Expansion of the Hybrid Model of Social Orientation and Ultimatum Games : From 2-person to N-person relations

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Expansion of the Hybrid Model of Social Orientation and Ultimatum Games: From 2-person to N-person relations

Toshiaki Doi

Abstract：It has already been shown that the decision making of a person in ultimatum games can be explained and predicted very well by the hybrid model of social orientation (Doi, 2009; Doi, 2010). However, the hybrid model was developed only for 2-person relations. Social settings, in which more than two persons are involved, are considered to provide a variety of situation where might be possible to observe interesting interactions that do not appear in 2-person relations. Some researchers have already investigated N-person ultimatum games. The purpose of this paper is to show how the hybrid model can be extended to N-person relations, and how the decision making of people in N-person ultimatum games can be explained and predicted by the extended hybrid model. The analysis also reveals that perceptions of the other person's motivational state play an important role in the decision making of the allocator in ultimatum games. Interpretation of the motivational component of the decision making process varies depending on factors in the social settings, especially the number of people involved, and this is shown and discussed.

Key words：social orientation, equality seeking, ultimatum game, allocation, decision making, indifference curve, cooperative, competitive, individualistic, altruistic

1. Geometrical model of social orientation and ultimatum games

People's decision makings in social settings rarely follow the rationality of economics, that is, the maximization of one's own interest. People usually consider the other person's interest in addition to their own, when they have to make a decision. Social orientation is a concept that describes how a person takes the other person's interest into consideration, compared with his/her own. Griesinger & Livingstone (1973) showed that the social orientation of a person can be elegantly expressed geometrically, as an orientation θ in a 2-dimensional X-Y space, as shown in Fig.1. Axes X and Y represent the payoffs to oneself and the other person, respectively. Choice alternatives are expressed by points (x_i, y_i), and a person is supposed to hold a particular θ uniformly in this space. When a person is given several alternatives, he/she is supposed to choose the alternative that gives the highest m_i (coordinate value on axis M). Axis M is given by $y = \tan\theta \cdot x$ in X-Y space and m_i is given by the following equation.

$$m_i = x_i \cos\theta + y_i \sin\theta \qquad (1)$$

m_i is the weighted sum of two people's payoffs x_i and y_i, and $\cos\theta$ and $\sin\theta$ are the weights given to them. When the m_i values of two alternatives are the same, a person is indifferent to them. Which alternative gives the highest m_i varies depending on social orientation θ. That is, the social

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orientation of a person determines what he/she should pursue when he/she has to choose one from among several alternatives. Therefore, θ can be regarded as a motivational component of his/her decision making process, and m_i can be interpreted as the subjective values of the given alternatives. Equation (1) gives a straight line in *X-Y* space, and m_i of any point (x_i, y_i) on this line takes the same value. Therefore, this line, which is perpendicular to axis *M*, can be regarded as an indifference curve.

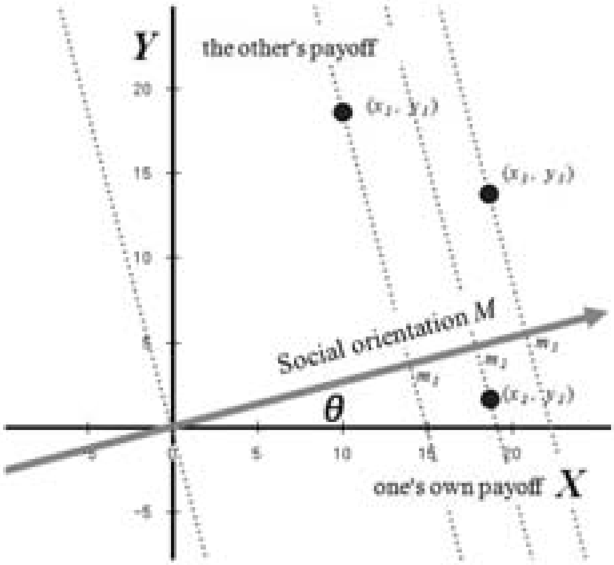

Fig.1 The geometrical expression of social orientation and indifference curves for 2-person relations.

Formally, social orientation θ can take any numerical value. Every person is supposed to own his/her unique orientation θ. Different values of θ give different weights to the payoffs to oneself and the other, and they lead people to make different decisions. Therefore, the geometrical model of social orientation can deal with individual differences in people's choice attitudes. Several social orientations θ are usually called by the specific names shown in Table.1. Each name represents a characteristic choice attitude, which corresponds to a concern for the other's payoff compared with one's own.

The geometrical model of social orientation has been applied universally to research on decision making in interdependent situations. How the decision making of a person for the given 2x2 payoff matrices can be predicted and explained based on the geometrical model has been explicitly shown (Doi, 1984; 1990; 1994). However, it has become clear that the geometrical model has its limitations in explaining the decision making of a person in some social settings. The experimental research on ultimatum games (Güth, Schmittberger, & Schwarze, 1982) or dictator games (Forsythe et al., 1994), for

Table.1 Interrelationship among social orientations θ, evaluation formulas, and their interpretations

social orientation θ	evaluation formula for the given alternatives m_i	maximizing	interpretation
90°	y_i	other's payoff	altruistic
45°	$\frac{1}{\sqrt{2}}(x_i + y_i)$	one's own and the other's payoff	cooperative
0°	x_i	one's own payoff	individualistic
-45°	$\frac{1}{\sqrt{2}}(x_i - y_i)$	difference of one's own payoff from the other's	competitive
-90°	$-y_i$	(minimizing) other's payoff	aggressive

example, clearly articulated that people have a strong preference for equal sharing of the given payoff.

Ultimatum games are decision situations where two persons are given a certain amount of payoff, but they have to divide it between them. One person, called the allocator, has to propose how to divide the given payoff between them. Then, the other person, called the recipient, has to decide if he/she accepts the proposed allocation or not. If the recipient accepts it, both he/she and the allocator can receive a portion of the given payoff according to the proposed allocation. If the recipient refuses to accept it, neither can get any payoff. The experimental studies on ultimatum games have repeatedly shown that people did not follows the rational decision principle in the economics sense, that is, the maximization of one's own payoff. The modal proposition was usually a 50-50 division of the given payoff (Güth, Schmittberger, & Schwarze, 1982; Güth, 1995; Carmer and Thaler, 1995; Carmer, 2003). Such experimental results are interpreted as indicating that people tend to avoid an unfair division of interests, or to prefer equality of received interest.

