A FAMILY OF CLASSES IN NESTED CHAIN ABACUS AND RELATED GENERATING FUNCTIONS

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Abstrak

Model abakus telah digunakan secara meluas untuk mewakili pemetakan bagi sebarang integer positif. Walau bagaimanapun, tiada kajian yang telah dilakukan untuk membangunkan manik abakus terkait dalam perwakilan bergraf bagi objek diskrit. Untuk mengatasi masalah keterkaitan, kajian ini tertumpu kepada pencirian n-objek terkait yang dikenali sebagai n-omino terkait, seterusnya menjana abakus rantai tersarang. Selanjutnya, sifat konsep teori bagi abakus rantai tersarang dibangunkan. Di samping itu, tiga jenis penjelmaan berbeza yang penting dalam pembinaan famili kelas turut dihasilkan. Fungsi penjana turut dirumuskan berdasarkan kelas ini dengan menggunakan pengangkaan objek kombinatorik (ECO). Dalam kaedah ECO, setiap objek diperoleh daripada objek yang lebih kecil dengan membuat pengembangan setempat. Pengembangan setempat ini dihuraikan dengan cara yang mudah melalui petua turutan. Kemudian petua turutan boleh diterjemahkan menjadi persamaan fungsian untuk fungsi penjana. Kesimpulannya, kajian ini berjaya menghasilkan perwakilan bergraf baru bagi abakus rantai tersarang yang dapat diaplikasikan dalam grid terhingga penjubinan.

Abstract

Abacus model has been employed widely to represent partitions for any positive integer. However, no study has been carried out to develop connected beads of abacus in graphical representation for discrete objects. To resolve this connectedness problem this study is oriented in characterising n - connected objects knows as n connected ominoes, which then generate nested chain abacus. Furthermore, the theoretical conceptual properties for the nested chain abacus are being formulated. Along the construction, three different types of transformation are being created that are essential in building a family of classes. To enhance further, based on theses classes, generating functions are also being formulated by employing enumeration of combinatorial objects (ECO). In ECO method, each object is obtained from smaller object by making some local expansions. These local expansions are described in a simple way by a succession rule which can be translated into a function equation for the generating function. In summary, this stud has succeeded in producing novel graphical representation of nested chain abacus, which can be applied in tiling finite grid.

Keywords: abacus, partition, n-connected omnioes, generating function

.

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List of Symbols

$\mu^{(e,r)}$	Connected Partition with e Columns and r Rows
$\mathfrak{N}$	Nested Chin Abacus
SR	Set-Row
SC	Set-Column
P_{ρ}^{Rec}	Sequence of Rectangular Nested Chain Abacus
P_{ρ}^{Rec-h}	Sequence of Rectangle Path Nested Chain Abacus
$SNC2$	Single Nested Chain Abacus Transformation
SNC	Singular Nested Chain Abacus Transformation
MNC	Multiple nested Chain Abacus
$\mathfrak{D}_{single}$	Singular Transformation Class
$\mathfrak{D}_{outer}$	Single Transformation Class, if $i = 1$
$\mathfrak{D}_{inner-i}$	Single Transformation Class, if $i > 1$
$\mathfrak{D}_{inner}$	Multi Transformation Classes
b_i	Number of Beads Positions in Chain i
$b_i^{'}$	Number of Empty Bead Positions in Chain i
$\mathfrak{I}^2$	Classes of Nested Chain Abacus with two Columns
$\mathfrak{N}_c$	Class of Nested Chin Abacus w.r.t columns
$\mathfrak{N}_r$	Class of Nested Chin Abacus w.r.t rows

Declaration Associated With This Thesis

Journal

1. Mohommed, E. F., Ahmad, N., Ibrahim, H. (2016). Intersection of Main James Abacus Diagram for the Outer Chain Movement with Length [1, 0, 0...]. Journal of Telecommunication, Electronic and Computer Engineering (JTEC), 8(8), 51-56 (scopus).
2. Mohommed, E. F., Ahmad, N., Ibrahim, H., Mahmood, A. S. (2016). Nested Chain Movement of length 1 of Beta Number in James Abacus Diagram. Global Journal of Pure and Applied Mathematics, 12(4), 2953-2969 (scopus).
3. Mohommed, E. F., Ibrahim, H. Ahmad, N. (2017). Enumeration of n-connected objects inscribed in an abacus. Journal of Algebra, Number Theory Applications, (scopus).
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proceeding

1. Mohommed, E. F., Ibrahim, H., Mahmood, A. S., Ahmad, N. (2015, December). Embedding chain movement in James diagram for partitioning beta number. In AIP Conference Proceedings (Vol. 1691, No. 1, p. 040019). AIP Publishing.
2. Mohommed, E. F., Ibrahim, H., Ahmad, N., Mahmood, A. (2016, August). Embedding the outer chain movement for main partition of β-number with length [1, 0, 0,]. In AIP Conference Proceedings (Vol. 1761, No. 1, p. 020076). AIP Publishing.

CHAPTER ONE

INTRODUCTION

1.1 Introduction

The theory of partition is a fundamental area of number theory, it is concerning the representation of integer as sum of other integers. The theory of partition has been applied in many different areas such as combinatorics, statistical and particle physic. The partitions can be graphically represented with diagrams such as Ferrers diagram and Young diagram. Agraphical representation of partition is important in the partition theory because it can design and facilitate a visual structure of any shape in the form of discrete object. Henceforth, this thesis focuses on the use of graphical illustration of partition to develop a new design structure of connected ominoes. The beauty of this construction is further extended to be used in tiling fnite grid.

1.2 Graphical Representation of Partition

Diagrams are used to represent a partition of any positive integer. Since 1800s, the famous diagrams are the Ferrers diagram and the Young diagram (Benjamin & Quinn, 2003; Hardy & Wright, 1979). On the other hand, a James diagram or known as *e*-abacus uses a β-number to represent a sequence of non-decreasing integer numbers (Gyoja et al., 2010). Next, the concept of partition and graphical representation of the partition are reviewed.

Definition 1.2.1. *(Andrews, 1998) A partition of a positive integer, t, is a finite non-increasing sequence of non-negative integers* $(\mu_1, \mu_2, \ldots, \mu_n)$ *such that* $\sum_{i=1}^{n} \mu_i = t$ *and n is the number of parts of any partition.*

Example 1.2.2. $(5,3,3,2,1), (5,5,2,2), (6,4,2,1,1), \cdots$ *are partitions of* $t = 14$. *If* $\mu = (5,3,3,2,1)$, *then* $n = 5$.

