Integrating Pricing And Distribution Decisions In Multiple Markets

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ABSTRACT

This paper deals with the simultaneous optimization of prices and shipment quantities in a supply network when the supplier has the market power to set prices, thereby influencing demand directly. We focus on the distribution stage of the supply chain where the firm's products are shipped from several locations (plants, warehouses) to various independent markets, and address the following questions: (i) what is the best price at each market?, and (ii) what is the best distribution plan given these prices? The combined problem can be modeled as a nonlinear optimization problem. For its solution, we propose an iterative linear programming approach that utilizes shadow price information from a series of successive transportation problems. To evaluate the heuristic's effectiveness, we compare it with a "brute-force" enumeration using a grid-search. The grid-search is implemented on a spreadsheet with a programming loop to facilitate repeated invocation of the transportation problem solver routine.

INTRODUCTION

Facing ever-increasing competition in today's markets, companies now realize that improvements of internal operations and cost reduction measures alone are not adequate to stay competitive. Beyond the implementation of manufacturing management philosophies and technologies such as just-in-time manufacturing, total quality management, and enterprise resource planning, companies in recent years have broadened their attention to all linked business processes and activities required to meet customers' requirements. As a result, supply chain management has become a major focus of today's business management in order to further reduce costs, increase market share, and improve profits.

A successful supply chain requires strong linkages of all stages of the chain, both between organizations and within a particular organization. This paper is concerned with the distribution stage of the supply chain where a firm's products are shipped from several supply locations (plants, warehouses) to be sold in various independent markets. The standard transportation linear programming model is usually adequate to solve the problem of minimizing the cost of satisfying total demand at all markets subject to supply limitations at the sources [Chopra and Meindl (2004)]. In conventional formulations of this problem, the supply and demand quantities are assumed to be *fixed* and *known in advance*. Such an assumption may be valid for commodity products, but when the firm has market power and is able to influence demand through price setting, the static demand assumption is unrealistic. The marketing literature abounds with models for matching pricing strategies to markets [e.g. Duke (1994)] but our focus here is on the effect of the price on demand and, consequently, its effect on an operational decision. In a previous study, Dökmeci (1998) considered the effect of a uniform price (across several markets) on the *facility location* decision. Here, we examine the effect of price on *demand allocation* given that the locations of the supply facilities have already been well established. Also, in contrast to the Dökmeci paper, we assume that different prices may be set at different markets.

The key issues in the demand allocation problem when prices can vary are: (i) what is the best set of prices for a given set of markets? (*the pricing subproblem*), and (ii) what is the best shipping plan at these prices? (*the distribution subproblem*). In this paper, we provide a mathematical programming formulation for the problem of simultaneously optimizing prices and shipment quantities when demand is a function of the price set by the decision maker. Since the incorporation of variable pricing causes the loss of linearity in the model, one cannot use standard linear programming (LP) techniques to solve it. We propose, instead, a heuristic methodology that is based on solving a series of fixed-demand

transportation subproblems to iteratively adjust prices and demands in the broader model. Starting at an arbitrary but feasible price vector, the heuristic uses shadow price information gleaned from the optimal solution to the corresponding transportation problem to modify the market demands. The transportation problem is then re-solved with these new demand quantities in order to generate a fresh set of shadow prices, and the cycle repeated until there is no further improvement in the objective function value. To evaluate the efficacy of our approach, we compare it with a "brute-force" grid search strategy. The grid search is implemented on Microsoft Excel enhanced with programming extensions provided by VBA (Visual Basic for Applications). VBA enables the use of a programming loop to repeatedly invoke the LP solver, and to keep track of the best solution.

The rest of this paper is organized in the following sequence: formulation of the model, development of the heuristic methodology, numerical example, grid search, and concluding comments.