2. Equality seeking and the hybrid model of social orientation

The geometrical model of social orientation cannot explain the equality seeking tendency of people in ultimatum games. However, decision making based on equality seeking itself can be geometrically expressed in *X-Y* space. If a person wants to achieve equal sharing of the given interest, he/she should choose the alternative (x_i, y_i) that minimize d_i (the absolute difference between the interests to 2 persons) given in the following equation.

$$d_i = |x_i - y_i| \tag{2}$$

Naturally, d_i is given by $(x_i - y_i)$ when $x_i > y_i$, and by $(y_i - x_i)$ when $x_i < y_i$. When $x_i = y_i$, $d_i = 0$. That is, a person evaluates the given alternatives of choice (x_i, y_i) differently, depending on where they are situated in *X-Y* space. Substituting $\theta = \frac{3}{4}\pi = 135°$ or $\theta = \frac{-1}{4}\pi = -45°$ into equation (1), minimization of d_i can be achieved by maximization of m_i, that is,

when $x_i > y_i$, m_i is given by

$$m_i = x_i \cos\frac{3}{4}\pi + y_i \sin\frac{3}{4}\pi$$
$$= \frac{(-x_i + y_i)}{\sqrt{2}} = \frac{-d_i}{\sqrt{2}} \tag{3}$$

and when $x_i < y_i$, m_i is given by

$$m_i = x_i \cos\frac{-1}{4}\pi + y_i \sin\frac{-1}{4}\pi$$
$$= \frac{(x_i - y_i)}{\sqrt{2}} = \frac{-d_i}{\sqrt{2}} \tag{4}$$

Then, it can be said that a person based on equality seeking activates two different social orientations, that is, $\theta = \frac{3}{4}\pi = 135°$ when $x_i > y_i$ and $\theta = \frac{-1}{4}\pi = -45°$, when $x_i < y_i$. The decision attitude described by (3) and (4) was named the geometrical model of equality seeking (Doi, 2009, 2010).

The hybrid model of social orientation

Both social orientation and equality seeking are considered to play an influential role as a motivational component in the decision making process of a person in a variety of social situations. Under the assumption that a person's decision making is affected by both his/her social orientation and equality seeking simultaneously, Doi (2007) proposed the hybrid model of social orientation. This model was obtained by a linear combination of the geometrical model of social orientation and equality seeking. According to this model, subjective values m_i of the alternatives of choice (x_i, y_i) are described by the following two equations.

When $x_i \geq y_i$, m_i is given by

$$m_i = a(x_i \cos\theta_M + y_i \sin\theta_M) + (1-a)\left(x_i \cos\tfrac{3}{4}\pi + y_i \sin\tfrac{3}{4}\pi\right) \quad (5),$$

and when $x_i \leq y_i$, m_i is given by

$$m_i = a(x_i \cos\theta_M + y_i \sin\theta_M) + (1-a)\left(x_i \cos\tfrac{-1}{4}\pi + y_i \sin\tfrac{-1}{4}\pi\right) \quad (6).$$

The first terms of the right side in (5) and (6) are identical and they express the decision making component based on the social orientation θ_M of a person (θ_M is used instead of θ to distinguish the hybrid model from the geometrical model). The second terms are different in (5) and (6) and they express the decision making component based on equality seeking.

Parameter a in (5) and (6) takes numerical values between 0 and 1. It expresses how strongly or weakly each component affects the decision making process of a person. When $a = 1$, equations (5) and (6) are identical to equation (1), that is, the geometrical model of social orientation. When $a = 0$, (5) and (6) are identical to equations (3) and (4), respectively, that is, equality seeking. Therefore, a person makes a decision based only on his/her social orientation θ_M when $a = 1$, and only on equality seeking when $a = 0$. When $0 < a < 1$, a person is supposed to make a decision based partly on social orientation and partly on equality seeking, simultaneously. The closer the value of a to 1, the more the decision making of a person is affected by social orientation θ_M. On the other hands, the closer the value of a gets to 0, the more the decision making is affected by equality seeking. Therefore, the hybrid model is considered to be genuine integration of the geometrical models of social orientation and equality seeking. Equations (5) and (6) can be transformed into the following equations.

$$m_i = x_i\left(a \cdot \cos\theta_M - \tfrac{(1-a)}{\sqrt{2}}\right) + y_i\left(a \cdot \sin\theta_M + \tfrac{(1-a)}{\sqrt{2}}\right) \quad (7)$$

$$m_i = x_i\left(a \cdot \cos\theta_M + \tfrac{(1-a)}{\sqrt{2}}\right) + y_i\left(a \cdot \sin\theta_M - \tfrac{(1-a)}{\sqrt{2}}\right) \quad (8)$$

These two equations express two different straight lines, in *X-Y* space. When m_i in (7) and (8) take the same value, two lines connect necessarily on the line given by $y = x$, resulting in a polygonal line, as shown in Fig.2. Since m_i is a subjective value, this polygonal line can be regarded as an indifference curve. The shape and the position of the indifference curve are determined by θ_M and a. However, the angle points always situate on the line $y = x$. This indifference curve determines which alternative a person should choose from among the given alternatives. This indifference curve divides a whole space into two areas, upper right and lower left in *X-Y* space. Any point in the upper right area always gives higher m_i than any point in the lower left area. Then a person should choose (x_3, y_3) from the three alternatives shown in Fig.2.

As discussed earlier, the indifference curve given by (1) is perpendicular to axis M that is given by social orientation θ. Then, there should be social orientations that give the two indifference curves (7)

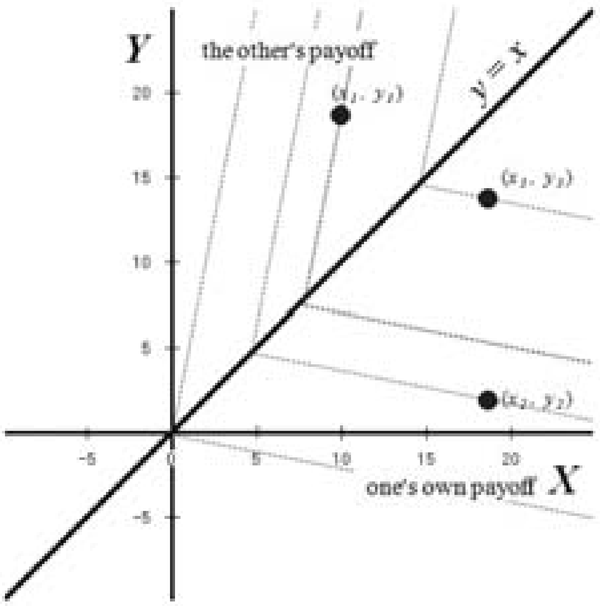

Fig.2 Indifference curves of the hybrid model for 2-person relations.

