CORE
Metadata, citation and similar papers at core.ac.uk
Provided by Universiti Utara Malaysia: UUM eTheses

NEW SEQUENTIAL AND PARALLEL DIVISION FREE METHODS FOR DETERMINANT OF MATRICES

SHARMILA KARIM

DOCTOR OF PHILOSOPHY

UNIVERSITI UTARA MALAYSIA

2013

Permission to Use

In presenting this thesis in fulfilment of requirements for a postgraduate degree from Universiti Utara Malaysia, I agree that the University Library may make it freely available for inspection. I further agree that permission for copying of this thesis in any manner, in whole or in part, for scholarly purposes may be granted by my supervisor(s) or, in their absence, by the Dean of the Awang Had Salleh Graduate School. It is understood that any copying or publication or use of this thesis or parts thereof for financial gain shall not be allowed without my written permission. It is also understood that due recognition shall given to me and to Universiti Utara Malaysia for any scholarly use which may be made of any material from my thesis.

Requests for permission to copy or to make other use of materials in this thesis, in whole or in part, should be addressed to:

Dean of Awang Had Salleh Graduate School
UUM College of Arts and Sciences
Universiti Utara Malaysia
06010 Sintok
Kedah Darul Aman

Abstrak

Penentu berperanan penting dalam kebanyakan aplikasi aljabar linear. Pencarian penentu menggunakan kaedah pembahagian bukan bebas akan menghadapi masalah sekiranya pemasukan matriks diwakili dalam ungkapan nisbah atau polinomial dan juga apabila kesilapan titik apungan wujud. Bagi mengatasi masalah ini, kaedah pembahagian bebas digunakan. Dua kaedah pembahagian bebas yang biasa digunakan dalam pencarian penentu adalah pendaraban silang dan pengembangan kofaktor. Walau bagaimanapun, pendaraban silang yang menggunakan Petua Sarrus hanya berhasil untuk matriks berperingkat kurang atau sama dengan tiga, sedangkan apabila berhadapan dengan matriks yang bersaiz besar, pengembangan kofaktor memerlukan pengiraan yang terlalu panjang dan rumit. Oleh itu, kajian ini berusaha membangunkan kaedah berjujukan dan kaedah selari yang baharu untuk mencari penentu bagi matriks. Kajian ini juga berhasrat untuk mengitlakkan Petua Sarrus bagi sebarang peringkat matriks segi empat sama berpandukan pilih atur yang diperolehi menggunakan set penjana. Dua strategi diperkenalkan bagi menjana set penjana yang berlainan iaitu operasi kitaran dan operasi saling tukar dua unsur. Beberapa hasil teori dan sifat matematik dalam penjanaan pilih atur dan penentuan penentu turut dibina bagi menyokong kajian ini. Keputusan berangka menunjukkan masa pengiraan kaedah baharu yang dicadangkan adalah lebih baik jika dibandingkan dengan kaedah sedia ada. Masa pengiraan kaedah berjujukan baharu yang dibangunkan tertakluk kepada penjanaan set penjana. Oleh demikian, dua strategi selari dibangunkan untuk menyelaraskan algoritma ini bagi mengurangkan masa pengiraan. Keputusan berangka turut menunjukkan bahawa kaedah selari berupaya mengira penentu lebih cepat berbanding kaedah berjujukan, khususnya apabila tugas diagihkan dengan sama rata. Kesimpulannya, kaedah baharu yang telah dibangunkan boleh diguna sebagai alternatif yang berdaya saing dalam pencarian penentu bagi matriks. .

Kata kunci: Penentu, Pilih atur, Set penjana, Kaedah tanpa pembahagi, Kaedah jujukan dan selari

Abstract

A determinant plays an important role in many applications of linear algebra. Finding determinants using non division free methods will encounter problems if entries of matrices are represented in rational or polynomial expressions, and also when floating point errors arise. To overcome this problem, division free methods are used instead. The two commonly used division free methods for finding determinant are cross multiplication and cofactor expansion. However, cross multiplication which uses the Sarrus Rule only works for matrices of order less or equal to three, whereas cofactor expansion requires lengthy and tedious computation when dealing with large matrices. This research, therefore, attempts to develop new sequential and parallel methods for finding determinants of matrices. The research also aims to generalise the Sarrus Rule for any order of square matrices based on permutations which are derived using starter sets. Two strategies were introduced to generate distinct starter sets namely the circular and the exchanging of two elements operations. Some theoretical works and mathematical properties for generating permutation and determining determinants were also constructed to support the research. Numerical results indicated that the new proposed methods performed better than the existing methods in term of computation times. The computation times in the newly developed sequential methods were dominated by generating starter sets. Therefore, two parallel strategies were developed to parallelise this algorithm so as to reduce the computation times. Numerical results showed that the parallel methods were able to compute determinants faster than the sequential counterparts, particularly when the tasks were equally allocated. In conclusion, the newly developed methods can be used as viable alternatives for finding determinants of matrices.

