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**MODE DIVISION MULTIPLEXING ZERO FORCING  
EQUALISATION SCHEME USING LU FACTORIZATION**



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## Abstrak

Rangkaian optik dianggap sebagai rangkaian tulang belakang utama yang mengendalikan trafik internet di seluruh dunia. Pada masa kini, trafik internet telah mempunyai pertumbuhan tahunan yang besar disebabkan oleh penambahan peranti-peranti yang terangkai. Teknologi semasa dalam rangkaian optik dipercayai tidak akan mampu untuk mengendalikan pertumbuhan ini dalam masa terdekat sehingga baru-baru ini, teknik pemultipleksan dalam komunikasi optik bergantung kepada teknik modulasi di mana polarisasi, amplitud dan frekuensi isyarat yang digunakan sebagai pembawa data yang utama. Dalam teknik ini, mod cahaya adalah dianggap sebagai kesan tidak diinginkan menyebabkan penyebaran mod. Sebaliknya, pemultipleksan pembahagian mod (MDM) telah diperkenalkan sebagai pendekatan pemultipleksan yang bergantung kepada penggunaan mod cahaya untuk manfaat meningkatkan produk kapasiti jarak jauh rangkaian optik.

Seperti mana-mana teknologi baru, banyak masalah menghalangnya daripada dipiawaikan dan digunakan secara komersil. Salah satu isu utama MDM adalah gandingan mod, yang merupakan fenomena inventible berlaku apabila tenaga daripada satu mod pemindahan ke satu mod yang lain semasa pembiakan mereka sepanjang gentian optik menyebabkan gangguan antara simbol (ISI) meningkatkan kadar ralat bit (BER) dan mengurangkan prestasi sistem secara keseluruhan. Skim penyamaan yang berbeza telah dicadangkan setakat ini sebagai cubaan untuk mengurangkan kesan mod gandingan kepada isyarat optik MDM. Walau bagaimanapun, mereka mengalami kerumitan perkomputan tinggi dan bergantung kepada latihan isyarat dalam menganggarkan saluran optik yang meningkatkan muatan kekal. Teknik ini bergantung terutamanya kepada algoritma min kuasa dua terkecil (LMS) dan algoritma kuasa dua terkecil rekursif (RLS). Tujuan kajian ini adalah untuk memperkenalkan penyamaan tanpa paksaan berasaskan LU untuk MDM. Kajian sebelum ini dalam domain radio pada pelbagai input pelbagai output (MIMO) dan pemultipleksan pembahagian frekuensi ortogon (OFDM) menunjukkan bahawa skema-skema tanpa paksaan mempunyai kerumitan perkomputan yang rendah berbanding dengan skema-skema semasa kerana mereka menyamakan isyarat tanpa isyarat latihan, sekali gus mengurangkan muatan kekal. Semua idea terdahulu memberi motivasi kepada kajian ini untuk menyesuaikan diri dengan pendekatan ini dalam komunikasi optik.

Kajian ini menggunakan Metodologi Penyelidikan Reka bentuk (DRM) empat-peringkat. Data awal dikumpulkan dari simulator optik, diproses dan digunakan untuk mendapatkan fungsi pemindahan (H) daripada sistem. Kemudian ia digunakan untuk membangunkan skema penyamaan dalam MATLAB. Eksperimentasi pada penyamaan tanpa paksaan berasaskan LU menunjukkan kerumitan  $O(N)$  yang lebih rendah daripada RLS yang mempunyai  $O(N^2)$  dan lebih cepat daripada LMS kerana LMS memerlukan purata 0.0126 saat untuk memproses manakala isyarat tanpa paksaan berasaskan LU memerlukan 0.0029 saat sahaja. Sebaliknya, penyamaan yang dicadangkan mengurangkan kelewatan penyebaran masa saluran, menyebabkan tiga kali kenaikan dalam kapasiti saluran MDM dan kerumitan pengiraan lebih rendah. Sumbangan utama kajian ini adalah pengurangan kerumitan pengiraan skema penyamaan di MDM. Penggunaan skema ini dalam sistem MDM sebenar boleh menghasilkan kos lebih efektif dan pemprosesan isyarat digital (DSP) yang lebih kecil untuk MDM dan boleh mempercepatkan kerja pada

penyeragaman MDM untuk digunakan secara komersial sebagai teknik pemultipleksan untuk rangkaian komunikasi optik.



## Abstract

Optical networks is considered as the main backbone networks that handled the Internet traffic worldwide. Currently, the Internet traffic has had huge annual growth due to the increment in connected devices. At this rate, it is believed that the current technology in optical network will not able to handle this growth in the near future. Till recently, multiplexing techniques in the optical communication rely on modulation techniques where polarization, amplitude and frequency of the signal are used as the main data carrier. In these techniques, light modes are considered as an undesired effect causing modal dispersion. In contrast, mode division multiplexing (MDM) was introduced as a multiplexing approach which relies on the utilization of the light modes for the benefit of increasing the capacity-distance product of the optical network.

