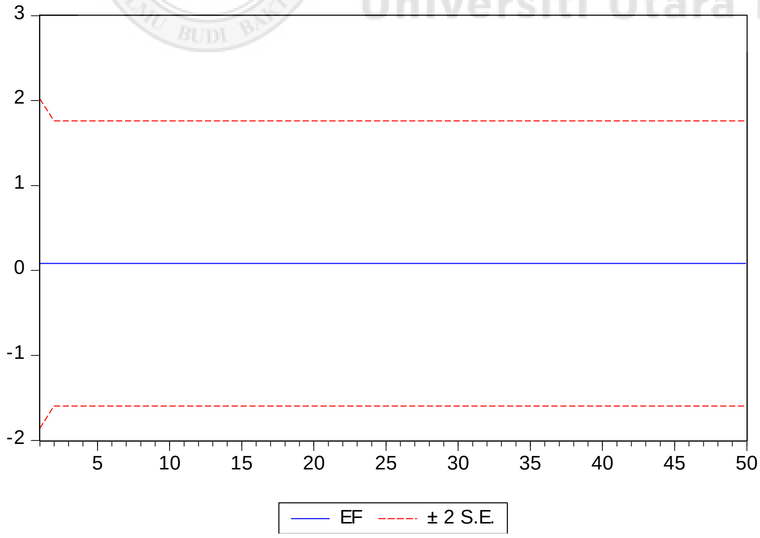
Dependent Variable: Y
Method: Least Squares
Date: 08/15/17 Time: 12:04
Sample: 1 200
Included observations: 200
Convergence achieved after 5 iterations
MA Backcast: 0

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.071917	0.071774	-1.001992	0.3176
MA(1)	0.015207	0.071082	0.213932	0.8308

R-squared	0.000192	Mean dependent var	-0.071946
Adjusted R-squared	-0.004858	S.D. dependent var	0.997487
S.E. of regression	1.001224	Akaike info criterion	2.964712
Sum squared resid	197.9632	Schwarz criterion	2.938023
Log likelihood	-293.7614	Hannan-Quinn criter.	2.860989
F-statistic	0.038025	Durbin-Watson stat	2.002003
Prob(F-statistic)	0.845594		

Inverted MA Roots	-.02

10. MA forecast N=200(0.01 low leverage and 0.7 moderate skewness)



Forecast: EF
Actual: E
Forecast sample: 1 50
Included observations: 50

Root Mean Squared Error	0.814519
Mean Absolute Error	0.642980
Mean Abs. Percent Error	228.9763
Theil Inequality Coefficient	0.902757
Bias Proportion	0.000004
Variance Proportion	0.999232
Covariance Proportion	0.000764

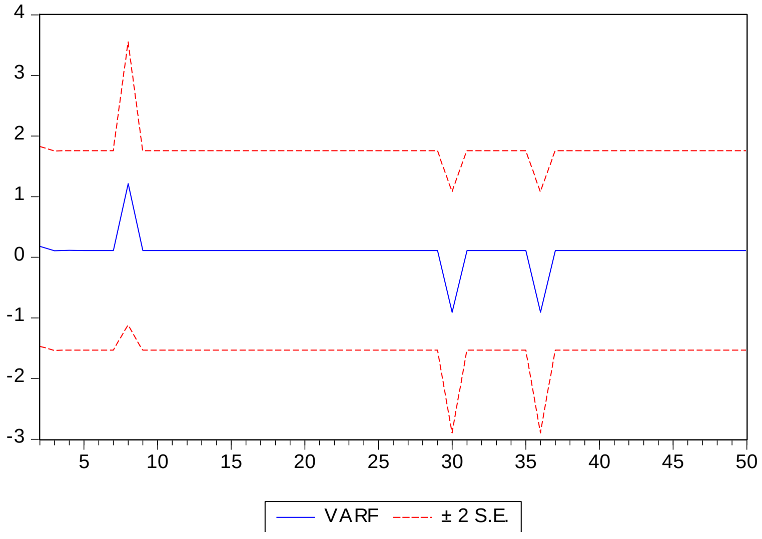
11. VAR estimation N=200(0.01 low leverage and 0.7 moderate skewness)

Vector Autoregression Estimates
Date: 08/15/17 Time: 12:32
Sample (adjusted): 2 200
Included observations: 199 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	Y	X15	X40
Y(-1)	0.076830	-0.006286	-0.024217
	(0.07300)	(0.00778)	(0.01336)
	[1.05250]	[-0.80789]	[-1.81295]
X15(-1)	0.279240	-0.009248	0.045712
	(0.67450)	(0.07190)	(0.12342)
	[0.41400]	[-0.12862]	[0.37036]
X40(-1)	0.054526	0.013259	0.012550
	(0.39414)	(0.04201)	(0.07212)
	[0.13834]	[0.31559]	[0.17401]
C	0.064551	-0.013498	0.021204
	(0.07464)	(0.00796)	(0.01366)
	[0.86483]	[-1.69655]	[1.55246]
R-squared	0.007633	0.003729	0.016833
Adj. R-squared	-0.007634	-0.011598	0.001708
Sum sq. resids	209.8620	2.384442	7.027122
S.E. equation	1.037408	0.110580	0.189833
F-statistic	0.499965	0.243300	1.112896
Log likelihood	-287.6567	157.8530	50.31222
Akaike AIC	2.931223	-1.546262	-0.465449
Schwarz SC	2.997420	-1.480065	-0.399252
Mean dependent	0.067286	-0.013568	0.019095
S.D. dependent	1.033471	0.109944	0.189995

Determinant resid covariance (dof adj.)	0.000452
Determinant resid covariance	0.000425
Log likelihood	-74.66667
Akaike information criterion	0.871022
Schwarz criterion	1.069613

12. VAR forecast N=200(0.01 low leverage and 0.7 moderate skewness)



Forecast: VARF	
Actual: P	
Forecast sample: 1 50	
Adjusted sample: 2 50	
Included observations: 49	
Root Mean Squared Error	0.780438
Mean Absolute Error	0.587197
Mean Abs. Percent Error	279.4167
Theil Inequality Coefficient	0.709514
Bias Proportion	0.000005
Variance Proportion	0.513424
Covariance Proportion	0.486571

13. EGARCH estimation N=200(0.01 low leverage and 1.2 high skewness)

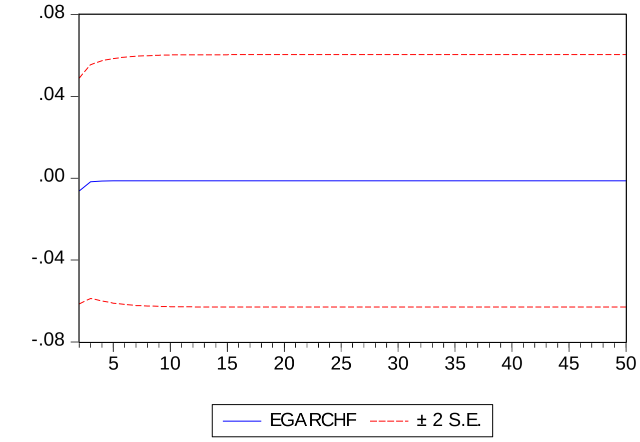
Dependent Variable: Y
Method: ML - ARCH (Marquardt) - Student's t distribution
Date: 08/15/17 Time: 13:51
Sample (adjusted): 2 200
Included observations: 199 after adjustments
Convergence achieved after 36 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)
*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000166	0.002246	0.074121	0.9409
AR(1)	0.075662	0.061852	1.223273	0.2212
Variance Equation				
C(3)	-1.695535	0.955355	-1.774770	0.0759
C(4)	-0.281114	0.151754	-1.852437	0.0640
C(5)	0.244701	0.097572	2.507886	0.0121
C(6)	0.725740	0.145519	4.987254	0.0000
T-DIST. DOF	21.06627	32.22258	0.653774	0.5133

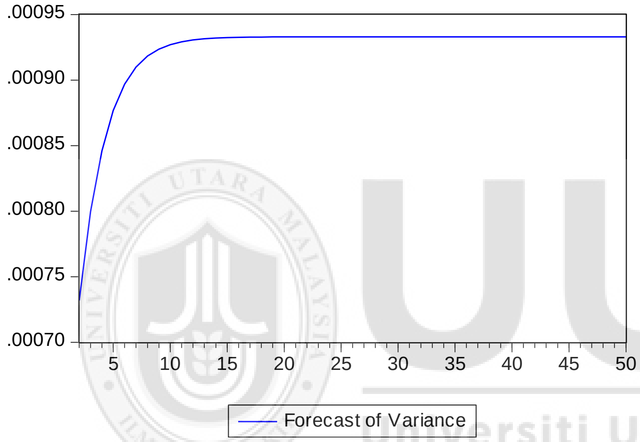
R-squared	-0.002631	Mean dependent var	-0.001434
Adjusted R-squared	-0.007721	S.D. dependent var	0.030628
S.E. of regression	0.030746	Akaike info criterion	-4.151830
Sum squared resid	0.186223	Schwarz criterion	-4.035985
Log likelihood	420.1071	Hannan-Quinn criter.	-4.104945
Durbin-Watson stat	2.026715		

Inverted AR Roots	.08

14. EGARCH forecast N=200(0.01 low leverage and 1.2 high skewness)



Forecast: EGARCHF	
Actual: G	
Forecast sample: 1 50	
Adjusted sample: 2 50	
Included observations: 49	
Root Mean Squared Error	0.035634
Mean Absolute Error	0.027621
Mean Abs. Percent Error	108.7905
Theil Inequality Coefficient	0.956648
Bias Proportion	0.000003
Variance Proportion	0.960584
Covariance Proportion	0.039413



15. MA estimation N=200(0.01 low leverage and 1.2 high skewness)

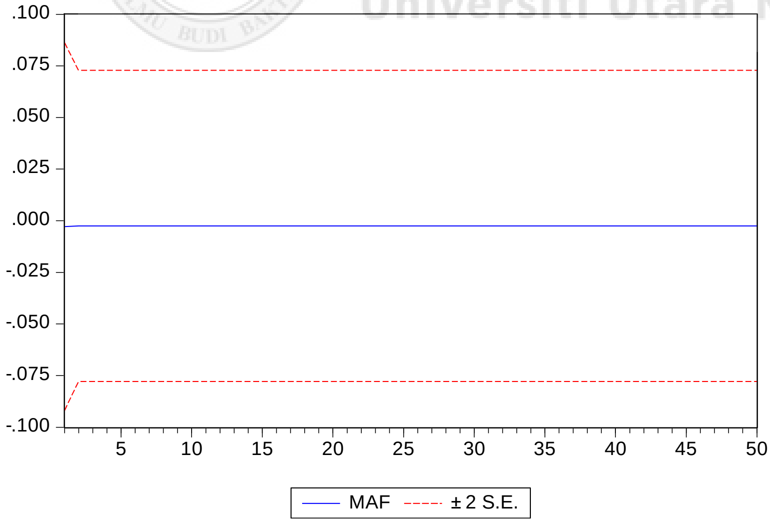
Dependent Variable: Y
Method: Least Squares
Date: 08/15/17 Time: 13:55
Sample: 1 200
Included observations: 200
Convergence achieved after 6 iterations
MA Backcast: 0

