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**PARAMETRIC MIXTURE MODEL OF THREE COMPONENTS FOR
MODELLING HETEROGENEOUS SURVIVAL DATA**

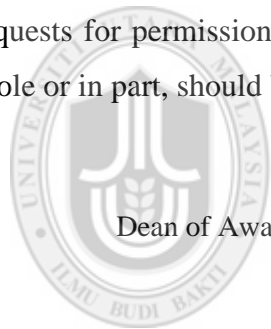


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Abstrak

Kajian yang lepas menunjukkan model kemandirian campuran dua komponen mencatatkan prestasi yang lebih baik berbanding model kemandirian berparameter klasik tulen. Namun terdapat juga keperluan yang penting bagi model kemandirian campuran tiga komponen kerana tingkah laku data kemandirian heterogen yang lazimnya merangkumi lebih dari dua taburan. Oleh itu dalam kajian ini dua model bagi tiga komponen telah dibina. Model 1 adalah model kemandirian campuran berparameter tiga komponen bertaburan Gamma dan Model 2 adalah model kemandirian campuran berparameter tiga komponen bertaburan Eksponen, Gamma dan Weibull. Kedua-dua model telah dianggar menggunakan Pemaksimuman Jangkaan (EM) dan pengesahan prestasi model melalui kajian simulasi dan empirikal. Simulasi telah diulang 300 kali dengan mengambil kira tiga saiz sampel berbeza: 100, 200, 500; tiga peratus penapisan yang berbeza: 10%, 20%, 40%; dan dua set kebarangkalian bercampur secara: menaik (10%, 40%, 50%) dan secara menurun (50%, 30%, 20%). Beberapa set data sebenar telah digunakan dalam kajian empirikal dan perbandingan model-model telah dilaksanakan. Model 1 telah dibandingkan dengan model kemandirian berparameter klasik tulen, model kemandirian berparameter campuran dua dan empat komponen bertaburan Gamma. Model 2 telah dibandingkan dengan model kemandirian berparameter klasik tulen dan model kemandirian berparameter campuran tiga komponen bertaburan sama. Persembahan grafik, *log likelihood* (LL), Kriteria Maklumat Akaike (AIC), Min Ralat Kuasa Dua (MSE) dan Punca Min Ralat Kuasa Dua (RMSE) telah digunakan bagi menilai prestasi. Dapatan simulasi menunjukkan bahawa kedua-dua model mencatatkan prestasi yang baik pada saiz sampel yang besar, peratus tertapis yang kecil dan pada kebarangkalian bercampur secara menaik. Kedua-dua model menghasilkan ralat yang kecil berbanding dengan model kemandirian jenis lain dalam kajian empirikal. Ini menunjukkan bahawa kedua-dua model yang dibina adalah lebih tepat dan merupakan pilihan yang lebih baik untuk menganalisis data kemandirian heterogen.

Kata kunci: data survival, heterogen, tiga komponen, eksponen, Gamma, Weibull, Pengmaksimuman Jangkaan

Abstract

Previous studies showed that two components of survival mixture model performed better than pure classical parametric survival model. However there are crucial needs for three components of survival mixture model due to the behaviour of heterogeneous survival data which commonly comprises of more than two distributions. Therefore in this study two models of three components of survival mixture model were developed. Model 1 is three components of parametric survival mixture model of Gamma distributions and Model 2 is three components of parametric survival mixture model of Exponential, Gamma and Weibull distributions. Both models were estimated using the Expectation Maximization (EM) and validated via simulation and empirical studies. The simulation was repeated 300 times by incorporating three different sample sizes: 100, 200, 500; three different censoring percentages: 10%, 20%, 40%; and two different sets of mixing probabilities: ascending (10%, 40%, 50%) and descending (50%, 30%, 20%). Several sets of real data were used in the empirical study and models comparisons were implemented. Model 1 was compared with pure classical parametric survival model, two and four components parametric survival mixture models of Gamma distribution, respectively. Model 2 was compared with pure classical parametric survival models and three components parametric survival mixture models of the same distribution. Graphical presentations, log likelihood (LL), Akaike Information Criterion (AIC), Mean Square Error (MSE) and Root Mean Square Error (RMSE) were used to evaluate the performance. Simulation findings revealed that both models performed well at large sample size, small percentage of censoring and ascending mixing probabilities. Both models also produced smaller errors compared to other type of survival models in the empirical study. These indicate that both of the developed models are more accurate and provide better option to analyse heterogeneous survival data.

Keywords: survival data, heterogeneous, three components, Exponential, Gamma, Weibull, Expectation Maximization.

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Glossary of Terms

E1_E2_E3 PARAMETRIC SURVIVAL MIXTURE MODEL OF
EXPONENTIAL_EXPONENTIAL_EXPONENTIAL
DISTRIBUTIONS.

E_G_W PARAMETRIC SURVIVAL MIXTURE MODEL OF
EXPONENTIAL_GAMMA_WEIBULL DISTRIBUTIONS.

G1_G2_G3 PARAMETRIC SURVIVAL MIXTURE MODEL OF
GAMMA_GAMMA_GAMMA DISTRIBUTIONS.

W1_W2_W3 PARAMETRIC SURVIVAL MIXTURE MODEL OF
WEIBULL_WEIBULL_WEIBULL DISTRIBUTIONS.



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List of Abbreviations

AIC	AKAIKE INFORMATION CRITERION
EEG	EXTENDED EXPONENTIAL-GEOMETRIC
EM	EXPECTATION MAXIMIZATION
KM	KAPLAN-MEIER
LL	LOG LIKELIHOOD
MCM	MIXTURE CURE MODELS
ML	MAXIMUM LIKELIHOOD
MSE	MEAN SQUARE ERROR
MTIWD	MIXTURE OF TWO INVERSE WEIBULL DISTRIBUTION
RMSE	ROOT MEAN SQUARE ERROR

CHAPTER ONE

INTRODUCTION

1.1 Background of the Study

Survival data analysis is the analysis of time to occurrence of a particular event of interest. The data are usually related to clinical studies of human, or laboratory studies of animal, or studies to test the life time of some devices. Major applications are in the areas of human clinical studies and industrial life testing (Kalbfleisch & Prentice, 2002).

The event of interest in clinical studies could be death, remission, or some other clinical events. The event of interest could be time taken to learning a new skill, exit from unemployment, divorce of a couple or failure of a device, to mention a few. The variable of interest, the time to occurrence of particular event T , which is a positive random variable, should clearly be defined in the study at hand. The start and end with the length of the time period in-between corresponding to T , should also be clearly defined prior to the commencement of the study (Lee & Wang, 2003).

Generally, in survival analysis, some individuals or objects do not experience the event of interest for one reason or the other, either they are lost to follow up during the period of the study or they do not experience the event until the end of the study. In such situation, the information about this particular individual will not be exactly known, and such individuals are referred to as censored observations or censored times.

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