As per any new technology, it is still facing a lot of problems preventing it from being commercially standardized and used. One of the main MDM issues is the mode coupling, which is an inevitable phenomena occurs when the energy of one mode transfers to another mode during their propagation throughout the optical fibre causes inter-symbol interference (ISI), increasing the bit error rate (BER) and reducing the overall system performance. Different equalization schemes have been proposed so far attempting to mitigate the effect of mode coupling on the MDM optical signal. However, they suffer from high computational complexity and rely on training signals in estimating the optical channel which increases the overhead payload. These technique mainly rely on Least Mean Squared (LMS) and Recursive Least Squared (RLS) algorithms. The purpose of this study is to introduce a Zero Forcing LU-based equalization scheme for MDM. Previous research in the radio domain on multiple-input multiple output (MIMO) and orthogonal frequency division multiplexing (OFDM) demonstrated that zero forcing schemes have low computational complexity compared to current schemes as they equalize the signal without training signals, thus reducing the overhead payload. All of the previous points motivate the work of this study to adapt this approach in optical communications.

The study adopts the four stages of the Design Research Methodology (DRM). The initial data was collected from the optical simulator, processed and used to derive the transfer function (H) of the system. Then it was used to develop the equalization scheme in MATLAB. The experimentation on Zero Forcing LU based equalization scheme shows  $O(N)$  complexity which is lower than RLS which has  $O(N^2)$  and faster than LMS, in fact, LMS needs an average of 0.0126 seconds to process the signal while ZF LU-based needs 0.0029 seconds only. On the other hand, the proposed equalization reduces the time delay spread of the channel, resulting three times increment in the capacity of the MDM channel and even lower computational complexity. The main contribution of this study is the reduction of the computational complexity of the previous equalization schemes in MDM. Applying this scheme in real MDM systems can produce more cost effective and smaller digital signal processing (DSP) parts for MDM equipment and can accelerate the work on the standardization of MDM for being commercially used as a multiplexing technique for optical communication networks.

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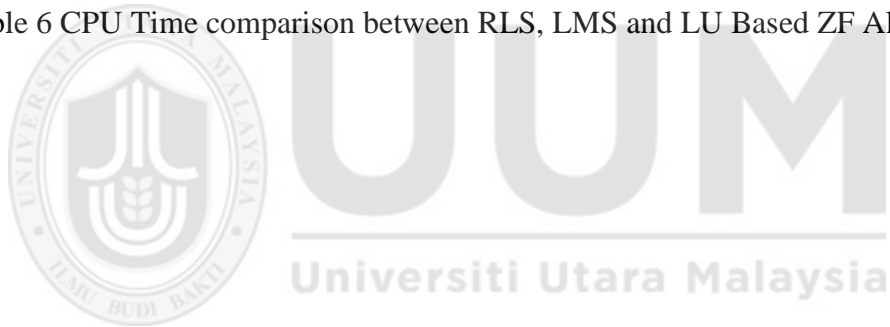
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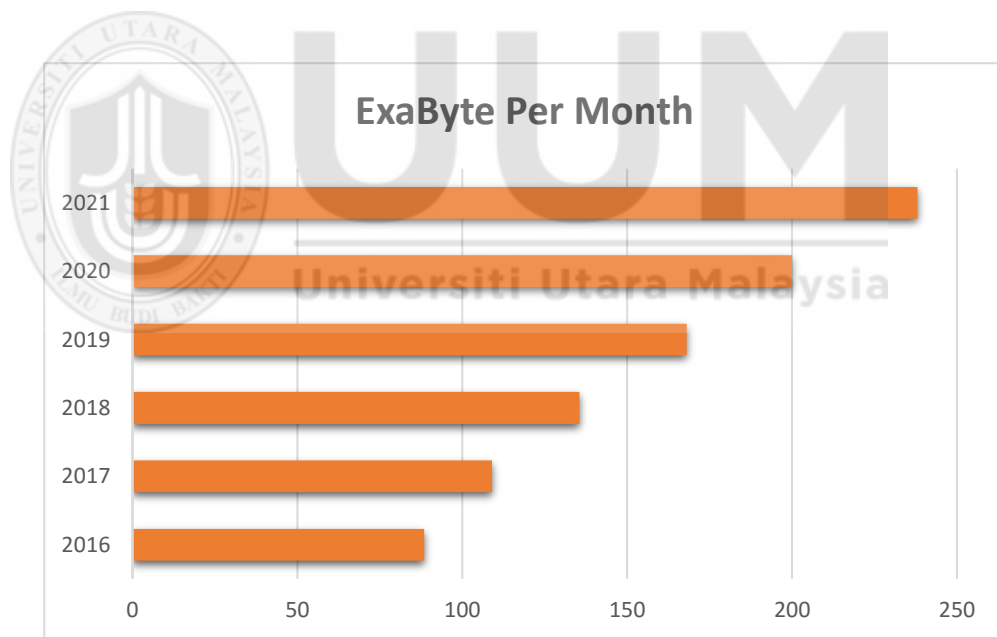
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## Chapter One: Introduction

### 1.1 Research Overview

Two decades since the appearance of the modern commercial Internet in the mid of 1990s, and the worldwide network traffic has increased by an estimated rate exceeding 50% each year [1]. This dramatic growth comes from the appearance of high-definition video streaming, cloud computing, mobile networking, mobile gaming and many other web 2.0 applications. The forecast of the global traffic growth rate for the coming five years is shown in Figure 1 below [2].



*Figure 1 Global Internet Traffic Forecast [2]*

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