FORMULATION

We begin with the following notation for a standard transportation problem with a profit maximization objective:

I set of sources $\{1,2,\dots m\}$ indexed by i

J set of markets $\{1,2,\dots n\}$ indexed by j

S_i supply at the ith source

D_j demand at the jth market

c_{ij} cost of transporting one unit of product from the ith source to the jth market

P_j price of the product at the jth market

X_{ij} quantity shipped from the ith source to the jth market

When prices, and hence demands, are fixed the standard linear programming transportation model can be written as:

(P1)   *Maximize* $Z_1 = \sum_{j \in J} P_j D_j - \sum_{i \in I} \sum_{j \in J} c_{ij} X_{ij}$ (1)

     subject to:

$$\sum_{j \in J} X_{ij} \leq S_i \qquad \forall \ i \tag{2}$$

$$\sum_{i \in I} X_{ij} = D_j \qquad \forall \ j \tag{3}$$

$$X_{ij} \geq 0 \qquad \forall \ i,j \tag{4}$$

Now suppose that D_j is a function of the price charged at the jth market, i.e. $D_j = f_j(P_j)$, with a linear functional form:

$$D_j = a_j - b_j P_j \qquad \forall \ j \tag{5}$$

$$P_j^{\,L} \;\leq\; P_j \;\leq\; P_j^{\,U} \qquad \forall \; j \tag{6}$$

where $a_j, b_j > 0$ for all j. The model can then be recast as:

$$Maximize \quad Z_2 = \sum_{j \in J} P_j \,(a_j - b_j P_j) - \sum_{i \in I} \sum_{j \in J} c_{ij}\, X_{ij} \tag{7}$$

subject to:

$$\sum_{j \in J} X_{ij} \;\leq\; S_i \qquad\qquad \forall \; i \tag{8}$$

(P2)
$$\left[\sum_{i \in I} X_{ij}\right] \;+\; b_j\, P_j \;=\; a_j \qquad\qquad \forall \; j \tag{9}$$

$$X_{ij} \;\geq\; 0 \qquad \forall \; i, j \tag{10}$$

$$P_j^{\,L} \;\leq\; P_j \;\leq\; P_j^{\,U} \qquad \forall \; j \tag{11}$$

Note that while model **(P1)** is a linear program, model **(P2)** is a linearly constrained program with: (i) a quadratic objective function due to the $P_j^{\,2}$ term in eq. (7), and (ii) simple bounds on the variables (eq. (11)). Note also that the standard transportation model structure of the constraints is lost due to the additional P_j variables in eq. (9).

HEURISTIC METHOD

This section gives the technical details of the proposed heuristic for solving model **(P2)**. The method attempts to find the best market demands (and corresponding market prices) in model **(P2)** by using shadow price information from model **(P1)**.

Method

According to linear programming theory, the shadow price of a constraint is defined as the change in objective value per unit change in the RHS-value of that constraint. Let the shadow price of the jth market demand constraint in the standard transportation model, i.e., eq. (3) in model **(P1)**, be SP_j, with a reported range of validity of $[D_j^{\min}, D_j^{\max}]$. Suppose now that demand at this market is increased (within the above range of validity) by δ_j, the change in objective function value can be computed as:

$$\Delta Z_1 = SP_j * \delta_j \tag{12}$$

Note that this expression includes the amount $P_j\, \delta_j$ representing the increase in revenue due to the increased demand, and is computed on the basis of a *fixed* market price of P_j. However, since price must decrease in order to induce this increase in demand (eq. (5)), the amount $P_j\, \delta_j$ is an overestimate of the true increase in revenue and an

adjustment should be made to the computation of ΔZ_1 in eq (12) to account for this. Let P'_j be the price when demand is $D_j + \delta_j$. From eq. (5):

$$D_j + \delta_j = a_j - b_j P'_j$$

$$b_j P'_j = a_j - D_j - \delta_j$$

$$P'_j = P_j - (\delta_j / b_j) \tag{13}$$

Let the true increase in revenue as a result of a price change from P_j to P'_j be ΔR. ΔR is given by:

$$\Delta R \quad = \quad P'_j (D_j + \delta_j) - P_j D_j$$

$$= \quad [P_j - (\delta_j / b_j)] (D_j + \delta_j) - P_j D_j \qquad \text{[from eq. (13)]}$$

$$= \quad P_j \delta_j - (\delta_j / b_j)(D_j + \delta_j) \tag{14}$$

The change in the objective function value, eq. (12), should then read:

$$\Delta Z_1 \quad = \quad SP_j * \delta_j + \text{true increase in revenue} - \text{erroneous estimate of increase in revenue}$$

$$= \quad SP_j * \delta_j + \Delta R - P_j \delta_j$$

$$= \quad SP_j * \delta_j + P_j \delta_j - (\delta_j / b_j)(D_j + \delta_j) - P_j \delta_j \qquad \text{[from eq. (14)]}$$

$$= \quad SP_j * \delta_j - (\delta_j / b_j)(D_j + \delta_j) \tag{15}$$

We define an *adjusted shadow price* (ASP) by letting δ_j, the change in the RHS value of the constraint, be equal to 1. Thus,

$$ASP_j \quad = \quad SP_j - (1/b_j)(D_j + 1) \tag{16}$$

From eqs. (5) and (6), the range of possible demand values at the jth market is in the range: $[a_j - b_j P_j^U, a_j - b_j P_j^L]$. Combining this range with the range of validity of the corresponding shadow price, $[D_j^{\min}, D_j^{\max}]$, the *effective* lower and upper limits of demand in market *j* for ASP_j are as follows (see also Figure 1).

$$[\max \{D_j^{\min}, a_j - b_j P_j^U\}, \min\{D_j^{\max}, a_j - b_j P_j^L\}] \tag{17}$$

The proposed heuristic method uses these *ASP*s to iteratively modify market demands within their effective ranges. The steps are as follows:

- *Step 1:* In model **(P1)**, set each market's price to the midpoint of its permitted range. Solve the resulting transportation LP.
- *Step 2:* Use the shadow price information from the LP to compute *ASP*s [eq. (16)]. Choose the market with the largest absolute value of *ASP* and adjust it's demand to the effective upper limit of validity if the chosen *ASP* is *positive*, or effective lower limit of validity if the *ASP* is *negative* [eq. (17)].
- *Step 3:* Solve the modified transportation problem. If the objective function value has improved, go to Step 2; else, STOP.

Figure 1: Effective Limits Of Validity For Adjusted Shadow Prices
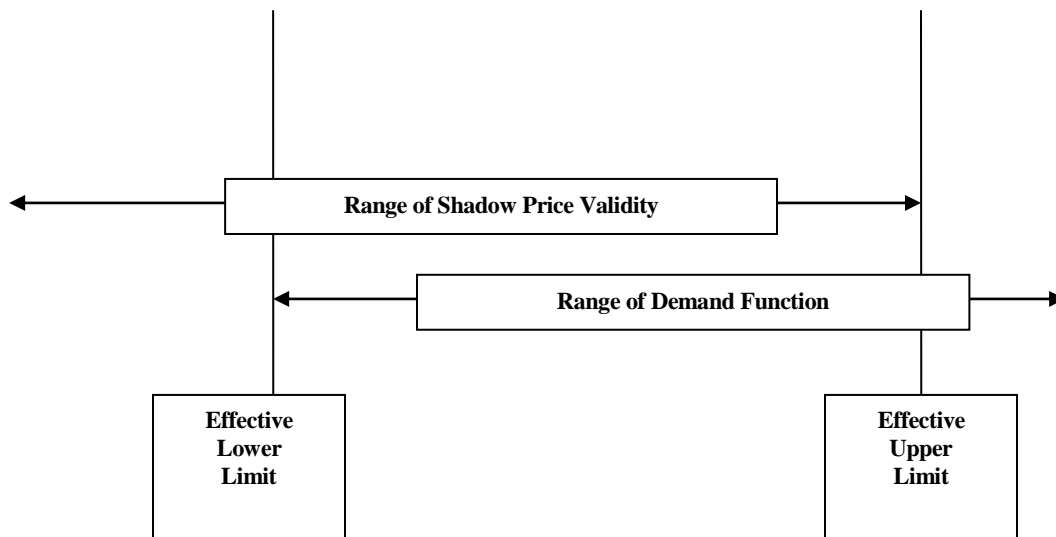
Numerical Example