The contents of the thesis is for internal user only

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APPENDIX A

GENERATING FUNCTION W.R.T CHAINS

Input Rows and columns

r=input('Input the number of rows:');

e=input('Input the number of columns:');

* Classify cases

* Case 1(If r < e and r is odd, then p1=e-r + 1 p2=2p1+6)

if r<e mod(r,2)==1

P1=e-r+1;

P2=2*P1+6;

end * Case 2(If e < r and e is odd. then p1= r-e + 1 and p2 = 2 p1 + 6)

if e<r mod(e,2)==1

P1=r-e+1;

P2=2*P1+6;

end * Case 3(If e < r and e is odd. then p1= r-e + 1 and p2 = 2 p1 + 6)

if e==r mod(r,2)==1 mod(e,2)==1

P1=1;

P2=8;

end

* Case 4(If r < e and r is even then p1= 2r + 2e - 4 (2c- 1)=2r-2e+4, where c= r/2 and p2=p1 + 8.)

if r<=e mod(r,2)==0

P1=2*r-2*e+4;

P2=P1+8;

end

* Case 5(If e <= r and e is even then p1= 2r + 2e - 4 (2c- 1)=2e-2r+4, where c=e/2 and p2 = p1 + 8)

```
if e<=r mod(e,2)==0

P1=2*e-2*r+4

P2=P1+8

end

** Compute the generating function syms x

syms y

* f(x,y) fprintf('================f(x,y)=================')

f(x,y)=-exp(1/(8*x*y^8)) * int(y^(P2 - 9) *exp(-1/8 *x*y^8), y)

· Generatingfunctionf(x)
fprintf('============= Generatingfunctionf(x) =================

)

f(x, 1)

· PolynomialFrom
ff(1) = 1;ff(2) = P2;

forn = 3 : 10

ff(n) = ff(n-1) * (P2 + 8);

end

* Writethisresult

fori = 1 : 10

fprintf('f(*d) = *d', i, ff(i))

end
```

APPENDIX B

TILLING ALGORITHM W.R.T ROW

theta$_c ol = 25;$

$theta_r ow = 25;$

$H1 = 3;$

$r = 5;$

$e = 5;$

$Computetheuppero f s, s^I, s^{II}$

$s_l im = (ceil(theta_r ow/r)) - 1;$

$s1_l im = (ceil((theta_c ol - e)/H1)) + 3;$

$s2_l im = ceil(theta_c ol/H1);$

$Setcolors$

$color2 = [255, 217, 102]/255;$

$color1 = [131, 59, 10]/255;$

$\cdot$ $Generatinginitialabacus$

$*temp = ceil(rand(1, e) * (r - num_b ead + 1));$

$temp = [2, 3, 2, 1, 2];$

$rect = zeros(theta_r ow, theta_c ol);$

$f ori = 1 : r$

$rect(i, temp(i) : temp(i) + H1 - 1) = 1;$

end

$Draw_s hape(rect, theta_r ow, theta_c ol, color1, color2)$

$title(^I InitialRectangle^I)$

$H = max(temp);$

$L = temp(1, 1);$

$L1 = temp(1, end);$

$p = H1;$

$p1 = L1 - L;$

$\cdot IfL1 > L, thenapplicatethemapping1andmapping2, mapping3$
$ifL1 > L$

$form = 1 : r$

$forj = 1 : e$

$fors = 1 : s_lim$

$ifrect(m, j) == 1and(j + s * p1) > 0and(j + s * p1) <= theta_colandmod(s, 2)$

$1rect(m + s * r, j + s * p1) = 0;$

$elseifrect(m, j) == 0and(j + s * p1) > 0and(j + s * p1) <= theta_coland$

$mod(s, 2) == 1$

$rect(m + s * r, j + s * p1) = 1;$

$elseifrect(m, j) == 1and(j + s * p1) > 0and(j + s * p1) <= theta_coland$

$mod(s, 2) == 0$

$rect(m + s * r, j + s * p1) = 1;$

$elseifrect(m, j) == 0and(j + s * p1) > 0and(j + s * p1) <= theta_coland$

$mod(s, 2) == 0$

$rect(m + s * r, j + s * p1) = 0;$

end

end

end

end

$Draw_shape(rect, theta_row, theta_col, color1, color2)$

$title(^IRectangleafterMapping1^I)$

$form = 1 : r$

$rect(m + s * r, j + s * p1 + s1 * p) = 1;$

$elseifrect(m, j) == 0and(m + s * p1 + s1 * p) > 0and$

$(m + s * p1 + s1 * p) <= theta_colandmod(s, 2) == 1andmod(s1, 2) == 1$

$rect(m + s * r, j + s * p1 + s1 * p) = 0;$

end

end

end

end

end

$Draw_shape(rect, theta_row, theta_col, color1, color2)$

$title(^I Rectangleaf terMapping2^I)$

end

$i f L1 > L$

$f orm = 1 : r$

$f or j = 1 : e$

$f ors = 1 : s_lim$

$ifrect(m, j) == 1and(j + s * p1) > 0and(j + s * p1) <= theta_coland$

$mod(s, 2) == 1$

$rect(m + s * r, j + s * p1) = 0;$

$elseifrect(m, j) == 0and(j + s * p1) > 0and(j + s * p1) <= theta_coland$

$mod(s, 2) == 1$

$rect(m + s * r, j + s * p1) = 1;$

$elseifrect(m, j) == 1and(j + s * p1) > 0and(j + s * p1) <= theta_coland$

$mod(s, 2) == 0$

$rect(m + s * r, j + s * p1) = 1;$

$elseifrect(m, j) == 0and(j + s * p1) > 0and(j + s * p1) <= theta_coland$

$mod(s, 2) == 0$

$rect(m + s * r, j + s * p1) = 0;$

end

end

end

end

$Draw_shape(rect, theta_row, theta_col, color1, color2)$

$title({}^{I}RectangleafterMapping1^{I})$

$form = 1 : r$

$forj = H : e$

$fors = 0 : s_lim$

$fors1 = 1 : s1_lim$

$ifrect(m, j) == 1and(m + s * p1 + s1 * p) > 0and(m + s * p1 + s1 * p) <= theta_col$