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.001634	0.002257	-0.724188	0.4698
MA(1)	0.039377	0.072363	0.544166	0.5869

R-squared	0.001377	Mean dependent var	-0.001612
Adjusted R-squared	-0.003667	S.D. dependent var	0.030654
S.E. of regression	0.030710	Akaike info criterion	-4.118510
Sum squared resid	0.186734	Schwarz criterion	-4.085527
Log likelihood	413.8510	Hannan-Quinn criter.	-4.105163
F-statistic	0.272943	Durbin-Watson stat	1.961176
Prob(F-statistic)	0.601948		

Inverted MA Roots	-.04

16. MA forecast N=200(0.01 low leverage and 1.2 high skewness)



Forecast: MAF	
Actual: G	
Forecast sample: 1 50	
Included observations: 50	
Root Mean Squared Error	0.036549
Mean Absolute Error	0.028429
Mean Abs. Percent Error	112.5024
Theil Inequality Coefficient	0.932325
Bias Proportion	0.000033
Variance Proportion	0.997941
Covariance Proportion	0.002026

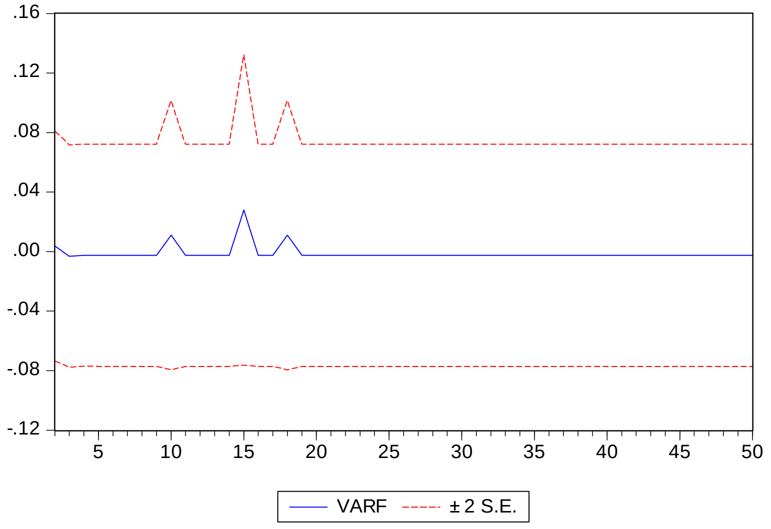
17. VAR estimation N=200(0.01 low leverage and 1.2 high skewness)

Vector Autoregression Estimates
Date: 08/15/17 Time: 15:46
Sample (adjusted): 2 200
Included observations: 199 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	Y	X1	X13
Y(-1)	0.036652	0.129652	-0.321973
	(0.07260)	(0.95597)	(0.40856)
	[0.50485]	[0.13562]	[-0.78807]
X1(-1)	0.001338	-0.015487	-0.009272
	(0.00544)	(0.07162)	(0.03061)
	[0.24602]	[-0.21624]	[-0.30293]
X13(-1)	0.000220	-0.047227	-0.025913
	(0.01270)	(0.16724)	(0.07147)
	[0.01734]	[-0.28239]	[-0.36256]
C	-0.001317	-0.051668	-0.029205
	(0.00223)	(0.02941)	(0.01257)
	[-0.58968]	[-1.75698]	[-2.32376]
R-squared	0.001588	0.000735	0.004251
Adj. R-squared	-0.013772	-0.014638	-0.011068
Sum sq. resids	0.185440	32.15383	5.872918
S.E. equation	0.030838	0.406068	0.173544
F-statistic	0.103387	0.047833	0.277485
Log likelihood	411.9752	-101.0028	68.16507
Akaike AIC	-4.100253	1.055305	-0.644875
Schwarz SC	-4.034056	1.121502	-0.578678
Mean dependent	-0.001434	-0.049749	-0.027638
S.D. dependent	0.030628	0.403128	0.172592

Determinant resid covariance (dof adj.)	4.72E-06
Determinant resid covariance	4.44E-06
Log likelihood	379.2361
Akaike information criterion	-3.690815
Schwarz criterion	-3.492223

18. VAR forecast N=200(0.01 low leverage and 1.2 high skewness)



Forecast: VARF	
Actual: K	
Forecast sample: 1 50	
Adjusted sample: 2 50	
Included observations: 49	
Root Mean Squared Error	0.035432
Mean Absolute Error	0.027026
Mean Abs. Percent Error	114.1529
Theil Inequality Coefficient	0.867052
Bias Proportion	0.000015
Variance Proportion	0.744255
Covariance Proportion	0.255730

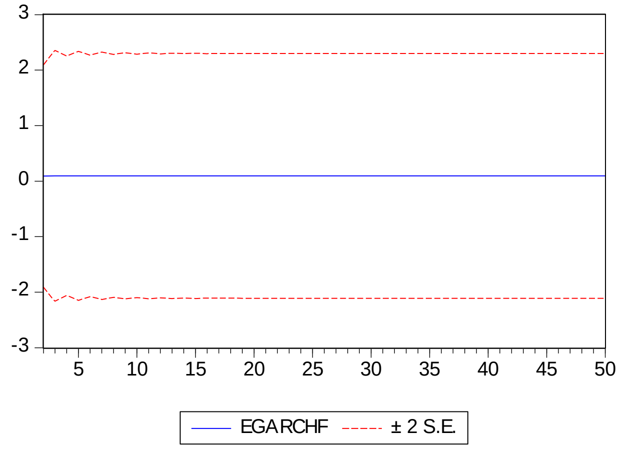
19. EGARCH estimation N=200 (0.05 moderate leverage and 0.5low skewnessness)

Dependent Variable: Y
Method: ML - ARCH (Marquardt) - Student's t distribution
Date: 08/15/17 Time: 17:38
Sample (adjusted): 2 200
Included observations: 199 after adjustments
Convergence achieved after 26 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)
*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

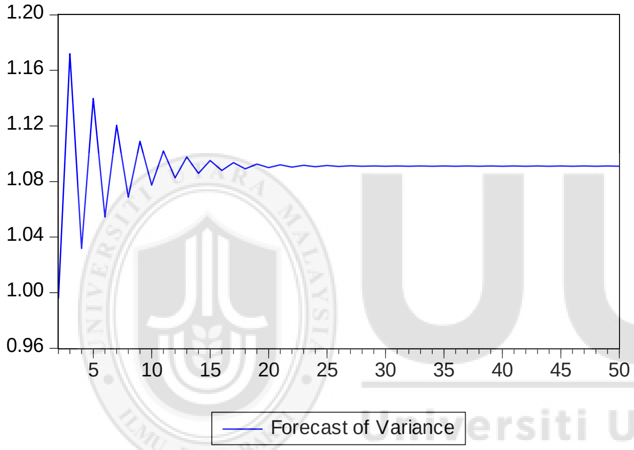
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.042887	0.078677	0.545110	0.5857
AR(1)	-0.077203	0.069344	-1.113333	0.2656
Variance Equation				
C(3)	-0.131057	0.064260	-2.039459	0.0414
C(4)	0.209730	0.113059	1.855045	0.0636
C(5)	-0.059643	0.078296	-0.761763	0.4462
C(6)	0.960517	0.045534	21.09467	0.0000
T-DIST. DOF	5.123388	1.792701	2.857916	0.0043

R-squared	0.009150	Mean dependent var	0.020159
Adjusted R-squared	0.004120	S.D. dependent var	1.430806
S.E. of regression	1.427856	Akaike info criterion	3.431959
Sum squared resid	401.6382	Schwarz criterion	3.547804
Log likelihood	-334.4799	Hannan-Quinn criter.	3.478844
Durbin-Watson stat	2.052602		
Inverted AR Roots	-.08		

20. EGARCH forecast N=200 (0.05 moderate leverage and 0.5low skewness)



Forecast: EGARCHF	
Actual: A	
Forecast sample: 1 50	
Adjusted sample: 2 50	
Included observations: 49	
Root Mean Squared Error	1.225355
Mean Absolute Error	0.907402
Mean Abs. Percent Error	97.75381
Theil Inequality Coefficient	0.929123
Bias Proportion	0.002232
Variance Proportion	0.997028
Covariance Proportion	0.000739



21. MA ESTIMATION N=200 (0.05 moderate leverage and 0.5 low skewness)

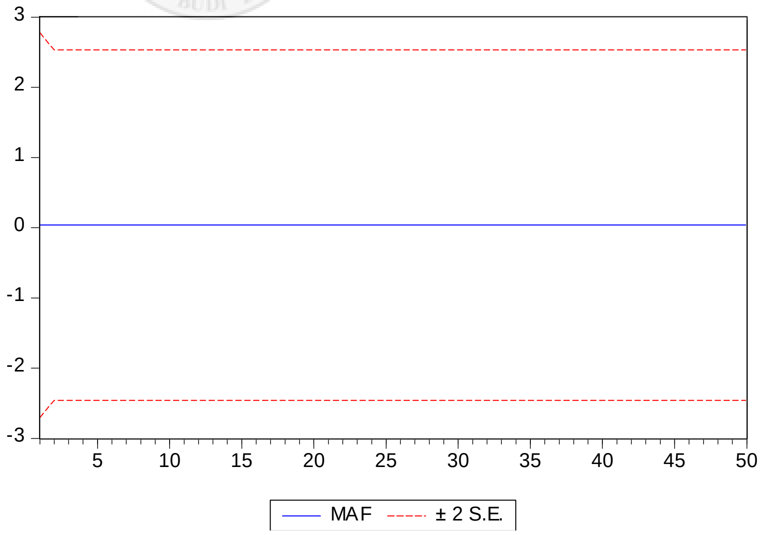
Dependent Variable: Y
Method: Least Squares
Date: 08/15/17 Time: 17:43
Sample: 1 200
Included observations: 200
Convergence achieved after 7 iterations
MA Backcast: 0

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.016384	0.088393	0.185355	0.8531
MA(1)	-0.122339	0.070562	-1.733789	0.0845

