The Effect Of Government Debt Quantity Shocks On The Term Structure Of Interest Rates

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ABSTRACT

In this paper, the effect of the maturity composition of marketable public debt on the term structure of interest rate is explored. The research has shown that this effect is relatively small. Unlike previous research, the yield changes around the quantity shocks are analyzed in relation to these shocks. Our results show that yields respond significantly to the auctioning of new bonds. The announcements of auctions do not have any impact on yields. A two-factor affine yield model is used to explain the relationship between quantity shocks in public debt and term structure of interest rates. The parameters are estimated using Generalized Method of Moments. While the relationship between quantities and yields is weak, yields can be related to the event of the auctioning process.

1. INTRODUCTION

This paper investigates the effect of government debt quantity shocks on interest rates. There has been a long debate about the effects of the maturity composition of marketable interest bearing public debt on interest rates. Historically, the upward sloping term structure curve is observed more often than the downward sloping yield curve. Only in the 1980s, were some persistent downward sloping curves observed.

Government policy makers often thought to take advantage of the upward sloping term structure. It has been argued that if the average maturity of the public debt could be shortened, the cost of the debt would be decreased. The success of such a policy is not guaranteed. First of all, shortening the average maturity of the debt means that the Treasury needs to issue debt in more frequent intervals which will increase transaction costs. At the same time, it will expose the Treasury to interest rate risk. This kind of government policy may fail if investors can substitute the decreasing amount of long-term debt with some other alternative securities. If there is a market segmentation, this kind of policy change may have the intended effects. Otherwise, the market will adjust itself and the government will not be able to save any money by replacing the short-term securities with the long-term securities.

A positive relationship between quantities and yields could wipe out any intended gains from shortening the average maturity of government debt. Such a relationship also has some implications on the hypothesis of the term structure of interest rates, especially on the market segmentation and preferred habitat hypotheses. Several techniques are used to determine whether there is a relationship between quantities and yields and the nature of the relationship.

First, an event study methodology is used to determine a pattern in yield changes around the auctions. The yield changes are greater on an auction day compared to other important dates such as an announcement date and an issue date. Therefore, the auction dates are used to measure the yield changes in response to quantity shocks.

Second, we estimate the relationship between yield changes and quantity shocks using Ordinary Least Square (OLS) models. In these regressions, the quantity changes in a given type of Treasury security are used as an independent variable. This variable is paired one by one with each yield changes across the term structure. The OLS estimations show that 13-week and 26-week bills and 30-year bonds have a wide spread effect on the term structure.

Quantity shocks on these securities significantly affect almost all yields.

OLS models show the unstructured relationship between quantity and yield changes. By imposing more structure on this relationship, we can learn more about the relationship. This relationship may change with the maturity of the security. One can use an interest rate model to test the existence of maturity dependent effects of a quantity shock. The Affine yield models are one of these models. These models are popular because of their tractability. In an affine yield model, yields are affine functions of state variables. State variables in turn are affine functions of their own drifts and variances. The quantity shock is the first state variable. The short rate is added as the second state variable. The Affine yield models help detect some of the significant relationships that are not captured by OLS models.

In our final analysis, instead of trying to measure the effect of quantity shocks on each yield, the effects are measured on three factors extracted from yields. These factors are "level", "slope", and "curvature" factors. Any significant relationship between quantity shocks and slope factor could be evidence of market segmentation. The quantity shocks of 13-week and 26-week bills negatively affect the slope factor but the quantity shocks of 7-year and 10-year notes positively affect the slope factor.

In this comprehensive study, it is found that the yield movements are significant on an auction day but not on announcement or issue days. These yield changes are not usually related to the net quantity changes but only related to the event of an auction. That is, the informational effect of an auction is more important than the quantity effect of an auction.

The potential implications of debt policy on the economy have drawn a lot of interest from researchers. Unfortunately, the overall results of this extensive research are not conclusive in regards to either relevance or irrelevance of the public debt composition. Modigliani and Sutch (1967), one of the first papers that analyze the subject, does not find any relationship between yields and quantities of bonds. More recent research by Hoelscher (1983), Schirm-Sheehan-Ferri(1989), Wallace and Warner (1996) support the findings of Modigliani and Sutch. A number of studies, however, reach a different conclusion. Bekdache (2001), Hakim and Rashidian (2000), Simon (1994, 1991), Park (1999), Chopin-Dickens-Gilley (1997), Roley (1982), and Friedman (1992) show evidence of a relationship between yields and quantities of Treasury bonds.

Most studies to date have used monthly or quarterly data. Simon (1994, 1991), however is an exemption. Simon uses daily data and finds the announcement of new cash bills increases the yield difference between cash management bills and adjacent maturity bills. The author shows that the relative supplies of 13-week and 12-week Treasury bills (the difference between the outstanding amount of 13-week bills and 12-week bills) significantly affect their yield differentials. Simon investigates the relationship between the supply of debt and yield spreads at only very short end of the term structure.

In this paper, we use all auctions and all yields to examine the relationship between the quantity of Treasury securities and their yields. Unlike any previous studies, the effects of a quantity change of a given Treasury security on its own yield and all other yields are measured for all Treasury securities. This is the first study to explore the effect of government debt quantity shocks on the term structure of daily interest rates using all bonds, and multiple techniques.

In this study, 4,497 daily yield changes from January 5, 1982 to December 31, 1999 are used. Quantity shocks are measured as the net changes of outstanding interest-bearing marketable public debt because of new auctions during the same period. The number of auctions (quantity shocks) in this period varies depending on the type of securities.

The next section describes the data used. The methodologies are described in the third section. Empirical findings are also summarized in that section. The paper concludes with a summary of the major findings and their limitations.

2. DATA DESCRIPTION

We divide the data in this study into two distinct categories: yield data (i.e. term structure data) and maturity composition data (i.e. quantities). Both categories are described in details below.

2.1 The Term Structure Data

A true term structure (as opposed to a yield to maturity) curve is the relationship between the time to maturity and the yield for default free pure discount bonds. In practice, long maturity pure discount bonds are rare or not available, and as such, a true term structure is not generally directly observable. Consequently, we must infer what term structure is consistent with observable traded coupon bonds. Unfortunately, there is no universally accepted way to derive the term structure from traded coupon bonds.

Most of the previous studies used the McCulloch and Kwon (1993) data. Some other popular term structure derivation methods are the Smoothed Fama and Bliss (SFB), the Fisher-Nychka-Zervos Cubic Spline (1995), and the Nelson-Siegel (1987).

This study employs the Unsmoothed-Fama-Bliss (UFB) term structure data. Among several term structure data, Bliss (1997) claims that the UFB fits the actual data best among all methods tested. Fama and Bliss (1987) first use this technique to create term structure data. This data is later updated by Bliss (1997b). Our data is updated until year 2000. Daily observations of bond returns are extracted from the CRSP files. The daily yields for the following maturities are created from these observations: 3, 6, 12, 24, 36, 60, 84, 120, 180, 240, and 360 months.

2.2 Maturity Composition Data

The maturity composition of the Treasury debt refers to the dollar market value of bills, notes, and bonds at each investment horizon. The composition at each maturity length is given either as absolute dollar terms or as a percentage of the total debt. While a change in the maturity structure can occur when either new debt is issued or existing debt matures, we focus on Treasury auction dates, as these are readily identifiable. In addition, a Treasury auction is a more conspicuous event and is more likely to contain information than a change caused by the maturity of an arbitrarily defined investment horizon. If there is any surprise concerning the amount of the auction, it should be reflected in yields around the event of auction. Using daily data allows us to examine such effects.

2.2.1 Public Debt

Our data consist of 4,497 daily observations of interest bearing, publicly held, marketable debt from January 5, 1982 to December 31, 1999[1]. This category of debt includes Treasury bills, notes, and bonds. Most of the treasury securities are auctioned on a regular schedule. However, cash management bills are auctioned whenever the Treasury needs additional funds. The public debt substantially increased during our sample period of 18 years. The federal debt was 6,153,295 million dollars in June 2002; of this, government accounts held $2,662,925 million, with the remaining $3,490,370 million held by the public. Of the total debt, marketable debt was $3,036,922 million or about 49.00 percent. Treasury bills, notes, bonds, and TIPS were $822,439 (13.00%), $1,474,296 (24.00%), $592,604 (9.60%), and $147,482 (2.40%) million, respectively. The marketable interest bearing public debt in January 1981 was only $626,752 million compared to today's level of just above 3 trillion dollars. The public debt has continued to increase for most of our sample period. It reached a peak of $3,515,860 million dollars on April 16, 1997. Since 1997, this total has decreased slightly.

During the sample period, average maturity of the debt increased slightly from 3 years and 10 months to 5 years and 8 months. Average maturity increased steadily from 1981 to 1988 then stayed relatively constant at about 6 years until 1993. It briefly dropped until 1996 and has since returned approximately to its 1987 level.

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[1] The treasury discloses the auctions starting from 1980. We lost almost two years of data to easily estimate the total quantities of debt and partially to filter very high interest rates in early 80s.