$andmod(s, 2) == 0andmod(s1, 2) == 1$

$rect(m + s * r, j + s * p1 + s1 * p) = 0;$

$elseifrect(m, j) == 0and(m + s * p1 + s1 * p) > 0and$

$(m + s * p1 + s1 * p) <= theta_colandmod(s, 2) == 0andmod(s1, 2) == 1$

$rect(m + s * r, j + s * p1 + s1 * p) = 1;$

$elseifrect(m, j) == 1and(m + s * p1 + s1 * p) > 0and$

$(m + s * p1 + s1 * p) <= theta_colandmod(s, 2) == 0andmod(s1, 2) == 0$

$rect(m + s * r, j + s * p1 + s1 * p) = 1;$

$elseifrect(m, j) == 0and(m + s * p1 + s1 * p) > 0and (m + s * p1 + s1 * p) <$

$0andmod(s1, 2) == 0$

$rect(m + s * r, j + s * p1 + s1 * p) = 0;$

$elseifrect(m, j) == 1and(m + s * p1 + s1 * p) > 0and$

$(m + s * p1 + s1 * p) <= theta_colandmod(s, 2) == 1andmod(s1, 2) == 0$

$rect(m + s * r, j + s * p1 + s1 * p) = 0;$

$elseifrect(m, j) == 0and(m + s * p1 + s1 * p) > 0and$

$(m + s * p1 + s1 * p) <= theta_colandmod(s, 2) == 1andmod(s1, 2) == 0$

$rect(m + s * r, j + s * p1 + s1 * p) = 1;$

$elseifrect(m, j) == 1and(m + s * p1 + s1 * p) > 0and$

$(m + s * p1 + s1 * p) <= theta_{c}olandmod(s, 2) == 1andmod(s1, 2) == 1$

$rect(m + s * r, j + s * p1 + s1 * p) = 1;$

$elseif rect(m, j) == 0and(m + s * p1 + s1 * p) > 0and$

$(m + s * p1 + s1 * p) <= theta_{c}olandmod(s, 2) == 1andmod(s1, 2) == 1$

$rect(m + s * r, j + s * p1 + s1 * p) = 0;$

end

end

end

end

end

$Draw_{s}hape(rect, theta_{r}ow, theta_{c}ol, color1, color2)$

$title({}^{I}RectangleafterMapping2^{I})$

$form = 1 : r$

$forj = 1 : H1$

$fors = 1 : s1_{l}im$

$fors2 = 1 : s2_{l}im$

$forj = H : e$

$fors = 0 : s_{l}im$

$fors1 = 1 : s1_{l}im$

$ifrect(m, j) == 1and(m + s * p1 + s1 * p) > 0and(m + s * p1 + s1 * p) <= theta_{c}ol$

$andmod(s, 2) == 0andmod(s1, 2) == 1$

$rect(m + s * r, j + s * p1 + s1 * p) = 0;$

$elseif rect(m, j) == 0and(m + s * p1 + s1 * p) > 0and$

$(m + s * p1 + s1 * p) <= theta_{c}olandmod(s, 2) == 0andmod(s1, 2) == 1$

$rect(m + s * r, j + s * p1 + s1 * p) = 1;$

$elseif rect(m, j) == 1and(m + s * p1 + s1 * p) > 0and$

$(m + s * p1 + s1 * p) <= theta_{c}olandmod(s, 2) == 0andmod(s1, 2) == 0$

$rect(m + s * r, j + s * p1 + s1 * p) = 1;$

$elseif rect(m, j) == 0 and (m + s * p1 + s1 * p) > 0 and$

$(m + s * p1 + s1 * p) <= theta_colandmod(s, 2) == 0andmod(s1, 2) == 0$

$rect(m + s * r, j + s * p1 + s1 * p) = 0;$

$elseif rect(m, j) == 1 and (m + s * p1 + s1 * p) > 0 and$

$(m + s * p1 + s1 * p) <= theta_colandmod(s, 2) == 1andmod(s1, 2) == 0$

$rect(m + s * r, j + s * p1 + s1 * p) = 0;$

$elseif rect(m, j) == 0 and (m + s * p1 + s1 * p) > 0 and$

$(m + s * p1 + s1 * p) <= theta_colandmod(s, 2) == 1andmod(s1, 2) == 0$

$rect(m + s * r, j + s * p1 + s1 * p) = 1;$

$elseif rect(m, j) == 1 and (m+s*p1+s1*p) > 0 and (m+s*p1+s1*p) <$

$1andmod(s1, 2) == 1$

$rect(m + s * r, j + s * p1 + s1 * p) = 1;$

$elseif rect(m, j) == 0 and (m + s * p1 + s1 * p) > 0 and$

$(m + s * p1 + s1 * p) <= theta_colandmod(s, 2) == 1andmod(s1, 2) == 1$

$rect(m + s * r, j + s * p1 + s1 * p) = 0;$

end

end

end

end

end

$Draw_shape(rect, theta_row, theta_col, color1, color2)$

$title({}^{I}RectangleafterMapping2^{I})$

$form = 1 : r$

$forj = 1 : H1$

$fors = 1 : s1_lim$

$fors2 = 1 : s2_lim$

$f\,unctionDraw_shape(M, a, b, color1, color2)$

$f\,igure$

$axis([0b0a])$

$holdon$

$f\,ori = 1 : a$

$f\,orj = 1 : b$

$i\,f\,M(i, j) == 1$

$rectangle(^IPosition^I, [\,j - 1, a - i, 1, 1],^I\ FaceColor^I, color1); elsei\,f\,M(i, j) == 0$

$rectangle(^IPosition^I, [j - 1, a - i, 1, 1],^IFaceColor^I, color2);$

end

end

end

end

end

APPENDIX C

TILLING ALGORITHM W.R.T COLUMN

Initial conditions $theta_{c}ol = 30;$

$theta_{r}ow = 30;$

$d1 = 3;$

$r = 5;$

$e = 5;$

$s1_{l}im = ceil(theta_{c}ol/e) - 1;$

$s_{l}im = ceil((r - theta_{r}ow)/d1);$

$s2_{l}im = ceil(theta_{c}ol/e) - 1;$

$color1 = [56, 85, 34]/255;$

$color2 = [156, 194, 228]/255;$

$temp = [2, 1, 2, 3, 2];$

$rect = zeros(theta_{r}ow, theta_{c}ol);$

$f ori = 1 : e$

$rect(temp(i) : temp(i) + d1 - 1, i) = 1;$
end

$Draw_{s}hape(rect, theta_{r}ow, theta_{c}ol, color1, color2)$
$title(^{I}InitialRectangle^{I})$