Tables 1 and 2 give the data for a sample problem. The price-elastic demand functions for Markets 1 and 2 are depicted in Figure 2 for illustration.

Table 1: Demand Function Data For Sample Problem

Market	Market 1	Market 2	Market 3	Market 4
Demand Function				
Intercept	6020	4020	2020	1520
Slope	-400	-200	-120	-50
Price Range				
Minimum	$10	$8	$9	$11
Maximum	11	10	11	13

Table 2: Unit Transportation Costs And Supply Limits For Sample Problem

Source	Market 1	Market 2	Market 3	Market 4	Supply
Source 1	$3	$4	7	11	5000
Source 2	4	3	5	8	6000
Source 3	5	2	4	7	2500

Figure 2: Demand Functions At Markets 1 And 2
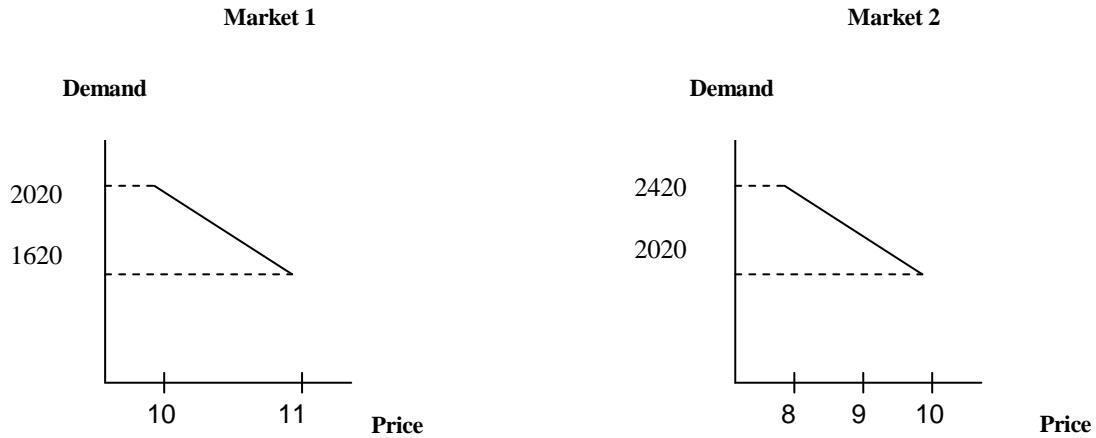
First, the price at each market is set at the midpoint of the corresponding price range (e.g., for Market 1, it is set at $10.50). The resulting demands at the four markets are 1820, 2220, 820, and 920, respectively. Hence, we solve the following transportation problem:

Source	Market 1	Market 2	Market 3	Market 4	Supply
Source 1	$3	$4	$7	$11	5000
Source 2	4	3	5	8	6000
Source 3	5	2	4	7	2500
Demand	1820	2220	820	920	
Price	$10.50	$9.00	$10.00	$12.00	

The optimal objective function value is $ 37,250. The optimal flows are shown below:

Source	Market 1	Market 2	Market 3	Market 4	Supply
Source 1	1820	0	0	0	5000
Source 2	0	0	820	640	6000
Source 3	0	2220	0	280	2500
Demand	1820	2220	820	920	

Sensitivity information reported by the LP solver is as follows:

Cell	Final Value	Shadow Price	Constraint R.H. Side	Allowable Decrease	Allowable Increase
Source 1 Shipped	1820	0	5000	3180	1E+30
Source 2 Shipped	1460	0	6000	4590	1E+30
Source 3 Shipped	2500	1	2500	280	640
Received Market 1	1820	7.5	1820	1820	3180
Received Market 2	2220	6	2220	640	280
Received Market 3	820	5	820	820	4540
Received Market 4	920	4	920	640	4540

From this information, we can use eq. (16) compute the adjusted shadow prices and effective limits of validity as follows:

Market	LP-Reported Shadow Price	ASP	Effective Lower Limit of Demand	Effective Upper Limit of Demand
Market 1	7.5	2.95	1620	2020
Market 2	6	-5.10	2020	2420
Market 3	5	-1.84	700	940
Market 4	4	-14.42	870	970

From the above calculations, we see how profit changes at each market with one unit change in delivered quantity. Since Market 4 has the biggest absolute value for *ASP*, we choose Market 4 for demand modification. For this market, note that the effective range of *ASP* was computed as: [*max*{920–640, 1520 – 50×13}, *min*{920+4540, 1520–50×11}] = [870,970]. Since ASP_4 is negative, D_4 should be decreased as much as possible. Therefore, we set D_4 to its effective lower limit of 870 and re-solve the transportation problem, and repeat the cycle. The iterations of the heuristic are:

Iteration 1: Objective Value $37,250

Market	Current Price	Current Demand	Shadow Price	*ASP*	Remark
Market 1	$10.50	1820	7.5	2.95	
Market 2	9.00	2220	6	-5.10	
Market 3	10.00	820	5	-1.84	
Market 4	12.00	920	4	-14.42	Set $D_4 = 870$ (at $P_4 = \$13$) for the next iteration

Iteration 2: Objective Value $37,920

Market	Current Price	Current Demand	Shadow Price	*ASP*	Remark
Market 1	$10.50	1820	7.5	2.95	
Market 2	9.00	2220	6	-5.10	Set $D_2 = 2020$ ($P_2 = \$10$)
Market 3	10.00	820	5	-1.84	
Market 4	13.00	870	5	-12.42	Blocked (already at the lower limit)

At Iteration 2, Market 4 had the highest absolute value for *ASP*. Since this value (-12.42) is negative, it indicates that we should decrease the demand at Market 4 to its effective lower limit. However, this demand quantity is already at its effective lower limit of 870 and, hence, is *blocked*. So we proceed with the next best *ASP* (Market 2 with -5.10). The effective lower limit at this market is 2020, so we set D_2 to 2020 (corresponds to a price of $10).

Iteration 3: Objective Value $38,740

Market	Current Price	Current Demand	Shadow Price	*ASP*	Remark
Market 1	$10.50	1820	7.5	2.95	Set $D_1 = 2020$ ($P_1 = \$10$)
Market 2	10.00	2020	7	-3.10	Blocked
Market 3	10.00	820	5	-1.84	
Market 4	13.00	870	5	-12.42	Blocked

At Iteration 3, both Markets 2 and 4 are blocked, so we are forced to choose the third best *ASP* (Market 1). We increase the demand at Market 1 (its *ASP* is positive) to its effective upper limit of 2020 (this corresponds to a price of $10).

Iteration 4: Objective Value $39,230

Market	Current Price	Current Demand	Shadow Price	*ASP*	Remark
Market 1	$10.00	2020	7	1.95	Blocked
Market 2	10.00	2020	7	-3.10	Blocked
Market 3	10.00	820	5	-1.84	Set $D_3 = 700$ $(P_3 = \$11)$
Market 4	13.00	870	5	-12.42	Blocked

At Iteration 4, the only available choice is Market 3 whose demand should be reduced to 700 (at a price of $11).

