2-Level-Service

Nick T. Thomopoulos, Illinois Institute of Technology

ABSTRACT

2-level-service occurs when an inventory system has two or more locations and one location is the source to another location in the network. The source location receives its stock from a supplier and -- when called upon -- replenishes the stock to another location, here called the 2-level-service location. This paper shows how to control the inventory at each location and generates table values on inventory levels for a range of scenarios

INTRODUCTION

Consider an inventory network with two or more stocking locations. Each part has a supplier who serves as the source to the network. When replenish stock is needed, the supplier is notified and stock is shipped to the network, accordingly. Some locations in the network are replenished from the supplier and some perhaps from another location in the network. In this paper, the former location has 1-level-service and the latter has 2-level-service. This goal of this paper is to measure the inventory needs between locations when 2-level-service is in use.

Consider two locations (1 and 2) and each has its own customer base and responsibility to fill their customers demands. Suppose location 1 is larger in storage size than location 2; and may have a larger customer base. On certain parts, location 1 may buy stock from the supplier for the needs of both locations (1 and 2). Location 1 holds the location 2 stock in its storage space until location 2 calls for replenish stock. This way, the supplier is the source for location 1; and location 1 is the source for location 2. The lead time from the supplier to location 1 is identified as L1, and the lead time from location 1 to location 2 is L2. On most situations L1 is much larger than L2, perhaps, L1 = 2 months and L2 = 2 days.

In the above situation, location 1 is replenished in a 1-level-service way and location 2 is replenished in a 2-level-service way. A location is here said to have 1-level-service (1LS) when the source is the supplier. A location is here identified with 2-level-service (2LS) when the source to the location is yet another location that receives replenish stock from the supplier. In general, all locations have 1-level-service unless their source is yet another location.

INVENTORY MEASURES

Two of the key measures on the performance in an inventory system are the amount of on-hand inventory and the service level. The service level SL is typically measured as the ratio of (demand filled) over (total demand). The on-hand inventory is the available inventory. The on-hand inventory is conveniently grouped into two partitions: cycle stock and safety stock. The cycle stock is the portion of stock carried to meet the average flow of demands as planned by the forecasts over the future time periods. The safety stock is the stock carried to meet the uncertainty associated with the forecast of the demands. The uncertainty in demands is measured by the forecast error. The typical measure of the forecast errors is the standard deviation of the one month ahead forecast error and is denoted here as σ.

SAFETY STOCK AND SERVICE LEVEL

References [1], [2] and [3] show how to compute the safety stock to yield the service level goal as desired by the management. On an individual part, the data used to determine the safety stock is listed below:

SL = desired service level
F = average monthly forecast
L = lead time to procure the part
Q = the size of the order quantity
σ = the standard deviation of the one-month ahead forecast error

For the analysis of this paper, references [2] and [3] show how the order size and forecast error are converted to units of the average monthly forecast as shown below:

M = Q/F = months supply
cov = σ/F = coefficient of variation

This way, the data is independent from the forecast size and is defined in relative terms. The data to determine the safety stock is now reduced to the following:

SL = desired service level
L = lead time in months
M = the order size in months supply
cov = the coefficient of variation

SERVICE LEVEL (SL)

The service level is defined as SL = (demand filled / total demand) and when measured in this way it is sometimes referred as the percent fill. In this paper, the examples cover three service level settings: 0.90, 095 and 0.97.

THE ORDER SIZE (Q)

The order size for a part and location is labeled as Q; and for convenience, this is listed in months supply and labeled as M. M is a measure of the order size in months supply with respect to the average demand (or forecast) of the part and location. When d = average monthly demand then Q = M×d.

LEVEL OF SERVICE (LS)

This paper concerns 1-level-service and 2-level-service by location. The basic scenario of this paper has LS =1LS for location 1 and LS =2LS for location 2.

HOME DEMAND (dh)

The home demand for a location is the regular demand from the location territory base. The average home demand per month (dh) is listed in relative terms. For the basic scenario, dh1 = 1.00 represents the average 1-month demand for location 1. The home demand in location 2 is listed in a relative way to the home demand in location 1. When location 2 has dh2 = 0.1, the average month home demand for location 2 is 10 percent of the home demand for location 1. So in the examples of this paper, dh1 = 1.00 and dh2 = 0.10 and 0.25.

EFFECTIVE DEMAND (de)

When location 2 has 2LS, location 1 must order stock from the supplier to accommodate the home demands for both locations 1 and 2. This demand is here called the effective demand (de) and becomes de1 = (dh1 + dh2) = (1.0 + 0.1) = 1.1 when dh2 = 0.1; and de1 = (1.00 + 0.25) = 1.25 when dh2 = 0.25. The effective demand for location 2 remains as the home demand, i.e., de2 = dh2.