2.2.2 Shocks To Public Debt

We examine twelve different Treasury instruments based on their maturities. Cash management bills are auctioned on an "as needed" basis. In our sample, their maturities varied from 2 days to 335 days. During the sample period, these bills were auctioned 150 times. The smallest amount issued was $2 billion in 1981, and the largest amount was $42 billion in February 1999. The average size of Cash bill auctions during this period was $11.55 billion. Corresponding figures for more regularly auctioned Treasury instruments are shown in Table 1.

Table 1
Key Historical Auction Figures From 02/01/1981 to 12/31/1999
(in Billions of Dollars)

Security	Number of Auctions	The Largest Auction	The Smallest Auction	Average Size
Cash Bills	150	42.00	2.00	11.55
12-week T-Bill	985	16.20	3.00	9.07
24-week T-Bill	985	16.44	3.00	9.20
52-week T-Bill	246	21.00	4.00	12.28
2-year Notes	228	22.00	4.50	13.80
3-year Notes	73	23.31	1.00	14.20
4-year Notes	38	9.65	3.37	7.44
5-year Notes	140	19.80	1.00	10.91
7-year Notes	49	10.55	3.33	7.10
10-year Notes	85	16.00	1.00	10.20
20-year Notes	18	4.76	1.75	3.47
30-year Bonds	67	12.90	2.30	8.64

Notes: The data are obtained from Treasury Bulletin. The numbers in the first column indicate the number of time each Treasury securities are auctioned between 1981 and 2000. The numbers in the next two columns indicate the size of the largest and the smallest auctions in billions of dollars for a particular security category. The last column indicates the average size of the auction in billions of dollars for a particular security category.

3. METHODOLOGY

3.1 Event Study

While our data contain 4,497 yield changes, the numbers of quantity changes are relatively small. In modeling the relationship between these quantity and yield changes, the data on the yield side needs to be reduced. To measure the effect of a shock on daily yield changes, any day can be used in a regular auction cycle. This could be done for certain securities that are auctioned every week but it would be pretty difficult for most securities that are auctioned in 3-month cycles. If the quantity shocks have any effect, this effect should be around the event, that is, auction. In an efficient market, this effect should take place immediately. Therefore, an event study could help to determine whether there is an unusual yield change around an auction. The Treasury first announces, then auctions, and finally issues a treasury security. Therefore, any of these three days can be a good candidate to be an event day.

For 13-week and 26-week bills we test the null hypothesis that the yield changes are identical for all business days of a week. The nonparametric statistics reject the hypothesis[2]. The yield changes are different for all five days. The yield changes are highest on Mondays, which are auction days. Other securities are issued once a month or longer. The comparison of yield changes reveals that the change on an auction day is usually greater compared to the change on announcement and issue dates. A further yield change comparison between the auction day and the changes before and after a day of an auction day shows that the yield change on an auction day is greater. We find

[2] The results are not reported here but are available from authors upon request.

similar results for most bonds except 3-year and 20-year bonds. The yield changes in 5-year bonds are greatest one day after an auction. It is also tested whether the auctioning of a certain kind of securities affects the yields of other kind. The auctioning of 13-week and 26-week bills affects the yields of almost all other securities. We also compare the absolute values of yield changes for different securities. The yield changes of 30-year bonds are greater compared to yield changes of any other treasury securities. That is, they have the greatest volatility among all bonds.

3.2 OLS Models

We run ordinary regressions between 12 quantity variables (Cash-bills, 13, 26, 52-week bills, 2, 3, 4, 5, and 7-year notes, and 10, 20, and 30-year bonds) and 10 yield variables[3]. Among these 120 regressions only a few of them have significant constants and quantity variables simultaneously. The quantity shocks in 5, 7, and 10-year notes affect both the slope and constant variables in several regressions. On the other hand, the 13-week and 26-week bills significantly affect the constant factor in many regressions. Quantity shocks on these two bills significantly shift the yields on short to medium term securities. The effect is pretty widespread. The results are reported in Table 2.

Unfortunately, it is not possible to put all 12 independent variables into one equation. We can only combine the quantity shocks of 13-week and 26-week bills in one equation since we can observe them on the same day. Other quantity shocks appear at different times.

We repeat these 120 regressions for the yield changes in ten consecutive days before and after an auction. We discover some interesting movements in yields. The yield movements in the 21-day span suggest that the auction day is actually not a significant day for most yield changes in response to most quantity shocks. However, the auction day is the most significant day if we combine all the quantity shocks and compare all the days in the 21-day window. That is, for a given day and a given quantity shock, it is more likely to see a significant yield change in any of the securities on an auction day than any other day.

The 30-year bonds and 13-week and 26-week bills are benchmark securities since quantity shocks on these securities cause a lot of movements all over the yield curve. Another important conclusion can be drawn from these results. The event of an auction (regardless of the amount of the auction) is much more important than the actual quantity of the auction.

We draw this conclusion based on consistently significant constants with much larger magnitude compared to the usually insignificant quantity variable with a relatively small magnitude in the OLS models. The same regressions are estimated again without a constant. In that case, 10-year notes emerge as the most important market mover.

As we stated earlier, it is not possible to combine different types of shocks in one equation. On the other hand, these 10 different equations might be related. Therefore, the next logical step would be to put these equations into one system. The Seemingly Unrelated Regressions (SUR) and the Generalized Method of Moment (GMM) techniques can be employed to capture the cross reactions from different equations in a system.

The following simple model is employed:

$$e_{ni,t+1} = \Delta y_{ni,t+1} - \delta_i \Delta x_{it} .$$

In this model, the whole relationship between yield changes ($\Delta y_{ni,t+1}$) and quantity changes (Δx_{it}) are compacted into a single parameter δ_i. Since this model is a linear model, we can estimate the parameters by SUR. The results are presented in Table 3.

[3] The maturities of cash bills are different for each issue. The 4-year yields are not usually included in term structure data. Therefore, these two yields are not included in our yield data set.

The results from SUR are almost identical with the results from simple regression. The quantity shocks on the long-term bonds seem to affect the yields on short-term bills and long-term bonds but not the yields on the intermediate-term notes. Again 10-year bonds are important market movers.

On any given day there may not be any change in the quantity of any bond or there could be auctions for 1, 2, or 3 different bonds. In our sample, there is no change on 2,637 days out of 4,497 days, 1 change on 893 days, 2 changes on 945 days, and 3 changes on 23 days. Therefore, the maximum number of usable observations in our analysis is limited by the number of auctions. Essentially, we omit a lot of data. The quantity changes are zero if there is no auction or maturity on most days. We plug in those zeros in our data and rerun the estimations above. This must be legitimate because actually the quantity changes are zero on non-auction days. Only the significant coefficients are presented in Table 4.

Table 4
The Effect of The Quantity Shocks on the Whole Sample

Yield	**Quantity Shocks**									
Changes	**13-Week**	**26-Week**	**52-Week**	**3-Year**	**4-Year**	**5-Year**	**7-Year**	**10-Year**	**20-Year**	**30-year**
13 Week	4.09E-09	5.21E-09					-4.14E-09	-2.01E-09		
	(2.09E-09)**	(2.13E-09)**					(2.13E-09)*	(1.02E-09)**		
26 Week	3.81E-09	4.78E-09			-5.72E-09		-4.09E-09	-1.98E-09		
	(2.02E-09)*	(2.05E-09)**			(2.54E-09)**		(2.06E-09)**	(9.86E-10)**		
2 Year						-1.95E-09	-4.15E-09			
						(6.19E-10)**	(1.90E-09)**			
7 Year			-6.14E-09					1.97E-09	-1.01E-08	
			(3.30E-09)*					(1.04E-09)*	(5.78E-09)*	
10 Year							4.63E-09			-4.85E-09
							(2.65E-09)**			(1.40E-09)**
20 Year				-2.75E-09						
				(1.24E-09)**						
30 Year								2.10E-09		
								(1.16E-09)*		

Notes: The table reports the coefficients of the various quantity shocks on the yield changes. Numbers in the first row are the the coefficents. Numbers in parenthesis are standard errors of the coefficient efficients. We report only the significant coefficients. ***, **, and * Indicate significance at the 1%, 5%, and 10% level.

The 13-week and 26-week bills affect their own yields significantly. The relationship between the quantity shocks and yield changes is positive. The quantity shocks on 7-year and 10-year bonds negatively affect the yields of 13-week and 26-week bills, and each other.

3.3 The Affine Yield Model

In this section, the motivation is to put more structure into the relationship between quantities and yields if such a relationship exists. A discrete version of the affine yield models is employed in this section. We follow Campbell, Lo, and MacKinlay (1997) and Fleming and Remolona (1999) for the notation and the form for this model.

The bond price can be expressed as a function of the stochastic discount factor and next period's bond price:

$$P_{nt} = E_t\,(M_{t+1}P_{n-1,t+1}), \tag{1}$$

where, P_{nt} ($P_{n-1,t+1}$) is the price of an n-period ($n-1$ period) zero-coupon bond at time t ($t+1$). M_{t+1} is the stochastic discount factor. Assuming the distributions of P_{nt} and M_{t+1} are jointly conditionally lognormal, we can take the logarithm of both sides of equation (1) and yield:

$$p_{nt} = \mathrm{E}_t\,(m_{t+1} + p_{n-1,t+1}) + \tfrac{1}{2}\,\mathrm{Var}_t\,(m_{t+1} + p_{n-1,t+1}), \tag{2}$$

where, the lowercase letters represent the logarithm of the uppercase counterpart.

