Decision making of a person in the ultimatum games: theoretical analysis based on the hybrid model of social orientation

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Decision Making of a Person in the Ultimatum Games : Theoretical Analysis Based on the Hybrid Model of Social Orientation

Toshiaki Doi

Abstract : Decision analysis of the ultimatum game based on the hybrid model was carried out. The hybrid model of social orientation is composed of two major decision attitudes, that is, social orientation and equality-seeking. It is an expansion of the classical geometrical model of social orientation, but it contains equality-seeking motive, which is considered to play an influential role in the decision making process in the ultimatum games. According to the hybrid model, the decision making of a person is described by two parameters, θ_M (social orientation) and a (weight to θ_M). It was clearly shown how the decision making by both the allocator and the recipient in the ultimatum games can be explained and predicted by the hybrid model. Furthermore, it was also made clear that it captures the salient feature of the decision making by the allocators and the recipients found in the empirical studies of the ultimatum games. It is expected to promote the development of research on people's decision-making behavior, not only in the ultimatum games but also other decision problems in the interdependent situations.

Key Words : social orientation, equality seeking, ultimatum game, decision making, decomposed game, matrix game.

1 The geometrical model of social orientation and equality-seeking motive

The geometrical model of social orientation (Griesinger & Livingstone, 1973) is an expression of the tendency how a person takes the other person's interest into consideration compared with his/her own interest, when he/she has to make a choice in the interdependent situations. The social orientation of a particular person is expressed by the orientation θ in the 2 dimensional $X-Y$ space, as shown in Fig.1. Axis X and Y represent person's own and other's payoff respectively. An alternative of choice is expressed as a point (x_i, y_i) in $X-Y$ space. A person is supposed to keep a particular orientation θ uniformly in this space. When a person has to choose one from among several alternatives (points) in $X-Y$ space, he/she is supposed to choose the alternative that gives the highest value of m_i, according to the geometrical model. The value of m_i is given by the next equality.

$$m_i = x_i \cos\theta + y_i \sin\theta \qquad (1)$$

$\cos\theta$ and $\sin\theta$ are the weights given to two person's payoffs. Variable m_i can be interpreted as the subjective values of the given alternatives for a person who has to make a choice. Furthermore, equality (1) gives a straight line in $X-Y$ space, and any point (x_i, y_i) on this line gives the same value of m_i. Then the line given by equality (1) can be regarded as an indifference curve. Further, this

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indifference curve, which is straight in reality, is perpendicular to axis M. Since, m_i is determined unambiguously by the social orientation θ of a person, social orientation θ can be interpreted to represent the choice attitude of a person.

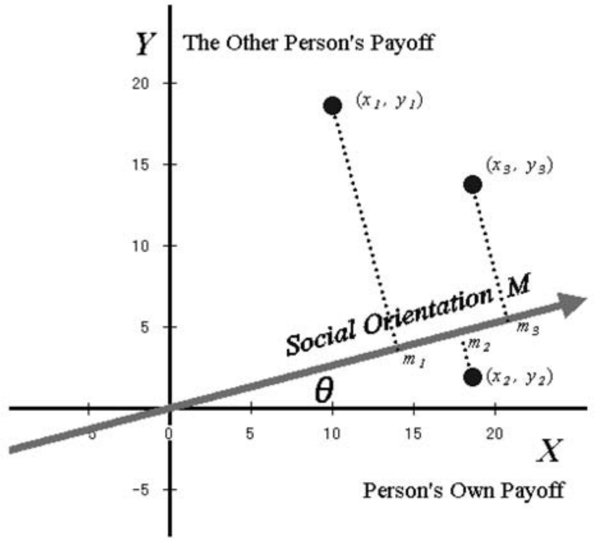

Fig. 1 Geometrical model of social orientation

θ can take any numerical value, therefore the geometrical model can express a variety of social orientations, and some of them are called altruistic, cooperative, individualistic, competitive, or aggressive orientation. However, the geometrical model does not contain the choice attitude based on equality-seeking motive, which is considered to play an influential role in the people's decision making in the interdependent situations, in general. A person, who makes a choice based on equality-seeking, is considered to choose the alternative that minimizes d_i (the absolute difference between the payoffs to two persons), which is given by the next equality.