Keywords: Determinant, Permutation, Starter sets, Division free method, Sequential and parallel methods

Acknowledgement

First for all, I would like to express my deep sense of gratitude to Allah S.W.T for His Merciful and Kindness in giving me a chance to complete this study. I am highly indebted to Prof. Dr. Zurni bin Omar, Assoc. Prof. Dr. Haslinda binti Ibrahim, and Assoc. Prof. Dr. Khairil Iskandar bin Othman for their invaluable help and guidance during the course of research. I highly appreciated them for constantly encouraging me by giving their critics on my work. I am grateful to them for having given me the support and confidence.

I am also indebted to the Government of Malaysia (Ministry of Higher Education), and Universiti Utara Malaysia for sponsoring this research.

I would like to thank my family. I could not have made it to this level without their love and prayers. Gratitude to my parents, Haji Karim bin Karon and Hajjah Besah binti Nordin, and my siblings. They always believed in me and have always supported me.

Last but not least, I would like to thanks all my friends including Hasimah binti Sapiri, Azizah binti Mohd Rohni, Idariyana binti Mohd Seran, Fatimah Bibi binti Hamzah, Teh Raihana Nazirah binti Roslan, Hashibah binti Hamid, Hartima binti Abu Talib and INSPEM, UPM especially to Miss Hazwanie Abdullah for helping me in conducting HPC machines and INSPEM's staff for giving me a convenient office to work in.

Declaration Associated With This Thesis

Karim, S., Ibrahim, H., Omar, Z., Othman. K. I.& Sulaiman, M. (2009). Generating permutation based on starter sets, Eletronics *Proceeding of 5th Asian Mathematical Conference*,1, 90-96. available url: http://www.mat.usm.my/AMC 20200920 Proceedings/Pure/P134.PDF

Karim, S., Ibrahim, H., Omar, Z., Othman. K. I., & Sulaiman, M.(2009). New Recursive Method for Generating Permutation. *Prosiding Simposium Kebangsaan Sains Matematik ke-17*, 829-835.

Karim, S., Omar, Z. & Ibrahim, H. (2011). New Scheme in Determining nth Order Diagrams for Cross Multiplication Method via Combinatorial Approach. *Proceeding of World Academy of Science Engineering and Technology*, Issue 78, 641-644.

Karim, S., Ibrahim, H., Omar, Z., & Othman, K.I. (2011). Parallel Strategy for Starter Sets to List All Permutation Based on Cycling Restriction. *Proceeding of The Third International Conference Engineering and Technology*, 251-256.

Karim, S., Ibrahim, H.,& Omar, Z. (2011). New division free algorithm for finding the determinant, Far East Journal of Mathematical Sciences, 49, 43-55.

Karim, S., Omar, Z. & Ibrahim, H. (2011). Integrated strategy for generating permutation. International Journal of Contemporary Mathematical Sciences, Vol. 6, no. 24, 1167 - 1174.

Table of Contents

Permission to Use i

Abstrak ii

Abstract iii

Acknowledgement iv

Declaration Associated With This Thesis v

Table of Contents vi

List of Figures x

List of Tables xii

Appendices xiv

Glossary of Terms xv

CHAPTER ONE INTRODUCTION TO DETERMINANT METHODS 1

1.1 Background of the Study 1

1.1.1 A survey of Determinant Methods 3

1.1.2 Problem Statement 11

1.2 Research Question 12

1.3 Objective of the Study 13

1.4 Methodology 13

1.5 Significance of the Study 15

1.6 Scope of the Study 16

1.7 Assumption and Limitation of the Study 16

1.8 Thesis Outline 16

CHAPTER TWO PERMUTATION, DETERMINANT, PARALLEL COMPUTING AND ITS RELATED STUDIES 18

2.1 Introduction . . . 18
2.2 Permutation . . . 18
2.3 Determinant of Matrix . . . 19
2.4 Parallel Computing . . . 21
2.4.1 Parallel Computing Platform . . . 23
2.4.2 Parallel Computer Sunfire 1280 Architecture . . . 26
2.4.3 Performance of Parallel Algorithms . . . 27
2.4.3.1 Speedup . . . 28
2.4.3.2 Efficiency . . . 28
2.4.3.3 Scalability . . . 28
2.4.3.4 Cost . . . 29
2.4.3.5 Big O notation . . . 29
2.4.4 Performance Influence Factors . . . 30
2.5 Related Studies on Permutation and Determinant . . . 30
2.5.1 Permutation Generation Techniques . . . 31
2.5.2 Sequential Algorithm For Generating Permutation . . . 34
2.5.3 Parallel Algorithm For Generating Permutation . . . 36
2.5.4 Finding the Determinant Using n! Permutations . . . 40
2.5.5 Division Free Parallel Algorithm for Finding Determinant . . . 41
2.6 Summary . . . 42