The change in the supply of a particular bond should affect the stochastic discount factor. We use the change in the supply of the 12 securities as factors to explain the stochastic discount factor. These shocks could be due to either new issues (auctions) or the expiration of the current issues. The Treasury usually replaces the maturing bond with new auctions. Therefore, in most cases these two shocks happen simultaneously. Officially, the issue date of a new bond corresponds to the maturity date of the old bond. Most of the quantity related yield changes take place around the auction date, not on the issue date. In this case, two big shocks are very close to each other (one due to auctioning of new bonds and the other due to the maturing of old bonds). One may consider combining these two shocks and taking the net change as the factor shock.

However, there are cases in which there is either only an auction or a maturity event but not both. Therefore, to be consistent, we could either exclude such cases or use only one event. Elimination seems very arbitrary; therefore, we only consider auctions as factor shocks[4]. Consistent with the literature, let's assume that the stochastic discount factor is a linear function of several state variables

(3) $$-m_{t+1} = x_{1t} + \sum_{j=1}^{K} x_{2jt} + x_{1t}^{\gamma}\beta_1 e_{1,t+1} + \sum_{j=1}^{K} \beta_{2j} e_{2j,t+1} \,,$$

where, x_{1t} represents the short rate and , x_{2jt} represents quantity shocks[5]. We separate the error term into two parts. The first error term e_1 represents unexpected shocks to the short rate. This error term is multiplied by a certain power of the short rate itself. The assumption about the gamma term is different for each one-factor model. It is assumed zero in Vasicek (1977) and ½ in Cox, Ingersol, and Ross (1985) models. Chan, Karolyi, Longstaff, and Sanders (1992)' estimate of gamma is 1.5. We impose 0.5 and also try other values, it does not make a significant difference in results.

Finally, the e_{2s} represents quantity shocks. The short rate itself is a mean reverting process.

(4) $$x_{1,t+1} = (1-\phi)\mu + \phi x_{1t} + x_{1t}^{\gamma} e_{1,t+1} + \sum_{j=1}^{K} \beta_{3j} e_{2j,t+1} \,,$$

where, ϕ is the persistence parameter and μ is the constant mean.

The variance of the short rate is $\sigma_{x1}^2 = x_{1t}^{2\gamma}\sigma_1^2 + \sum_{j=1}^{K}\beta_{3j}^2\sigma_{2j}^2$, $Cov(e_{1t}, e_{2jt}) = 0$ for any j and t, and

(5) $$x_{2j,t+1} = x_{2jt} + e_{2j,t+1}.$$

The amount of the public debt doesn't change unless there is a new auction or maturity. Therefore, equation (5) is the most reasonable representation of the quantity process. The yield function will be as follows.

(6) $$y_{nit} = \frac{1}{n}\left(A_n + B_n x_{1t} + \sum_{j=1}^{K} C_n x_{2ijt}\right),$$

(7) $$\Delta y_{nit} = \frac{1}{n}\left(B_n \Delta x_{1t} + \sum_{j=1}^{K} C_n \Delta x_{2ijt}\right),$$

[4]When maturities are used as a quantity shock, yield movements are not very responsive.

[5] We also try a one-factor version of this model, that is, without a short rate. One factor model reasonably fits to the model for the 5-year notes. But for others, model couldn't estimate a reasonable number for the persistence parameter which needs to be less than 1 but close to 1, according to the literature.

where, it can be shown that[6]

(8) $$B_n = 1 + \phi B_{n-1} - \frac{1}{2}\sigma_1^2 x_{1t}^{2\gamma-1}(\beta_1 + B_{n-1})^2,$$

(9) $$C_n = n,$$

(10) $$A_n = A_{n-1} + B_{n-1}(1-\phi)\mu - \frac{1}{2}\sum_{j=1}^{K}\sigma_{2j}^2\left[(\beta_{2j} + C_n)^2 + B_{n-1}^2\beta_{3j}^2 + B_{n-1}C_{n-1}\beta_{3j} + 2B_{n-1}\beta_{2j}\beta_{3j}\right].$$

A shock to any of the 11 factors together with the fluctuations in short rate will predict a change in the yields of all bonds.

(11) $$y_{in,t+1} - y_{\text{int}} = \frac{1}{n}\sum_{j=1}^{K}\left(\beta_{3ijn}B_n + C_n\right)\delta_{ijn}\Delta x_{2ij,t+1} + \frac{1}{n}B_n\alpha_{in}\left(\phi_i\Delta x_{1t} + x_{1t}^{\gamma}e_{1,t+1} - x_{1,t-1}^{\gamma}e_{1t} - \sum_{j=1}^{K}\beta_{3ijn}e_{2ijt}\right),$$

where δ_{jn} and α_{jn} are scaling parameters. $\Delta x_{1,t+1}$ is the function of its own lag and the changes in two error terms. The change in the second factor is essentially equal to the second error term. Since, the second factor never changes in two consecutive days, the second error term in time t is always zero. It is sensible to put together the direct effect of the second factor on the yields and its indirect effect through short rate (the second error term) as it is shown by equation (11). However, it is pretty difficult to estimate the model in that form since we don't know how to split the error in the first factor into error 1 and error 2. Therefore, instead we split the direct and indirect effect of the second term into two parts. This somehow weakens the potential coefficient associated with a quantity shock.

(12) $$y_{in,t+1} - y_{\text{int}} = \frac{1}{n}\sum_{j=1}^{K}\left(C_n\right)\delta_{ijn}\Delta x_{2ij,t+1} + \frac{1}{n}B_n\alpha_{in}\left(\Delta x_{1,t+1}\right).$$

The observed yield change is almost certainly will be different from the one predicted by equation (12). Therefore, we estimate

(13) $$y_{in,t+1} - y_{\text{int}} = \Delta y_{in,t+1} = \frac{1}{n}\sum_{j=1}^{K}\left(C_n\right)\delta_{ijn}\Delta x_{2ij,t+1} + \frac{1}{n}B_n\alpha_{in}\left(\Delta x_{1,t+1}\right) + \varepsilon_{in,t+1},$$

where $\varepsilon_{in,t+1}$ is a measurement error that is orthogonal to the factor shocks.

We estimate all the parameters using the GMM. The moment conditions vector has 14 elements: 9 for the errors in equation (13), and the remaining 5 are related with the first and second moments of two factors used in our model.

The instrument vector Z(3) consists of a constant, factor shocks, and the short rate.

(14) $$f_{it}(\theta) = Z_{it} \otimes M_{it}.$$

where, $\otimes$ shows the Kronecker product of the two matrices.

[6] The actual derivations of equations (8), (9), and (10) are not shown here but are available upon request.

Using this basic model we estimate parameter vector θ_T. The parameters for each quantity shocks are estimated separately since quantity changes do not coincide except for 13-week and 26-week bills. Our null hypothesis is $E[f_{it}(\theta)] = 0$.

We replace this expectation with its estimate using our available sample,

(15) $$g_T(\theta) = \frac{1}{T}\sum_{t=1}^{T} f_t(\theta),$$

where, *T* is the number of days in the sample when there is a change in the supply of a particular bond. This number can be anywhere from 14 to 942[7] depending on the type of the bond.

The number of moment conditions in our model is always greater than the number of parameters to be estimated. Therefore, the system is over-identified. The GMM method allows one to use a weighting vector to decrease the system to solve for the parameters. Depending on the weighting vector chosen, the estimates will differ. However, if the system is exactly identified, then the vector chosen will not affect the estimation of the parameters.

(16) $$h_T(\theta) = a_T g_T(\theta)$$

where $h_T(\theta)$ represents the weighted sample moment conditions. It is desirable to have $h_T(\theta)$ to be close to zero. The parameters that minimize the following criterion function will be GMM estimators.

(17) $$J_T(\theta) = |h_T(\theta)|^2 = g'_T(\theta) a'_T a_T g_T(\theta).$$

Hansen (1982) proved that if $a_T \xrightarrow{P} a_0, \theta_T \xrightarrow{P} \theta_0$ and $\frac{\partial g_T(\theta_T)}{\partial \theta} \xrightarrow{P} d_0$ the optimal choice of the weighting matrix is

(18) $$W_T = a'_0 a_0 = S_T^{-1},$$

where, $S_T = E[f_t(\theta) f'_t(\theta)]$. This weighting matrix is used if $f_{it}(\theta)$ is not serially correlated. If these moment conditions are serially correlated, an additional term will be added to this weight matrix. For example Newey and West (1987) suggest the following adjustment to the weighting matrix for serial correlation.