FORECAST (F)

For convenience in this paper, the average forecast per month will be the same as the effective demand per month, i.e., F = de.

COEFFICIENT OF VARIATION (cov)

The standard deviation of the forecast error (σ) is also listed in months supply by use of the coefficient-of-variation (cov). For brevity in this paper, the computations are developed when cov = 0.30 for the home demands at location 1. Thereby, cov1 = 0.30 is for location 1 and the associated standard deviation is σ_1 = cov1×dh1. The standard deviation σ_2 at location 2 is obtained by the relation $\sigma_2 = \sqrt{dh}\,\sigma_1$. The corresponding measure for location 2 is cov2 = cov1/$\sqrt{dh}$. Note when location 2 has effective demand de2 = dh1+dh2, the associated standard deviation is $\sigma e_2 = \sqrt{\sigma_1^2 + \sigma_2^2}$. -- and the corresponding cov = σe_2/de2.

LEAD TIME (L)

Two lead times are noted in this paper, one for each location. L1 will represent the lead time for location 1 and this is the lead time from the source supplier to location 1. L2 will represent the lead time for location 2 to receive its stock from its source. When location 2 has 1LS, the source for location 2 is the same as the source for location 1, and thereby L2 = L1. When location 2 has 2LS, the source for location 2 is location 1 and the lead time L2 is generally much smaller than L1. In the examples of this paper, the lead times from the source supplier to location 1 are L1 = 1, 2, 3, 4 months. When location 2 has 1LS, L2 = L1. When location 2 has 2LS, L2 = 0.1 and 0.5 months are used in the analysis. Note L2 = 0.1 month is about 3 days and L2 = 0.5 months is about 15 days.

LOCATION DATA

To carry out the computations, the following data is needed by location: de, σ, M, L and LS. For location 1, these are noted as: de1, σ_1, M1, L1 and LS = 1LS; and for location 2 they are de2, σ_2, M2, L2, LS = 1LS or 2LS. Because dh1 = 1.00 = one month demand, all the computation are carried out in months supply relative to dh1 = 1.00.

CYCLE STOCK (cs)

The average cycle stock is cs = Q/2 , but since M = 1, then Q = Mde = 1de = de, and so the average cycle stock becomes cs = de/2. In the examples of this paper, de1 = 1.00, 1.10 or 1.25, and thereby cs1 = 0.50, 0.55 or 0.625, respectively. When location 2 has de2 = 0.10 and 0.25, then cs2 = 0.05 and 0.125, respectively.

SAFETY STOCK (ss)

The safety stock for a location is computed to yield the desired service level SL. In this paper, SL= 0.90, 0.95 and 0.97. To find the safety stock that yields this service level, three steps are taken. First the partial expectation e(k) is found by the relation $e(k) = (1\text{-SL})Q/\sigma_L = (1\text{-SL})M/(\sqrt{L}\,cov)$ where $\sigma_L = \sqrt{L}\sigma$ is the lead time standard deviation. Second, the safety factor k is obtained from a table lookup as in references [2], [3]. Third, the safety stock needed becomes $ss = k\sigma_L$. In this way, the safety stock is obtained for each location.

TOTAL STOCK (ts)

The average total stock (ts) by location can now be computed. This is merely the sum of the safety stock and the average cycle stock (ts = ss + cs). The total stock for locations 1 and 2 are denoted as ts1 and ts2, respectively. The total stock for both locations is TS = ts1 + ts2.

TABLES

Table 1 lists the total stock in months supply for locations 1 and 2 for the various scenarios in review. In all situations, the cov = 0.30 and M = 1 month. The table results are given when the service level is SL = 0.90, 0.95 and 0.97. The home demand for locations 1 and 2 are: (dh1 = 1.00, dh2 = 0.10) and (dh1 = 1.00, dh2 = 0.25). The level of service for location 1 is 1LS and for location 2 it is 1LS and 2LS. The lead time from the supplier (L1) ranges from 1 to 4 months. The lead time (L2) from location 1 to location 2 is set at 0.10 and 0.50 months.

Note (in Table 1) when locations 1 and 2 have 1LS, the two locations receive their replenishments from the supplier and both locations have the same lead time, whereby L2 = L1. When location 1 has 1LS and location 2 has 2LS, the total stock goes up for location 1 and goes down for location 2. In this latter situation, the effective demand at location 1 is de1 = dh1 + dh2 and it is de2 = dh2 at location 2. Here, location 1 is replenished by the supplier and location 2 by location 1.