CHAPTER THREE DEVELOPING NEW SEQUENTIAL ALGORITHMS FOR LISTING ALL PERMUTATIONS USING STARTER SETS 43

3.1 Introduction . . . 43
3.2 Preliminary Definitions . . . 43
3.3 Algorithm Development for Permutation . . . 46
3.3.1 Circular Operation Strategy . . . 46
3.3.1.1 Starter Sets Generation Under Circular Operation . . . 46
3.3.1.2 Permutation Generation under Circular and Reversing Operation . . . 48

3.3.1.3 Circular Algorithm 51
3.3.2 Exchange Operation Strategy 52
3.3.2.1 Starter Sets Generation Under Exchange Operation . . 52
3.3.2.2 Permutation Generation Under Circular and Reversing Operations 54
3.3.2.3 Exchange Algorithm 57
3.4 Theoretical Results 58
3.5 Numerical Results 63
3.5.1 Computational Complexity of Permutation Algorithm 66
3.5.1.1 Pseudocode of Circular Operation Strategy under Recursion (PERMUT1) 66
3.5.1.2 Pseudocode of Exchange Operation Strategy under Recursion(PERMUT2) 68
3.5.1.3 Pseudocode of Circular Operation Strategy under Iteration (PERMUTIT3) 70
3.6 Summary 72

CHAPTER FOUR DEVELOPING NEW SEQUENTIAL DIVISION FREE METHOD FOR DETERMINANT 73

4.1 Introduction 73
4.2 Preliminary Definitions 73
4.3 A Division Free Method Development for Finding Determinant 77
4.4 General Algorithm for Finding Determinant Using Permutation 90
4.5 Theoretical Results 95
4.6 Numerical Results of Division Free Algorithms 100
4.7 Summary 109

CHAPTER FIVE DEVELOPING NEW PARALLEL METHODS FOR DETERMINANT 110

5.1 Introduction 110
5.2 Preliminary Definitions 111
5.3 Parallelization Across the Time (ATT) 111

5.3.1 Parallel Algorithm for Permutation 112
5.3.2 Parallel Algorithm for Finding Determinant 116
5.4 Parallelization Across The Method (ATM) 121
5.4.1 Derivation of Parallel Algorithm for Permutation Generation . . . 121
5.4.1.1 Initial Starter Sets Generation 122
5.4.1.2 Starter Sets Generation from Initial Starter Sets 125
5.4.1.3 Permutation Generation 129
5.4.2 Parallel Algorithm for Permutation Generation 133
5.4.3 Parallel Algorithm for Finding the Determinant 135
5.5 Theoretical Results for Across The Method Algorithm 139
5.6 Performance of Parallel Algorithm for Permutation Generation 143
5.6.1 Numerical Results of Across The Time Permutation Algorithm . . 143
5.6.2 Numerical Results of Across The Method Permutation Algorithm 155
5.7 Performance of Parallel Algorithm for Determinant Method 163
5.7.1 Numerical Results of Parallel Across The Time Determinant Algorithm 164
5.7.2 Numerical Analysis of Across The Method Determinant Algorithm 181
5.7.3 Comparison Between Execution Time of Sequential, Across the Time and Across the Method Program for Finding the Determinant 188
5.8 Summary 190

CHAPTER SIX CONCLUSION AND RECOMMENDATIONS 191
6.1 Summary 192
6.2 Future Work 193

REFERENCES 195

List of Figures

2.1 General MPI Program Structure 25
2.2 List of 4! Permutations Based on Langdon Technique 32
2.3 All Pivots for $n = 5$ 32
2.4 Repetition Permutations over Pivot $[1, 3, 2, 4, 5]$ and $[1, 5, 4, 2, 3]$ 33
2.5 Starter Sets and its Equivalence 33

3.1 List of Starter Sets for $n = 4$ 47
3.2 Starter Sets by Performing CO over Last Three Elements 47
3.3 List of Starter Sets for $n = 5$ 48
3.4 List of 4! Permutations 48
3.5 List of 5! Permutations 49
3.6 List of Starter Sets for $n = 4$ 53
3.7 Starter Sets from the Exchange of the $(n - 2)$th Element 53
3.8 All Starter Set for $n = 5$ 54
3.9 List of 4! Permutations 54
3.10 List of 5! Permutations 55

5.1 Parallel Computational Graph with Master-Slave Approach 133
5.2 Speedup versus Number of Processors for PERMUT1 with 12 Initial Starter Sets 146
5.3 Speedup versus Number of Processors for PERMUT1 with 60 Initial Starter Sets 146
5.4 Efficiency versus Number of Processors for PERMUT1 with 12 Initial Starter Sets 147
5.5 Efficiency versus Number of Processors for PERMUT1 with 60 Initial Starter Sets 147
5.6 Speedup versus Number of Processors for PERMUT2 with 12 Initial Starter Sets 150