(19) $$\tilde{S}_T = \hat{\Omega}_{0,T} + \sum_{j=1}^{m}\{1-[j/(1+m)]\}(\hat{\Omega}_{j,T} + \hat{\Omega}_{j,T}), \text{ where } \Omega_{j,T} = (1/T)\sum_{t=j+1}^{T}[f_t(\theta) f'_{t-j}(\theta)].$$

This weighting matrix (W_T) enables us to find an estimator vector with the minimum asymptotic covariance matrix. The estimation process is iterative. It is necessary to know the weighting matrix W_T to minimize the criterion function as expressed in equation (17). On the other hand, parameters (θ) are required to calculate the weighting matrix. Therefore, the usual way to proceed is to start with an arbitrary weighting matrix (the common practice is to use an identity matrix), estimate the parameters, then calculate the weighting matrix for second round estimation of the parameters. This iterative process is continued until the estimates converge to a predetermined criterion. The optimal GMM estimator θ_T* will have the following covariance matrix:

[7] These numbers are different than the number of auctions listed in Table 1. We shortened the sample for consistency.

(20) $$V = \frac{1}{T}\left(d_0' S_T^{-1} d_0\right)^{-1}.$$

This covariance matrix is used to assess the significance of each estimated parameter. In all the models we test, the number of orthogonality conditions is greater than the numbers of parameters estimated. These over-identifying restrictions could be tested using the optimal value of the expression in equation (17). That is, using W_T as the weighting matrix:

(21) $$[\sqrt{T} g_T(\theta)]' \hat{S}^{-1}{}_T [\sqrt{T} g_T(\theta)].$$

The test statistic is the *T* times the optimal value of the objective function. This random number converges to a Chi-square distribution with *r-q* (the number of over-identifying restrictions) degrees of freedom. If this statistic is greater than its critical value, the model is misspecified. The results of this model are reported in Table 5 on pages 80-81.

The 13-week yields (the proxy for the short rate) are highly persistent. The model estimates the persistency parameter between 0.987 and 1.010 depending on the types of quantity shocks. The fixed mean of this series is estimated anywhere from 4.89 to 8.10. The last two negative estimates are not significant. The sample mean of the short rate is 6.31. The sample standard deviation is 2.21. The estimates of this parameter are between -0.0187 and 0.0330. The estimates of the standard deviations (σ_{2i}) for the quantity shocks are almost identical to their sample variances. The effects of quantity shocks on short-term rate are estimated with parameter β_{3i}. This parameter seems to be significant across the board. The quantity shocks of 13-week and 26-week bills, 5, 7, and 30-year bonds change the short rate in the same direction and the others in the opposite direction. The general effects of each quantity shocks are highlighted next.

For Cash Bills, the only significant quantity related coefficient is the 26-week bills. The coefficient is positive. That is, an increase in the quantity of cash bills drives the yields of 26-week bills up. For 13-week and 26-week Bills, only 52-week bills are significantly affected by these shocks. For 52-week Bills, those shocks immediately affect the yields on 7-year and 30-year bonds. Both yields go down in response to a positive quantity shock. For 2-year Notes, the yields on 52-week bills and 2-year notes respond positively to the quantity shocks in 2-year notes. For 3-year Notes, the response to a positive shock in 3-year notes is negative from very short-term bonds (26-week bills) and very long-term bonds (20 and 30-year bonds). The response is positive from the similar maturity bonds (e.g. 2-year notes). For 5-year Notes, those shocks seem to negatively affect the short-end (26-week and 2-year) and long-end (10, 20, and 30-year bonds) of the yield curve. For 7-year Notes, the 26-week bills negatively, but 5 and 10-year notes are positively affected from those shocks. For 10-year Notes, the effects of those shocks are more widespread and the sign patterns are not as consistent as other bonds. That is, we do not observe the general pattern of negative signs on dissimilar bonds and positive signs on similar bonds. Lastly, for 30-year Bonds, the effect on 10-year note is very strong and negative.

There are 14 equations, 3 instruments and 19 over identifying restrictions. Given the J-statistics, all models are misspecified. However, despite the misspecification, the estimates of quantity coefficients still hold[8]. The estimations are robust against heteroskedasticity and autocorrelation. Moreover, a prewhitening process is also performed to eliminate the possible correlations among the moment conditions.

[8] The misspecification in the models can be avoided if only similar yields (in terms of maturity) are used together in the estimation instead of the whole yield structure. For example, for 52-week bills, if we use only 52-week bills, and 2 and 3-year notes we can obtain a well-specified model. For some quantity shocks, removing only very short or very long-term yields helps the model to be well specified. For other quantity shocks, we needed to use only a single yield to obtain a well-specified model. The J statistics do not depend on a particular specification of the model. For example, as long as two factors are used, the J-statistics are identical in a simplified model and an affine yield model. The J-statistics vary according to instruments used, the number of variables in the system, and the combination of moment conditions. Most importantly, the coefficients related to the quantity shocks do not depend on estimation technique or the combination of moment conditions. Therefore, the coefficient estimates presented in the paper still hold although the J-statistics suggest a misspecification.

By looking at the relationship between quantity shocks and yield changes on the auction days only, the significant relations are sparse. In an efficient market, one would expect the price adjustments to occur instantly. However, it would be interesting to look at the relations over a wider time span.

So far, the auction day is used as the event day. That is, the quantity shocks are materialized on the auction days. The auctions are announced beforehand. Moreover, after each auction the bonds are issued after a couple of days. Although those two events do not seem to be as important as an auction date, they still may affect the relationship. The gap among announcement, auction, and issue date varies for different bonds. However, almost all announcements happen in ten days prior to an auction and all issues materialize in ten days following an auction. The time span for those days actually corresponds to 4 weeks because those days are business days. The GMM model is employed to estimate the relations between quantity shocks and lagged and leaded yields. That is, a total of 21 tests are made for each quantity shock: 10 lagged yield changes, 10 leaded yield changes, and the auction day yield change. A similar analysis using the OLS estimations was presented earlier. These two models (the OLS and affine yield model) measure different things. The OLS with a constant will utilize the information value of the auction itself (the shifts that are captured by a regression constant and can not be related to the quantity of the auction). However, the affine yield model only measures the quantity effect of the auctioning process since by design it does not have a constant term. The short rate process has a drift term. Therefore, some of those yield shifts are probably captured by the changes in the short rate.

The most obvious result from the affine yield model is that the short rate is a significant factor in driving the other yields. Unlike the OLS results there aren't any clear patterns although there are many significant relationships between the quantity shocks and yields. The event of an auction carries much more weight than the quantity of the auction. At a given auction the change in quantity is relatively small. However, the auctioning process could be a mechanism to reflect the changing economic conditions at each auction.

3.4 Factor Models

Bliss (1997b) shows that three orthogonal factors can explain up to 97 % of the variation in changes of interest rates. These factors are defined as "level", "slope" and "curvature" factors. A shock to a level factor affects all interest rates evenly, and a shock to a slope factor affects the long-term and short-term interest rates differently. Finally, a shock to a curvature factor affects mostly intermediate term yields. If there is a strong relationship between slope factor and the changes in maturity composition, the maturity composition changes the term structure of interest rates. It will be more informative to examine the effect of maturity composition on these three factors. This approach is also useful to examine whether a particular treasury security is a good substitute for the other.

Here, we follow a similar approach proposed by Barret, Gosnell, and Heuson (2000) and Bliss. The technical procedure is adapted from Johnson and Wichern (1992). First, the factors are estimated. Then, these factors are regressed on maturity composition variables. Interest rates could be represented by a number of orthogonal factors.

(22) $$X_t - \mu = LF_t + \varepsilon_t$$

where, $E(X_t) = \mu$, $E(F_t) = 0$, $E(\varepsilon_t) = 0$, $E(F_t F'_t) = I$, $E(\varepsilon_t F'_t) = 0$, $E[\varepsilon_t \varepsilon_t] = \Psi$

The differenced yields (X_t) on 10 different treasury securities are used. These are 13, 26, and 52-week bills, 2,3,5,7, and 10-year notes, and 20 and 30-year bonds. *F* represents factors. The most significant three factors are extracted from these yields. *L* represents factor loadings. The variance-covariance matrix of X_t is defined as:

(23) $$\Sigma \equiv E\left[(X_t - \mu)(X_t - \mu)'\right] = LL' + \Psi .$$

Since factors are orthogonal to each other and to individual error terms, factors themselves and individual error terms do not show up in equation (23). Therefore, it is possible to estimate factor loadings *L* and individual variances Ψ. If we assume all idiosyncratic variances are zero, we can write this expression as:

(24) $$\underset{(p\times p)}{\Sigma} = \underset{(p\times p)}{L}\,\underset{(p\times p)}{L'} + \underset{(p\times p)}{0} = LL'$$

where the number of factors equal to the number of random variables. The reason to use the factor analysis is to decrease the number of factors that will explain the variance-covariance structure reasonably well. There are 10 random variables (yield changes) and three factors can explain the whole variation reasonably well. Therefore, the remaining 7 factors are ignored. Equation (23) is estimated using only the first three factors.

If idiosyncratic variances are allowed, $\Sigma \cong LL' + \Psi$, then Ψ can be calculated based on the difference between the variance-covariance factor and factor loadings: $\Psi \cong \Sigma - LL'$. Here, the off diagonal elements are set to zero since we assume that different error terms are not correlated.

3.4.1 Factor Rotation

If there is more than one factor involved, it is possible to have different loadings with the same covariance matrix.