$k = max(temp);$

$k1 = temp(1,\ 1);$

$k2 = temp(1, end);$

$p = d1;$

$p1 = k2 - k1;$

$i f k2 <= k1$

$f orm = 1 : r$

$f or j = 1 : e$

$for s1 = 1 : s1_lim$

$if rect(m, j) == 1 and (m + s1 * p1) > 0 and$

$(m + s1 * p1) <= theta_row and mod(s1, 2) == 1$

$rect(m + s1 * p1, j + s1 * e) = 0;$

$elseif rect(m, j) == 0 and (m + s1 * p1) > 0$

$and (m + s1 * p1) <= theta_row and mod(s1, 2) == 1$

$rect(m + s1 * p1, j + s1 * e) = 1;$

$elseif rect(m, j) == 1 and (m + s1 * p1) > 0$

$and (m + s1 * p1) <= theta_row and mod(s1, 2) == 0$

$rect(m + s1 * p1, j + s1 * e) = 1;$

$elseif rect(m, j) == 0 and (m + s1 * p1) > 0$

$and (m + s1 * p1) <= theta_row and mod(s1, 2) == 0$

$rect(m + s1 * p1, j + s1 * e) = 0;$

end

end

end

end

$Draw_shape(rect, theta_row, theta_col, color1, color2)$

$for m = r - p + 1 : r$

$for j = 1 : e$

$for s = 0 : 6$

$for s1 = 1 : 12$

$if rect(m, j) == 1 and (m + s * p1 + s1 * p) > 0$

$and (m + s * p1 + s1 * p) <= theta_col and mod(s, 2) == 0 and$

$mod(s1, 2) == 1$

$rect(m + s * p1 + s1 * p, j + s * e) = 0;$

$elseif rect(m, j) == 0 and$

$(m+s*p1+s1*p)>0and(m+s*p1+s1*p)<=theta_oland$

$mod(s,2)==0andmod(s1,2)==1$

$rect(m+s*p1+s1*p, j+s*e) = 1;$

$elseifrect(m, j) == 1and$

$(m+s*p1+s1*p)>0and(m+s*p1+s1*p)<=theta_oland$

$mod(s,2)==0andmod(s1,2)==0$

$rect(m+s*p1+s1*p, j+s*e) = 1;$

$elseifrect(m, j) == 0and$

$(m+s*p1+s1*p)>0and(m+s*p1+s1*p)<=theta_oland$

$mod(s,2)==0andmod(s1,2)==0$

$rect(m+s*p1+s1*p, j+s*e) = 0;$

$elseifrect(m, j) == 1and$

$(m+s*p1+s1*p)>0and(m+s*p1+s1*p)<=theta_oland$

$mod(s,2)==1andmod(s1,2)==0$

$rect(m+s*p1+s1*p, j+s*e) = 0;$

$elseifrect(m, j) == 0and$

$(m+s*p1+s1*p)>0and(m+s*p1+s1*p)<=theta_oland$

$mod(s,2)==1andmod(s1,2)==0$

$rect(m+s*p1+s1*p, j+s*e) = 1;$

$elseifrect(m, j) == 1and$

$(m+s*p1+s1*p)>0and(m+s*p1+s1*p)<=theta_oland$

$mod(s,2)==1andmod(s1,2)==1$

$rect(m+s*p1+s1*p, j+s*e) = 1;$

$elseifrect(m, j) == 0and$

$(m+s*p1+s1*p)>0and(m+s*p1+s1*p)<=theta_oland$

$mod(s,2)==1andmod(s1,2)==1$

$rect(m+s*p1+s1*p, j+s*e) = 0;$

end

end

end

end

end

$Draw_shape(rect, theta_row, theta_col, color1, color2)$

$title(^I Rectangleafter Mapping2^I)$

end

$if k2 > k1$

$for m = 1 : r$

$for j = 1 : e$

$for s1 = 1 : s1_lim$

$if rect(m, j) == 1 and (m + s1 * p1) > 0 and (m + s1 * p1) <= theta_row$

$and mod(s1, 2) == 1$

$rect(m + s1 * p1, j + s1 * e) = 0;$

$elseif rect(m, j) == 0 and (m + s1 * p1) > 0$

$and (m + s1 * p1) <= theta_row and mod(s1, 2) == 1$

$rect(m + s1 * p1, j + s1 * e) = 1;$

$elseif rect(m, j) == 1 and (m + s1 * p1) > 0$

$and (m + s1 * p1) <= theta_row and mod(s1, 2) == 0$

$rect(m + s1 * p1, j + s1 * e) = 1;$

$elseif rect(m, j) == 0 and (m + s1 * p1) > 0$

$and (m + s1 * p1) <= theta_row and mod(s1, 2) == 0$

$rect(m + s1 * p1, j + s1 * e) = 0;$

end

end

end

end

$Draw_shape(rect, theta_row, theta_col, color1, color2)$

$form = r - p + 1 : r$

$forj = 1 : e$

$fors = 0 : 6$

$fors1 = 1 : 12$

$ifrect(m, j) == 1and(m + s * p1 + s1 * p) > 0and$

$(m+s*p1+s1*p) <= theta_colandmod(s, 2) == 0and$

$mod(s1, 2) == 1$

$rect(m + s * p1 + s1 * p, j + s * e) = 0;$

$elseifrect(m, j) == 0and$

$(m+s*p1+s1*p) > 0and(m+s*p1+s1*p) <= theta_coland$

$mod(s, 2) == 0andmod(s1, 2) == 1$

$rect(m + s * p1 + s1 * p, j + s * e) = 1;$

$elseifrect(m, j) == 1and$

$(m+s*p1+s1*p) > 0and(m+s*p1+s1*p) <= theta_coland$

$mod(s, 2) == 0andmod(s1, 2) == 0$

$rect(m + s * p1 + s1 * p, j + s * e) = 1;$

$elseifrect(m, j) == 0and$

$(m+s*p1+s1*p) > 0and(m+s*p1+s1*p) <= theta_coland$

$mod(s, 2) == 0andmod(s1, 2) == 0$

$rect(m + s * p1 + s1 * p, j + s * e) = 0;$

$elseifrect(m, j) == 1and$

$(m+s*p1+s1*p) > 0and(m+s*p1+s1*p) <= theta_coland$

$mod(s, 2) == 1andmod(s1, 2) == 0$

$rect(m + s * p1 + s1 * p, j + s * e) = 0;$

$elseifrect(m, j) == 0and$

$(m+s*p1+s1*p) > 0 and (m+s*p1+s1*p) <= theta_c ol and$

$mod(s,2) == 1 and mod(s1,2) == 0$

$rect(m+s*p1+s1*p, j+s*e) = 1;$

$elseif rect(m, j) == 1 and$

$(m+s*p1+s1*p) > 0 and (m+s*p1+s1*p) <= theta_c ol and$

$mod(s,2) == 1 and mod(s1,2) == 1$

$rect(m+s*p1+s1*p, j+s*e) = 1;$

$elseif rect(m, j) == 0 and$

$(m+s*p1+s1*p) > 0 and (m+s*p1+s1*p) <= theta_c ol and$

$mod(s,2) == 1 and mod(s1,2) == 1$

$rect(m+s*p1+s1*p, j+s*e) = 0;$

end

end

end

end

end

$Draw_s hape(rect, theta_r ow, theta_c ol, color1, color2)$

$title(^I Rectangle after Mapping 2^I)$

$for m = 1 : d1$

$for j = 1 : e$

$for s = 0 : 6$

$for s2 = 1 : 12$

$if rect(m, j) == 1 and (m+s*p1-s2*p) > 0 and (m+s*p1-s2*p) <= theta_c ol and$

$mod(s,2) == 0 and mod(s2,2) == 1 rect(m+s*p1-s2*p, j+s*e) = 0;$

$elseif rect(m, j) == 0 and (m+s*p1-s2*p) > 0 and$

$(m+s*p1-s2*p) <= theta_c ol and mod(s,2) == 0 and mod(s2,2) == 1$

$rect(m+s*p1-s2*p, j+s*e) = 1;$

$elseif rect(m, j) == 1 and (m + s * p1 - s2 * p) > 0 and$

$(m + s * p1 - s2 * p) <= theta_col and mod(s, 2) == 0 and mod(s2, 2) == 0$

$rect(m + s * p1 - s2 * p, j + s * e) = 1;$

$elseif rect(m, j) == 0 and (m + s * p1 - s2 * p) > 0 and$

$(m + s * p1 - s2 * p) <= theta_col and mod(s, 2) == 0 and mod(s2, 2) == 0$

$rect(m + s * p1 - s2 * p, j + s * e) = 0;$

$elseif rect(m, j) == 1 and (m + s * p1 - s2 * p) > 0 and$

$(m + s * p1 - s2 * p) <= theta_col and mod(s, 2) == 1 and mod(s2, 2) == 0$

$rect(m + s * p1 - s2 * p, j + s * e) = 0;$

$elseif rect(m, j) == 0 and (m + s * p1 - s2 * p) > 0 and$

$(m + s * p1 - s2 * p) <= theta_col and mod(s, 2) == 1 and mod(s2, 2) == 0$

$rect(m + s * p1 - s2 * p, j + s * e) = 1;$

$elseif rect(m, j) == 1 and (m + s * p1 - s2 * p) > 0 and$

$(m + s * p1 - s2 * p) <= theta_col and mod(s, 2) == 1 and mod(s2, 2) == 1$

$rect(m + s * p1 - s2 * p, j + s * e) = 1;$

$elseif rect(m, j) == 0 and (m + s * p1 - s2 * p) > 0 and$

$(m + s * p1 - s2 * p) <= theta_col and mod(s, 2) == 1 and mod(s2, 2) == 1$

$rect(m + s * p1 - s2 * p, j + s * e) = 0;$

end

end

end

end

end

$Draw_shape(rect, theta_row, theta_col, color1, color2)$

$title('Rectangle after Mapping 2')$

end

$function Draw_shape(M, a, b, color1, color2)$

$figure$

$axis([0b0a])$

$holdon$

$fori = 1 : a$

$forj = 1 : b$

$ifM(i, j) == 1$

$rectangle({}^{I}Position^{I}, [j-1, a-i, 1, 1], {}^{I}FaceColor^{I}, color1);$

$elseifM(i, j) == 0$

$rectangle({}^{I}Position^{I}, [j-1, a-i, 1, 1], {}^{I}FaceColor^{I}, color2);$

end

end

end

end

end

APPENDIX D

GENERATING N_C AND N_R NESTED CHAIN ABACUS

Code A

r=input('Enter a number of rows:');

e=input('Enter a number of columns:');

theta$_r ow = input(^I Enteranumberofcolmnsofonutputrectangle(r < theta_r ow) :^I)$

$theta_c ol = input(^I Enteranumberofcolmnsofonutputrectangle(e < theta_c ol) :^I);$

$d1 = input(^I Enterthesamenumberofbeadposition :^I);$

$con = Find_c onnected_a bacus(e, r-d1+1, r, e, d1, ^I col^I); num_i nitialrect = size(con, 1);$