Iteration 5: Objective Value $39,330

Market	Current Price	Current Demand	Shadow Price	*ASP*	Remark
Market 1	$10.00	2020	7	1.95	Blocked
Market 2	10.00	2020	7	-3.10	Blocked
Market 3	11.00	700	6	0.16	Set $D_3 = 940$ $(P_3 = \$9)$
Market 4	13.00	870	5	-12.42	Blocked

At Iteration 5, we reset the demand of Market 3 this time to its upper limit of 940 (at a price of $9).

Iteration 6: Objective Value $38,890

Market	Current Price	Current Demand	Shadow Price	*ASP*	Remark
Market 1	$10.00	2020	7	1.95	The method terminates here because the objective function value is worse than at the previous iteration.
Market 2	10.00	2020	7	-3.10	
Market 3	9.00	940	4	-3.85	
Market 4	13.00	870	5	-12.42	

This is the first iteration at which the objective function value has failed to improve, so the method is terminated. The best solution found is the one at the previous iteration (Iteration 5) with a price vector of ($10, $10, $11, $13) at the four markets. The corresponding demand quantities are: 2020, 2020, 700, and 870, respectively. The objective function value is $39,330. The progress of the heuristic is depicted in Figure 3.

GRID SEARCH

To evaluate the effectiveness of the heuristic method, we solve model **(P2)** using an alternative method—a "brute-force" enumeration using a grid search. At each grid point (price vector), the corresponding standard transportation problem **(P1)** is solved using Excel Solver [see e.g. Ragsdale (2004)] and the optimal objective value is recorded. Using integer step-sizes for the price at each market, there are 2 x 3 x 3 x 3 = 54 price vectors. The maximum among these 54 optimal transportation problem objective function values determines the optimal solution to **(P2)**. A final iteration then re-creates the optimal transportation model solution corresponding to the best price vector found. To carry out the grid search efficiently, we used VBA (Visual Basic for Applications) to provide a programming loop that enabled the repeated invocation of Solver and to keep track of the objective values at the intermediate steps. Details of VBA programming may be found in Albright (2001). While we have assumed linearity of the demand function for convenience, the search strategy can be employed for any type of function including ones defined only empirically. The results of the grid search (Table 3) indicate that the optimal solution is at price vector 27. This is precisely the same solution as the one obtained by the heuristic in only 6 iterations. The savings in computational effort due to the heuristic is, therefore, [54 − 6] / 54, or 89%, over a full search.
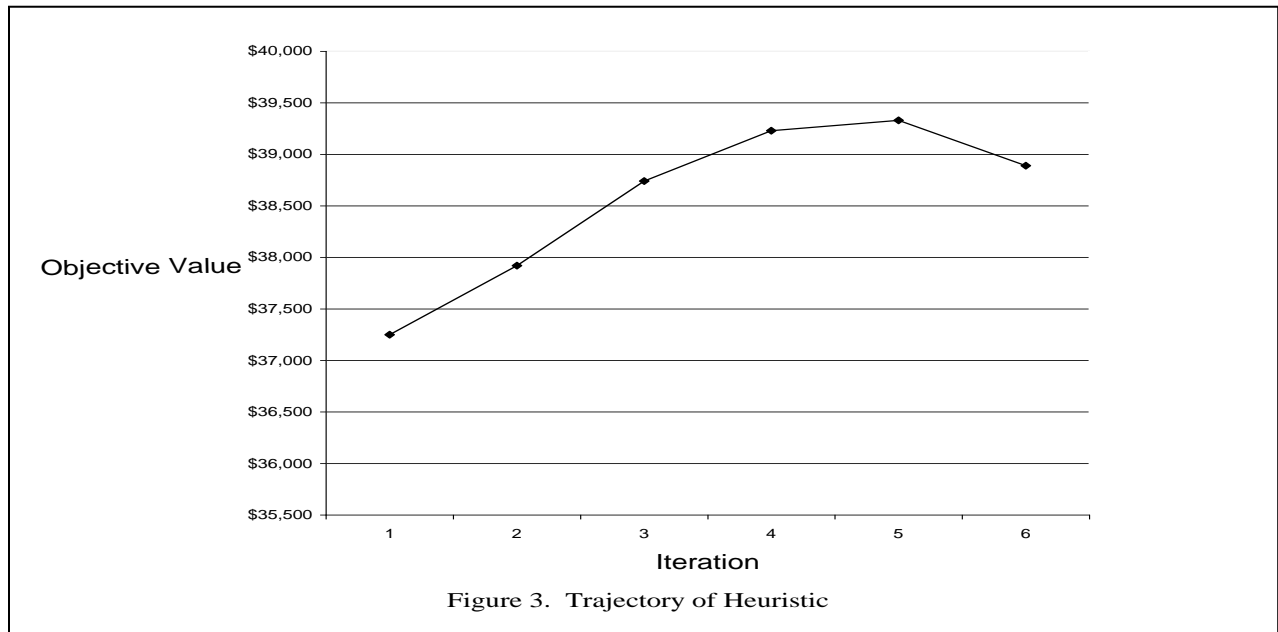
Figure 3. Trajectory of Heuristic