(25) $$X - \mu = LF + \varepsilon = LTT'F + \varepsilon = L^*F^* + \varepsilon$$

where T is an orthogonal matrix, that is $TT' = T'T = I$. $L^* = LT$ and $F^* = T'F$.

Here L^* and L represent different factor loadings from the same set of data. This property allows us to rotate factors, that is, multiply the loadings with an orthogonal matrix to make the interpretation easier.

The T matrix will have three free parameters: θ_1, θ_2, and θ_3. A nonlinear estimation technique is used to estimate the optimal values of those three parameters. The optimization criterion is to find the T matrix that will minimize the variation in the first factor so that it can be called level factor. By using these optimal loadings, factors themselves can be estimated with the following expression.

(26) $$F_t = (L'\Psi^{-1}L)^{-1}L'\Psi^{-1}(X_t - \mu)$$

where L is the vector of optimal factor loadings, X_t is the vector of daily yield changes of 10 Treasury securities. There are several term structure data which can be used to extract the loadings and factors. To be consistent with the prior work, Unsmoothed Fama-Bliss (UFB) term structure data is used.

The level factor should be close to a straight line (have equal loadings on each yields). Although it is not exactly a straight line, it still looks like a nearly leveled line. If any quantity shock has a significant positive relation with the level factor, an increase in quantity of that security will shift yields of all securities across the maturity spectrum. If any quantity shock has a significant positive relation with the curvature factor, an increase in the quantity of a security will boost the yields on 52-week bills, 2, 3, 5, 7, and 10-year notes. However, it will lower the yields on 13-week and 26- week bills and 20 and 30-year bonds. Finally, a positive loading on a slope factor combined with a positive shock in a security will shift the yields on all bills, 2, 3, 5-year notes downward and it will increase the yields on 7-year and longer maturity securities.

If a quantity shock significantly affects both the curvature and the slope factor, the combined effect is interesting. If the coefficients of both factors are positive, a positive shock will decrease the yields on 13-week and 26-week bills very dramatically since both factors have negative loadings on these securities. The yields on 52-week bills, 2, 3, and 5 year notes may not be affected at all since the curvature factor has positive loadings and the slope factor has negative loadings on these securities. Therefore, these two factors may cancel each other to a certain degree. The yields on 7-year and 10-year notes are going to increase dramatically since both factors have positive loadings in that spectrum. Finally, yields on 20-year and 30-year bonds may not be affected because of the opposite signs in two loadings.

Table 6
The Optimized Loadings, L*= LT

Yield	Level	Slope	Curvature
13-Week	0.6960	-0.5983	-0.1926
26-Week	0.8007	-0.5126	-0.0971
52-Week	0.7779	-0.3973	0.0780
2-Year	0.8180	-0.2297	0.2904
3-Year	0.8199	-0.1070	0.3306
5-Year	0.8452	-0.0315	0.3147
7-Year	0.7720	0.1314	0.3803
10-Year	0.6370	0.1039	0.4650
20-Year	0.7725	0.4540	-0.2479
30-Year	0.7798	0.4435	-0.2241

Notes: *L* represents factor loadings, *L** represents optimized factor loadings. *T* is an optimized orthogonal matrix that is used to minimize the variation in the first factor so that it can be called level factor.

One to one OLS estimates of the relationship between factors and quantity shocks are reported in Table 7. The quantity shocks of 13-week bills seem to significantly affect the movements of the slope and level factors. This suggests that positive shocks will raise the interest rates over the whole maturity spectrum. On the other hand, the coefficient on the second factor is negative which implies that the quantity shocks of 13-week bills will flatten the yield curve. According to the results of the ordinary regressions, constants associated with the quantity shocks of 13-week and 26-week bills are always significant. Therefore, it is not a surprise to find a significant relationship between these bills and the level factor. These results confirm the benchmark role of these bills especially the 13-week bills. The shocks of 2-year notes slightly affect all three factors, that is, increase the general level of all interest rates, steepen the slope of the yield curve, and increase the curvature. The combined overall effect is an increase in the yield of mid-term bonds.

The shocks of 5-year and 7-year bonds have significant negative coefficients on the curvature factor and significant positive constants on the same factor. The negative coefficients on the curvature factors will decrease the curvature. That is, the yields on mid-range bonds will decrease and yields on the short and long end of the curve will increase in response to a positive shock in any of these two bonds.

We also examine the relationship using a GMM technique. The results are reported in Table 8.

The GMM results seem more interesting[9]. First of all, none of the coefficients are significant on the level factor. Auctions of short-term bills have a negative coefficient on the slope factor. That is, a net increase in the supply of these bills will raise the yield on securities up to maturity of 5 years but will lower the yields on the longer-term bonds. In some of the previous results, significant positive relationships are detected between yield changes on these two bills and their own quantity shocks (see SUR estimation results). In the OLS estimates, the regression constant is positive and significant for all yields from 3 months to 5 years. The shocks of 3-year notes significantly affect both slope and curvature factors. The coefficient of the slope factor is negative while the coefficient of the curvature factor is positive. Therefore, there should be a significant positive relationship between the quantity shocks of 3-year securities and yield changes of 52-week bills, 2, 3, and 5-year notes.

These shocks should also negatively affect the yields on 20-year and 30-year bonds. In an ordinary regression, we find a significant negative relationship between 3-year shocks and 20-year yields. The results of the affine yield model confirm this prediction. The quantity shocks of 3-year notes shift the yields of very short-term (up to 6 months) and long-term (20 and 30 years) rates downward. The same shocks increase the yields of medium-term bonds.

[9] Naturally, the GMM estimations and OLS estimations are identical if the OLS regressions are estimated without a constant.

Table 7
The OLS Estimation of the Relationship Between Factors and Quantity Shocks

Quantity Shocks	Level	Slope	Curvature
Constant	0.0744	-0.0726	-0.0360
	(0.6345)	(-0.6016)	(-0.2461)
CashQ	-3.32E-09	1.17E-09	-1.79E-09
	(-0.4798)	(0.1641)	(-0.2077)
Constant	0.1434	-0.1707	-0.0353
	(3.8403)	(-4.3100)***	(-0.9699)
WK13Q	-2.35E-09	-4.35E-08	1.04E-08
	(7.05E-02)	(-1.2292)	(0.3196)
WK26Q	1.02E-08	-2.33E-08	-3.49E-09
	(2.99E-01)	(-0.6405)	(-0.1047)
Constant	-0.0964	-0.1520	-0.0449
	(-1.4818)	(-2.6775)	(-0.5403)
WK52Q	4.29E-10	1.98E-08	2.05E-08
	(0.0111)	(0.5834)	(0.4122)
Constant	0.1393	0.1218	0.1360
	(1.9776)**	(1.6737)*	(1.7687)*
YR2Q	-4.67E-09	-9.45E-09	-4.20E-09
	(-0.5693)	(-1.1159)	(-0.4695)
Constant	-0.1770	-0.0711	0.0815
	(-0.9349)	(-0.3822)	(0.6658)
YR3Q	4.55E-10	-1.10E-08	1.26E-08
	(0.0344)	(-0.5589)	(0.9776)
Constant	0.1146	0.0946	-0.1155
	(0.2957)	(0.2796)	(-0.2964)
YR4Q	-4.28E-08	1.47E-08	6.51E-08
	(-0.6011)	(0.2354)	(0.9077)
Constant	0.0501	-0.0898	0.4471
	(0.3162)	(-0.6205)	(2.7121)***
YR5Q	-1.02E-08	1.12E-08	-3.78E-08
	(-0.6667)	(0.8009)	(-2.3754)**
Constant	-0.0793	0.9586	1.8122
	(-0.1308)	(1.4561)	(2.7963)***
YR7Q	-7.30E-09	-1.13E-07	-2.40E-07
	(0.0693)	(-0.9917)	(-2.1399)**
Constant	0.0810	0.3830	0.1927
	(0.4951)	(1.3483)	(1.0213)
YR10Q	3.34E-10	5.85E-09	2.35E-11
	(0.0186)	(0.1879)	(0.0011)
Constant	-0.6023	0.3911	1.3393
	(-0.3058)	(0.2165)	(0.5139)
YR20Q	1.30E-07	1.08E-08	4.70E-07
	(0.2579)	(0.0233)	(-0.7058)
Constant	0.3405	0.7367	-0.6315
	(0.9000)	(1.6241)	(-1.5482)
YR30Q	-2.93E-08	-6.00E-08	3.69E-08
	(-0.7238)	(-1.2338)	(0.8448)

Notes: Level factor is a factor with similar loadings across the term structure. Slope factor has a negative loadings up to 5-year yields and positive loadings for the longer maturity yields. Curvature factor has negative loadings at very short and very long end of the yield curve but positive loadings at the middle. The number in the parenthesis is the corresponding t-values. ***, **, * indicate significance at the 1%, 5%, and 10% level..

The supply shocks of 7-year and 10-year notes have significant positive effect on both slope and curvature factors. That is, we may expect that the positive shocks on these notes will shift downward the yields of 13-week and 26-week bills. On the other hand, the yields on medium-term bonds (1 to 5 years) will not be affected as much since the loadings on these bonds are negative on the slope factor and positive on the curvature factor.