5.7 Speedup versus Number of Processors for PERMUT2 with 60 Initial Starter Sets . . . 150
5.8 Efficiency versus Number of Processors for PERMUT2 with 12 Initial Starter Sets . . . 151
5.9 Efficiency versus Number of Processors for PERMUT2 with 60 Initial Starter Sets . . . 151
5.10 Speedup versus Number of Processors for PERATM1 . . . 159
5.11 Speedup versus Number of Processors for PERATM2 . . . 159
5.12 Efficiency versus Number of Processors for PERATM1 . . . 160
5.13 Efficiency versus Number of Processors for PERATM2 . . . 160
5.14 Speedup versus Number of Processors for PERMUTDET1 with 12 Initial Starter Sets . . . 170
5.15 Speedup versus Number of Processors for PERMUTDET1 with 60 Initial Starter Sets . . . 170
5.16 Efficiency versus Number of Processors for PERMUTDET1 with 12 Initial Starter Sets . . . 171
5.17 Efficiency versus Number of Processors for PERMUTDET1 with 60 Initial Starter Sets . . . 172
5.18 Speedup versus Number of Processors for PERMUTDET2 with 12 Initial Starter Sets . . . 178
5.19 Speedup versus Number of Processors for PERMUTDET2 with 60 Initial Starter Sets . . . 179
5.20 Efficiency versus Number of Processors for PERMUTDET2 with 12 Initial Starter Sets . . . 179
5.21 Efficiency versus Number of Processors for PERMUTDET2 with 60 Initial Starter Sets . . . 180
5.22 Speedup versus Number of Processors for PDATM1 . . . 186
5.23 Speedup versus Number of Processors for PDATM2 . . . 186
5.24 Efficiency versus Number of Processors for PDATM1 . . . 187
5.25 Efficiency versus Number of Processors for PDATM2 . . . 187

List of Tables

2.1 Hardware Configuration of Sunfire 1280 27
2.2 Time Complexity of Heap, Ives, and Langdon Algorithm 35

3.1 The Computation Time of Recursive Algorithm (in seconds) 64
3.2 The Computation Time of Iterative Algorithm (in seconds) 64
3.3 The Computation Time Among New Algorithms (in seconds) 65
3.4 Comparison of Algorithm Order of Complexity 71

4.1 A pair of Main Diagonal and Secondary Diagonal Column Indices 75
4.2 The Computation Time of New Sequential Determinant Algorithm (in seconds) 100
4.3 The Computation Time Among Sequential Determinant Algorithm based on Permutation (in seconds) 101
4.4 The Comparison of Computation Time of New Algorithms to Cofactor Expansion (in seconds) 102
4.5 The Determinant Result from New Algorithms 104
4.6 The Comparison Order of Complexity of the New Algorithms to Cofactor Expansion and Permutation 106

5.1 The Number of Starter Sets Corresponding to k 113
5.2 The Difference of PERATM1 and PERATM2 in Starter Set Generation . 133
5.3 The Computation Time of PERMUT1 with 12 Initial Starter Sets (in seconds) 144
5.4 The Computation Time of PERMUT1 with 60 Initial Starter Sets (in seconds) 144
5.5 The Speedup of PERMUT1 with 12 and 60 Initial Starter Sets 145
5.6 The Efficiency of PERMUT1 with 12 and 60 Initial Starter Sets 145
5.7 The Computation Time of PERMUT2 with 12 Initial Starter Sets (in seconds) 148

5.8 The Computation Time of PERMUT2 with 60 Initial Starter Sets (in seconds) . . . 148
5.9 The Speedup of PERMUT2 with 12 and 60 Initial Starter Sets . . . 149
5.10 The Efficiency of PERMUT2 with 12 and 60 Initial Starter Sets . . . 149
5.11 The Computation Time of PERATM1 (in seconds) . . . 156
5.12 The Computation Time of PERATM2 (in seconds) . . . 156
5.13 The Speedup and Efficiency of PERATM1 . . . 157
5.14 The Speedup and Efficiency of PERATM2 . . . 158
5.15 The Computation Time of PERMUTDET1 with 12 Initial Starter Sets (in seconds) . . . 165
5.16 The Computation Time of PERMUTDET1 with 60 Initial Starter Sets (in seconds) . . . 166
5.17 The Speedup of PERMUTDET1 with 12 and 60 Initial Starter Sets . . . 168
5.18 The Efficiency of PERMUTDET1 with 12 and 60 Initial Starter Sets . . . 169
5.19 The Computation Time of PERMUTDET2 with 12 Initial Starter Sets (in seconds) . . . 173
5.20 The Computation Time of PERMUTDET2 with 60 Initial Starter Sets (in seconds) . . . 174
5.21 The Speedup of PERMUTDET2 with 12 and 60 Initial Starter Sets . . . 176
5.22 The Efficiency of PERMUTDET2 with 12 and 60 Initial Starter Sets . . . 177
5.23 The Computation Time of PDATM1 (in seconds) . . . 182
5.24 The Computation Time of PDATM2 (in seconds) . . . 183
5.25 The Speedup and Efficiency of PDATM1 . . . 184
5.26 The Speedup and Efficiency of PDATM2 . . . 185
5.27 The Computation Time for PERMUTDET1 under Circular Strategy (in seconds) . . . 189
5.28 The Computation Time for PERMUTDET2 under Exchange Strategy (in seconds) . . . 189
5.29 The Computation Time for the Sequential Algorithms and Across The Method Algorithms (in seconds) . . . 189