$$d_i = |x_i - y_i| \qquad (2)$$

Obviously, equality (1) does not include this choice attitude. However, decision making based on equality-seeking can be described by two equalities as described bellow. According to equality-seeking motive, a person is supposed to choose the alternative giving the highest value of m_i, which is given by the following equalities. When the given alternatives meet $x_i > y_i$, m_i is given by

$$m_i = x_i \cos\frac{3\pi}{4} + y_i \sin\frac{3\pi}{4} = \frac{-x_i + y_i}{\sqrt{2}} \qquad (3)$$

and, when $x_i < y_i$, m_i is given by

$$m_i = x_i \cos\frac{-\pi}{4} + y_i \sin\frac{-\pi}{4} = \frac{x_i - y_i}{\sqrt{2}} \qquad (4)$$

In other words, people are supposed to activate two different social orientation systems, depending on which area the alternatives of choice are situated in $X-Y$ space, either the upper left area or the lower right. That is, a person activate $\theta = \frac{3}{4}\pi = 135°$ when $x_i > y_i$, and $\theta = -\frac{1}{4}\pi = -45°$ when $x_i < y_i$. We call the decision attitude described by equalities (3) and (4) the geometrical model of equality-seeking, in this paper.

2 The hybrid model of social orientation

The hybrid model of social orientation (Doi, 2007) hypothesizes that two different decision attitudes discussed above are activated simultaneously. This model can be described by a linear combination of two geometrical models. The subjective values m_i of alternatives (x_i, y_i) of choice are described either by equalities (1) and (3), or by (1) and (4), depending on which area the alternatives are situated in X-Y space. When $x_i > y_i$, m_i is given by the equality (5) shown below.

$$m_i = a(x_i \cos\theta_M + y_i \sin\theta_M) + (1-a)(x_i \cos\frac{3}{4}\pi + y_i \sin\frac{3}{4}\pi) \quad (5)$$

When $x_i < y_i$, m_i is given by the equality (6) shown below.

$$m_i = a(x_i \cos\theta_M + y_i \sin\theta_M) + (1-a)(x_i \cos\frac{-1}{4}\pi + y_i \sin\frac{-1}{4}\pi) \quad (6)$$

The first terms in equalities (5) and (6) are identical and express the decision attitude based on the social orientation θ_M of a person (θ_M is used instead of θ in order to distinguish between the geometrical model and the hybrid model). It is supposed that the same value of θ_M is activated uniformly for any given alternative of choice independently of $x_i > y_i$ or $x_i < y_i$. The second terms are different between (5) and (6) and express the decision attitude based on equality seeking. A person is supposed to utilize equality (5) or (6) to make a choice, depending on whether the given alternative meets $x_i > y_i$ or $x_i < y_i$. Variable a in the equalities takes numerical values between 0 and 1, and it expresses how strongly or weekly equality seeking motive affects decision making of a person. When $a = 1$, both equalities (5) and (6) will become

$$m_i = (x_i \cos\theta_M + y_i \sin\theta_M) \quad (7)$$

Equality (7) is equivalent to equality (1), that is, the hybrid model is identical with the classical geometrical model of social orientation. When $a = 0$, equalities (5) and (6) will become the following equalities.

$$m_i = (x_i \cos\frac{3}{4}\pi + y_i \sin\frac{3}{4}\pi) \quad (8)$$

$$m_i = (x_i \cos\frac{-1}{4}\pi + y_i \sin\frac{-1}{4}\pi) \quad (9)$$

Equalities (8) and (9) are equivalent to (3) and (4), then, the hybrid model is identical with the geometrical model of equality-seeking, when $a = 0$.

When $0 < a < 1$, a person is supposed to make a choice based on partly social orientation and partly equality-seeking, simultaneously. The more the value of a is getting closer to 1, the more decision making of a person is affected by social orientation θ_M. Contrarily, the more the value of a is getting closer to 0, the more the decision making is affected by equality-seeking. Therefore, the hybrid model is considered to be a genuine integration of the geometrical model of social orientation and the geometrical model of equality seeking.