and (8), in *X-Y* space. Assigning θ_L and θ_N to such social orientations and l_i and n_i to the subjective values of the alternatives based on θ_L and θ_N, two equalities (7) and (8) can be transcribed as follows.

$$l_i = x_i \cos\theta_L + y_i \sin\theta_L \quad (9)$$

$$n_i = x_i \cos\theta_N + y_i \sin\theta_N \quad (10)$$

Mathematical analysis by Doi (2009, 2010) revealed that θ_L and θ_N can be interpreted as different modifications of the same θ_M. That is, a person owns a basic social orientation θ_M that is active at all times for any alternatives of choice, in *X-Y* space. However, this θ_M is modified by a equality seeking component, differently depending on whether $x_i \geq y_i$ or $x_i \leq y_i$. Social orientations θ_L and θ_N are resultant orientations from this modification. Therefore, social orientations θ_L and θ_N are not active simultaneously. Which one is active depends on where the given alternative situates in *X-Y* space. It can be said that θ_M is rather stable, but θ_L and θ_N are temporal. It seems likely that θ_M expresses a trait of social orientation, and θ_L and θ_N express a state of social orientation. In other words, the hybrid model of social orientation can be interpreted to be a model of a person who activates two different social orientations, depending on whether $x_i \geq y_i$ or $x_i \leq y_i$.

3. The application of the hybrid model to ultimatum games

Doi (2009, 2010) showed how the decision making of two persons in ultimatum games can be explained by applying the above indifference curve analysis. Ultimatum games, where 100 units are provided to 2 persons, can be expressed geometrically as shown in Fig.3. All the possible allocations situate necessarily on line A-B. What the allocator has to do is to choose one point from the line A-B. In response, the recipient has to decide if he/she accepts or rejects the proposed allocation. If he/she rejects it, neither person will get any units, which corresponds to the origin in *X-Y* space.

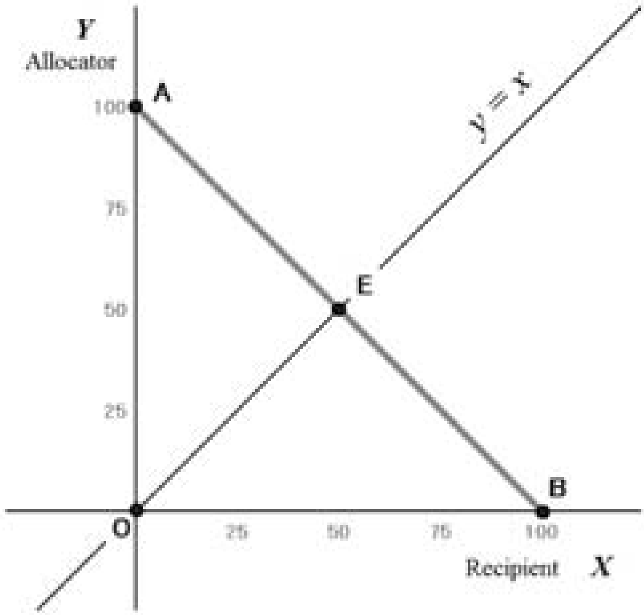

Fig.3 Geometrical expression of 2-person ultimatum games where two persons have to divide the given 100 units.

Decision making of the recipient can be explicitly described by his/her indifference curve that passes through the origin in the geometrical expression of the ultimatum game, as shown in Fig.4. It divides the whole X-Y space into 2 areas, acceptable and rejectable. The acceptable area is grayed in Fig.4. Any point in the acceptable area gives a higher value of m_i than the origin. That is, the recipient should accept the allocation if it situats in that area. Therefore, the indifference curve passing through the origin can be regarded as the definite criterion to judge if the recipient should accept the proposed allocation or not. This indifference curve is called FIC fundamental indifference curve (Doi, 2010).

allocator and 100 units to the recipients). The recipient does not feel happy, even if he/she gets all the given payoff. Furthermore, point E (50units to two persons equally) gives the highest m_i in the acceptable segment, in either Fig.4 or Fig.5. That is, the recipient should feel the highest satisfaction, when the allocator chooses point E as his/her allocation proposal. Since, the shape of FIC is determined by the two variables θ_M and a, we can predict unambiguously the decision making of a recipient in the ultimatum games, if those two variables are known.

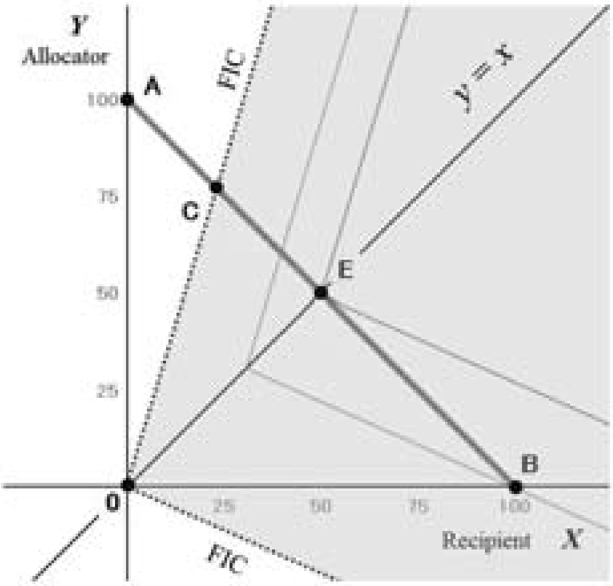

Fig.4 FIC and 2 indifference curves of the recipient, in 2-person ultimatum games (Example1)

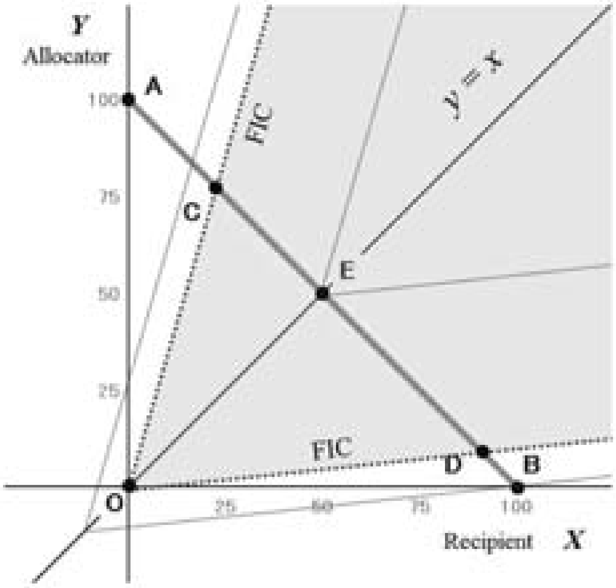

Fig.5 FIC and 2 indifference curves of the recipient, in 2-person ultimatum games (Example2)