List of Appendices

Appendix A Sequential Permutation Program 204

Appendix B Sequential Determinant Program 208

Appendix C Parallel Permutation Program 215

Appendix D Parallel Determinant Program 227

Appendix E Permutation and Determinant Program Output 243

Glossary of Terms

Abbreviation	Details
ATM	Across The Method
ATT	Across The Time
CO	Circular Operation
CP	Circular Permutation
D	Dimension
DFM	Division Free Method
GB	Giga Bytes
HPC	High Performance Computing
ID	IDentity
ISS	Initial Starter Sets
ISSG1	Initial Starter Sets Generator of first strategy
ISSG2	Initial Starter Sets Generator of second strategy
MPI	Message Passing Interface
nDFM	non Division Free Method
PDATM1	Permutation Determinant for Across The Method of circular strategy
PDATM2	Permutation Determinant for Across The Method of exchange strategy
PERATM1	Permutation for Across The Method of circular strategy
PERATM2	Permutation for Across The Method of exchange strategy
PERMUT1	Permutation of circular strategy
PERMUT2	Permutation of exchange strategy
PERMUTIT3	Permutation of iterative circular strategy
PERMUTDET1	Permutation determinant of circular strategy
PERMUTDET2	Permutation determinant of exchange strategy
PERMUTDETIT3	Permutation determinant of iterative circular strategy
RAM	Random Access Memory
RoCP	Reverse of Circular Permutation
SIMD	Single Instruction Multiple Data
SPU	Synergistic Processing Unit

CHAPTER ONE

INTRODUCTION TO DETERMINANT METHODS

1.1 Background of the Study

Matrices and determinants are the backbone of linear algebra (Bernstein, 2008). A determinant provides useful geometrical and algebraical information of a square matrix. Algebraically, a matrix has an inverse if and only if the determinant is not zero. This happens when the vectors are linearly independent. Meanwhile geometrically, the row entries of $n \times n$ matrix define the edges of a parallelepiped in n-dimensional space, of which the area and volume are the absolute value of the determinant of a square matrix for spaces R^2 and R^3 respectively.

The determinant has been the subject of study for over 200 years. The name determinant was introduced by Carl Friedrich Gauss (1777-1855) while discussing quadratic forms. The term determinant was used because it determined the properties of the quadratic form (O'Connor & Robertson, 1996). The theory of determinant was expanded gradually during the 18th century through the theory of equations in the work of Leibniz, Maclaurin, Cramer and Laplace (Rice & Torrence, 2006). Then it became an increasingly significant subject in the mathematical area by the 19th century.

The applications of determinant can be found in various areas for example in mathematical physics in which any solvable equation having a solution can be expressed as a determinant (Vein & Dale, 1999). The determinant is required in inverse kinematics singularity analysis of parallel manipulator which this manipulator is described as 6×6 transformation matrix (Luyang et al., 2006). Meanwhile from the statistical perspective, the determinant is used in normalizing the constant of the probability density function of the multivariate normal distribution, and is also involved in experimental design (Harville, 1997). In addition, the determinant is a beneficial tool in eigenvalue problems in which

The contents of the thesis is for internal user only

REFERENCES

Abeles, F.F. (2008). Dodgson condensation: The historical and mathematical development of an experimental method.*Linear Algebra and its Application*, 429, 429-438.

Abdi, H. (2007). The Eigen-decomposition: eigenvalues and eigenvectors.In: Neil Salkind (Ed.) (2007), *Encyclopedia of Measurement and Statistics*. Thousand Oaks (CA): Sage.

Akl, S. G., & Bruda, S. D. (2001). Improving a solution's quality through parallel processing. *Journal of Supercomputing*, 19, 221-233.

Akl, S. G., Meijer, H., & Stojmenovic, I. (1994). An optimal systolic algorithm for generating permutation in lexicographic order. *J. of Parallel and Distributed Computing*, 20(1), 84-91.

Akl, S. G., & Stojmenovic, I. (1992). A simple optimal systolic algorithm for generating permutations. In Albert Zomaya, *Parallel Processing Letter Parallel Computing: Paradigms and Application*(pp.639-670). London: International Thomson Computer Press.

Almasi, G. S., & Gottlieb, A. (1989). *Highly parallel computing*. Redwood City: Benjamin-Cummings publishers.

Alonso, L., & Schott, R. (1996). A parallel algorithm for the generation of a permutation and applications. *Theoretical Computer Science*, 159, 15-28.

Anderson, R. J. (1990). Parallel algorithms for generating random permutations on a shared memory machine, *ACM*, 95-102.

Anton, H. (2000). *Elementary linear algebra*(8th ed.). New York: John Wiley.

Anton, H., & Busby, R.C. (2002). *Contemporary linear algebra*. New York: John Wiley.