R-squared	0.011957	Mean dependent var	0.016314
Adjusted R-squared	0.006967	S.D. dependent var	1.428243
S.E. of regression	1.423259	Akaike info criterion	3.553725
Sum squared resid	401.0817	Schwarz criterion	3.586708
Log likelihood	-353.3725	Hannan-Quinn criter.	3.567073
F-statistic	2.396124	Durbin-Watson stat	1.973783
Prob(F-statistic)	0.123233		

Inverted MA Roots	.12

22. MA forecast N=200 (0.05 moderate leverage and 0.5 low skewness)



Forecast: MAF	
Actual: A	
Forecast sample: 1 50	
Included observations: 50	
Root Mean Squared Error	1.211821
Mean Absolute Error	0.895799
Mean Abs. Percent Error	96.24596
Theil Inequality Coefficient	0.969925
Bias Proportion	0.000000
Variance Proportion	0.999987
Covariance Proportion	0.000012

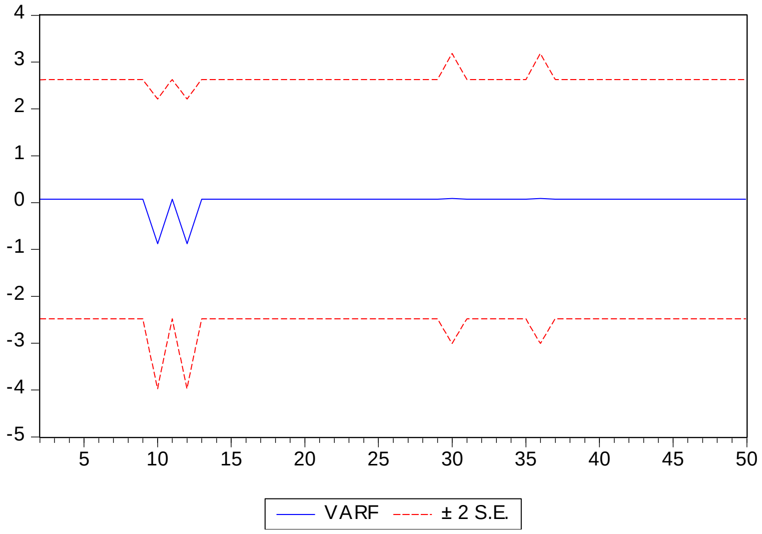
23. VAR estimation N=200 (0.05 moderate leverage and 0.5 low skewness)

Vector Autoregression Estimates
Date: 08/15/17 Time: 18:01
Sample (adjusted): 2 200
Included observations: 199 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	Y	X15	X30
Y(-1)	-0.096363	0.000771	0.003290
	(0.07151)	(0.00553)	(0.00733)
	[-1.34758]	[0.13935]	[0.44891]
X15(-1)	-0.907070	-0.016690	0.034672
	(0.92724)	(0.07175)	(0.09502)
	[-0.97825]	[-0.23261]	[0.36488]
X30(-1)	-0.091227	0.021311	-0.045206
	(0.70050)	(0.05420)	(0.07179)
	[-0.13023]	[0.39317]	[-0.62972]
C	0.011936	-0.014479	0.033519
	(0.10471)	(0.00810)	(0.01073)
	[0.11399]	[-1.78698]	[3.12372]
R-squared	0.014939	0.001082	0.004069
Adj. R-squared	-0.000216	-0.014286	-0.011253
Sum sq. resids	399.2914	2.390778	4.193420
S.E. equation	1.430961	0.110727	0.146645
F-statistic	0.985776	0.070386	0.265560
Log likelihood	-351.6592	157.5890	101.6801
Akaike AIC	3.574465	-1.543608	-0.981710
Schwarz SC	3.640662	-1.477411	-0.915513
Mean dependent	0.020159	-0.013568	0.031658
S.D. dependent	1.430806	0.109944	0.145827

Determinant resid covariance (dof adj.)	0.000533
Determinant resid covariance	0.000502
Log likelihood	-91.14510
Akaike information criterion	1.036634
Schwarz criterion	1.235225

24. VAR forecast N=200 (0.05 moderate leverage and 0.5 low skewness)



Forecast: VARF	
Actual: S	
Forecast sample: 1 50	
Adjusted sample: 2 50	
Included observations: 49	
Root Mean Squared Error	1.211501
Mean Absolute Error	0.875070
Mean Abs. Percent Error	92.30211
Theil Inequality Coefficient	0.855060
Bias Proportion	0.000001
Variance Proportion	0.730045
Covariance Proportion	0.269955

25. EGARCH estimation N=200(0.05 moderate leverage and 0.7moderate skewness)

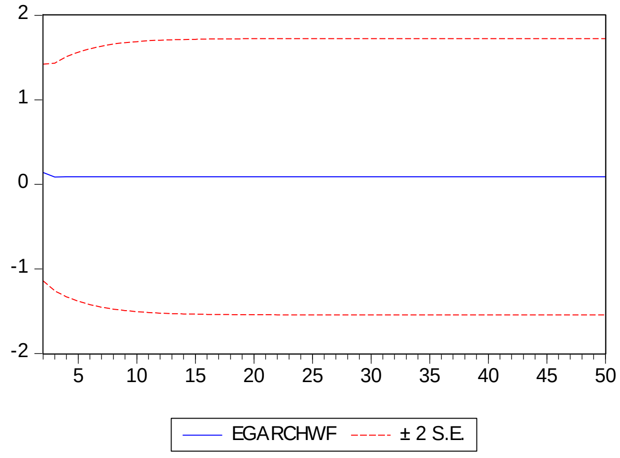
Dependent Variable: Y
Method: ML - ARCH (Marquardt) - Student's t distribution
Date: 08/13/17 Time: 08:53
Sample (adjusted): 2 200
Included observations: 199 after adjustments
Convergence achieved after 14 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1)))
+ C(5)
*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.059694	0.081798	0.729773	0.4655
AR(1)	0.112505	0.079966	1.406908	0.1595
		Variance Equation		
C(3)	-0.189064	0.211801	-0.892650	0.3720
C(4)	0.193144	0.171285	1.127612	0.2595
C(5)	-0.375537	0.104848	-3.581735	0.0003
C(6)	-0.235592	0.221063	-1.065719	0.2866
T-DIST. DOF	335.2047	11955.46	0.028038	0.9776

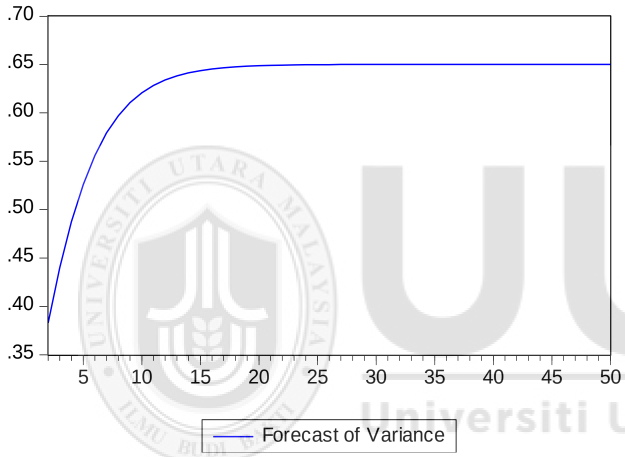
R-squared	0.005690	Mean dependent var	0.067286
Adjusted R-squared	0.000642	S.D. dependent var	1.033471
S.E. of regression	1.033139	Akaike info criterion	2.877968
Sum squared resid	210.2730	Schwarz criterion	2.993813
Log likelihood	-279.3579	Hannan-Quinn criter.	2.924854
Durbin-Watson stat	2.060796		

Inverted AR Roots	.11

26. EGARCH forecast N=200 (0.05 moderate leverage and 0.7 moderate skewness)



Forecast: EGARCHWF	
Actual: W	
Forecast sample: 1 50	
Adjusted sample: 2 50	
Included observations: 49	
Root Mean Squared Error	0.817929
Mean Absolute Error	0.643386
Mean Abs. Percent Error	241.2879
Theil Inequality Coefficient	0.894688
Bias Proportion	0.000022
Variance Proportion	0.983621
Covariance Proportion	0.016357



27. MA estimation N=200(0.05 moderate leverage and 0.7 moderate skewness)

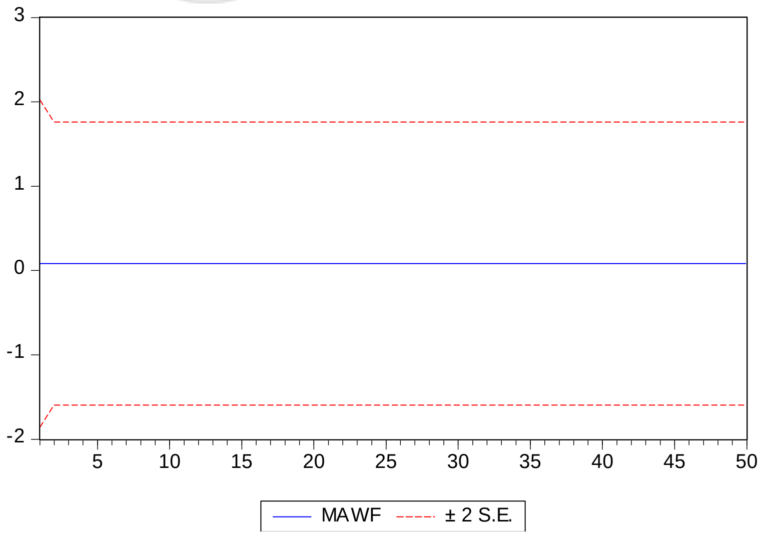
Dependent Variable: Y
Method: Least Squares
Date: 08/13/17 Time: 09:25
Sample: 1 200
Included observations: 200
Convergence achieved after 4 iterations
MA Backcast: 0

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.066745	0.078613	0.849032	0.3969
MA(1)	0.079587	0.071083	1.119625	0.2642

R-squared	0.006429	Mean dependent var	0.067154
Adjusted R-squared	0.001411	S.D. dependent var	1.030872
S.E. of regression	1.030145	Akaike info criterion	2.907225
Sum squared resid	210.1173	Schwarz criterion	2.940209
Log likelihood	-288.7225	Hannan-Quinn criter.	2.920573
F-statistic	1.281215	Durbin-Watson stat	1.993045
Prob(F-statistic)	0.259042		

Inverted MA Roots	-.08

28. MA forecast N=200 (0.05 moderate leverage and 0.7 moderate skewness)



Forecast: MAWF	
Actual: W	
Forecast sample: 1 50	
Included observations: 50	
Root Mean Squared Error	0.814555
Mean Absolute Error	0.643030
Mean Abs. Percent Error	228.9860
Theil Inequality Coefficient	0.902748
Bias Proportion	0.000005
Variance Proportion	0.999995
Covariance Proportion	0.000000