Line segment A-B in Fig.4 is divided into two line segments by FIC. Line segment B-C is an acceptable segment and A-C is a rejectable. Then, the recipient should accept any allocation from the acceptable segment B-C, and reject any from the rejectable segment A-C. However, the shape of FIC in Fig.4 is just one example. There are numerous shapes of FIC. Fig.5 is another example of FIC. Line segment A-B is divided into three line segments in Fig.5. The middle segment C-D is acceptable and two outside segments A-C and B-D are rejectable. The recipient is supposed to refuse the allocation corresponding to point B (no unit to the

Decision making of the allocator is more complicated, but it can be described by his/her FIC. Any point on the rejectable segment gives lower m_i than the origin in the X-Y space. That is, there is no use proposing an allocation from the rejectable segment. Then, the allocator is supposed to choose a point that gives the highest m_i from the acceptable line segment. However, it is crucial that the chosen point belongs to the acceptable line segment of the recipient. If not, the chosen point will be rejected by the recipient. As shown above, the FIC of the recipient is determined by his/her θ_M and a. That is, the allocator has to guess the FIC of the recipient, that is, his/her θ_M and α. Therefore, the allocator has to find the point that gives the highest m_i, from among the acceptable line of the recipient. The point giving the highest m_i has to be either one among three points, that is, point E or both ends of the acceptable line segment of the recipient. Fig.6 and Fig.7 show two examples of the decision making of the allocator. The highest m_i is given by point E ($x_i = y_i$) in Fig.6 and point C ($x_i > y_i$) in Fig.7.

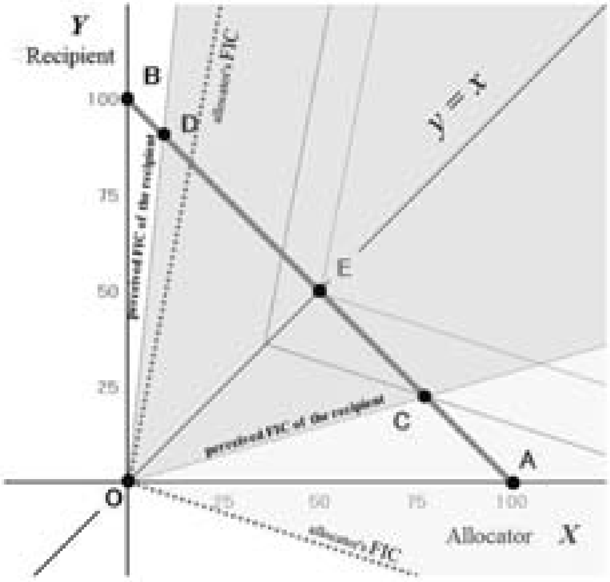

Fig.6　Allocator's FIC, 2 indifference curves, and the perceived FIC of the recipient, in 2-person ultimatum games (Example 1)

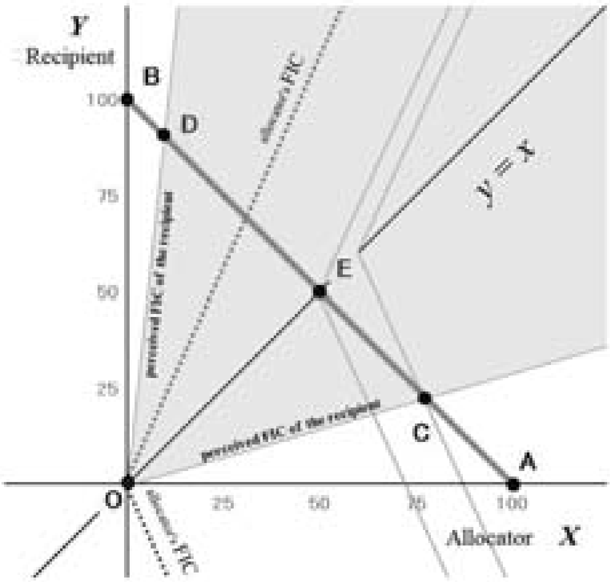

Fig.7　Allocator's FIC, 2 indifference curves, and the perceived FIC of the recipient, in 2-person ultimatum games (Example 2)

4. Expansion of the hybrid model to N-person relations.

The hybrid model of social orientation for two persons was created combining the classical geometrical model of social orientation and equality seeking. Then, first, we will show N-person versions of the geometrical model of social orientation and equality seeking, then we will combine them, in a manner similar to the hybrid model for two persons. An N-person extension of the geometrical model of social orientation has already been presented by Doi (1990). We will consider this model in detail.

The social orientation of a person and his/her decision making in an N-person situation are expressed geometrically in the N-dimensional space where axes X, Y_1, Y_2, ⋯, Y_{n-1} represent the payoff to oneself and $n-1$ others. Alternatives of choice are expressed by points (x_i, $y_{1,i}$, $y_{2,i}$, ⋯, $y_{k,i}$, ⋯, $y_{n-1,i}$) in this space. The social orientation of a person is expressed by axis M, and he/she is supposed to choose the alternative which gives the highest orthogonal projection m_i to axis M, in this space. The social orientation θ of a person corresponds to the angle between two axes M and X. The orthogonal projection m_i can be regarded as describing the weighted sum of the payoffs to self and $n-1$ others. Furthermore, theoretical analyses in this paper will be conducted only on the persons who perceive $n-1$ others indifferently, that is, the weights given to the payoffs to $n-1$ others are the same. This restriction is introduced in order to make follow-on discussions simple and clear.

When a person is indiscriminant to $n-1$ others, the orthogonal projection m_i is given by the next equation.

$$m_i = x_i\, cos\theta + \sum_{k=1}^{n-1} y_{k,i} \frac{sin\theta}{\sqrt{n-1}}$$

$$= x_i\, cos\theta + \frac{\sum_{k=1}^{n-1} y_{k,i}}{\sqrt{n-1}}\, sin\theta \qquad (11)$$

That is, equation (11) indicates that $\cos\theta$ is a weight given to a person's own payoff and $\frac{\sin\theta}{\sqrt{n-1}}$ is a weight given to $n-1$ other persons' payoffs. Introducing z_i given by the next equation,

$$z_i = \frac{\sum_{k=1}^{n-1} y_{k,i}}{\sqrt{n-1}} \quad (12),$$

equation (11) can be transformed into the following equation.

$$m_i = x_i \cos\theta + z_i \sin\theta \quad (13)$$

That is, the social orientation of a person and his/her decision making can be described geometrically, in the 2-dimensional *X-Z* space. The social orientation of a person is expressed as the direction of axis *M* in *X-Z* space. Axis *M* is given by the next equation.

$$z = \tan\theta \cdot x \quad (14)$$

Two-dimensional *X-Z* space exists inside of the N-dimensional space, and axis *Z* is given by the composition of unit vectors parallel to $n-1$ axes Y_k. That is, z_i can be interpreted to be representative of $n-1$ person's payoffs for the given alternative. When $n=2$, equation (11) becomes identical to equation (1). Then, equation (11) can be regarded as an expansion of the geometrical model of social orientation given by equation (1).