Aziz, A. I., Haron, N., Mehat, M., Jung, L.T., Mustapa, A. N., & Akhir, P. E. A. (2009). Solving traveling problem on cluster compute nodes. *International Journal of Computers*, 3(2), 260-269.

Bankier,J. D. (1961). The diagrammatic expansion of Determinants. *The American Mathematical Monthly*, 68(8), 788-790.

Bernstein, D. S. (2008). *Matrix mathematics*(2nd ed.). New Jersey: Princeton University Press.

Barisenko, A. A., Kalashnikov, V. V., Kulik, I. A., & Goryachev,O. E. (2008). Generation of permutations based upon factorial numbers, *IEEE*, 57-61.

Brawer, S. (1989). *Introduction to parallel programming*. Academic Press, Inc.

Bressoud, D. M., & Propp, J. (1999). How the alternating sign matrix conjecture was solved.*Notices of AMS*,46,637-646.

Brestscher, O. (2009). *Linear algebra with applications*(4th ed.). New Jersey:Prentice Hall International.

Burrage, K. (1995). *Parallel and sequential method for ordinary differential Equation*. New York: Oxford University Press Inc.

Chalmers, A. & Tidmus, J. (1996). *Practical parallel processing, an introduction to problem solving in parallel*. London : International Thomson Computer Press.

Cong, G., & Bader, D. A. (2006). An empirical analysis of parallel random permutations algorithm on SMPs. Technical Report, Georgia Institute of Technology.

Dodgson, C. L. (1866). Condensation of determinants, being a new and brief method for computing their arithmetic Values. *Proc. Roy. Soc.* Ser. A 15, 150-155.

Djamegni, C. T., & Tchuenté, M. (1997). A cost-optimal pipeline algorithm for permutation generation in lexicographic order. *J. of Parallel and Distributed Computing*, 44(2), 153-159.

Fadlallah, G., Lavoie, M., & Dessaint, L-A. (2000). Parallel computing environments

and methods. *IEEE*, 2-7.

Fike, C. T. (1975). A permutation generation method.*The Computer Journal*, 18(1), 21-22.

Gao, J., & Wang, D. (2003). Permutation generation: Two new permutation algorithms. Retrieved from http://arxiv.org/abs/cs/0306025. on September 2008.

Gentleman, W. M., & Johnson, S. C. (1974). The evaluation of determinants by expansion by minors and the general problem of substitution. *Mathematics of Computation*, 28(126), 543-548.

Gentleman, W. M., & Johnson, S. C. (1976). Analysis of algorithms, a case study: Determinants of matrices with polynomial entries. *ACM Transactions on Mathematical Software*, 2(3), 232-241.

Goldberg, D. (1991). What every computer scientist should know about floating point arithmetic,*ACM Computing Surveys*, 21(1),5-48.

Goldfinger, Y. (2008). Determinant by cofactor expansion using Cell processor. Retrieved on December, 2008 from http://mc2.umbc.edu/docs/goldfinger.pdf on 16/11/2008.

Grama, A., Gupta, A., Karypis, G. & Kumar, V. (2003). *Introduction to parallel computing*(2nd ed.). Essex: Addison-Wesley.

Griss, M. L. (1976). An efficient sparse minor expansion algorithm. *Proceedings of ACM annual conference*, Houston, Texas, United States, 429-434

Gropp, W., Lusk, E., & Skjellum, A. (1997). *Using MPI portable parallel programming with the message-passing interface*(2nd ed.). London: MIT Press.

Gupta, P., Agarwal, V., & Varshney, M. (2008). *Design and analysis of algorithms*. New Delhi: Prentice-Hall of India

Hajrizaj, D. (2009). New method to compute the determinant of a 3×3 matrix. *International J. Algebra*, 3, 211-219.

Hanly, J. R., & Koffman, E. B. (2004). *Problem solving and program design in C*

($4th$ ed.). Boston: Pearson Addison-Wesley.

Hanus, P. H. (1886). *An elementary treatise on the theory of determinants*. Boston: Gin and Company. Retrieved from http://archive.org/stream/elementarytreati00hanuuoft #page/4/mode/2up

Harville, D. A. (1997). Matrix algebra from a statistician's perspective. pp.179-208. New York : Springer-Verlag.

Hasiung, C. Y., & Mao, G. Y. (1998). *Linear algebra*. London: World Scientific Publishing.

Heap, B. R. (1963). Permutations by interchanges. *Computer J.* 6, 293-294.

Horowitz, E., Sahni, S., & Rajasekaran, S. (2008). *Computer algorithms/C++* (2nd ed.). New Jersey: Silicon Press.

Horowitz, E., & Sahni, S. (1975). On Computing the Exact Determinant of Matrices with Polynomial Entries.*Journal of the ACM*, 38-50.

Huang,T. C., Shiu, L.C., & Huang, J. H. (2001). Efficient local memory sequence generation for data parallel programs using permutation.*Journal of Systems Achitecture*, 47, 505-515.

Hussain, S. J., & Ahmed, G. (2005). A comparative study and analysis of PVM and MPI for parallel and distributed systems. *IEEE*, 183-187.