$if num_i nitialrect == 0$

$error(^I Error : Thereisnoconnectedabacuseforthese parameters.^I)$

end

$temp = con(randi([1, num_i nitialrect], 1), :)$

$rect = zeros(theta_r ow, theta_c ol);$

$fori = 1 : e$

$rect(temp(i)/2 + 1 : temp(i)/2 + d1, i) = 1;$

end

$s_l im = ceil(theta_r ow/r) - 1;$

$s1_l im = ceil(theta_c ol/e);$

$s2_l im = ceil(theta_c ol/e);$

$s_c olor1 = [56, 85, 34]/255;$

$s_c olor2 = [156, 194, 228]/255;$

$k = min(temp);$

$k1 = temp(1, 1)$

$k2 = temp(1, end)$

$p = d1;$

$p1 = k2 - k1;$

$rect = zeros(theta_row, theta_col);$

$f ori = 1 : e$

$rect(temp(i) : temp(i) + d1 - 1, i) = 1;$

end

$Draw_shape(rect, theta_row, theta_col, color1, color2)$

$title(^I InitialRectangle^I)$

if k2>k1

for m=1:r

for j=1:e

for s1=1:s1$_l im$

$i f rect(m, j) == 1 and(m + s1 * p1) > 0 and(m + s1 * p1) <= theta_row$

$and mod(s1, 2) == 1$

$rect(m + s1 * p1, j + s1 * e) = 0;$

$elsei f rect(m, j) == 0 and(m + s1 * p1) > 0$

$and(m + s1 * p1) <= theta_row and mod(s1, 2) == 1$

$rect(m + s1 * p1, j + s1 * e) = 1;$

$elsei f rect(m, j) == 1 and(m + s1 * p1) > 0$

$and(m + s1 * p1) <= theta_row and mod(s1, 2) == 0$

$rect(m + s1 * p1, j + s1 * e) = 1;$

$elsei f rect(m, j) == 0 and(m + s1 * p1) > 0$

$and(m + s1 * p1) <= theta_row and mod(s1, 2) == 0$

$rect(m + s1 * p1, j + s1 * e) = 0;$

end

end

end

end

$Draw_shape(rect, theta_row, theta_col, color1, color2)$

$form = r - p + 1 : r$

$forj = 1 : e$

$fors = 0 : 6$

$fors1 = 1 : 12$

$ifrect(m, j) == 1and(m + s * p1 + s1 * p) > 0and$

$(m + s * p1 + s1 * p) <= theta_colandmod(s, 2) == 0and$

$mod(s1, 2) == 1$

$rect(m + s * p1 + s1 * p, j + s * e) = 0;$

$elseifrect(m, j) == 0and$

$(m + s * p1 + s1 * p) > 0and(m + s * p1 + s1 * p) <= theta_coland$

$mod(s, 2) == 0andmod(s1, 2) == 1$

$rect(m + s * p1 + s1 * p, j + s * e) = 1;$

$elseifrect(m, j) == 1and$

$(m + s * p1 + s1 * p) > 0and(m + s * p1 + s1 * p) <= theta_coland$

$mod(s, 2) == 0andmod(s1, 2) == 0$

$rect(m + s * p1 + s1 * p, j + s * e) = 1;$

$elseifrect(m, j) == 0and$

$(m + s * p1 + s1 * p) > 0and(m + s * p1 + s1 * p) <= theta_coland$

$mod(s, 2) == 0andmod(s1, 2) == 0$

$rect(m + s * p1 + s1 * p, j + s * e) = 0;$

$elseifrect(m, j) == 1and$

$(m + s * p1 + s1 * p) > 0and(m + s * p1 + s1 * p) <= theta_coland$

$mod(s, 2) == 1andmod(s1, 2) == 0$

$rect(m + s * p1 + s1 * p, j + s * e) = 0;$

$elseifrect(m, j) == 0and$

$(m+s*p1+s1*p) > 0and(m+s*p1+s1*p) <= theta_coland$

$mod(s,2) == 1andmod(s1,2) == 0$

$rect(m+s*p1+s1*p, j+s*e) = 1;$

$elseifrect(m, j) == 1and$

$(m+s*p1+s1*p) > 0and(m+s*p1+s1*p) <= theta_coland$

$mod(s,2) == 1andmod(s1,2) == 1$

$rect(m+s*p1+s1*p, j+s*e) = 1;$

$elseifrect(m, j) == 0and$

$(m+s*p1+s1*p) > 0and(m+s*p1+s1*p) <= theta_coland$

$mod(s,2) == 1andmod(s1,2) == 1$

$rect(m+s*p1+s1*p, j+s*e) = 0;$

end

end

end

end

end

$Draw_shape(rect, theta_row, theta_col, color1, color2)$

$title(^IRectangleafterMapping2^I)$

$form = 1 : d1$

$forj = 1 : e$

$fors = 0 : 6$

$fors2 = 1 : 12$

$ifrect(m, j) == 1and(m+s*p1-s2*p) > 0and(m+s*p1-s2*p) <= theta_coland$

$mod(s,2) == 0andmod(s2,2) == 1rect(m+s*p1-s2*p, j+s*e) = 0;$

$elseifrect(m, j) == 0and(m+s*p1-s2*p) > 0and$

$(m+s*p1-s2*p) <= theta_colandmod(s,2) == 0andmod(s2,2) == 1$

$rect(m+s*p1-s2*p, j+s*e) = 1;$

$elseif rect(m,j) == 1 and (m+s*p1-s2*p) > 0 and$

$(m+s*p1-s2*p) <= theta_col and mod(s,2) == 0 and mod(s2,2) == 0$

$rect(m+s*p1-s2*p, j+s*e) = 1;$

$elseif rect(m,j) == 0 and (m+s*p1-s2*p) > 0 and$

$(m+s*p1-s2*p) <= theta_col and mod(s,2) == 0 and mod(s2,2) == 0$

$rect(m+s*p1-s2*p, j+s*e) = 0;$

$elseif rect(m,j) == 1 and (m+s*p1-s2*p) > 0 and$

$(m+s*p1-s2*p) <= theta_col and mod(s,2) == 1 and mod(s2,2) == 0$

$rect(m+s*p1-s2*p, j+s*e) = 0;$