θ can take any numerical values in *X-Z* space. When $\theta = 0°$, equation (13) gives $m_i = x_i$. That is, maximizing m_i is identical to maximizing x_i. A person is supposed to make a decision, without paying any attention to how his/her decision will affect other person's payoffs. Then, this orientation $\theta = 0°$ can be regarded as ***individualistic*** social orientation. On the other hand, when $\theta = \frac{1}{2}\pi = 90°$, equation (13) gives $m_i = z_i$. That is, maximizing z_i is equivalent to maximizing $y_{k,i}$. Therefore, a person is supposed to make a decision to maximize the payoffs to $n-1$ others, neglecting his/her own payoff. Then, this orientation $\theta = \frac{1}{2}\pi = 90°$ can be interpreted as ***altruistic*** social orientation.

There are two more orientations worth noting. When $\theta = \tan^{-1}\sqrt{n-1}$, equation (11) becomes as follows;

$$m_i = \frac{1}{\sqrt{n}}\left(x_i + \sum_{k=1}^{n-1} y_{k,i}\right) \quad (15).$$

A person is supposed to maximize the sum of the payoff to everybody including him/herself. That is, orientation $\theta = \tan^{-1}\sqrt{n-1}$ can be interpreted as ***cooperative*** social orientation. Furthermore, when $\theta = \tan^{-1}\frac{-1}{\sqrt{n-1}}$, equation (11) will be

$$m_i = \frac{\sqrt{n-1}}{\sqrt{n}}\left(x_i - \frac{\sum_{k=1}^{n-1} y_{k,i}}{n-1}\right) \quad (16).$$

A person is supposed to maximize the difference between the payoffs to him/herself and the other persons. Therefore, orientation $\theta = \tan^{-1}\frac{-1}{\sqrt{n-1}}$ can be regarded as ***competitive*** social orientation.

Obviously, the values of $\theta = \tan^{-1}\sqrt{n-1}$ and $\theta = \tan^{-1}\frac{-1}{\sqrt{n-1}}$ vary according to *n*. In general, the direction of axis *M*, that corresponds to a particular ratio of the weights given to the payoff to oneself and $n-1$ others, varies depending on the number of people involved in the social settings. Table.2 shows how values of θs that correspond to five social orientations change according to *n*. The increase of n does not affect θ of *altruistic* and *individualistic* orientations. *Altruistic orientation* is always given by $\theta = 90°$, and individualistic orientation, by $\theta = 0°$. However, the more n increases, the closer θ of *cooperative orientation* approaches to $90°$, that is, *altruistic orientation*, and competitive orientation to $0°$, *individualistic orientation*. Therefore, the difference between altruistic and cooperative orientations and the difference between individualistic and competitive orientations are likely to disappear when n is extremely large. However, interestingly enough, *cooperative* and *competitive* orientations are at right angles to each other, regardless of n.

Five social orientations are distinctively different geometrically when $n = 2$. However, in research on social orientation, altruistic and cooperative orientations have quite often been classified into *pro-social orientation*, and individualistic and competitive orientations to *proself orientation*. The above analysis seems to support theoretically the validity of such a classification.

Equality seeking in N-person relations

As we have seen, it was made clear how the geometrical model of social orientation can be expanded to N-person relations. It is now necessary to consider how equality seeking can be expressed geometrically in *X-Z* space. The essence of equality seeking would be defined as equal sharing of the given payoffs among n persons, in the same way as happens in equality seeking between 2 persons. We suppose that a person making decisions based on equality seeking in N-person relations tries to minimize the absolute difference between his/her own payoff and $n - 1$ other's average payoff, that is, $\left|x_i - \frac{\sum_{k=1}^{n-1} y_{ki}}{n-1}\right|$. This minimization can be achieved by two specific social orientations $\theta = tan\theta^{-1}\frac{-1}{\sqrt{n-1}}$

Table.2　Interrelationship among θ of a person, attributes of social orientation, and the number of persons, in N-person relations. This table is effective for the allocator in case of N-person ultimatum games.

interpretation of θ			altruiste	cooperative	individualistic	competitive	aggressive
evaluation formula for the given alternatives			y_i	$\frac{1}{\sqrt{n}}\left(x_i+\sum_{k=1}^{n-1} y_{ki}\right)$	x_i	$\frac{\sqrt{n-1}}{\sqrt{n}}\left(x_i-\frac{1}{n-1}\sum_{k=1}^{n-1} y_{ki}\right)$	$-y_i$
θ	n	$n-1$	90°	$\tan^{-1}\sqrt{n-1}$	0°	$\tan^{-1}\frac{-1}{\sqrt{n-1}}$	-90.0°
	2	1	90°	45.0°	0°	-45.0°	-90.0°
	3	2	90°	54.7°	0°	-35.3°	-90.0°
	4	3	90°	60.0°	0°	-30.0°	-90.0°
	10	9	90°	71.6°	0°	-18.4°	-90.0°
	20	19	90°	77.1°	0°	-12.9°	-90.0°
	40	39	90°	80.9°	0°	-9.1°	-90.0°
	60	59	90°	82.6°	0°	-7.4°	-90.0°
	80	79	90°	83.6°	0°	-6.4°	-90.0°
	100	99	90°	84.3°	0°	-5.7°	-90.0°
	1000	999	90°	88.2°	0°	-1.8°	-90.0°

that corresponds to two sets of (cosθ and sinθ), that is, ($\cos\theta = \frac{-\sqrt{n-1}}{\sqrt{n}}$, $\sin\theta = \frac{1}{\sqrt{n}}$) and ($\cos\theta = \frac{\sqrt{n-1}}{\sqrt{n}}$, $\sin\theta = \frac{-1}{\sqrt{n}}$). Substituting these sets into equation (11), we obtain the followings.

$$m_i = \frac{\sqrt{n-1}}{\sqrt{n}}\left(-x_i + \frac{\sum_{k=1}^{n-1} y_{k,i}}{n-1}\right)$$
$$= -\frac{\sqrt{n-1}}{\sqrt{n}}x_i + \frac{1}{\sqrt{n}}z_i \qquad (17)$$

$$m_i = \frac{\sqrt{n-1}}{\sqrt{n}}\left(x_i - \frac{\sum_{k=1}^{n-1} y_{k,i}}{n-1}\right)$$
$$= \frac{\sqrt{n-1}}{\sqrt{n}}x_i - \frac{1}{\sqrt{n}}z_i \qquad (18)$$

Equation (17) is effective when $x \geq \frac{1}{\sqrt{n-1}}z$, and (18) is effective when $x \leq \frac{1}{\sqrt{n-1}}z$. Those alternatives that give zero difference situate on the axis given by $z = \sqrt{n-1} \cdot x$, in X-Z space. When $n = 2$, equations (17) and (18) become to equations (3) and (4), respectively. Therefore, a set made up of equation (17) and equation (18) can be regarded as the geometrical model of equality seeking for N-person relations.