Equalities (5) and (6) can be transformed into the equalities (10) and (11), respectively.

$$m_i = x_i(a \cdot \cos\theta_M - \frac{(1-a)}{\sqrt{2}}) + y_i(a \cdot \sin\theta_M + \frac{(1-a)}{\sqrt{2}}) \quad (10)$$

$$m_i = x_i(a \cdot \cos\theta_M + \frac{(1-a)}{\sqrt{2}}) + y_i(a \cdot \sin\theta_M - \frac{(1-a)}{\sqrt{2}}) \quad (11)$$

Obviously, these two equalities express different straight lines, and they can be regarded as indifference curves. The lines by (10) give indifference curves in the area satisfying $x_i > y_i$, and the lines by (11) in $x_i < y_i$. As discussed earlier, the indifference curve (1) is perpendicular to axis M that is given by social orientation. Then, it seems reasonable to assume that there should be such social orientations that are perpendicular to the two lines given by (10) and (11). If so, the hybrid model is interpreted to suppose that a person activate two different social orientation systems, depending on whether $x_i > y_i$ or $x_i < y_i$. Assigning θ_L to the social orientation and l_i to the subjective values of alternatives of choice when $x_i > y_i$, and θ_N and n_i when $x_i < y_i$, the indifference curves is given by the following equalities.

$$l_i = x_i \cos\theta_L + y_i \sin\theta_L \tag{12}$$

$$n_i = x_i \cos\theta_N + y_i \sin\theta_N \tag{13}$$

Since equalities (12) and (13) correspond to equalities (10) and (11), l_i should be equal to m_i given by (10), and n_i should be equal to m_i given by (11). Equalities (10) and (11) are described by θ_M and a, whereas (12) by θ_L and (13) by θ_N. Thus, it is possible to interpret that a person owns basic social orientation θ_M but it is modified by a differently depending on either $x_i > y_i$ or $x_i < y_i$. θ_L and θ_N can be regarded as the social orientation θ_M modified by a.

According to the geometrical model given by (1), the decision making of a person is determined based on his/her θ (social orientation), that is, the choice behavior reflects unambiguously one's social orientation. Then, it is possible to assess θ based on the the choice behaviors for a variety of alternatives. Such method to assess θ was theoretically derived from equality (1), and it was properly validated (Doi, 1984, 1990, 1994). However, the expression of the hybrid model by (5) and (6) indicates that decision making of a person is determined by both θ_M (basic social orientation) and a (weight to θ_M). That is, the choice behavior does not reflects directly either θ_M or a. Therefore, θ_M and a cannot be assessed based on the choice behaviors of a person. However, according to (12) and (13), it is possible to assess θ_L and θ_N based on the choice behaviors of a person by the method derived from equality (1). Since, equalities (12) and (13) have to correspond to equalities (10) and (11) respectively, the following equalities can be derived from them.

$$\tan\theta_L = \frac{a \cdot \sin\theta_M + \frac{(1-a)}{\sqrt{2}}}{a \cdot \cos\theta_M - \frac{(1-a)}{\sqrt{2}}} \tag{14}$$

$$\tan\theta_N = \frac{a \cdot \sin\theta_M - \frac{(1-a)}{\sqrt{2}}}{a \cdot \cos\theta_M + \frac{(1-a)}{\sqrt{2}}} \tag{15}$$

Further, equalities (14) and (15) can be transformed into the following equalities respectively.

$$a = \frac{\tan\theta_L + 1}{\sqrt{2}\tan\theta_L \cdot \cos\theta_M + \tan\theta_L - \sqrt{2}\sin\theta_M + 1} \tag{16}$$

$$a = \frac{-(\tan\theta_N + 1)}{\sqrt{2}\tan\theta_N \cdot \cos\theta_M - \tan\theta_N - \sqrt{2}\sin\theta_M - 1} \tag{17}$$