Table 8
The GMM Estimation of the Relationship Between Factors and Quantity Shocks

Quantity Shocks	Level	Slope	Curvature	J-Statistics
Cash Bill	5.89E-11	-2.13E-09	-3.43E-09	
	(-0.0152)	(-0.6637)	(-0.7182)	0.0077
13-Week	-1.07E-09	-4.51E-08	1.01E-08	
	(-0.0480)	(-2.2737)**	(-0.5414)	0.0396
26-Week	2.73E-08	-4.36E-08	-7.70E-09	
	(1.2205)	(-1.9611)**	(-0.3651)	0.0396
52-Week	-2.16E-08	-1.49E-08	1.02E-08	
	(-0.6814)	(-0.5511)	(-0.2801)	0.0480
2-Year	5.69E-09	-3.89E-10	5.91E-09	
	(-1.1198)	(-0.0851)	(-0.8086)	0.0354
3-Year	-8.33E-08	-1.63E-08	1.88E-08	
	(-1.0213)	(-1.7692)*	(1.9535)*	0.0305
4-Year	-2.49E-08	2.95E-08	4.69E-08	
	(-1.2857)	(1.4644)	(1.7930)*	0.0039
5-Year	-5.75E-09	3.23E-09	1.82E-09	
	(-1.1585)	(0.7884)	(0.3407)	0.0476
7-Year	-2.06E-08	4.72E-08	6.27E-08	
	(-1.2759)	(2.4818)**	(3.4222)***	0.2381
10-Year	8.29E-09	4.35E-08	1.90E-08	
	(-1.1393)	(3.8012)***	(2.2487)**	0.0472
20-Year	-2.23E-08	1.10E-07	-1.32E-07	
	(-0.4054)	(1.6729)	(-1.4628)	0.2068
30-Year	5.69E-09	1.58E-08	2.80E-08	
	(0.6632)	(1.5556)	(3.0741)***	0.0301

Notes: Level factor is a factor with similar loadings across the term structure. Slope factor has a negative loadings up to 5-year yields and positive loadings for the longer maturity yields. Curvature factor has negative loadings at very short and very long end of the yield curve but positive loadings at the middle. The first number is the estimated coefficients from the GMM estimations. The number in the parenthesis is the corresponding t-values. ***, **, and * indicate significance at the 1%, 5% and 10% level.

Depending on the magnitude of these two factors the yields of these bonds may not change because the positive response to one factor will be cancelled with a negative response to the other. The same is true for 30-year and 20-year bonds. On the other hand, the same shocks should dramatically increase the yields on 7-year and 10-year bonds since these two have positive loadings on both factors. From the OLS estimations, we find a negative significant constant for 13-week bills in response to quantity shocks of 7-year bonds. Although other yields change according to the predicted direction, they are not significant. From the SUR estimations, both 13-week and 26-week bills have negative significant coefficients in response to 7-year quantity shocks. The affine yield model shows that the relationship between 7-year note and six-month rate is negative, as predicted.

The shocks in 10-year bonds somewhat produced the expected results. First of all, we detect the significant negative yield changes in 13-week and 26-week bills. Moreover, the significant positive changes in the yields of 7-week and 10-year notes confirm our expectations.

Finally, the 30-year bonds significantly affect only the curvature factor. This suggests that a positive shock in 30-year bonds will increase their own yields given the negative loadings of curvature factor on 30-year yields and the negative relation between the curvature factor and 30-year quantity shocks. The yields of 10-year bonds are significantly affected by the quantity shocks in 30-year bonds. This relationship is negative because of the combination of the negative curvature loadings on 30-year bonds and the positive relationship between curvature factor and 10-year yields. This predicted relationship between the quantity shocks of 30-year bonds and 10-year yields is confirmed in all models without a constant.

4. CONCLUSION

This paper explores the dynamic relationship between the quantities of U.S. Treasury Securities and their yields. We examine the quantity effect of public debt on the term structure of interest rates on an extensive time line. The effect of each quantity shock is analyzed before and after the event of an auction. The previous studies examined this relationship at a certain time. However, we explore the timing of this effect.

The results of event study suggest that the auction date is the most significant event for the yield changes compared to the issue date and the announcement date. The issue date is also a significant event for some yield changes. The announcement day does not have much impact on yields.

The OLS estimations show that 13-week and 26-week bills together with 30-year bonds are benchmark rates. The quantity shocks of these securities tend to move the whole term structure. The effect of quantity shocks of 30-year bonds is very strong on 10-year bonds. The event of auctioning carries much more weight than the actual quantity changes. That is, the movements in interest rates are pretty volatile around auctions but they cannot always be related to the quantities.

The affine yield models show that the short rate has a very significant effect on all yields. The number of significant relationships between quantity variables and yield variables increases considerably compared to simpler models used before.

We gain some valuable insights from our factor analysis. Three orthogonal factors are extracted from 10 yields. These factors are rotated so that the first factor has leveled loadings on all interest rates. This factor is called the level factor. The other two factors are called slope and curvature factors. The slope factor has negative loadings on yields from 3 months to 5 years, and positive loadings for the longer term yields. The curvature factor has negative loadings on 3 to 6-month yields, and also on 20 to 30-year yields. It is loaded positively in between. The level, slope, and curvature factors explain most of the variations (80 percent) in yields. The GMM estimates show that none of the quantity shocks significantly affect the level factor. The quantity shocks of 13-week and 26-week bills negatively affect the slope factor but the quantity shocks of 7-year and 10-year notes positively affect the slope factor. The 3, 7, 10, and 30-year shocks positively affect the curvature factor. These results have some implications on the movements of yields in response to quantity shocks. A quantity shock in 13-week and 26-week bills changes 3-month and 6-month yields. The OLS estimations show that the positive shocks on 13-week and 26-week bills shift up the yields for all maturities. However, the magnitude is considerably higher for 3-month and 6-month yields. The quantity shocks in 7-year and 10-year bonds should increase the yields on 7-year and 10-year securities. This is also confirmed from the OLS regressions. The quantity shocks in 30-year bonds significantly affect the 10-year yields.

Overall, there is some evidence that the yield changes are related to the quantity shocks. The volatility of yields increases around the auction date. However, it is very difficult to precisely quantify the relationship between yields and quantities. The event of an auction impacts yields. However, we cannot detect a consistent quantity related effect on yields.

A comprehensive Roley (1982) and Friedman (1992) type model may help to estimate the relationship more precisely by integrating the demand and supply sides of the market. Of course, such a model should also consider the substitution effect of debt securities issued by private sector, municipalities, and foreign governments. To a lesser degree stock market can be an alternative for potential investors.

Since we cannot find a clear quantity effect, this study does not support the market segmentation hypothesis. The new information is reflected in yields after each auction. This information effect is uneven on the term structure. These results at least refute the pure form of the expectation hypothesis.

The lack of evidence for a quantity effect could be due to the smooth changes in government debt. A sudden large change in quantity may affect yields. We know at least that smooth debt changes will not cause large yield changes.

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Table 2

Ordinary Least Square Estimation of the Relationship Between Yield Changes and Quantity Shocks