Expansion of the hybrid model to N-person relations

It was made clear that social orientation and equality seeking in N-person relations can be geometrically expressed in 2-dimensional space, in the same manner as 2-person relations. Then, the expansion of the hybrid model to N-person relations can be carried out straightforwardly, as follows.

When $x \geq \frac{1}{\sqrt{n-1}}z$, m_i is given by

$$m_i = a(x_i\,cos\theta_M + z_i sin\theta_M) +$$
$$(1-a)\left(-\frac{\sqrt{n-1}}{\sqrt{n}}x_i + \frac{1}{\sqrt{n}}z_i\right) \quad (19).$$

And when $x \leq \frac{1}{\sqrt{n-1}}z$, m_i is given by

$$m_i = a(x_i\,cos\theta_M + z_i sin\theta_M) +$$
$$(1-a)\left(\frac{\sqrt{n-1}}{\sqrt{n}}x_i - \frac{1}{\sqrt{n}}z_i\right) \quad (20).$$

These two equations can be transformed to

$$m_i = x_i\left(a\cdot\cos\theta_M - \frac{\sqrt{n-1}(1-a)}{\sqrt{n}}\right) +$$
$$z_i\left(a\cdot\sin\theta_M + \frac{(1-a)}{\sqrt{n}}\right) \quad (21)$$

$$m_i = x_i\left(a\cdot\cos\theta_M + \frac{\sqrt{n-1}(1-a)}{\sqrt{n}}\right) +$$
$$z_i\left(a\cdot\sin\theta_M - \frac{(1-a)}{\sqrt{n}}\right) \quad (22)$$

When $n = 2$, equations (19) and (20) become (5) and (6), and equations (21) and (22) become (7) and (8). Therefore, a pair of equation (19) and (20), or (21) and (22), can be definitely regarded as the expansion of the hybrid model to N-person relations. The essential part of the hybrid model, that a person is supposed to make a decision based on social orientation and equality seeking simultaneously, is kept perfectly, and it can be expressed geometrically in 2-dimensional space, regardless of n.

5. The application of the hybrid model to N-person ultimatum games

In 2-person ultimatum games, the recipient can decide by him/herself if he/she accepts or rejects a proposed allocation, without any restrictions. However, N-person ultimatum games are in some degree different in this point. There are $n-1$ recipients. The problem is how to make a decision to accept or reject the proposed allocation, and who makes the decision. There could be a variety of ways to make this decision. For example, the decision might be made by one person as a leader or a delegate of $n-1$ recipients, or by rule of majority, or by anonymous agreement, and so on. In any case, if the decision to accept the proposal is made, every recipient gets his/her fair share, and if a decision to reject is made, nobody gets any payoff. That is, the sum or the average of $n-1$ person's payoffs is considered to play an important part in making a decision to accept or reject a proposed allocation. The analysis of N-person relations made it clear that $z_i = \frac{\sum_{k=1}^{n-1} y_{k,i}}{\sqrt{n-1}}$ can be representative of $n-1$ person's payoffs in N-dimensional space. It is interesting that $z_i = \frac{\sum_{k=1}^{n-1} y_{k,i}}{\sqrt{n-1}}$ lies just in between the sum and the average of $n-1$ person's payoffs.

N-person ultimatum games, from the allocator's perspective, can be expressed geometrically in 2 dimensional X-Z space, in the same manner as 2-person ultimatum games. An N-person ultimatum game, where 100 units are given to be divided, is expressed geometrically, as shown in Fig.8. All the possible allocations are expressed by the points on line segment A-B. The allocation corresponding to point E means that all the people (allocator and $n-1$ recipients) share equal amounts of payoff, that is, $\frac{100}{n}$.

The decision analysis on 2-person ultimatum games based on the hybrid model, can be straightforwardly applied to the allocator's decision making in N-person ultimatum games, and it can be shown geometrically in X-Z space, as shown in Fig.8. The expression is quite similar to that of the allocator in 2-person ultimatum games. The way to determine the allocation is basically the same, regardless of the value of n. The allocator is supposed to determine his/her proposal based on his own FIC (θ_M and a) and his/her perceived FIC of the recipients. However the meaning of θ_M varies according to n, as shown in Table.2.

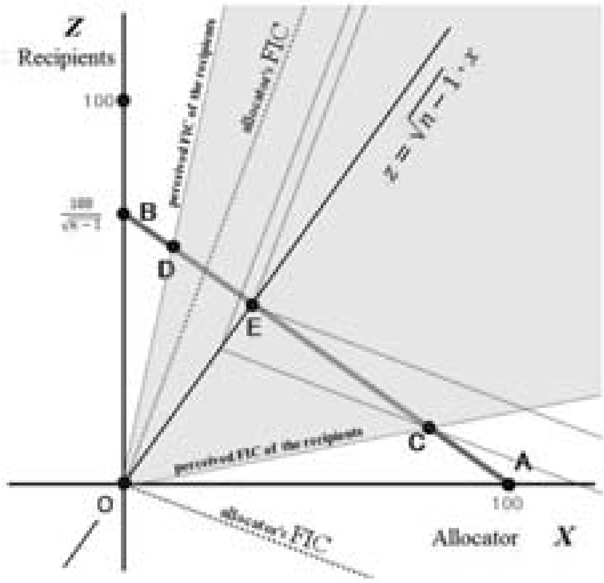

Fig.8　Allocator's FIC, 2 indifference curves, and the perceived FIC of the recipients, in N-person ultimatum games (from allocator's perspective)