$elseif rect(m,j) == 0 and (m+s*p1-s2*p) > 0 and$

$(m+s*p1-s2*p) <= theta_col and mod(s,2) == 1 and mod(s2,2) == 0$

$rect(m+s*p1-s2*p, j+s*e) = 1;$

$elseif rect(m,j) == 1 and (m+s*p1-s2*p) > 0 and$

$(m+s*p1-s2*p) <= theta_col and mod(s,2) == 1 and mod(s2,2) == 1$

$rect(m+s*p1-s2*p, j+s*e) = 1;$

$elseif rect(m,j) == 0 and (m+s*p1-s2*p) > 0 and$

$(m+s*p1-s2*p) <= theta_col and mod(s,2) == 1 and mod(s2,2) == 1$

$rect(m+s*p1-s2*p, j+s*e) = 0;$

end

end

end

end

end

$Draw_shape(rect, theta_row, theta_col, color1, color2)$

$title('Rectangle after Mapping 2')$

end

$function Draw_shape(M, a, b, color1, color2)$

```
figure
axis([0b0a])
holdon
fori = 1 : a
forj = 1 : b
ifM(i, j) == 1
rectangle('Position',[j −1,a−i, 1, 1],'FaceColor',color1);
elseifM(i, j) == 0
rectangle('Position',[j−1,a−i, 1, 1],'FaceColor',color2);
end
end
end
end
end
```


Code B

```
r=input('Enter a number of rows:');
e=input('Enter a number of columns:');
theta_row = input('Enteranumberofcolmnsofonutputrectangle(r < theta_row) :');
theta_col = input('Enteranumberofcolmnsofonutputrectangle(e < theta_col) :');
H1 = input('Enterthesamenumberofbeadposition :');
con = Find_connected_abacus(r,e−H1 + 1,r,e,H1,'row');
num_initialrect = size(con,1);
ifnum_initialrect == 0
error('Error : Thereisnoconnectedabacusefortheseparameters.')
end
```

$temp = con(randi([1, num_initialrect], 1), :);$

$rect = zeros(theta_row, theta_col);$

$for i = 1 : r$

$rect(i, temp(i)/2 + 1 : temp(i)/2 + H1) = 1;$

end

$s_lim = ceil(theta_row/r) - 1;$

$s1_lim = ceil(theta_col/e);$

$s2_lim = ceil(theta_col/e);$

$s_color1 = [131, 59, 10]/255;$

$s_color2 = [255, 217, 102]/255;$

$H = min(temp);$

$L = temp(1, 1);$

$L1 = temp(1, end);$

$p = e - H + 1;$

$p1 = L - L1;$

$figure$

$Draw_shape(rect, s_color1, s_color2)$

$title(^I InitialRectangle^I)$

$\cdot IfL1 <= L, applicatethemapping1andmpping1$

$ifL1 > L$

$form = 1 : r$

$forj = 1 : e$

$fors = 1 : s_lim$

$ifrect(m, j) == 1and(j + s * p1) > 0and(j + s * p1) <= theta_coland$

$mod(s, 2) == 1rect(m + s * r, j + s * p1) = 0;$

$elseifrect(m, j) == 0and(j + s * p1) > 0and(j + s * p1) <= theta_coland$

$mod(s, 2) == 1$

$rect(m+s*r, j+s*p1) = 1;$

$elseifrect(m, j) == 1and(j+s*p1) > 0and(j+s*p1) <= theta_coland$

$mod(s, 2) == 0$

$rect(m+s*r, j+s*p1) = 1;$

$elseifrect(m, j) == 0and(j+s*p1) > 0and(j+s*p1) <= theta_coland$

$mod(s, 2) == 0$

$rect(m+s*r, j+s*p1) = 0;$

end

end

end

end

$Draw_shape(rect, theta_row, theta_col, color1, color2)$

$title({}^{I}RectangleafterMapping1^{I})$

$form = 1 : r$

$rect(m+s*r, j+s*p1+s1*p) = 1;$

$elseifrect(m, j) == 0and(m+s*p1+s1*p) > 0and$

$(m+s*p1+s1*p) <= theta_colandmod(s, 2) == 1andmod(s1, 2) == 1$

$rect(m+s*r, j+s*p1+s1*p) = 0;$

end

end

end

end

end

$Draw_shape(rect, theta_row, theta_col, color1, color2)$

$title({}^{I}RectangleafterMapping2^{I})$

end

$ifL1 > L$

$for m = 1 : r$

$for j = 1 : e$

$for s = 1 : s_l im$

$if rect(m, j) == 1 and (j + s * p1) > 0 and (j + s * p1) <= theta_c ol and$

$mod(s, 2) == 1$

$rect(m + s * r, j + s * p1) = 0;$

$elseif rect(m, j) == 0 and (j + s * p1) > 0 and (j + s * p1) <= theta_c ol and$

$mod(s, 2) == 1$

$rect(m + s * r, j + s * p1) = 1;$

$elseif rect(m, j) == 1 and (j + s * p1) > 0 and (j + s * p1) <= theta_c ol and$

$mod(s, 2) == 0$

$rect(m + s * r, j + s * p1) = 1;$

$elseif rect(m, j) == 0 and (j + s * p1) > 0 and (j + s * p1) <= theta_c ol and$

$mod(s, 2) == 0$

$rect(m + s * r, j + s * p1) = 0;$

end

end

end

end

$Draw_s hape(rect, theta_r ow, theta_c ol, color1, color2)$

$title({}^I Rectangle after Mapping 1^I)$

$for m = 1 : r$

$for j = H : e$

$for s = 0 : s_l im$

$for s1 = 1 : s1_l im$

$if rect(m, j) == 1 and (m + s * p1 + s1 * p) > 0 and (m + s * p1 + s1 * p) <= theta_c ol$