The right terms in equality (16) and (17) should be equivalent. Then, we will obtain the following equality.

$$\tan\theta_M = \frac{2\tan\theta_L \cdot \tan\theta_N + \tan\theta_L + \tan\theta_N}{2 + \tan\theta_L + \tan\theta_N} \tag{18}$$

The right term in equality (18) contains only θ_L and θ_N. Then, $\tan\theta_M$ can be calculated if θ_L and θ_N were obtained. Accordingly, basic social orientation θ_M can be obtained by the next equality.

$$\theta_M = \arctan\theta_M \tag{19}$$

If θ_M or $\tan\theta_M$ was assessed, then a will be obtained by either equality (16) or (17). When $\theta_L = \theta_N = \theta_M$ were obtained, we will get $a = 1$. On the other hand, when $\theta_L = 135°$ and $\theta_N =$

$-45°$ were obtained, we will get $a = 0$. Therefore, a seems to express how θ_L and θ_N are similar to or different from each other.

Our analysis so far has not included the alternatives satisfying $x_i = y_i$. However formally, the hybrid model described by (5) and (6) can be applied to those alternatives satisfying $x_i = y_i$. Two equalities (5) and (6) give the same value of m_i for those alternatives satisfying $x_i = y_i$. That is,

$$m_i = a \cdot x_i(\cos\theta_M + \sin\theta_M) = a \cdot y_i(\cos\theta_M + \sin\theta_M) \quad (20)$$

It is reasonable to say that a person can make his/her choice for any combination of alternatives in X-Y space. He/she should choose the alternative that gives the highest m_i, applying (5), (6) or (20) depending on which area each alternative is situated in the space. The indifference curves given by (10) and (11) are polygonal lines. It is described by the combination of the following two lines.

$$y = -\frac{\sqrt{2}a \cdot \cos\theta_M - 1 + a}{\sqrt{2}a \cdot \sin\theta_M + 1 - a}x + \frac{\sqrt{2}}{\sqrt{2}a \cdot \sin\theta_M + 1 - a}m \quad (21)$$

when $x_i > y_i$, in X-Y space, and

$$y = -\frac{\sqrt{2}a \cdot \cos\theta_M + 1 - a}{\sqrt{2}a \cdot \sin\theta_M - 1 + a}x + \frac{\sqrt{2}}{\sqrt{2}a \cdot \sin\theta_M - 1 + a}m \quad (22)$$

when $x_i < y_i$, in the space. Fig.2 shows the indifference curves that are given when a=0.5 and θ_M=30°. As previously discussed, any point on the same indifference curve gives the same value of m_i. The bended point of the indifference curve moves on axis E. The higher the bended point moves to, the higher value of m_i the indifference curve gives.

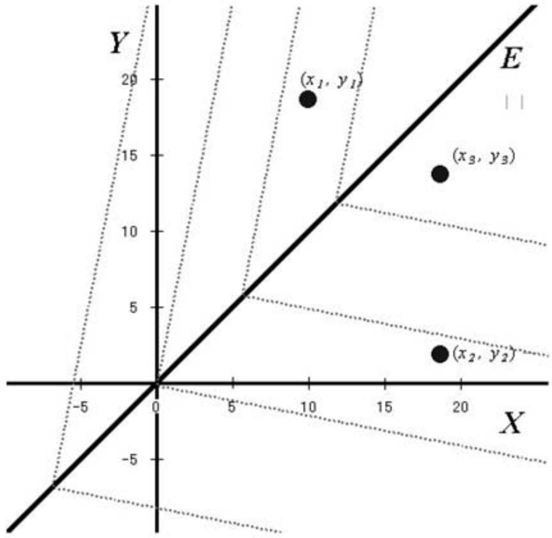

Fig. 2 Indifference curves given by the hybrid model when $a = 0.5$ and $\theta_M = 30°$

3 Theoretical analysis of the decision making of a person in the ultimatum games

Ultimatum game is a unique decision making problem where equality-seeking motive is considered to play an influential role in the decision making process of a person. Then, how well can the hybrid model of social orientation, which contains equality-seeking motive, explain the decision making of a person in the ultimatum games? The ultimatum game is a decision situation where two persons are given a certain amount of payoff, but they have to divide it between them. However, they cannot negotiate over the allocation of the given payoff. The behavioral choices given to the two persons are quite different. One person called the allocator has to propose the allocation of the given payoff. The other person called the recipient has to decide if he/she accepts the proposed allocation or not. If the recipient accepted it, both he/she and the allocator can get some payoff according to the proposed allocation. If the recipient refused to accept it, neither can get any payoff. The given payoff has to be simply confiscated.