Ibrahim, H., Omar, Z., & Rohni, A. M. (2010). New algorithm for listing all permutations. *Modern Applied Science* , 4(2), 89-94.

Iqbal, K. (1995). An algorithm for computation of determinants of polynomial matrices. Proceeding of the American Control Conferences, 2536-2537.

Iyer, M. (1995). Permutation generation using matrices. *Dr.Dobbs Journal*. Retrieved from http://www.ddj.com/184409671. on December 2008.

Johnson, L. W., Riess, R. D. & Arnold, J. T. (2002). *Introduction to linear algebra* (5th ed.). New York: Addison-Wesley Publishing Co.

Jones, M. A. (20055). Determinant Algorithm generation with numlists. *Dr.Dobbs Journal*. Retrieved from http://www.ddj.com/184402006. on 14 Februrary 2010.

Kaltofen, E. (1992). On computing Determinant of matrices without divisions. *ACM*, 324-349.

Khattar, D. (2010). *The Pearson guide to complete mathematics for AIEEE.* (3th ed.). India :Pearson Education.

Knuth, D. E. (2002). *The art of computer programming*, vol.4, New York: Addison-Wesley:

Kokosiński, Z. (1990). On generation of permutations through decomposition of symmetric groups into cosets. *BIT*, 30, 583-591.

Krattenthaler, C. (1999). Advanced determinant calculus. *Sminaire Lotharingien Combin. 42*,B42q, 1-67. Retrieved from http://www.mat.univie.ac.at/ slc/wpapers/s42kratt.pdf on September 3008.

Langdon, G. G. (1967). An Algorithm for generating permutations. *Communication of ACM*, 10(5), 298-299.

Lee, H. R., & Saunder, B. D. (1995). Fraction free gaussian elimination for sparse matrices, *Journal of Symbolic Computation*, 19(5), 393-402.

Leopald, C. (2001). *Parallel and distributed computing, a survey of models, paradigm, and approaches*. New York : John Wiley & Sons.

Levitin, A. (2007). *Introduction to the design & analysis of algorithms*(2nd ed.). New York: Addison-Wesley.

Lewis, T. G., & El-Rewini, H. (1992). *Introduction to parallel computing*. New Jersey: Prentice-Hall.

Li, K. (2010). Fast and highly scalable parallel computations for fundamental matrix problem on distributed memory systems. *The Journal Supercomputing*,271-297.

Li, Y. (2009a). An explicit construction of Gauss-Jordan elimination matrix. Retrived from http://arxiv.org/abs/0907.5038. on 17.Mac.2011.

Li, Y. (2009b). An effective hybrid algorithm for computing symbolic determinants. *Applied Mathematic and Computation*, 2495-2501.

Lin, C., & Snyder, L. (2009). *Principles of parallel programming*. Boston: Pearson

Addison-Wesley.

Lin, C. (1991). Parallel permutation generation on linear array. *International Journal of Computer Mathematic*, 38, 113-121.

Lipski, W., & Warsaw, J. (1979). More on permutation generation method. *Computing*, 357-365.

Luyang, L., Hongtao, W., & Jianying, Z. (2006). Architeture singularity analysis for a class of parallel manipulators. *Proceeding of International Technology and Innovation Conference 2006*, 2003-2006.

Mahajan, M. & Vinay, V. (1997). Determinant: Combinatorics, algorithms and complexity. *Chicago Journal of Theoretical Computer Science*, 730-738.

Mohd Saman, M. Y., & Evans, D. J. (1995). Top-down for finding optimal grain size of parallel tasks.*Pertanika J. Scince& Technology*,3(2),241-259.

Muir, T. (1933). *A treatise in the theory of determinants*. Ney York: Dover Publications.

Ord-Smith, R.J. (1970). Generating of Permutation Sequences: Part 1. *The Computer Journal*, 13, 152-155.

Osborn, R. (1960). Concerning fourth-order determinants. *The American Mathematical Monthly*, 67(7), 682-683.

O'Connor, J. J., & Robertson, E. F. (1996). Matrices and determinants. Retrieved from (http://www-history.mcs.st-andrews.ac.uk/HistTopics/ on September 2008.

Pacheco, P. S. (1997). *Parallel programming with MPI*. San Francisco: Morgan Kaufmann Publisher.

Pavlovic, S. V. (1961). On the generalisation of the Sarrus's Rule. *Mathematika I Fizika*, No.54,pp.19-23. Retrieved from http://pefmath2.etf.bg.ac.yu/files/40/54.pdf. on December, 2008.

Peng, T., Jian, M., & Dong-Ma, Z. (1999). Application of the simulated annealing algorithm to the combinatorial optimisation problem with permutation property:

An investigation of generation mechanism. *European Journal of Operational Research* , 81-94.

Perry, W. L. (1988). *Elementary linear algebra*. New York: McGraw Hill Inc.

Quinn, M. J. (2004). *Parallel programming in C with MPI and OpenMP*. New York : Mc Graw Hill.