$and mod(s, 2) == 0 and mod(s1, 2) == 1$

$rect(m + s * r, j + s * p1 + s1 * p) = 0;$

$elseif rect(m, j) == 0and(m + s * p1 + s1 * p) > 0and$

$(m + s * p1 + s1 * p) <= theta_colandmod(s, 2) == 0andmod(s1, 2) == 1$

$rect(m + s * r, j + s * p1 + s1 * p) = 1;$

$elseif rect(m, j) == 1and(m + s * p1 + s1 * p) > 0and$

$(m + s * p1 + s1 * p) <= theta_colandmod(s, 2) == 0andmod(s1, 2) == 0$

$rect(m + s * r, j + s * p1 + s1 * p) = 1;$

$elseif rect(m, j) == 0and(m + s * p1 + s1 * p) > 0and$

$(m + s * p1 + s1 * p) <= theta_colandmod(s, 2) == 0andmod(s1, 2) == 0$

$rect(m + s * r, j + s * p1 + s1 * p) = 0;$

$elseif rect(m, j) == 1and(m + s * p1 + s1 * p) > 0and$

$(m + s * p1 + s1 * p) <= theta_colandmod(s, 2) == 1andmod(s1, 2) == 0$

$rect(m + s * r, j + s * p1 + s1 * p) = 0;$

$elseif rect(m, j) == 0and(m + s * p1 + s1 * p) > 0and$

$(m + s * p1 + s1 * p) <= theta_colandmod(s, 2) == 1andmod(s1, 2) == 0$

$rect(m + s * r, j + s * p1 + s1 * p) = 1;$

$elseif rect(m, j) == 1and(m + s * p1 + s1 * p) > 0and$

$(m + s * p1 + s1 * p) <= theta_colandmod(s, 2) == 1andmod(s1, 2) == 1$

$rect(m + s * r, j + s * p1 + s1 * p) = 1;$

$elseif rect(m, j) == 0and(m + s * p1 + s1 * p) > 0and$

$(m + s * p1 + s1 * p) <= theta_colandmod(s, 2) == 1andmod(s1, 2) == 1$

$rect(m + s * r, j + s * p1 + s1 * p) = 0;$

end

end

end

end

end

$Draw_shape(rect, theta_row, theta_col, color1, color2)$

$title('Rectangle after Mapping 2')$

$for m = 1 : r$

$for j = 1 : H1$

$for s = 1 : s1_lim$

$for s2 = 1 : s2_lim$

$for j = H : e$

$for s = 0 : s_lim$

$for s1 = 1 : s1_lim$

$if rect(m, j) == 1 and (m + s * p1 + s1 * p) > 0 and (m + s * p1 + s1 * p) <= theta_col$

$and mod(s, 2) == 0 and mod(s1, 2) == 1$

$rect(m + s * r, j + s * p1 + s1 * p) = 0;$

$elseif rect(m, j) == 0 and (m + s * p1 + s1 * p) > 0 and$

$(m + s * p1 + s1 * p) <= theta_col and mod(s, 2) == 0 and mod(s1, 2) == 1$

$rect(m + s * r, j + s * p1 + s1 * p) = 1;$

$elseif rect(m, j) == 1 and (m + s * p1 + s1 * p) > 0 and$

$(m + s * p1 + s1 * p) <= theta_col and mod(s, 2) == 0 and mod(s1, 2) == 0$

$rect(m + s * r, j + s * p1 + s1 * p) = 1;$

$elseif rect(m, j) == 0 and (m + s * p1 + s1 * p) > 0 and$

$(m + s * p1 + s1 * p) <= theta_col and mod(s, 2) == 0 and mod(s1, 2) == 0$

$rect(m + s * r, j + s * p1 + s1 * p) = 0;$

$elseif rect(m, j) == 1 and (m + s * p1 + s1 * p) > 0 and$

$(m + s * p1 + s1 * p) <= theta_col and mod(s, 2) == 1 and mod(s1, 2) == 0$

$rect(m + s * r, j + s * p1 + s1 * p) = 0;$

$elseif rect(m, j) == 0 and (m + s * p1 + s1 * p) > 0 and$

$(m + s * p1 + s1 * p) <= theta_col and mod(s, 2) == 1 and mod(s1, 2) == 0$

$rect(m + s * r, j + s * p1 + s1 * p) = 1;$

$elseif rect(m, j) == 1 and (m+s*p1+s1*p) > 0 and (m+s*p1+s1*p) <=$

$1 and mod(s1, 2) == 1$

$rect(m + s * r, j + s * p1 + s1 * p) = 1;$

$elseif rect(m, j) == 0 and (m + s * p1 + s1 * p) > 0 and$

$(m + s * p1 + s1 * p) <= theta_c ol and mod(s, 2) == 1 and mod(s1, 2) == 1$

$rect(m + s * r, j + s * p1 + s1 * p) = 0;$

end

end

end

end

end

$Draw_s hape(rect, theta_r ow, theta_c ol, color1, color2)$

$title({}^I Rectangle after Mapping2^I)$

$for m = 1 : r$

$for j = 1 : H1$

$for s = 1 : s1_l im$

$for s2 = 1 : s2_l im$

$function Draw_s hape(M, a, b, color1, color2)$

$figure$

$axis([0 b 0 a])$

$hold on$

$for i = 1 : a$

$for j = 1 : b$

$if M(i, j) == 1$

$rectangle({}^I Position^I, [j - 1, a - i, 1, 1], {}^I FaceColor^I, color1); elseif M(i, j) == 0$

$rectangle({}^I Position^I, [j - 1, a - i, 1, 1], {}^I FaceColor^I, color2);$

end

end

end

end

end

Code C

function Draw$_{s}hape(M, a, b, color1, color2)$

$f\, igure$

$axis([0b0a])$

$holdon$

$f\, ori = 1 : a$

$f\, or j = 1 : b$

$i\, f M(i, j) == 1$

$rectangle({}^{I}Position^{I}, [j-1, a-i, 1, 1], {}^{I}FaceColor^{I}, color1);$

$elsei\, f M(i, j) == 0$

$rectangle({}^{I}Position^{I}, [j-1, a-i, 1, 1], {}^{I}FaceColor^{I}, color2);$

end

end

end

end

end

** Find all Initial rect

```
function com=create_combination(n, k)
for i = 1 : n
tmp = [];
for j = 1 : k
tmp = [tmp;ones(k^(n−i), 1) ∗ j];
end
rr = [];
for j = 1 : k^(i− 1)
rr = [rr;tmp];
end
com(:, i) = rr;
end
com = com− 1
end
```