The ultimatum game can be expressed geometrically in $X-Y$ space as shown in Fig.3. The proposed allocation is expressed as one point on the line segment A–B, where the sum of the payoff to the allocator and the recipient is constant. Point A represents the allocation where 100% to the allocator and nothing to the recipient. Point E represents the allocation where 50% to the two equally, and Point B, nothing to the allocator and 100% to the recipient. Point O, the origin of this space, represents the payoff to the two when the recipient refused the proposed allocation. What the allocator has to do in this space is to choose one point from the line segment A–B. If the allocator chose point P, then the recipient has to decide if he/she accept P or not, that is, to choose P or O, in the space. According to the hybrid model of social orientation, the decision making of the allocator and the recipient can be analyzed as follows.

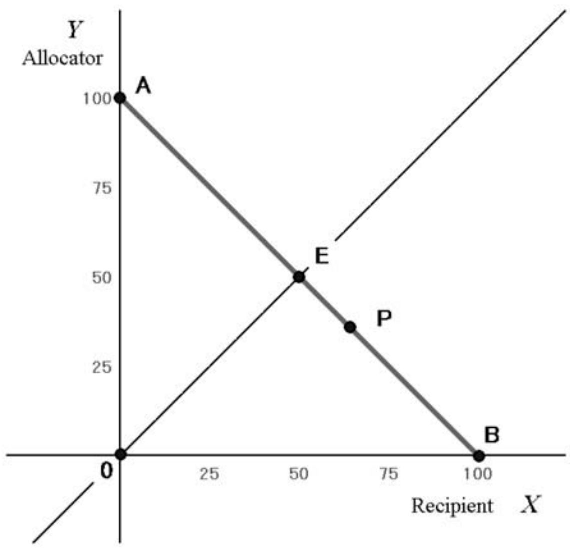

Fig. 3 Geometrical expression of the ultimatum games

The decision makings of the recipient

The decision making of the recipient can be predicted by the indifference curve given by recipient's θ_L and θ_N. The indifference curve that is passing through the origin O divides the whole $X-Y$ space into the two areas. The recipient should accept the proposed allocation if it is situated in the upper right area from this indifference curve, and he/she should reject it if it is situated in the lower left area. The upper right area can be called the acceptable area, and the lower left area, the rejectable area. Therefore, the indifference curve passing thorough the origin O can be regarded as the criterion to judge if the proposed allocation is acceptable or rejectable. We call this indifference curve FID (fundamental indifference curve). θ_L and θ_N that determine FID correspond unambiguously to a and θ_M of the recipient, as analyzed previously. Therefore, decision making of the recipient in the ultimatum game can be predicted explicitly, if his/her θ_M and a are given.

FID varies according to the combination of θ_L and θ_N. Three different examples of FID are shown in Fig.4 to Fig.6, where the acceptable area is grayed. FID shown in Fig.4 is given when $\theta_L < 90°$ and $\theta_N > 0°$. The entire line segment A–B is included in the acceptable area. Then, the recipient should accept any allocation proposed by the allocater, even if it was point A where all the payoff goes to the allocator and nothing is left for the recipient. While on the other hand, the acceptable line segment is much shorter in Fig.5, which is given when $\theta_L < 90°$ and $\theta_N < 0°$. The line segment A–C is not included in the acceptable area. Then the recipient should accept the proposed allocation if it was on the line segment C–B, and should reject it if on the line seg-