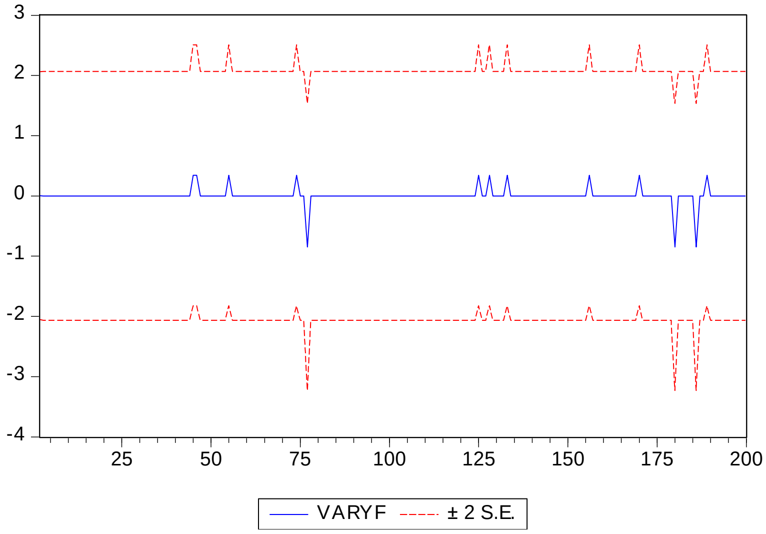
29. VAR estimation N=200(0.05 moderate leverage and 0.7 moderate skewness)

Vector Autoregression Estimates
Date: 08/13/17 Time: 09:38
Sample (adjusted): 2 200
Included observations: 199 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	Y	X15	X26
Y(-1)	0.075080	-0.006080	-0.000514
	(0.07192)	(0.00767)	(0.00459)
	[1.04398]	[-0.79224]	[-0.11205]
X15(-1)	0.267607	-0.010096	0.016639
	(0.67367)	(0.07189)	(0.04298)
	[0.39724]	[-0.14044]	[0.38714]
X26(-1)	0.856890	0.054471	0.052127
	(1.12323)	(0.11987)	(0.07166)
	[0.76288]	[0.45443]	[0.72742]
C	0.052642	-0.014092	0.014552
	(0.07612)	(0.00812)	(0.00486)
	[0.69154]	[-1.73473]	[2.99637]
R-squared	0.010489	0.004275	0.003536
Adj. R-squared	-0.004734	-0.011044	-0.011794
Sum sq. resids	209.2580	2.383136	0.851752
S.E. equation	1.035914	0.110550	0.066091
F-statistic	0.689007	0.279050	0.230638
Log likelihood	-287.3700	157.9076	260.2809
Akaike AIC	2.928341	-1.546810	-2.575687
Schwarz SC	2.994538	-1.480612	-2.509490
Mean dependent	0.067286	-0.013568	0.015075
S.D. dependent	1.033471	0.109944	0.065704

Determinant resid covariance (dof adj.)	5.63E-05
Determinant resid covariance	5.30E-05
Log likelihood	132.5011
Akaike information criterion	-1.211067
Schwarz criterion	-1.012475

30. VAR forecast N=200(0.05 moderate leverage and 0.7 moderate skewness)



Forecast: VARYF	
Actual: Y	
Forecast sample: 1 200	
Adjusted sample: 2 200	
Included observations: 199	
Root Mean Squared Error	1.024781
Mean Absolute Error	0.777904
Mean Abs. Percent Error	96.17985
Theil Inequality Coefficient	0.881693
Bias Proportion	0.003763
Variance Proportion	0.774255
Covariance Proportion	0.221982

31. EGARCH estimation N=200(0.05 moderate leverage and 1.2 high skewness)

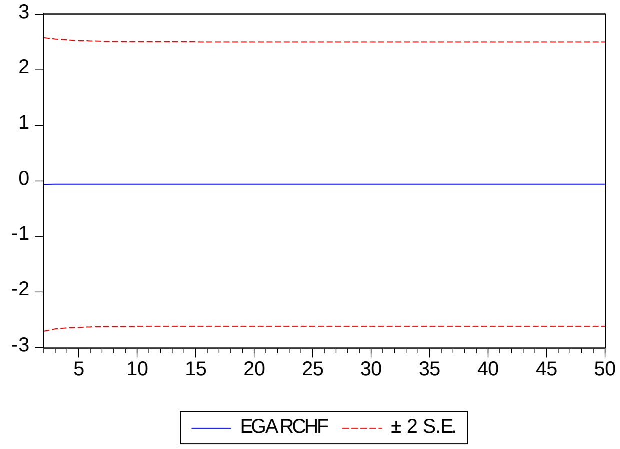
Dependent Variable: Y
Method: ML - ARCH (Marquardt) - Student's t distribution
Date: 08/15/17 Time: 20:28
Sample (adjusted): 2 198
Included observations: 197 after adjustments
Convergence achieved after 24 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)
*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.039898	0.080255	0.497144	0.6191
AR(1)	-0.053838	0.076905	-0.700062	0.4839
Variance Equation				
C(3)	-0.116174	0.180892	-0.642231	0.5207
C(4)	0.470653	0.239169	1.967872	0.0491
C(5)	-0.302460	0.138965	-2.176517	0.0295
C(6)	0.478044	0.307003	1.557132	0.1194
T-DIST. DOF	6.667645	3.488071	1.911556	0.0559

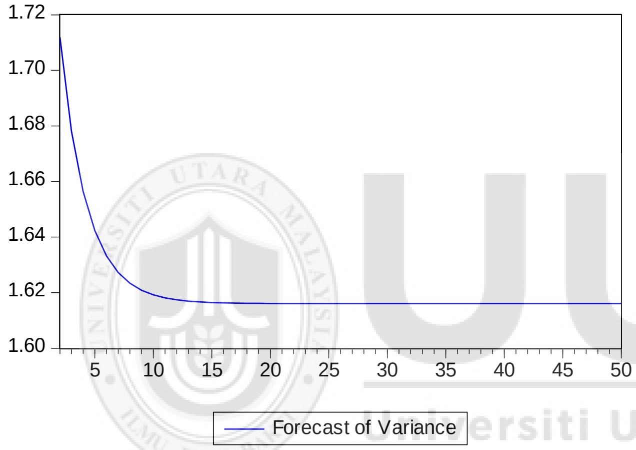
R-squared	-0.000733	Mean dependent var	0.137013
Adjusted R-squared	-0.005865	S.D. dependent var	1.293745
S.E. of regression	1.297533	Akaike info criterion	3.291818
Sum squared resid	328.3006	Schwarz criterion	3.408480
Log likelihood	-317.2440	Hannan-Quinn criter.	3.339043
Durbin-Watson stat	2.043412		

Inverted AR Roots	-.05

32. EGARCH Forecast N=200 (0.05 moderate leverage and 1.2 high skewness)



Forecast: EGARCHF	
Actual: B	
Forecast sample: 1 50	
Adjusted sample: 2 50	
Included observations: 48	
Root Mean Squared Error	1.310817
Mean Absolute Error	0.956724
Mean Abs. Percent Error	118.4882
Theil Inequality Coefficient	0.957897
Bias Proportion	0.000056
Variance Proportion	0.998823
Covariance Proportion	0.001122



33. MA estimation N=200(0.05 moderate leverage and 1.2 high skewness)

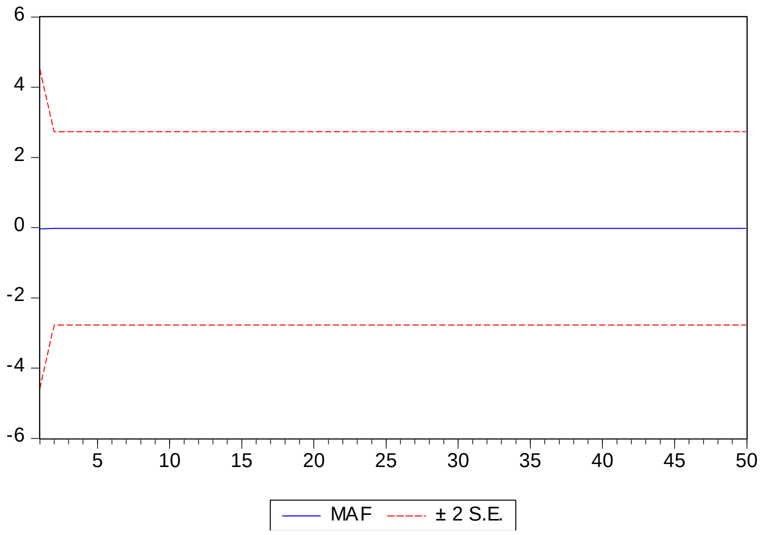
Dependent Variable: Y
Method: Least Squares
Date: 08/15/17 Time: 20:31
Sample (adjusted): 3 198
Included observations: 196 after adjustments
Convergence achieved after 5 iterations
MA Backcast: 2
Instrument specification: C
Lagged dependent variable & regressors added to instrument list

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.138510	0.084129	1.646390	0.1013
MA(1)	-0.091104	0.071729	-1.270117	0.2056

R-squared	0.007652	Mean dependent var	0.138906
Adjusted R-squared	0.002537	S.D. dependent var	1.296784
S.E. of regression	1.295138	Akaike info criteria	3.445014
Sum squared resid	325.4123	Schwarz criteria	3.494603
Log likelihood	-339.7712	Hannan-Quinn criter.	0.728171
F-statistic	0.728171	Durbin-Watson stat	1.986564
Prob(F-statistic)	0.393477		

Inverted MA Roots	.09

34. MA Forecast N=200(0.05 moderate leverage and 1.2 high skewness)



Forecast: MAF	
Actual: B	
Forecast sample: 1 50	
Included observations: 49	
Root Mean Squared Error	1.301238
Mean Absolute Error	0.948659
Mean Abs. Percent Error	103.4675
Theil Inequality Coefficient	0.983480
Bias Proportion	0.000922
Variance Proportion	0.996204
Covariance Proportion	0.002874