With respect to the decision making of recipients, there is a problem of how and who makes the decision to accept or reject the proposed allocation, as discussed above. No matter how the decision is made, a single decision to accept or reject the proposed allocation has to be made, considering the allocated payoff to $n-1$ persons. The payoff to $n-1$ persons is treated by a single parameter $\frac{\sum_{k=1}^{n-1} y_{k,i}}{\sqrt{n-1}}$, in the analysis of N-person relations in this paper. Therefore, it is possible to suppose that $n-1$ recipients make a decision considering $\frac{\sum_{k=1}^{n-1} y_{k,i}}{\sqrt{n-1}}$, as if they were a single person. The social orientation θ_M of this postulated agent is expressed geometrically in Z-Y space where Z stands for the agent and Y for the allocator. The subjective value of the given allocation is described by the following equation.

$$m_i = \frac{\sum_{k=1}^{n-1} x_{k,i}}{\sqrt{n-1}} \cos\theta + y_i \sin\theta$$
$$= \sum_{k=1}^{n-1} x_{k,i} \frac{1}{\sqrt{n-1}} + y_i \sin\theta \qquad (23)$$

Replacing $\frac{\sum_{k=1}^{n-1} x_{k,i}}{\sqrt{n-1}}$ by z_i, FIC of the postulated agent can be given in the next equations.

$$m_i = z_i\left(a \cdot \cos\theta_M - \frac{(1-a)}{\sqrt{n}}\right) + y_i\left(a \cdot \sin\theta_M + \frac{\sqrt{n-1}(1-a)}{\sqrt{n}}\right) \quad (24)$$

$$m_i = z_i\left(a \cdot \cos\theta_M + \frac{(1-a)}{\sqrt{n}}\right) + y_i\left(a \cdot \sin\theta_M - \frac{\sqrt{n-1}(1-a)}{\sqrt{n}}\right) \quad (25)$$

Equations (24) and (25) are effective when $y_i \geq \frac{1}{\sqrt{n-1}} z_i$ and $y_i \leq \frac{1}{\sqrt{n-1}} z_i$, respectively. N-person ultimatum games from the recipients' perspective and the recipients' FIC are expressed geometrically in *Z-Y* space, as shown in Fig.9. The decision making of the recipients can be predicted in the same manner as in 2-person ultimatum games. All the possible allocations of 100 units situate on line segment A-B, and the middle line segment C-D is included in the acceptable area. Then, recipients would accept the proposed allocation only when it comes from line segment C-D.

The space in Fig.9 is obtained by replacing the horizontal and vertical axes of the space in Fig.8. There is an interesting difference between the social orientations of the allocator and the postulated actor (recipients). The allocator's cooperative and competitive orientations get closer to altruistic and individualistic orientation respectively, according to the increase of n (number of persons). Conversely, the postulated actor's cooperative and competitive orientation gets close to individualistic and aggressive orientation. It seems that the meaning of θ is fairly different, depending on whether a person is facing many other persons or he/she is facing another person as a leader of a large group, according to the increase of n. Cooperative orientation becomes much the same as altruistic orientation in the former situation, but individualistic in the latter, when n is quite large. Competitive orientation becomes much the same as individualistic orientation in the former situation but aggressive in the latter. The interpretation of θ for the recipients in N-person ultimatum games is shown in Table.3. It seems very likely that the motivational component of the decision making process is affected by the number of persons involved in social settings.

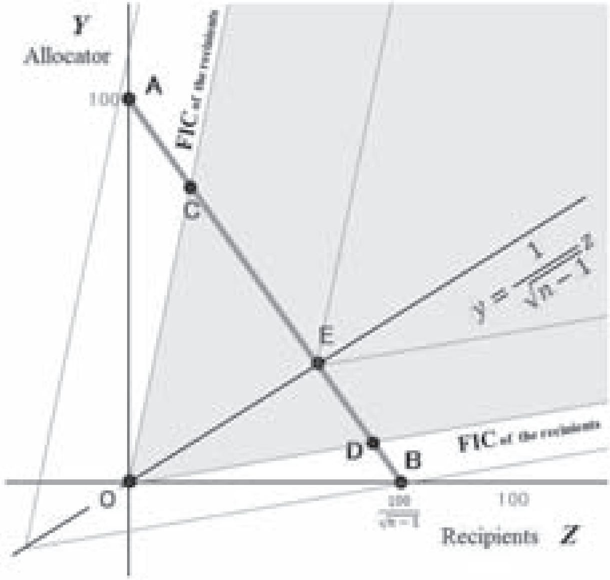

Fig.9 FIC of the recipients in N-person ultimatum games (from recipients f perspective)

6. Implications for the research

In this paper, it was argued that the decision making of the allocator and the recipient in ultimatum games can be explained and predicted based on

Table.3 Interrelationship among θ of a group of n-1 recipients, attributes of social orientation, and the number of persons, in N-person ultimatum games. This table is effective for a group of n-1 recipients.

interpretation of θ			altruiste	cooperative	individualistic	competitive	aggressive
evaluation formula for the given alternatives			y_i	$\frac{1}{\sqrt{n}}\left(\sum_{k=1}^{n-1} x_{ki} + y_i\right)$	x_i	$\frac{\sqrt{n-1}}{\sqrt{n}}\left(\frac{1}{n-1}\sum_{k=1}^{n-1} x_{ki} - y_i\right)$	$-y_i$
θ	n	$n-1$	90°	$\tan^{-1}\frac{1}{\sqrt{n-1}}$	0°	$\tan^{-1} -\sqrt{n-1}$	-90.0°
	2	1	90°	45.0°	0°	-45.0°	-90.0°
	3	2	90°	35.3°	0°	-54.7°	-90.0°
	4	3	90°	30.0°	0°	-60.0°	-90.0°
	10	9	90°	18.4°	0°	-71.6°	-90.0°
	20	19	90°	12.9°	0°	-77.1°	-90.0°
	40	39	90°	9.1°	0°	-80.9°	-90.0°
	60	59	90°	7.4°	0°	-82.6°	-90.0°
	80	79	90°	6.4°	0°	-83.6°	-90.0°
	100	99	90°	5.7°	0°	-84.3°	-90.0°
	1000	999	90°	1.8°	0°	-88.2°	-90.0°

the hybrid model of social orientation. It seems that the hybrid model can explain major research findings in ultimatum games quite well. It has been frequently observed that quite a few persons prefer equal division of the provided payoff. Most of the proposed allocations stuck around 50-50 division. The hybrid model can explain those research findings in ultimatum games considerably well, supposing that each person has his/her own decision criterion FIC (or θ_M and a). In addition, the analyses of ultimatum games by the hybrid model revealed the importance of the allocator's perception of the FIC of the recipient. Decision making by the allocator is determined by the interaction between his/her own FIC and the perceived FIC of the recipients. That is, if the allocator cannot perceive the recipient's choice attitude in an appropriate manner, he/she will not properly deal with the decision problems. Then, research on the perception of the other person's decision attitude would be quite important to understand the decision making process of a person.