Reek, K. A. (1998). *Pointers on C*. Massachusetts: Addison-Wesley.

Reffgen, A. (2003). The determinant in finite and infinite-dimensional vector spaces(Unpublished master dissertation). Matematiska institutionen, Lund University, Sweden.

Rezaee, H., & Rezaifar,O. (2007). A new approach for finding the determinant of matrices. *Applied Mathematics and Computation*,188, 1445-1454.

Rice, A., & Torrence, E. (2006). Lewis Carrol's condensation methods for evaluating determinants. Retrieved from http://www.MAA.ORG/MATHHORIZONS. 12-15.

Rolfe, T. J. (2008). The assignment problem: Exploring paralleism(1). *Bulletin of ACM SIG on Computer Science Education*.

Rote, G. (2001). Division-free algorithms for the determinant and the Praffian: Algebraic and combinatorial approaches. *Computational Discrete Mathematics*, 119-135.

Sasaki, T., & Kanada, Y. (1981). Parallelism in algebraic computation and parallel algorithms for symbolic linear systems. *Proceeding of the AMC on Symbolic and Algebraic Computation*, 160-167.

Sasaki, T., & Murao, H.(1982). Efficient Gaussian elimination method for symbolic determinants and linear systems. *ACM Trans. Math. Software* , 8/3, 277-289.

Scott, R. F. (2009). *A Treatise of the theory of determinants*. BiblioBazaar.

Schneider, H., & Barker, G. P. (1989). *Matrices and linear algebra*. New York: Dover publications inc.

Scneider, M. D., Steeg, M. & Young, H. F. (1982). *Linear algebra*. New York: Macmillan Publishing Co.

Sedgewick, R. (1977). Permutation generation methods. *Computing Surveys*, 9(2), 137-164.

Sedgewick, R. (2002). Permutation generation methods. Talks Dagstuhl Workshop on Data Structures, Wadern, Germany, February, 2002. Retrieved from http://www.cs.princeton.edu/ rs/talks/perms.pdf. on December, 2008.

Sengupta, R. (1997). Cancellation is exponentially powerful for computing the determinant. *Information Processing Letters*,62, 177–181.

Shin, D. W. (2002). The permutation algorithm for non-sparse matrix determinant in symbolic computation. *Proceeding on the 15th CISL winter workshop*, Japan. Retrieved from http://icat.snu.ac.kr:3333/ww/pdf/ww200210/pdf. on November 2008.

Simon, C. P., & Blume, L. E. (1994). Mathematics for economists. Chapter 7. W. W. Norton & Company: New York.

Smit, J. (1979). New recursive minor expansion algorithmd, a presentation om a comparable context. Lecture Notes In Computer Science, *Proceedings of the International Symposiumon on Symbolic and Algebraic Computation*, 72, 74-87.

Soltys, M. (2002). Berkowitz's algorithm and clow sequences. *The electronic Journal of Linear Algebra*, Vol.9, 42-54.

Standish, T. A. (1997). *Data structures, algorithms & software principles in C*. New York : Addison-Wesley Publishing.

Stojmenovic, I. (2006). Listing combinatoric objects in parallel. *The International Journal og Parallel, Emergent and Distributed Systems*, 21(2), 127-146.

Teimoori, H., Bayat, M., Amiri, A. & Sarijloo, E. (2005). A new parallel algorithm for evaluating the determinant of a matrix of order n[PowerPoint slides]. Retrieved from http://www.math.tu-berlin.de/EuroComb05/Talks/Poster. on December 2008,

Thongchiew, K. (2007). A computerize algorithm for generating permutation and it's application in determining a determinant. *Proc.of World Academy of Science,*

Engineering and Technology, 21 , 178-183.

Tsay, J. C., & Lee, W. P. (1994). An systolic design for generating permutations in lexicographic order. *Parallel Computing*, 20(3), 353-361.

Umeda, Y., & Sasaki, T. (2006). Computing determinants of rational functions. *AGM SIGSAM Bulletin*, 40(1), 2-8.

Vein, R., & Dale, P.(1999). *Determinants and their applications in mathematical physics*. New York: Springer Verlag.

Vieira, R. S. (2010). A new Algorithm for determinant evaluation-the reduction method. Retrieved from http://arxiv.org/ PS_cache/ arxiv/ pdf/ 1012/ 1012.1790v2.pdf. on 8 March 2011.

Viktorov, O. V. (2007). Permutation generation algorithm. *Asian Journal of Information Technology*, 6(9), 956-957.

Wells, M. B. (1961). Generations of permutations by transposition. *Math Comp.*, 15,192 -195.

Wilde, C. (1988). *Linear algebra*. Massachusetts:Addison-Wesley Publishing Co.

Wilkinson, B., & Allen, M. (2005). *Parallel programming. techniques and application using networked workstations and parallel computers*(2nd ed.). New Jersey : Pearson Prentice-Hall.

Zaks, S. (1984). A new algorithm for generation of permutations. *BIT*, 24, 196-204.