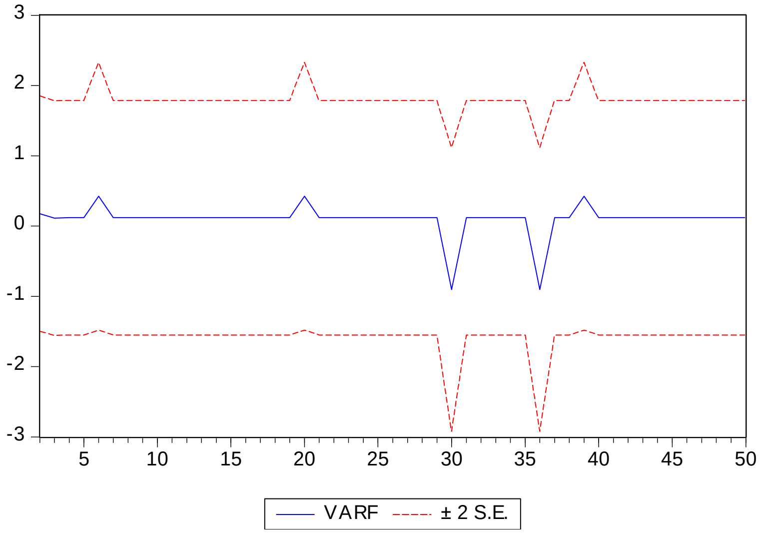
35. VAR estimationN=200(0.05 moderate leverage and 1.2 high skewness)

Vector Autoregression Estimates
Date: 08/15/17 Time: 18:01
Sample (adjusted): 2 200
Included observations: 199 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	Y	X15	X30
Y(-1)	-0.096363	0.000771	0.003290
	(0.07151)	(0.00553)	(0.00733)
	[-1.34758]	[0.13935]	[0.44891]
X15(-1)	-0.907070	-0.016690	0.034672
	(0.92724)	(0.07175)	(0.09502)
	[-0.97825]	[-0.23261]	[0.36488]
X30(-1)	-0.091227	0.021311	-0.045206
	(0.70050)	(0.05420)	(0.07179)
	[-0.13023]	[0.39317]	[-0.62972]
C	0.011936	-0.014479	0.033519
	(0.10471)	(0.00810)	(0.01073)
	[0.11399]	[-1.78698]	[3.12372]
R-squared	0.014939	0.001082	0.004069
Adj. R-squared	-0.000216	-0.014286	-0.011253
Sum sq. resids	399.2914	2.390778	4.193420
S.E. equation	1.430961	0.110727	0.146645
F-statistic	0.985776	0.070386	0.265560
Log likelihood	-351.6592	157.5890	101.6801
Akaike AIC	3.574465	-1.543608	-0.981710
Schwarz SC	3.640662	-1.477411	-0.915513
Mean dependent	0.020159	-0.013568	0.031658
S.D. dependent	1.430806	0.109944	0.145827

Determinant resid covariance (dof adj.)	0.000533
Determinant resid covariance	0.000502
Log likelihood	-91.14510
Akaike information criterion	1.036634
Schwarz criterion	1.235225

36. VAR forecast N=200(0.05 moderate leverage and 1.2 high skewness



Forecast: VARF	
Actual: P	
Forecast sample: 1 50	
Adjusted sample: 2 50	
Included observations: 49	
Root Mean Squared Error	0.791565
Mean Absolute Error	0.598730
Mean Abs. Percent Error	287.5479
Theil Inequality Coefficient	0.745013
Bias Proportion	0.000004
Variance Proportion	0.573649
Covariance Proportion	0.426347

37. GARCH estimation N=200(0.09 high leverage and 0.5 low skewness)

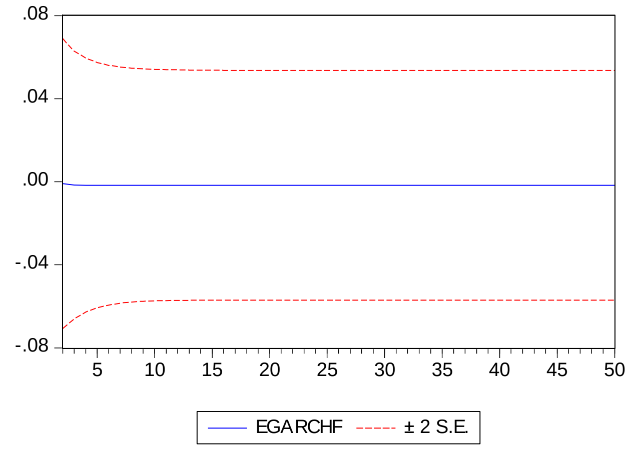
Dependent Variable: Y
Method: ML - ARCH (Marquardt) - Student's t distribution
Date: 08/16/17 Time: 07:05
Sample (adjusted): 2 200
Included observations: 199 after adjustments
Convergence achieved after 50 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)
*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000364	0.002049	0.177405	0.8592
AR(1)	0.100261	0.051974	1.929076	0.0537
		Variance Equation		
C(3)	-1.695209	0.857571	-1.976758	0.0481
C(4)	-0.364154	0.140392	-2.593830	0.0095
C(5)	0.228315	0.095214	2.397912	0.0165
C(6)	0.724207	0.127756	5.668674	0.0000
T-DIST. DOF	20.13125	28.31180	0.711055	0.4771

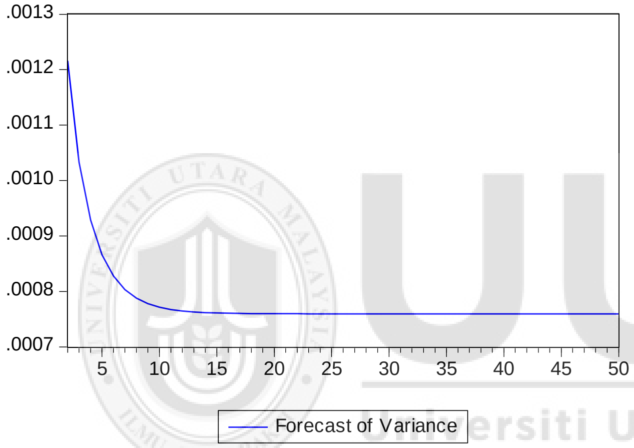
R-squared	-0.000624	Mean dependent var	-0.001484
Adjusted R-squared	-0.005704	S.D. dependent var	0.027825
S.E. of regression	0.027905	Akaike info criterion	-4.350280
Sum squared resid	0.153396	Schwarz criterion	-4.234435
Log likelihood	439.8529	Hannan-Quinn criter.	-4.303395
Durbin-Watson stat	2.011708		

Inverted AR Roots .10

38. GARCH forecast N=200(0.09 high leverage and 0.5 low skewness)



Forecast: EGARCHF	
Actual: Z	
Forecast sample: 1 50	
Adjusted sample: 2 50	
Included observations: 49	
Root Mean Squared Error	0.031946
Mean Absolute Error	0.024493
Mean Abs. Percent Error	119.6734
Theil Inequality Coefficient	0.949640
Bias Proportion	0.000031
Variance Proportion	0.990683
Covariance Proportion	0.009287



39. MA estimationN=200(0.09 high leverage and 0.5 low skewness)

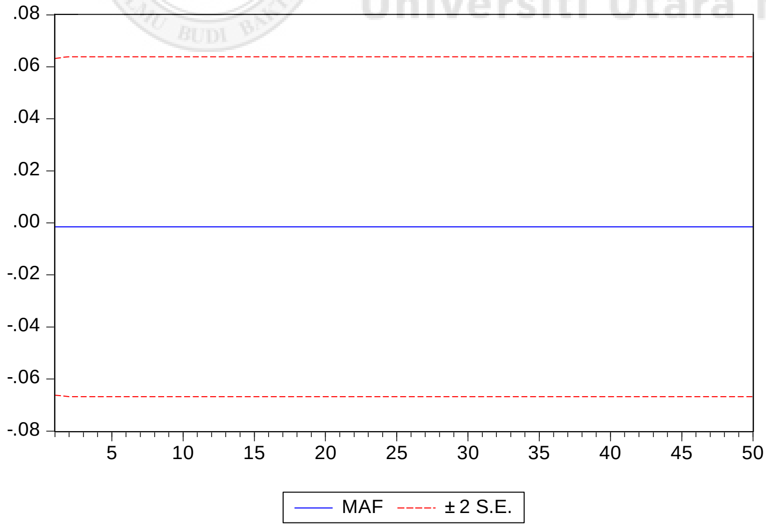
Dependent Variable: Y
Method: Least Squares
Date: 08/16/17 Time: 07:07
Sample: 1 200
Included observations: 200
Convergence achieved after 6 iterations
MA Backcast: 0

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.001682	0.002113	-0.795687	0.4272
MA(1)	0.073305	0.072104	1.016669	0.3106

R-squared	0.004641	Mean dependent var	-0.001644
Adjusted R-squared	-0.000386	S.D. dependent var	0.027847
S.E. of regression	0.027853	Akaike info criterion	-4.313832
Sum squared resid	0.153602	Schwarz criterion	-4.280849
Log likelihood	433.3832	Hannan-Quinn criter.	-4.300485
F-statistic	0.923289	Durbin-Watson stat	1.966967
Prob(F-statistic)	0.337785		

Inverted MA Roots	-.07

40. MA forecast N=200(0.09 high leverage and 0.5 low skewness)



Forecast: MAF
Actual: Z
Forecast sample: 1 50
Included observations: 50

Root Mean Squared Error	0.031678
Mean Absolute Error	0.024308
Mean Abs. Percent Error	116.9023
Theil Inequality Coefficient	0.953415
Bias Proportion	0.000000
Variance Proportion	0.999954
Covariance Proportion	0.000046

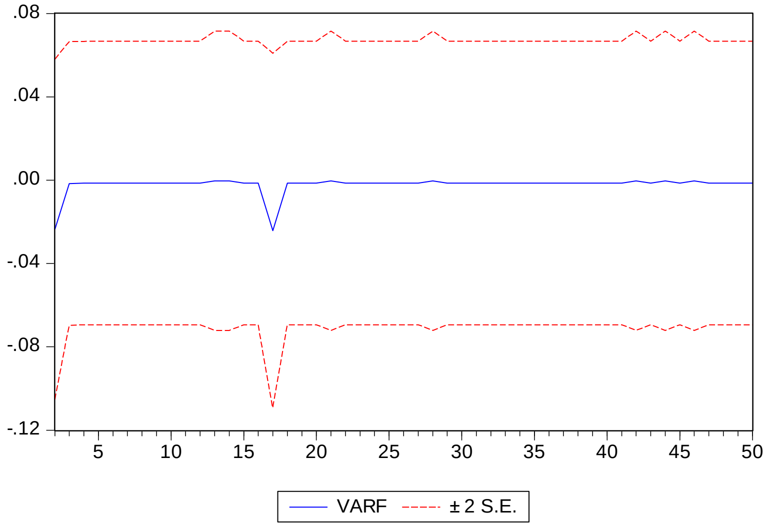
41. VAR estimation N=200(0.09 high leverage and 0.5 low skewness)