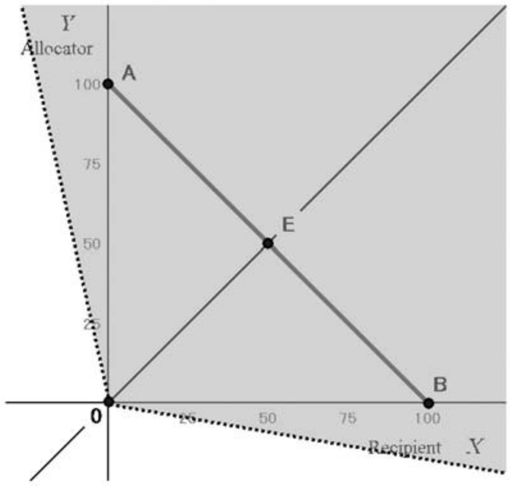

Fig. 4 FID of the recipient and the acceptable area for him/her (example-1)

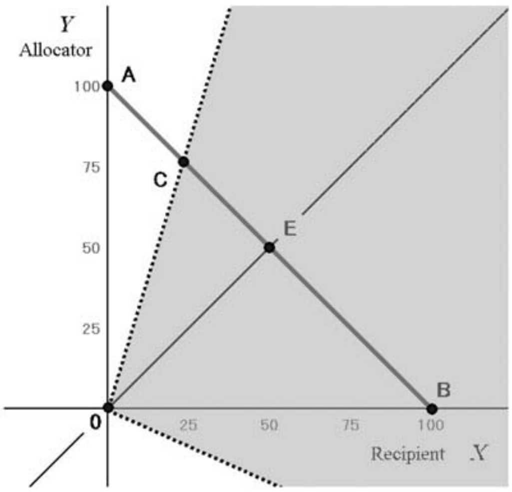

Fig. 5 FID of the recipient and the acceptable area for him/her (example-2)

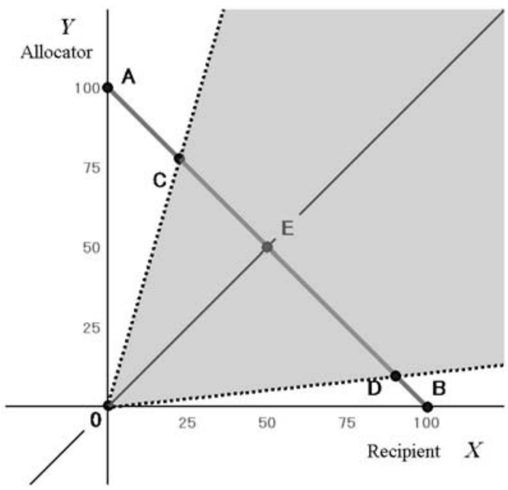

Fig. 6 FID of the recipient and the acceptable area for him/her (example-3)

ment A–C. The recipient in this case will refuse to accept point A, but accept point B where all the payoff goes to the recipient and nothing is left for the allocator. Further, the acceptable line segment is even shorter in Fig.6, which is given when $\theta_L > 90°$ and $\theta_N < 0°$. Only the line segment C–D is acceptable. Then, the recipient should refuse the proposed allocation if it was on the rejectable line segment A–C or D–B. Hence, the recipient is supposed to reject both point A and B.

Point C and D, which location is determined by FID, reflects the recipient's θ_L and θ_N. That is, if the location of point C and D were clarified experimentally, then the recipient's a and θ_M, can be assessed by equalities (18) and either (16) or (17). Giving the recipient various ultimatum games which allocation proposals correspond to the various points on the line segment A–B, if the pattern of the accepted points is congruent with the line segment C–D shown in Fig.6, the recipient's θ_L and θ_N can be assessed. Then the recipient's a and θ_M can be derived from them. If this pattern is congruent with the line segment shown in Fig.4 or Fig.5, only either one of θ_L or θ_N can be assessed. Thus, the recipient's a and θ_M cannot be assessed. It is concluded that recipient's θ_M and a can be assessed based on his/her decision making in the ultimatum games, only when his/her pattern of the accepted points meets the condition, that is, $\theta_L > 90°$ and $\theta_N < 0°$.