					Quantity Shocks						
Yield Changes	**CASHQNA**	**WK13QNA WK26QNA**	**WK52QNA**	**YR2QNA**	**YR3QNA**	**YR4QNA**	**YR5QNA**	**YR7QNA**	**YR10QNA**	**YR20QNA**	**YR30QNA**
Constant	6.10E-03	0.0200	-1.42E-03	-1.33E-03	-5.08E-03	8.73E-03	-6.02E-03	-0.1000	-0.0140	-0.0600	5.67E-03
	(0.6700)	(4.2800)	(-0.2300)	(-0.2200)	(-0.3700)	(0.2900)	(-0.5500)	(-2.3500)**	(-1.0000)	(-0.4300)	(0.2400)
Wk13dy	-3.73E-10	1.78E-09(0.5550)	-2.14E-09	-1.01E-10	5.65E-10	-2.88E-09	-3.01E-11	1.29E-08	-5.47E-10	1.02E-08	-9.16E-10
	(-0.6900)	2.35E-09(0.7100)	(-0.5800)	(-0.1400)	(0.3900)	(-0.5200)	(-0.0300)	(1.7200)*	(-0.3400)	-0.2992	(-0.3600)
Constant	8.10E-03	0.0200	-1.61E-03	-2.34E-03	-9.06E-03	-1.12E-03	5.07E-03	-0.0400	-0.0200	-0.0900	-0.0100
	(0.9007)	(5.1400)***	(-0.2900)	(-0.4400)	(-0.6600)	(-0.0500)	(0.5500)	(-1.3200)	(-1.1800)	(-0.6500)	(-0.5600)
Wk26dy	-2.60E-10	1.7E-09(0.5800)	-4.87E-10	4.13E-10	1.30E-10	-5.54E-09	-1.08E-09	2.38E-09	-6.95E-11	1.74E-08	9.02E-10
	(0.5300)	1.79E-09(0.5900)	(-0.1500)	(0.6700)	(0.0900)	(-1.2600)	(-1.2100)	(0.4700)	(-0.0400)	(0.4700)	(0.3800)
Constant	-6.78E-03	8.58E-03	1.70E-03	0.01284	-4.10E-03	2.06E-03	0.0180	-0.0430	7.31E-03	-0.0400	-5.13E-03
	(-0.7900)	(2.7600)***	(0.4100)	(2.5200)**	(-0.3800)	(0.1300)	(1.6900)*	(-1.4700)	(0.7200)	(-0.2800)	(-0.2200)
Wk52dy	5.33E-10	7.15E-10(0.2600)	-1.65E-09	9.44E-11	6.13E-10	-1.97E-09	-1.78E-09	5.13E-09	-5.15E-10	3.45E-09	-4.17E-10
	(1.0500)	-3.71E-10(-0.1300)	(-0.6600)	(0.1600)	(0.5300)	(-0.6800)	(-1.7200)*	(1.000)	(-0.4600)	(0.1000)	(-0.1700)
Constant	0.0190	0.0089	-3.22E-03	0.0100	6.72E-04	7.09E-04	-5.40E-03	-0.0160	-6.22E-03	-0.0100	9.15E-03
	(2.0200)**	(3.3600)***	(-0.7000)	(1.7300)*	(0.0600)	(0.0300)	(-0.3900)	(-0.4300)	(-0.5100)	(-0.0800)	(0.3900)
Yr2dy	-8.89E-10	1.33E-09(0.5570)	-2.36E-10	-2.31E-10	8.27E-10	-2.09E-09	-1.47E-09	-1.40E-09	1.49E-09	-7.48E-10	-7.04E-10
	(-1.5300)	2.84E-10(0.1200)	(-0.0900)	(-0.3300)	(0.7000)	(-0.5000)	(-1.1000)	(-0.2100)	(1.1100)	(-0.0200)	(-0.2800)
Constant	-2.98E-03	0.0052	-3.83E-03	0.0140	-6.95E-03	-5.73E-03	0.0100	0.0598	-1.15E-03	0.0410	-0.0170
	(-0.3000)	(1.8900)**	(-0.7900)	(2.4800)**	(-0.7600)	(-0.2700)	(0.8000)	(1.8400)*	(-0.0900)	(0.2500)	(-0.6700)
Yr3dy	1.66E-10	8.39E-10(0.3434)	2.40E-09	-5.88E-10	2.29E-10	1.88E-09	-1.01E-09	-1.09E-08	-5.64E-10	-1.32E-08	1.87E-09
	(0.2800)	1.29E-10(0.0500)	(0.8300)	(-0.8500)	(0.2400)	(0.4900)	(-0.8200)	(-1.9300)*	(-0.400)	(-0.3200)	(0.6700)

Notes: Each column shows the effect of the quantity shocks in each Treasury securities. Each row shows the yield changes (dependent variable) in a particular security in response to 12 different quantity shocks (independent variables). Only 13-26 week bills are auctioned simultaneously. Therefore, those two shocks are put in a single equation (two independent variables). The rest of the equations are one to one regressions with a constant. The first number is the estimated coefficient and the number in the parenthesis is the t-values. ***, **, and * indicate significance at the 1%, 5%, and 10% level.

Table 3

Seemingly Unrelated Regressions (SUR) Estimates of the Relationships Between Quantity Shocks and Yield Changes

	Quantity Shocks											
Yield Changes	**CASH**	**13-WEEK**	**26-WEEK**	**52-WEEK**	**2-YEAR**	**3-YEAR**	**4-YEAR**	**5-YEAR**	**7-YEAR**	**10-YEAR**	**20-YEAR**	**30-YEAR**
13-Week	-9.60E-11	1.92E-09	4.18E-09	-2.50E-09	-2.00E-10	1.80E-10	-1.50E-09	-5.60E-10	-4.14E-09	-2.00E-09	-4.20E-09	-3.30E-10
	(-0.2800)	(0.5900)	(1.2700)	(-0.7300)	(-0.3700)	(0.1800)	(-0.5400)	(-1.3600)	(-2.000)**	(-2.8000)***	(-0.8900)	(-0.4800)
26-Week	1.08E-10	1.84E-09	3.80E-09	-8.54E-10	2.39E-10	-5.51E-10	-5.72E-09	-6.31E-10	-4.08E-09	-1.98E-09	-6.54E-09	-3.69E-10
	(0.3400)	(0.6200)	(1.2600)	(-0.2800)	(0.5000)	(-0.5500)	(-2.5600)**	(-1.7900)*	(-3.0400)**	(-2.4800)**	(-1.2200)	(-0.5700)
52-Week	2.25E-10	7.92E-10	6.51E-10	-1.26E-09	1.05E-09	3.01E-10	-1.65E-09	-1.74E-10	-2.09E-09	2.03E-10	-6.17E-09	-9.45E-10
	(0.7000)	(0.2800)	(0.2300)	(-0.5500)	(2.2800)**	(0.3700)	(-1.1200)	(-0.4200)	(-1.5500)	(0.4100)	(-1.2500)	(-1.3900)
2-Year	1.55E-11	1.41E-09	1.35E-09	-9.70E-10	5.43E-10	8.77E-10	-1.98E-09	-1.95E-09	-4.15E-09	8.74E-10	-3.65E-09	2.37E-10
	(0.0416)	(0.5900)	(0.5600)	(-0.3825)	-1.0059	-1.0788	(-0.9417)	(-3.7087)***	(-2.4379)**	(1.4900)	(-0.6931)	(0.3432)
3-Year	3.13E-11	8.86E-10	7.46E-10	1.52E-09	5.03E-10	-2.93E-10	9.83E-10	-1.10E-10	-8.59E-10	-6.77E-10	-2.82E-09	6.61E-11
	(0.0800)	(0.3600)	(0.3000)	(0.5700)	(0.9400)	(-0.4300)	(0.5000)	(-0.2300)	(-0.5700)	(-1.0900)	(-0.4800)	(0.0800)
5-Year	-4.56E-10	-3.00E-10	2.99E-09	7.19E-10	5.06E-11	-3.72E-10	1.29E-11	5.70E-11	1.67E-09	4.07E-10	-8.14E-09	-2.05E-10
	(-1.1500)	(-0.1300)	(1.2800)	(0.2700)	(0.0930)	(-0.5000)	(0.000)	(0.1100)	(1.0300)	(0.6200)	(-1.5400)	(-0.2500)
7-Year	-2.28E-10	1.16E-10	1.85E-09	-6.14E-09	2.33E-10	-4.79E-10	4.28E-10	-1.37E-10	1.12E-09	1.97E-09	-1.01E-08	-3.26E-10
	(-0.6200)	(0.0500)	(0.7300)	(-1.7900)*	(0.4300)	(-0.6300)	(0.2100)	(-0.2700)	(0.4400)	(2.2200)**	(-1.1600)	(-0.3200)
10-Year	-4.37E-10	-8.78E-10	7.04E-10	-5.40E-09	2.64E-10	-2.12E-10	-2.62E-09	-2.80E-10	4.63E-09	1.63E-09	-2.73E-09	-4.85E-09
	(-1.0500)	(-0.2600)	(0.2100)	(-1.0700)	(0.3400)	(-0.2200)	(-0.7100)	(-0.5600)	(1.2900)	(2.3300)**	(-0.4200)	(-4.1700)***
20-Year	-2.17E-10	-2.93E-09	5.61E-10	-1.81E-09	1.00E-10	-2.75E-09	-1.71E-09	-6.48E-10	-9.72E-10	1.76E-09	5.71E-09	2.01E-09
	(-0.4200)	(-0.9700)	(0.1800)	(-0.4800)	(0.1790)	(-2.7800)***	(-0.4100)	(-1.1400)	(-0.2400)	(1.7600)*	(0.8200)	(1.4800)
30-Year	8.52E-11	-1.52E-09	-7.33E-10	-3.85E-09	2.62E-10	-8.07E-10	-2.16E-09	-4.77E-10	-1.84E-09	2.10E-09	6.53E-09	1.02E-09
	(0.2000)	(-0.5200)	(-0.2500)	(-1.1800)	(0.4800)	(-0.9600)	(-0.5300)	(-0.8400)	(-0.6200)	(2.1000)**	(0.8900)	(0.7400)

Notes: Each row shows the yield changes (dependent variable) in a particular security in response to 12 different quantity shocks (independent variables). Only 13-26 week bills are auctioned simultaneously. Therefore, those two shocks are put in a single equation (two independent variables). The rest of the equation are one to one regression without a constant. Each column shows the effect of the quantity shocks in each treasury securities. The first number is the estimated coefficient and the number in parenthesis is the t-values. ***, **, * indicate significance at the 1%, 5%, and 10% level.