Vector Autoregression Estimates
Date: 08/16/17 Time: 07:32
Sample (adjusted): 2 200
Included observations: 199 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	Y	X23	X28
Y(-1)	0.066716	-0.046404	0.393600
	(0.07254)	(0.06841)	(0.21978)
	[0.91970]	[-0.67835]	[1.79087]
X23(-1)	0.041009	0.131080	0.190500
	(0.07520)	(0.07092)	(0.22785)
	[0.54530]	[1.84829]	[0.83607]
X28(-1)	-0.000226	0.015224	-0.043920
	(0.02344)	(0.02211)	(0.07103)
	[-0.00962]	[0.68862]	[-0.61836]
C	-0.001086	-0.006854	0.018723
	(0.00210)	(0.00198)	(0.00638)
	[-0.51622]	[-3.45349]	[2.93660]
R-squared	0.005724	0.022677	0.020568
Adj. R-squared	-0.009572	0.007642	0.005499
Sum sq. resids	0.152423	0.135548	1.399161
S.E. equation	0.027958	0.026365	0.084706
F-statistic	0.374228	1.508229	1.364973
Log likelihood	431.4840	443.1586	210.8957
Akaike AIC	-4.296322	-4.413654	-2.079354
Schwarz SC	-4.230125	-4.347457	-2.013157
Mean dependent	-0.001484	-0.007538	0.016080
S.D. dependent	0.027825	0.026466	0.084940

Determinant resid covariance (dof adj.)	3.88E-09
Determinant resid covariance	3.65E-09
Log likelihood	431.48
Akaike information criterion	-0.7956
Schwarz criterion	-0.5974

42. MA estimation N=200(0.09 high leverage and 0.5 low skewness)



Forecast: VARF	
Actual: U	
Forecast sample: 1 50	
Adjusted sample: 2 50	
Included observations: 49	
Root Mean Squared Error	0.032216
Mean Absolute Error	0.024667
Mean Abs. Percent Error	142.3351
Theil Inequality Coefficient	0.859572
Bias Proportion	0.000010
Variance Proportion	0.750625
Covariance Proportion	0.249365

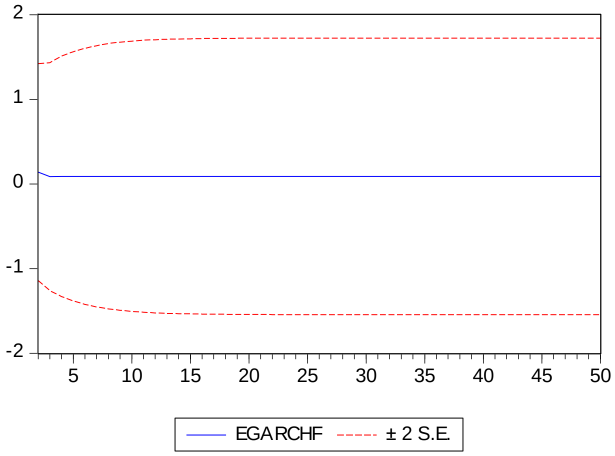
43. GARCH estimation N=200(0.09 high leverage and 0.5 low skewness)

Dependent Variable: Y
Method: ML - ARCH (Marquardt) - Student's t distribution
Date: 08/16/17 Time: 08:36
Sample (adjusted): 2 200
Included observations: 199 after adjustments
Convergence achieved after 14 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)
*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

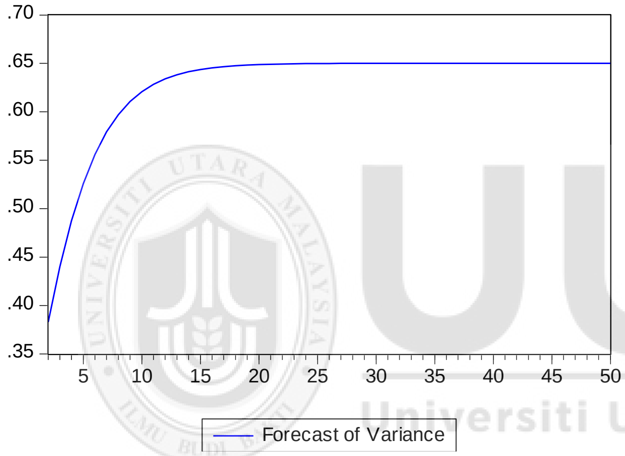
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.059694	0.081798	0.729775	0.4655
AR(1)	0.112505	0.079966	1.406903	0.1595
Variance Equation				
C(3)	-0.189063	0.211803	-0.892639	0.3721
C(4)	0.193143	0.171286	1.127604	0.2595
C(5)	-0.375537	0.104848	-3.581717	0.0003
C(6)	-0.235592	0.221064	-1.065715	0.2866
T-DIST. DOF	334.9883	11940.08	0.028056	0.9776

R-squared	0.005690	Mean dependent var	0.067286
Adjusted R-squared	0.000642	S.D. dependent var	1.033471
S.E. of regression	1.033139	Akaike info criterion	2.877969
Sum squared resid	210.2730	Schwarz criterion	2.993813
Log likelihood	-279.3579	Hannan-Quinn criter.	2.924854
Durbin-Watson stat	2.060795		
Inverted AR Roots	.11		

44. EGARCH forecast N=200(0.09 high leverage and 0.7 moderate skewness)



Forecast: EGARCHF	
Actual: A	
Forecast sample: 1 50	
Adjusted sample: 2 50	
Included observations: 49	
Root Mean Squared Error	0.817929
Mean Absolute Error	0.643386
Mean Abs. Percent Error	241.2879
Theil Inequality Coefficient	0.894688
Bias Proportion	0.000022
Variance Proportion	0.983621
Covariance Proportion	0.016357



45. MA estimation N=200 (0.09 high leverage and 0.7 moderate skewness)

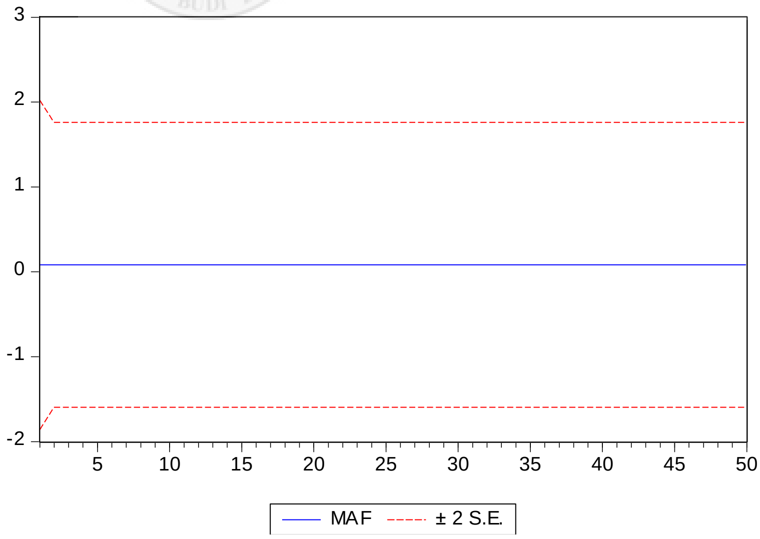
Dependent Variable: Y
Method: Least Squares
Date: 08/16/17 Time: 08:44
Sample: 1 200
Included observations: 200
Convergence achieved after 4 iterations
MA Backcast: 0

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.066745	0.078613	0.849032	0.3969
MA(1)	0.079587	0.071083	1.119625	0.2642

R-squared	0.006429	Mean dependent var	0.067154
Adjusted R-squared	0.001411	S.D. dependent var	1.030872
S.E. of regression	1.030145	Akaike info criterion	2.907225
Sum squared resid	210.1173	Schwarz criterion	2.940209
Log likelihood	-288.7225	Hannan-Quinn criter.	2.920573
F-statistic	1.281215	Durbin-Watson stat	1.993045
Prob(F-statistic)	0.259042		

Inverted MA Roots	-.08

46. MA forecast N=200(0.09 high leverage and 0.7 moderate skewness)



Forecast: MAF	
Actual: A	
Forecast sample: 1 50	
Included observations: 50	
Root Mean Squared Error	0.814519
Mean Absolute Error	0.642980
Mean Abs. Percent Error	228.9763
Theil Inequality Coefficient	0.902757
Bias Proportion	0.000004
Variance Proportion	0.999232
Covariance Proportion	0.000764

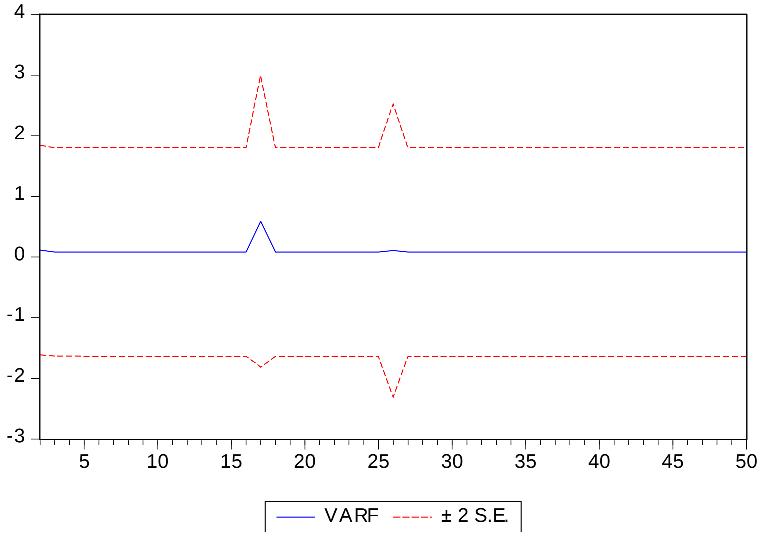
47. VAR estimation N=200 (0.09 high leverage and 0.7 moderate skewness)

Vector Autoregression Estimates
Date: 08/16/17 Time: 09:03
Sample (adjusted): 2 200
Included observations: 199 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	Y	X28	X33
Y(-1)	0.061035	0.001933	-0.000611
	(0.07120)	(0.00492)	(0.00985)
	[0.85728]	[0.39286]	[-0.06206]
X28(-1)	1.694881	-0.023155	-0.041185
	(1.03855)	(0.07179)	(0.14363)
	[1.63196]	[-0.32254]	[-0.28674]
X33(-1)	0.831829	-0.011992	-0.020462
	(0.52070)	(0.03599)	(0.07201)
	[1.59752]	[-0.33319]	[-0.28414]
C	0.029177	0.010386	0.020969
	(0.07421)	(0.00513)	(0.01026)
	[0.39317]	[2.02472]	[2.04315]
R-squared	0.031837	0.001649	0.000879
Adj. R-squared	0.016942	-0.013710	-0.014492
Sum sq. resids	204.7434	0.978283	3.916153
S.E. equation	1.024679	0.070830	0.141714
F-statistic	2.137472	0.107384	0.057179
Log likelihood	-285.1998	246.4997	108.4866
Akaike AIC	2.906531	-2.437183	-1.050117
Schwarz SC	2.972728	-2.370986	-0.983920
Mean dependent	0.067286	0.010050	0.020101
S.D. dependent	1.033471	0.070349	0.140698