Code D

** Find the connected Abacus

```
function con=Find_connected_abacus(n, k, r, e, H, str)
con = [];
· Findallpossibleinitialrects
all = create_combination(n, k);
· ∗Findtheconnectedabacus
· Algorithmforcolumn
ifstrcmp(str,'col')
fornn = 1 : size(all, 1)
temp = all(nn, :);
```

$flag = 1;$

$rectisconnectedabacus)$

$rect = zeros(r, e);$

$f ori = 1 : e$

$rect(temp(i) + 1 : temp(i) + H, i) = 1;$

end

$\cdot * Testtheconnection$

$t = sum(rect^{I});$

$f ori = 1 : r$

$f lag = f lag * (t(1, i) >= 1);$

end

$f ori = 1 : e - 1$

$A = temp(i) : temp(i) + H - 1;$

$B = temp(i + 1) : temp(i + 1) + H - 1;$

$C = intersect(A, B);$

$f lag = f lag * (isempty(C));$

end

$i f f lag == 1$

$con = [con; temp];$

end

APPENDIX E

CHAIN TRANSFORMATION

File Number one

```
clc
kkk=0;
clear;
nt=0;
v=1;
global Tmat;
r = input(' numbers of rows ')
c = input(' numbers of columns ')
mat = ones(r,c);
Tmat=mat;
r,c
= size(mat) ;
ch=21;
path=0;
tn=0;
fl=[r,c] ;
tmpv=min(fl)/2;
tpn1=ceil(tmpv)
while path <tpn1
path=path+1
v=v-.15
si=path;sj=path;
```

```
i=si;j=sj;
vv=path;
n=0;
sti=2;
stj=2;
str=('r-si-1');
nr=r-si;
nc=c-sj;
vs=1/ch;
vs=0;
nt=nt+1;
while 1
if (i==si ) (j==sj )( n>0)
break
end
n=n+1;
if ( tn>=totn)
break
end
mat(i,j)=v;
tn=tn+1;
pt(tn,4)=j;
pt(tn,3)=i;
pt(tn,1)=tn;
pt(tn,2)=path;
if ((i<nr+1)(sti>0))
i=i+1;
```

```
else if((j<nc+1)(stj>0));
j=j+1;
sti=-1;
else
if (i>si)
i=i-1;
stj=-1;
else if (j>sj)
j=j-1;
if ((i==si)(j==sj)) sti=1;stj=1;
end
end
end
end
end
i;
j;
A1 = path;
A2 = n;
end
end
s=size(pt);
c=s(1);
cp=1
i=1;
clc
k=1;
```

```
pathStart(1)=1;
for i=1:s
if (i>1)
if pt(i,2) =pt(i-1,2) k=k+1;
pathStart(k)=i;
end
end
end
k=k+1;
pathStart(k)=s(1)
k(1:tpn1)=0;
clc
p01=1
No=0;
pt(:,5)=1;
totalN ss
=size(pt)
tpn=max(pt(:,2))
for i=1:tpn
i
pn(i)=input('ÚÏÏ ÇáÚäÇÕÑ áááãÓÇÑ ')
end
for p=1:length(pn)
for i=1:totalN
if(pt(i,2)==p)(k(p)<pn(p)) pt(i,5)=11;
k(p)=k(p)+1;
end
```

```
ii=pt(i,3);
jj=pt(i,4);
mat(ii,jj)= pt(i,5);
end
end
Ax=0
ii=0;
jj=0;
**ccc=length(x1)
Ax(1:pathStart(2)-1,1:5,1:3) =-1
k=0
for i=1:tpn
i1=pathStart(i)
if i<s(1)
end
if (i==tpn)
i2=pathStart(i+1)
else
i2=pathStart(i+1)-1
end
* x1=pt(pathStart(1):pathStart(2)-1,:)
x=pt(i1:i2,:)
k=k+1;
rrr=size(x)
pL(k)=rrr(1)
Ax(1:pL(k),:,i)=x;
* Ax(1:length(x),:,i)=x;
```

```
end
Ax(1:pL(i),:,i)
clc
** x1=Ax(1:pL(1),:,1);
** x2=Ax(1:pL(2),:,2);
** x3=Ax(1:pL(3),:,3);
s=0
N=0
mTem=0
i=1
global Nx1
Nx1=0;
LoopF(i,Ax,pL,mat)
F3
```


File Number Two

```
global Nx1
global Tmat
for t=1:sxv
ii=xt(t,3);
jj=xt(t,4);
tt=xt(t,5)
mat(ii,jj)= tt;
end
```

```
mat
r,c
= size(mat) ;
if  (isequal(mTem,mat))
dlmwrite('Rtxt',mat,'-append','delimiter',' ','roffset',1);
*************************************************************
imagesc((1:c)+0.5,(1:r)+0.5,mat);
colormap(winter);
axis equal ;
N=N+1 ;
set(gca,'XTick',1:(c),'YTick',1:(r),...
'XLim',[1 c+1],'YLim',[1 r+1],...
'GridLineStyle','-','XGrid','on','YGrid','on');
rndd1 = 1
rndd2 = 1
Nx1=Nx1+1
Tmat(:,:,Nx1)=mat;
s=sprintf('000
saveas(gcf,s);
******************************************************************* end
mTem=mat;
```

File Number Three

```
a b w
=size(Tmat)
Tmat2=Tmat(:,:,:)
```

```
Tmat3=Tmat(:,:,1)
k=0;
m1=Tmat(:,:,1);
m2=Tmat2(:,:,1);
kk=1
for i=1:w
t=1
t=0
for j=i+1:w
kk=kk+1
m1=Tmat(:,:,i);
m2=Tmat2(:,:,j);
if Tmat(:,:,i)== Tmat2(:,:,j)
t=1;
end
if k==150
nnn=2
end
end
if (t==0)
k=k+1
Tmat3(:,:,k)= Tmat(:,:,i)
```

File Number Four

```
function it=LoopF(i,Ax,pL,mat)

global Nx1;

Nx1=Nx1*1

LpL=length(pL)

if i>LpL

return;

end

pLt=pL(i);

xt=Ax(1:pLt,:,i);

sx(i)=size(xt,1);

sxv=sx(i)

mTem=0;

N=0;

clc

for j=1:sxv

sh=1;

Y1 = circshift(xt(:,5),sh);

xt(:,5)=Y1;

F2

Ax(1:pL(i),5,i)=Y1;

************************************************************** tt=i+1

Ax2=Ax;pL2=pL;mat2=mat;

LoopF(tt,Ax2,pL2,mat2)

end

end
```

APPENDIX F

GENERATING FUNCTION

```
clc;clear all;close all;

key=3;

tmp=key;

fprintf('===================================')

for n=1:7

tmp1=[];

for i=1:size(tmp,2)

tmp1=[tmp1,ones(1,tmp(i)-1)*tmp(i),tmp(i)+2];

end

tmp=[];tmp=tmp1;

fn=size(tmp,2);

fprintf('f

(end
```