Table 5

The GMM Estimates of the Parameters of the Two-Factor Affine Yield Model

Quantity Shocks

	CASH	**13-WEEK**	**26-WEEK**	**52-WEEK**	**2-YEAR**	**3-YEAR**	**5-YEAR**	**7-YEAR**	**10-YEAR**	**30-YEAR**
Φ	1.0050	1.0132	1.0130	0.9910	0.9950	0.9958	1.0020	0.9870	0.9986	0.9988
	(1357)***	(740)***	(740.00)***	(976.00)***	(871.00)***	(1265.00)***	(1654.00)***	(456.00)***	(2313.00)***	(944.00)***
μ	5.7954	5.0170	5.0170	5.9359	5.9664	6.3356	8.0860	4.8957	-7.5541	-8.8045
	(21.20)***	(26.00)***	(26.00)***	(24.28)***	(19.41)***	(11.80)***	(11.60)***	(11.50)***	(-1.760)*	(-0.60)
β_3	-2.12E-11	1.88E-09	1.88E-09	-1.76E-09	-2.68E-10	-2.01E-10	7.60E-11	9.22E-10	-2.19E-11	1.49E-09
	(-13.10)***	(8.33)***	(8.33)***	(-11.60)***	(-21.70)***	(-13.60)***	(16.45)***	(9.60)***	(-15.86)***	(7.10)***
σ_1	-0.0227	0.0335	0.0335	0.0309	0.0214	0.0253	0.0166	0.0197	-0.0187	0.0161
	(-31.00)***	(46.30)***	(46.30)***	(38.56)***	(31.88)***	(28.50)***	(17.30)***	(14.10)***	(-25.64)***	(33.63)***
σ_2	1.6940	1.2990	1.2770	1.6740	0.8600	0.9470	1.0370	0.5760	0.9120	0.9340
	(28.00)***	(21.00)***	(21.00)***	(25.94)***	(53.38)***	(37.20)***	(52.45)***	(20.70)***	(66.70)***	(32.13)***
δ-	5.09E-13	6.10E-13	7.58E-13	3.65E-13	9.15E-13	-2.22E-12	-1.52E-12	-5.69E-12	3.85E-12	-2.24E-13
26week	(2.10)**	(0.26)	(0.26)	(0.14)	(1.61)	(-2.30)**	(-2.80)***	(-4.00)***	(2.80)***	(-0.38)
α-	0.0810	0.0870	0.0870	0.0340	0.0460	0.1410	0.0150	0.0510	0.1730	0.0900
26week	(7.30)***	(46.00)***	(46.00)***	(6.83)***	(1.96)**	(18.90)***	-0.85	(8.90)***	(7.77)***	(13.17)***
δ-	6.25E-13	-1.13E-12	0.1293	-5.34E-13	3.03E-12	4.04E-13	4.56E-13	9.93E-14	-6.93E-12	-2.47E-12
52week	(1.80)*	(-0.30)	(26.87)***	(-0.14)	(4.52)***	(0.55)	(0.76)	(0.06)	(-4.55)***	(-2.71)***
α-	0.0006	-5.44E-12	-5.44E-12	0.0738	0.0414	0.1760	0.1244	0.1066	-0.2785	0.0353
52week	(0.02)	(-1.40)	(-1.40)	(6.97)***	(0.93)	(12.60)***	(3.18)***	(5.47)***	(-5.18)***	(1.87)*
δ-2year	5.17E-13	1.18E-12	-2.19E-12	-2.10E-12	1.81E-12	2.09E-12	-4.40E-12	-6.40E-12	6.62E-12	6.94E-13
	(1.03)	(0.35)	(-0.66)	(-0.55)	(1.99)**	(2.34)**	(-3.90)***	(-1.63)	(3.28)***	(0.51)
α-2year	0.7329	0.2110	0.2110	0.0300	0.2270	0.2860	0.2690	0.1850	0.3120	0.0160
	(6.60)***	(17.10)***	(17.10)***	(1.96)**	(1.91)*	(21.40)***	(2.45)**	(2.40)**	(2.54)**	(0.37)

Notes: Delta parameter (δ) is the coefficient of the quantity shock and the alpha parameter (α) is the coefficient of the short rate. DRC is the determinant residual covariance. Each column represents the quantity shocks of the specified bonds. The parameters that listed in the first column are estimated independently for each quantity shocks. Therefore, the effect of a particular quantity shock on yields can be observed in the that column. J-statistics are the objective value of the function given by equation (17). ***, **, and * indicate significance at the 1%, 5%, and 10% level

Table 5 (Continued)

The GMM Estimates of the Parameters of the Two-Factor Affine Yield Model

	Quantity Shocks									
	CASH	**13-WEEK**	**26-WEEK**	**52-WEEK**	**2-YEAR**	**3-YEAR**	**5-YEAR**	**7-YEAR**	**10-YEAR**	**30-YEAR**
δ-3yr	2.02E-13	-3.46E-14	-3.37E-12	6.14E-12	1.27E-12	-1.14E-12	-3.34E-13	-3.44E-12	-3.30E-12	9.52E-14
	(0.34)	(-0.01)	(-0.91)	(1.20)	(1.22)	(-1.10)	(-0.36)	(-1.09)	(-1.66)*	(0.06)
α-3yr	0.2680	0.2900	0.2900	0.1420	-0.1400	0.3990	-0.0110	-0.0570	-0.1580	-0.0590
	(1.87)*	(18.00)***	(18.00)***	(5.32)***	(-0.88)	(23.30)***	(-0.09)	(-0.46)	(-0.82)	(-0.75)
δ-5yr	-1.22E-12	-3.16E-12	3.24E-12	3.44E-12	3.27E-13	-1.42E-12	-3.97E-13	7.11E-12	-5.50E-12	-6.78E-13
	(-2.40)**	(-0.82)	(0.93)	(0.66)	(0.32)	(-1.13)	(-0.37)	(2.31)**	(-1.87)*	(-0.44)
α-5yr	0.1710	0.4530	0.4530	0.1790	0.3460	0.7890	-0.3680	0.2240	-1.2320	-0.1200
	(0.71)	(15.75)***	(15.75)***	(2.65)***	(1.65)*	(23.80)***	(-1.59)	(1.83)*	(-2.96)***	(-0.93)
δ-7yr	-6.21E-13	-1.49E-12	1.18E-12	-1.42E-11	6.37E-13	-1.20E-12	-1.39E-12	4.82E-12	-4.20E-12	-9.90E-13
	(-1.25)	(-0.34)	(0.29)	(-1.90)*	(0.52)	(-0.88)	(-1.20)	(0.79)	(-1.48)	(-0.75)
α-7yr	0.0630	0.4940	0.4940	0.4920	-0.0250	-0.3610	-0.9370	0.2150	-2.5170	-0.1340
	(0.21)	(12.85)***	(12.85)***	(3.21)***	(-0.07)	(-3.70)***	(-2.39)**	(0.43)	(-3.14)***	(-0.66)
δ-10yr	-8.59E-13	-4.64E-12	-2.84E-12	-1.37E-11	3.60E-13	-9.91E-13	-2.87E-12	1.21E-11	2.73E-12	-1.36E-11
	(-1.30)	(-0.90)	(-0.61)	(-1.62)	(0.31)	(-1.00)	(-1.96)*	(2.40)**	(+1.17)	(-6.39)***
α-10yr	2.7380	0.8550	0.8550	0.3200	-1.3900	1.6350	-2.7710	-0.1290	-0.6690	-0.3380
	(4.10)***	(11.20)***	(11.2)***	(1.29)	(-1.43)	(22.20)***	(-4.60)***	(-0.29)	(-0.88)	(-0.61)
δ-20yr	-4.01E-13	-9.88E-12	-2.23E-12	-3.73E-12	5.41E-14	-7.79E-12	-3.60E-12	-2.10E-12	-5.57E-12	4.36E-12
	(-0.90)	(-2.05)**	(-0.45)	(-0.77)	(0.06)	(-4.15)***	(-2.80)***	(-0.40)	(-1.68)*	(1.96)*
α-20yr	3.12	1.354	1.354	0.648	-1.667	1.291	-4.755	0.215	-7.778	-5.554
	(2.08)**	(7.67)***	(7.67)***	(2.16)**	(-1.68)*	(5.41)***	(-3.60)***	(0.14)	(-2.74)***	(-4.73)***
δ-30yr	3.93E-13	-6.10E-12	-6.13E-12	-9.56E-12	5.28E-13	-2.44E-12	-3.74E-12	4.72E-12	-4.93E-12	1.58E-12
	(0.71)	(-1.40)	(-1.48)	(-2.55)**	(0.60)	(-1.84)*	(-2.75)***	(0.80)	(-1.45)	(0.73)
α-30yr	3.6260	2.1890	2.1890	0.8550	-2.2440	2.3760	-9.5840	5.3240	-12.0000	-8.4970
	(1.29)	(9.19)***	(9.19)***	(2.04)**	(-1.52)	(6.45)***	(-4.88)***	(3.20)***	(-2.78)***	(-4.10)***
DRC	2.37E-12	1.57E+14	1.57E+14	1.48E-13	7.53E-14	1.12E-14	7.49E-19	1.10E-15	2.93E-16	4.4e1-15
J-stat	0.9000	0.5800	0.5800	0.4400	0.9500	1.1000	0.9900	1.2800	1.0000	1.0500
(p-values)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Notes: Delta parameter (δ) is the coefficient of the quantity shock and the alpha parameter (α) is the coefficient of the short rate. DRC is the determinant residual covariance. Each column represents the quantity shocks of the specified bonds. The parameters that listed in the first column are estimated independently for each quantity shocks. Therefore, the effect of a particular quantity shock on yields can be observed in the that column. J-statistics are the objective value of the function given by equation (17). ***, **, and * indicate significance at the 1%, 5%, and 10% level.