Determinant resid covariance (dof adj.)	0.000104
Determinant resid covariance	9.75E-05
Log likelihood	71.81507
Akaike information criterion	-0.601156
Schwarz criterion	-0.402565

48. VAR forecast N=200 (0.09 high leverage and 0.7 moderate skewness)



Forecast: VARF	
Actual: P	
Forecast sample: 1 50	
Adjusted sample: 2 50	
Included observations: 49	
Root Mean Squared Error	0.815445
Mean Absolute Error	0.635168
Mean Abs. Percent Error	229.9929
Theil Inequality Coefficient	0.865169
Bias Proportion	0.000001
Variance Proportion	0.839008
Covariance Proportion	0.160991

49. EGARCH estimation N=200 (0.09 high leverage and 1.2 high skewness)

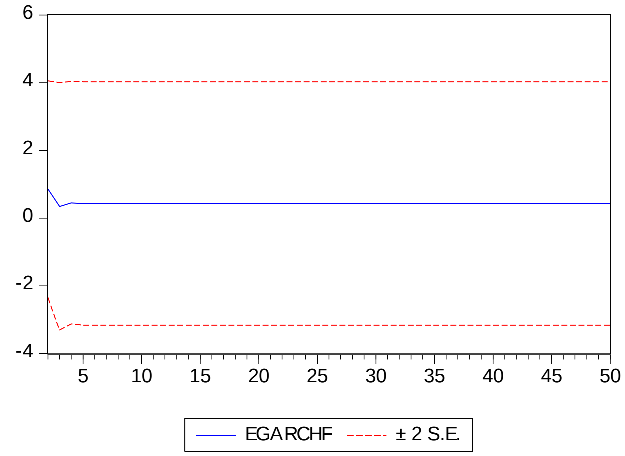
Dependent Variable: Y
Method: ML - ARCH (Marquardt) - Student's t distribution
Date: 08/16/17 Time: 09:24
Sample (adjusted): 2 200
Included observations: 199 after adjustments
Convergence achieved after 17 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(3) + C(4)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(5)
*RESID(-1)/@SQRT(GARCH(-1)) + C(6)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.020993	0.092883	0.226011	0.8212
AR(1)	-0.099867	0.075201	-1.328010	0.1842
Variance Equation				
C(3)	-0.051734	0.071413	-0.724428	0.4688
C(4)	0.142972	0.114151	1.252477	0.2104
C(5)	-0.001957	0.077030	-0.025411	0.9797
C(6)	0.923481	0.110526	8.355313	0.0000
T-DIST. DOF	7.895627	5.181577	1.523788	0.1276

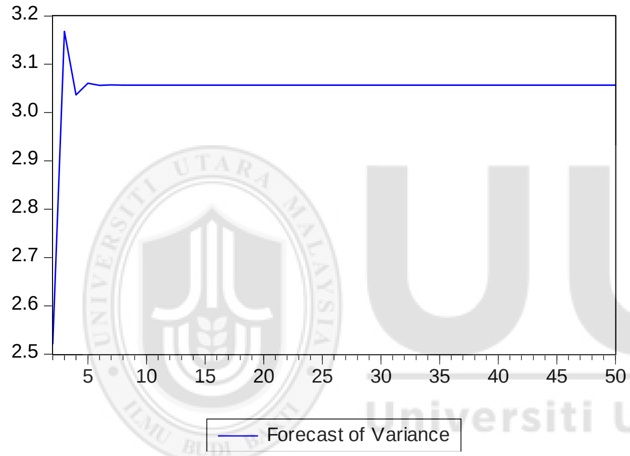
R-squared	0.010973	Mean dependent var	0.029784
Adjusted R-squared	0.005953	S.D. dependent var	1.502179
S.E. of regression	1.497701	Akaike info criterion	3.620747
Sum squared resid	441.8925	Schwarz criterion	3.736592
Log likelihood	-353.2643	Hannan-Quinn criter.	3.667632
Durbin-Watson stat	2.021259		

Inverted AR Roots	-.10

50. EGARCH forecast N=200 (0.09 high leverage and 1.2 high skewness)



Forecast: EGARCHF	
Actual: B	
Forecast sample: 1 50	
Adjusted sample: 2 50	
Included observations: 49	
Root Mean Squared Error	1.795642
Mean Absolute Error	1.265990
Mean Abs. Percent Error	381.1213
Theil Inequality Coefficient	0.791686
Bias Proportion	0.004807
Variance Proportion	0.935559
Covariance Proportion	0.059634



51. MA estimation N=200 (0.09 high leverage and 1.2 high skewness)

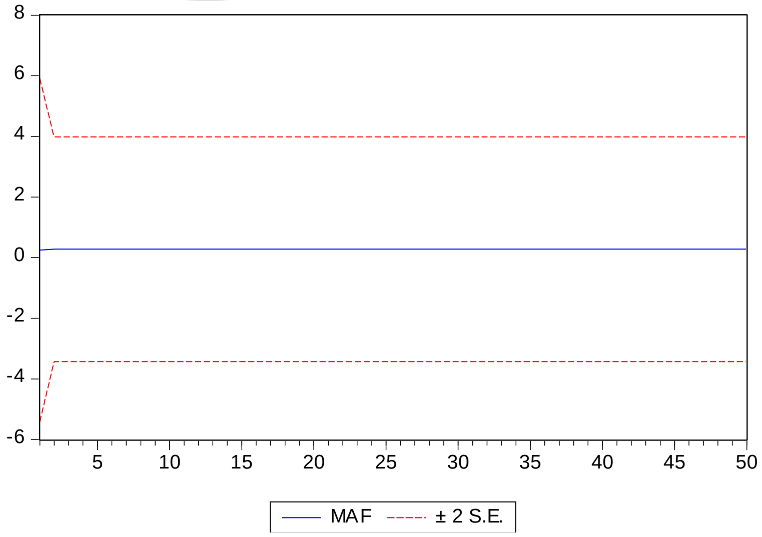
Dependent Variable: Y
Method: Least Squares
Date: 08/16/17 Time: 09:26
Sample: 1 200
Included observations: 200
Convergence achieved after 7 iterations
MA Backcast: 0

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.026671	0.090165	0.295803	0.7677
MA(1)	-0.145768	0.070557	-2.065941	0.0401

R-squared	0.014861	Mean dependent var	0.027471
Adjusted R-squared	0.009886	S.D. dependent var	1.498757
S.E. of regression	1.491331	Akaike info criterion	3.647164
Sum squared resid	440.3653	Schwarz criterion	3.680147
Log likelihood	-362.7164	Hannan-Quinn criter.	3.660512
F-statistic	2.986887	Durbin-Watson stat	1.953649
Prob(F-statistic)	0.085500		

Inverted MA Roots	.15

52. MA forecast N=200 (0.09 high leverage and 1.2 high skewness)



Forecast: MAF	
Actual: B	
Forecast sample: 1 50	
Included observations: 50	
Root Mean Squared Error	1.798919
Mean Absolute Error	1.301575
Mean Abs. Percent Error	272.7882
Theil Inequality Coefficient	0.857448
Bias Proportion	0.000000
Variance Proportion	0.995753
Covariance Proportion	0.004246

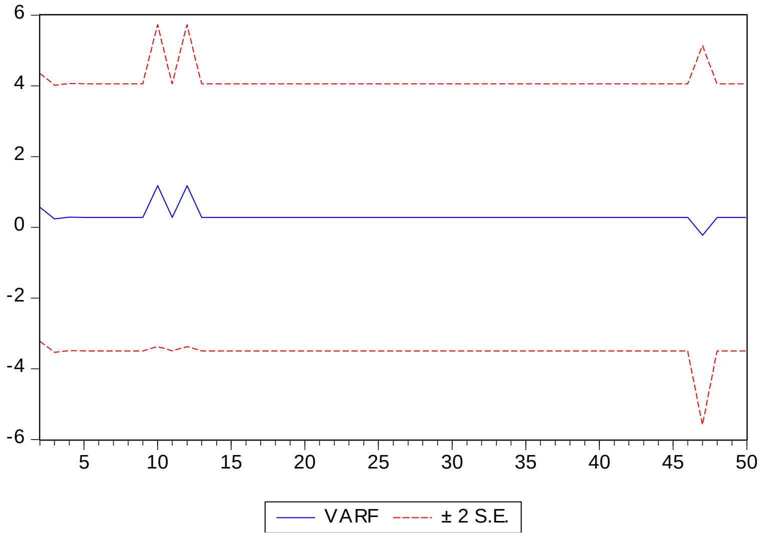
53. VAR estimation N=200 (0.09 high leverage and 1.2 high skewness)

Vector Autoregression Estimates
Date: 08/16/17 Time: 09:40
Sample (adjusted): 2 200
Included observations: 199 after adjustments
Standard errors in ( ) & t-statistics in [ ]

	Y	X30	X35
Y(-1)	-0.102344	-0.001940	0.005068
	(0.07105)	(0.00695)	(0.00572)
	[-1.44047]	[-0.27899]	[0.88634]
X30(-1)	-0.068993	-0.047167	-0.020079
	(0.73120)	(0.07156)	(0.05884)
	[-0.09436]	[-0.65913]	[-0.34123]
X35(-1)	-1.129529	-0.027219	-0.012485
	(0.88841)	(0.08695)	(0.07149)
	[-1.27140]	[-0.31305]	[-0.17463]
C	0.047915	0.033524	0.012731
	(0.10933)	(0.01070)	(0.00880)
	[0.43824]	[3.13304]	[1.44700]
R-squared	0.019188	0.003169	0.004604
Adj. R-squared	0.004099	-0.012166	-0.010710
Sum sq. resids	438.2219	4.197208	2.837929
S.E. equation	1.499097	0.146711	0.120638
F-statistic	1.271653	0.206670	0.300654
Log likelihood	-360.9161	101.5903	140.5292
Akaike AIC	3.667499	-0.980807	-1.372152
Schwarz SC	3.733696	-0.914610	-1.305955
Mean dependent	0.029784	0.031658	0.012060
S.D. dependent	1.502179	0.145827	0.119997