The ultimatum games are unique decision making situations. It provides us a useful research framework to study various psychological aspects of people's decision making in social settings. Recently, several researchers began to pay attention to ultimatum games where more than two persons are involved, that is, N-person ultimatum games (Güth & van Damme,1998; Knets & Camerer, 1995). Social settings, where more than two parties interact with each other, are ubiquitous. There seem to exist more interesting phenomenon in N-person situations, which do not appear in 2-person situations. For example, a variety of problems caused by the negotiation between leader and followers can be seen in any organization or work group. A person has to make a decision against a group of many people in some situations, and he/she has to make a decision against a single person, as a delegate of a group of people in other situations. N-person ultimatum games would provide a useful research frame work for a variety of such social settings.

The analyses conducted based on the hybrid model in this paper, showed clearly that the decision making of the allocator and recipients in N-person ultimatum games can be explicitly explained and predicted in the same manner as 2-person ultimatum games. Two major components of the hybrid model, social orientation and equality seeking, can reveal how people evaluate the given alternatives to make a decision. In addition, our analysis in this paper also made clear that the perception or cognition of other people's motivational state also plays an important role in decision making, in some social settings. The hybrid model together with ultimatum games are expected to provide a new research perspective and an effective research tool for a variety of problems in our society, and to contribute to the research on personal decision making in various social settings.

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Appendix

Equation (11), which describes the subjective values of choice alternatives in N-person relations, can be obtained as follows.

First, let us consider 3-person relations where $n = 3$. Choice alternatives are expressed by points $(x_i, y_{1,i}, y_{2,i})$ in three dimensional X-Y_1-Y_2 space. x_i represents the payoff to self, and $y_{1,i}$ and $y_{2,i}$ the payoffs to two other persons. A person is supposed to choose the choice alternative $(x_i, y_{1,i}, y_{2,i})$ that gives the highest coordinate value m_i on axis M in this space, in the same way as 2-person relations. The coordinate value m_i is considered to reflect the weighted sum of the payoffs to self and two other persons. When a person gives the same weight to the payoffs to two other persons, axis M is supposed to be included in the 2 dimensional space formed by axes X and Y_M that locates midway between two axes Y_1 and Y_2. The coordinate values of choice alternatives (x_i, y_{1i}, y_{2i}) on axis Y_M are given by $\frac{y_{1,i}+y_{2,i}}{\sqrt{2}}$, and axes X and Y_M are mutually orthogonal. Introducing θ to express the angle between two axes X and M, the coordinate values m_i of choice alternatives (x_i, y_{1i}, y_{2i}) on axis M are expressed by $x_i \cos\theta + \frac{y_{1,i}+y_{2,i}}{\sqrt{2}} \sin\theta$, in a manner similar to 2-person relations. That is,

$$m_i = x_i \cos\theta + y_{1,i}\frac{\sin\theta}{\sqrt{2}} + y_{2,i}\frac{\sin\theta}{\sqrt{2}}.$$

Obviously, it describes the weighted sum of the payoffs to 3 persons. Then, the angle θ between two axes X and M in X-Y_M space can be regarded as reflecting the social orientation of a person in 3-person relations.

4-person relations ($n = 4$) can be described in the same way as 3-person relations shown above. Choice alternatives are expressed by points $(x_i, y_{1,i}, y_{2,i}, y_{3,i})$ in 4 dimensional X-Y_1-Y_2-Y_3 space. A person is supposed to choose the choice alternative $(x_i, y_{1,i}, y_{2,i}, y_{3,i})$ that gives the highest coordinate value m_i on axis M in this space. When a person gives the same weight to the payoffs to three other persons, axis M is supposed to be included in the 2-dimensional space formed by axes X and Y_M that locates in the center of three axes Y_1, Y_2 and Y_3. The coordinate values of choice alternatives $(x_i, y_{1,i}, y_{2,i}, y_{3,i})$ on axis Y_M are given by $\frac{y_{1,i}+y_{2,i}+y_{3,i}}{\sqrt{3}}$, and axes X and Y_M are mutually orthogonal. Introducing θ to represent the angle between two axes X and M, the coordinate values m_i of choice alternatives $(x_i, y_{1,i}, y_{2,i}, y_{3,i})$ on axis M are expressed by $x_i \cos\theta + \frac{y_{1,i}+y_{2,i}+y_{3,i}}{\sqrt{3}} \sin\theta$, that is,

$$m_i = x_i \cos\theta + y_{1,i}\frac{\sin\theta}{\sqrt{3}} + y_{2,i}\frac{\sin\theta}{\sqrt{3}} + y_{3,i}\frac{\sin\theta}{\sqrt{3}}.$$

Then, the angle θ between two axes X and M in X-Y_M space can be regarded as reflecting the social orientation of a person in 4-person relations.

Based on the above reasoning for 3- and 4-person relations, the social orientation of a person in N-person relations can be described in 2 dimensional space formed by axes X and Y_M, when he/she gives the same weight to the payoffs to $n-1$ other persons. Axis Y_M situates in the center of $n-1$ axes Y_1, Y_2, Y_3, ·····, and Y_{n-1} in the space, and axes X and Y_M are mutually orthogonal. The coordinate values of choice alternatives $(x_i, y_{1,i}, y_{2,i}, \cdots, y_{n-1,i})$ on axis Y_M are given by $\frac{y_{1,i}+y_{2,i}+\cdots+y_{n-1,i}}{\sqrt{n-1}}$. Introducing θ to express the angle between two axes X and M, the coordinate values m_i of choice alternatives $(x_i, y_{1i}, y_{2,i}, \cdots, y_{n-1,i})$ on axis M are described by $x_i \cos\theta + \frac{y_{1,i}+y_{2,i}+\cdots+y_{n-1,i}}{\sqrt{n-1}} \sin\theta$, in a manner similar to 2-person relations, that is,

$$m_i = x_i \cos\theta + y_{1,i}\frac{\sin\theta}{\sqrt{n-1}} + y_{2,i}\frac{\sin\theta}{\sqrt{n-1}} + \cdots + y_{n-1,i}\frac{\sin\theta}{\sqrt{n-1}}$$

$$= x_i \cos\theta + \sum_{k=1}^{n-1} y_{k,i}\frac{\sin\theta}{\sqrt{n-1}}.$$

It clearly expresses the weighted sum of the payoffs to n persons, and the weights given to $n-1$ other persons' payoffs are the same. Then, the angle θ between two axes X and M in 2 dimensional X-Y_M space can be genuinely regarded as reflecting the social orientation of a person in N-person relations.