The decision makings of the allocator

The unique aspect of ultimatum games is that either the allocator or the recipient cannot get any payoff if the recipient refused the proposed allocation. The assignment for the allocator is to choose one point from the line segment A–B as the allocation proposal. But, it has to be accepted by the recipient. Therefore, it is extremely important for the allocator to confirm if the point that he/she has chosen is situated on the acceptable line segment for the recipient or not. If the chosen point by the allocator was situated on the rejectable line segment for the recipient, he/she will not get any payoff. As previously discussed, whether a particular point is acceptable or not, is determined by FIC that reflects recipient's θ_L and θ_N, or recipient's a and θ_M. Then, it is crucial for the allocator to determine the allocation proposal how he/she perceives recipient's FIC, or more specifically his/her a and θ_M. If the assessment of the recipient's FIC were made, the acceptable line segment of the recipient can be determined. Then he/she has to choose one point from the acceptable line segment, which gives the highest subjective value m_i, according to his/her own indifference curve system that is given by his/her own θ_M and a. If his/her perception of the recipient's FIC is reasonable, the recipient will accept his/her proposal of the allocation, and he/she will obtain the maximum satisfaction. Then the both allocator's own θ_M and a and his/her perception of the recipient's FIC, that is θ_M and a, are crucial in order to explain the decision making by the allocator in the ultimatum games.

If the perceived FIC by the allocator was similar to the one shown in Fig.6, then he/she has to choose one point from the line segment C–D. Theoretical analysis predicts that the allocator should choose from among three points C, D, and E, as the allocation proposal, according to his/her own indifference curve system. Fig.7 to Fig.9 show three examples of the allocator's indifference curves. Three indifference curves that pass through point C, E, and B are drawn in each Fig. The indifference curves in Fig.7 are likely obtained when a is closer to 0, that is when equality-seeking motive plays influential role in decision making process. As previously stated, the higher the bended point of the indifference curve locates on axis E, the higher value of m_i the indifference curve gives. Therefore, the indifference curve passing through point M gives the highest m_i in Fig.7.

The Indifference curves shown in Fig.8 and Fig.9 are likely obtained when a is rather closer to 1, that is when equality-seeking motive does not influence decision making process much. Accordingly, the bended angle of the indifference curves is much shallower than those in Fig.7. The highest m_i is given by the indifference curve passing through point C in Fig.8, and point D in Fig.9. Therefore, the allocation proposal by the allocator can be predicted if both his/her own θ_M and a and his/her perception of the recipient's FIC are given. The allocator's perception of recipient's FIC can be obtained fairly easily, simply asking the allocator to judge whether various points from the line segment A–B would be accepted by the recipient or not. However, the chosen point by the allocator does not reflect his/her own θ_M and a directly. Variety of θ_M and a leads the allocator to the one from among point C, E, or D on the acceptable line segment. Therefore, it is not possible to assess θ_M and a based on his/her decision making in the ultimatum games.

4 Final remarks

The analysis we have conducted reveals how the decision making by both the allocator and the recipient in the ultimatum games can be explained and predicted by the hybrid model

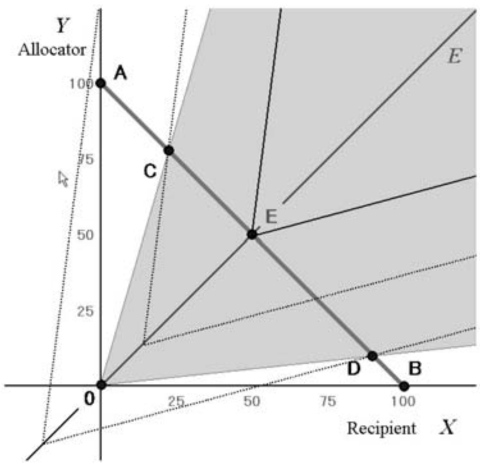

Fig. 7 Three indifference curves of the allocator (example-1) passing through three points C, E, and D, when his/her FIC is given by Fig. 6