Determinant resid covariance (dof adj.)	0.000702
Determinant resid covariance	0.000661
Log likelihood	-118.5458
Akaike information criterion	1.312018
Schwarz criterion	1.510609

54. VARforecast N=200 (0.09 high leverage and 1.2 high skewness)



Forecast: VARF	
Actual: P	
Forecast sample: 1 50	
Adjusted sample: 2 50	
Included observations: 49	
Root Mean Squared Error	1.788754
Mean Absolute Error	1.295542
Mean Abs. Percent Error	281.9767
Theil Inequality Coefficient	0.815128
Bias Proportion	0.000001
Variance Proportion	0.800990
Covariance Proportion	0.199008

Appendix I

Code for CWN Estimation

The pseudo code for the R software functions for CWN.

```
library(MTS)
library(mnormt)
function (da, q = 1, include.mean = T, fixed = NULL, beta = NULL,
    sebeta = NULL, prelim = F, details = F, thres = 2)
{
    if (!is.matrix(da))
        da = as.matrix(da)
    nT = dim(da)[1]
    k = dim(da)[2]
    if (q < 1)
        q = 1
    kq = k * q
    THini <- function(y, x, q, include.mean) {
        #############
        ############
        ############
       ############
        nT = dim(y)[1]
        k = dim(y)[2]
        ist = 1 + q
        ne = nT - q
        if (include.mean) {
            xmtx = matrix(1, ne, 1)
        }
        else {
            xmtx = NULL
        }
        ymtx = y[ist:nT, ]
        for (j in 1:q) {
```

```
        xmtx = cbind(xmtx, x[(ist - j):(nT - j), ])
      }
      xtx = crossprod(xmtx, xmtx)
      xty = crossprod(xmtx, ymtx)
      xtxinv = solve(xtx)
      beta = xtxinv %*% xty
      resi = ymtx - xmtx %*% beta
      sse = crossprod(resi, resi)/ne
      dd = diag(xtxinv)
      sebeta = NULL
      for (j in 1:k) {
        se = sqrt(dd * sse[j, j])
        sebeta = cbind(sebeta, se)
      }
      THini <- list(estimates = beta, se = sebeta)
    }
    if (length(fixed) < 1) {
      m1 = VARorder(da, p=1, result = FALSE)
      porder = m1$aicor
      if (porder < 1)
        porder = 1
###############
###############
      x = m2$residuals
      m3 = THini(y, x, q, include.mean)
      beta = m3
      sebeta = m3
      nr = dim(beta)[1]
      if (prelim) {
        fixed = matrix(0, nr, k)
        for (j in 1:k) {
          tt = beta[, j]/sebeta[, j]
          idx = c(1:nr)[abs(tt) >= thres]
          fixed[idx, j] = 1
```

```
        }
    }
    if (length(fixed) < 1) {
        fixed = matrix(1, nr, k)
    }
}
else {
    nr = dim(beta)[1]
}
par = NULL
separ = NULL
fix1 = fixed
VMAcnt = 0
ist = 0
if (include.mean) {
    jdx = c(1:k)[fix1[1, ] == 1]
    VMAcnt = length(jdx)
    if (VMAcnt > 0) {
        par = beta[1, jdx]
        separ = sebeta[1, jdx]
    }
    TH = -beta[2:(kq + 1), ]
    seTH = sebeta[2:(kq + 1), ]
    ist = 1
}
else {
    TH = -beta
    seTH = sebeta
}
for (j in 1:k) {
    idx = c(1:(nr - ist))[fix1[(ist + 1):nr, j] == 1]
    if (length(idx) > 0) {
        par = c(par, TH[idx, j])
        separ = c(separ, seTH[idx, j])
```

```
    }
}
ParMA <- par
LLKvma <- function(par, zt = zt, q = q, fixed = fix1, include.mean = include.mean) {
    k = ncol(zt)
    nT = nrow(zt)
    mu = rep(0, k)
    icnt = 0
    VMAcnt <- 0
    fix <- fixed
    iist = 0
    if (include.mean) {
        iist = 1
        jdx = c(1:k)[fix[1, ] == 1]
        icnt = length(jdx)
        VMAcnt <- icnt
        if (icnt > 0)
            mu[jdx] = par[1:icnt]
    }
    for (j in 1:k) {
        zt[, j] = zt[, j] - mu[j]
    }
    kq = k * q
    Theta = matrix(0, kq, k)
    for (j in 1:k) {
        idx = c(1:kq)[fix[(iist + 1):(iist + kq), j] == 1]
        jcnt = length(idx)
        if (jcnt > 0) {
            Theta[idx, j] = par[(icnt + 1):(icnt + jcnt)]
            icnt = icnt + jcnt
        }
    }
    TH = t(Theta)
    if (q > 1) {
```

```
    tmp = cbind(diag(rep(1, (q - 1) * k)), matrix(0,
        (q - 1) * k, k))
    TH = rbind(TH, tmp)
}
mm = eigen(TH)
V1 = mm
P1 = mm
v1 = Mod(V1)
ich = 0
for (i in 1:kq) {
    if (v1[i] > 1)
        V1[i] = 1/V1[i]
    ich = 1
}
if (ich > 0) {
    P1i = solve(P1)
    GG = diag(V1)
    TH = Re(P1 %*% GG %*% P1i)
    Theta = t(TH[1:k, ])
    ist = 0
    if (VMAcnt > 0)
        ist = 1
    for (j in 1:k) {
        idx = c(1:kq)[fix[(ist + 1):(ist + kq), j] ==
          1]
        jcnt = length(idx)
        if (jcnt > 0) {
          par[(icnt + 1):(icnt + jcnt)] = TH[j, idx]
          icnt = icnt + jcnt
        }
    }
}
at = mFilter(zt, t(Theta))
sig = t(at) %*% at/nT
```

```
    ll = dmvnorm(at, rep(0, k), sig)
    LLKvma = -sum(log(ll))
    LLKvma
  }
  cat("Number of parameters: ", length(par), "\n")
  cat("initial estimates: ", round(par, 4), "\n")
  lowerBounds = par
  upperBounds = par
  npar = length(par)
  mult = 2
  if ((npar > 10) || (q > 2))
    mult = 1.2
  for (j in 1:npar) {
    lowerBounds[j] = par[j] - mult * separ[j]
    upperBounds[j] = par[j] + mult * separ[j]
  }
  cat("Par. Lower-bounds: ", round(lowerBounds, 4), "\n")
cat("Par. Upper-bounds: ", round(upperBounds, 4), "\n")
  if (details) {
    fit = nlminb(start = ParMA, objective = LLKvma, zt = da,
      fixed = fixed, include.mean = include.mean, q = q,
      lower = lowerBounds, upper = upperBounds, control = list(trace = 3))
  }
  else {
    fit = nlminb(start = ParMA, objective = LLKvma, zt = da,
      fixed = fixed, include.mean = include.mean, q = q,
      lower = lowerBounds, upper = upperBounds)
  }
  epsilon = 1e-04 * fit$par
  npar = length(par)
  Hessian = matrix(0, ncol = npar, nrow = npar)
  for (i in 1:npar) {
    for (j in 1:npar) {
      x1 = x2 = x3 = x4 = fit
```

```
        x1[i] = x1[i] + epsilon[i]
        x1[j] = x1[j] + epsilon[j]
        x2[i] = x2[i] + epsilon[i]
        x2[j] = x2[j] - epsilon[j]
        x3[i] = x3[i] - epsilon[i]
        x3[j] = x3[j] + epsilon[j]
        x4[i] = x4[i] - epsilon[i]
        x4[j] = x4[j] - epsilon[j]
        Hessian[i, j] = (LLKvma(x1, zt = da, q = q, fixed = fixed,
          include.mean = include.mean) - LLKvma(x2, zt = da,
          q = q, fixed = fixed, include.mean = include.mean) -
          LLKvma(x3, zt = da, q = q, fixed = fixed, include.mean = include.mean) +
          LLKvma(x4, zt = da, q = q, fixed = fixed, include.mean = include.mean))/(4 *
          epsilon[i] * epsilon[j])
  }
}
est = fit$par
cat("Final   Estimates: ", est, "\n")
se.coef = sqrt(diag(solve(Hessian)))
tval = fit$par/se.coef
matcoef = cbind(fit$par, se.coef, tval, 2 * (1 - pnorm(abs(tval))))
dimnames(matcoef) = list(names(tval), c(" Estimate", " Std. Error",
  " t value", "Pr(>|t|)"))
cat("\nCoefficient(s):\n")
printCoefmat(matcoef, digits = 4, signif.stars = TRUE)
cat("---", "\n")
cat("Estimates in matrix form:", "\n")
icnt = 0
ist = 0
cnt = NULL
if (include.mean) {
  ist = 1
  cnt = rep(0, k)
  secnt = rep(1, k)
```

```
    jdx = c(1:k)[fix1[1, ] == 1]
    icnt = length(jdx)
    if (icnt > 0) {
        cnt[jdx] = est[1:icnt]
        secnt[jdx] = se.coef[1:icnt]
        cat("Constant term: ", "\n")
        cat("Estimates: ", cnt, "\n")
    }
}
cat("MA coefficient matrix", "\n")
TH = matrix(0, kq, k)
seTH = matrix(1, kq, k)
for (j in 1:k) {
    idx = c(1:kq)[fix1[(ist + 1):nr, j] == 1]
    jcnt = length(idx)
    if (jcnt > 0) {
        TH[idx, j] = est[(icnt + 1):(icnt + jcnt)]
        seTH[idx, j] = se.coef[(icnt + 1):(icnt + jcnt)]
        icnt = icnt + jcnt
    }
}
icnt = 0
for (i in 1:q) {
    cat("MA(", i, ")-matrix", "\n")
    theta = t(TH[(icnt + 1):(icnt + k), ])
    print(theta, digits = 3)
    icnt = icnt + k
}
zt = da
if (include.mean) {
    for (i in 1:k) {
        zt[, i] = zt[, i] - cnt[i]
    }
}
```

```
    at = mFilter(zt, t(TH))
    sig = t(at) %*% at/nT
    cat(" ", "\n")
    cat("Residuals cov-matrix:", "\n")
    print(sig)
    dd = det(sig)
    d1 = log(dd)
    aic = d1 + 2 * npar/nT
    bic = d1 + log(nT) * npar/nT
    cat("----", "\n")
    cat("aic= ", aic, "\n")
    cat("bic= ", bic, "\n")
    Theta = t(TH)
    if (include.mean) {
        TH = rbinds(cnt, TH)
        seTH = rbind(secnt, seTH)
    }
VMA<list(data=da,MAorder=q,cnst=include.mean,coef=TH,se=seTH,residuals=at,Sigma=sig,
aic=aic,bic=bic)
```