Table 3: Results Of Grid Search

Price Vector	Price At				Objective Function Value
	Market 1	**Market 2**	**Market 3**	**Market 4**	
1	$10	$8	$9	$11	$35,410
2	10	8	9	12	36,180
3	10	8	9	13	36,850
4	10	8	10	11	35,750
5	10	8	10	12	36,520
6	10	8	10	13	37,190
7	10	8	11	11	35,850
8	10	8	11	12	36,620
9	10	8	11	13	37,290
10	10	9	9	11	36,630
11	10	9	9	12	37,400
12	10	9	9	13	38,070
13	10	9	10	11	36,970
14	10	9	10	12	37,740
15	10	9	10	13	38,410
16	10	9	11	11	37,070
17	10	9	11	12	37,840
18	10	9	11	13	38,510
19	10	10	9	11	37,450
20	10	10	9	12	38,220
21	10	10	9	13	38,890
22	10	10	10	11	37,790
23	10	10	10	12	38,560
24	10	10	10	13	39,230
25	10	10	11	11	37,890
26	10	10	11	12	38,660
27	10	10	11	13	39,330
28	11	8	9	11	34,230

29	11	8	9	12	35,000
30	11	8	9	13	35,670
31	11	8	10	11	34,570
32	11	8	10	12	35,340
33	11	8	10	13	36,010
34	11	8	11	11	34,670
35	11	8	11	12	35,440
36	11	8	11	13	36,110
37	11	9	9	11	35,450
38	11	9	9	12	36,220
39	11	9	9	13	36,890
40	11	9	10	11	35,790
41	11	9	10	12	36,560
42	11	9	10	13	37,230
43	11	9	11	11	35,890
44	11	9	11	12	36,660
45	11	9	11	13	37,330
46	11	10	9	11	36,270
47	11	10	9	12	37,040
48	11	10	9	13	37,710
49	11	10	10	11	36,610
50	11	10	10	12	37,380
51	11	10	10	13	38,050
52	11	10	11	11	36,710
53	11	10	11	12	37,480
54	11	10	11	13	38,150

CONCLUSION

In this paper, we have developed a heuristic approach to the problem of simultaneously determining prices and shipment quantities in a supply network consisting of multiple sources and markets. The heuristic is based on shadow price information that can be obtained from a linear programming solution of the underlying transportation model. By analyzing the shadow prices, a trajectory can be chosen for improving the objective function value. The method is illustrated on a numerical example that demonstrates the potential for huge savings in computational effort over a full search. This approach can easily be extended to any type of demand function: linear, nonlinear, discrete, or even empirical. It can also be extended to the supply side to incorporate production functions.

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