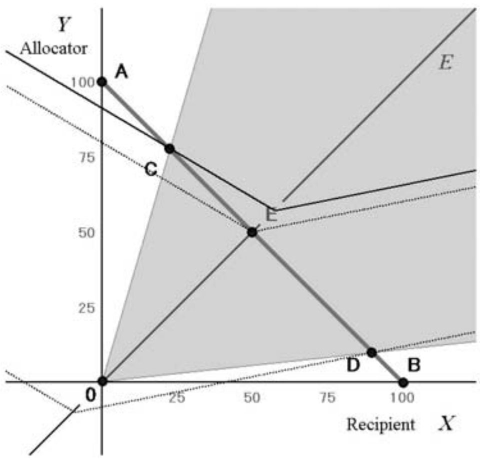

Fig. 8 Three indifference curves of the allocator (example-2) passing through three points C, E, and D, when his/her FIC is given by Fig. 6

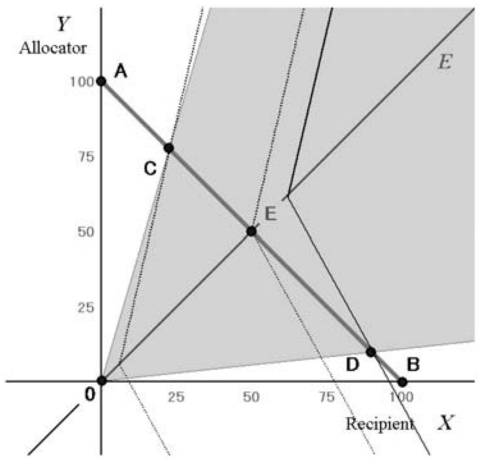

Fig. 9 Three indifference curves of the allocator (example-3) passing through three points C, E, and D, when his/her FIC is given by Fig. 6

of social orientation. The recipient's decision making can be explained clearly and predicted if his/her θ_M and a were known, and the allocator's decision making if his/her θ_M and a and his/her perception of the recipient's FIC (or θ_M and a) were given. However, our analysis also reveals that there are limitations to assess θ_M and a of the allocator and the recipient based on their choice behaviors in the ultimatum games. It is necessary to assess θ_M and a of the allocator and the recipient based on the choice behaviors in other decision making situation other than the ultimatum games. A methodology to assess θ_M and a based on the choice behaviors of a person in the decomposed game was proposed, and it was confirmed that θ_M assessed by this method can predict choice behaviors of a person in the matrix games considerably (Doi, 2007). This assessment methodology utilizing decomposed game is expected to be helpful for explaining and predicting the choice behavior of the allocators and the recipients in more detail.

The empirical research of the ultimatum games shows that the allocators tend to propose the allocation given by point C, M, or D in the experiment. They seldom propose the allocation corresponding to point A or B. That is, their allocation pattern seems to accord with that shown in Fig.7. Some recipients tend to refuse to receive the proposed allocation where they could obtain 100% of the payoff, that is, point B in X-Y space. The recipient's decision makings on acceptance of the proposed allocation in empirical studies seem to correspond to the acceptable area shown in Fig.6. Therefore, the hybrid model of social orientation seems to capture the characteristic tendencies of decision making by the allocators and the recipients, and it is expected to promote the development of research on people's decision-making behavior in the ultimatum games.

As shown previously, the hybrid model can explain choice behavior of a person in the matrix games considerably well (Doi, 2007). The classical geometrical model of social orientation can explain choice behaviors of a person in the matrix game fairly well, but it is incompetent for the ultimatum games. The crucial deficiency of the geometrical model is that it cannot deal with equality-seeking motive of a person. Since the hybrid model is an expansion of the geometrical model, the hybrid model can deal with any decision problem that the geometrical model can deal with. In addition, the hybrid model can deal with other decision problems such as the ultimatum games that the geometrical model is powerless to deal with. Therefore, the hybrid model might give a powerful research framework to investigate people's choice behavior in a variety of decision problems in the interdependent situations, and is expected to promote the development of research in this area.

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