Table 2 is concerned with the total stock sum of locations 1 and 2. Using the notation ts1 and ts2 for the total stock of locations 1 and 2, respectively, then TS = ts1 + ts2 is the sum of the total stock over the two locations. Let TS_1 = the total stock sum when location 2 has 1LS and let TS_2 = the total stock sum when location 2 has 2LS. This table lists the ratio of (TS_2 / TS_1) -- comparing when location 2 has 2LS over when it has 1LS. The comparisons use the same parameters of cov, M, SL, L1, dh1 and dh2. The only change is that location 2 has 2LS (instead of 1LS) and the lead time to location 2 changes from L2 = L1 to either L2 = 0.10 or 0.50 months.

Note (in Table 2) when L2 = 0.10 months, TS goes down by 2 to 16 percent depending on the combination of the SL, L1, and dh1, dh2. When L2 = 0.50, the ratio does not drop as much -- and sometimes the ratio is 1.00 or larger. A ratio larger than 1.00 indicates the total stock for the two locations is larger when 2-level-service is used in location 2. This only happens when L1 = 1.00 and L2 = 0.50 months -- indicating a 2-weekl decrease in the lead time for location 2.

SUMMARY

This paper introduces 2-level-service and the mathematical methods needed to control the inventory at the locations. The data by location includes the home demand, standard deviation, lead time, order size and desired service level The computations are the order quantity, safety stock, cycle stock and total stock. Table values are generated to measure the inventory sensitivity at each location and for the total inventory network when this system of stocking inventory is in use.

REFERENCES

1. Brown, R.G., *Smoothing, Forecasting and Prediction of Discrete Time Series,* Englewood Cliffs, NJ. Prentice Hall, Inc., 1962.
2. Thomopoulos, N.T., *Applied Forecasting Methods,* Englewood Cliffs, NJ. Prentice Hall, Inc., 1980.
3. Thomopoulos, N.T., *Strategic Inventory Management and Planning*, Carol Stream, Il. Hitchcock Publishing Company, 1990.

Table 1
Total stock (in months supply) by locations 1 and 2 when cov = 0.30, M = 1.0 and combinations of SL, (dh1, dh2), (L1, L2) and (1LS, 2LS).

SL	dh1	dh2	L1 \ L2	location 1 1LS	1LS	1LS	location 2 1LS 1.0	2LS 0.1	2LS 0.5
.90	1.00	0.10	1	0.54	0.58	0.58	0.13	0.05	0.10
.90	1.00	0.10	2	0.66	0.71	0.71	0.19	0.05	0.10
.90	1.00	0.10	3	0.77	0.82	0.82	0.24	0.05	0.10
.90	1.00	0.10	4	0.86	0.91	0.91	0.28	0.05	0.10
.90	1.00	0.25	1	0.54	0.64	0.64	0.22	0.13	0.17
.90	1.00	0.25	2	0.66	0.77	0.77	0.30	0.13	0.17
.90	1.00	0.25	3	0.77	0.88	0.88	0.37	0.13	0.17
.90	1.00	0.25	4	0.86	0.99	0.99	0.42	0.13	0.17
.95	1.00	0.10	1	0.68	0.73	0.73	0.17	0.07	0.12
.95	1.00	0.10	2	0.84	0.90	0.90	0.24	0.07	0.12
.95	1.00	0.10	3	0.98	1.04	1.04	0.29	0.07	0.12
.95	1.00	0.10	4	1.10	1.16	1.16	0.34	0.07	0.12
.95	1.00	0.25	1	0.68	0.81	0.81	0.27	0.14	0.21
.95	1.00	0.25	2	0.84	0.98	0.98	0.37	0.14	0.21
.95	1.00	0.25	3	0.98	1.13	1.13	0.46	0.14	0.21
.95	1.00	0.25	4	1.10	1.26	1.26	0.53	0.14	0.21
.97	1.00	0.10	1	0.77	0.83	0.83	0.19	0.08	0.14
.97	1.00	0.10	2	0.96	1.02	1.02	0.27	0.08	0.14
.97	1.00	0.10	3	1.12	1.18	1.18	0.33	0.08	0.14
.97	1.00	0.10	4	1.25	1.33	1.33	0.38	0.08	0.14
.97	1.00	0.25	1	0.77	0.91	0.91	0.31	0.16	0.24
.97	1.00	0.25	2	0.96	1.11	1.11	0.43	0.16	0.24
.97	1.00	0.25	3	1.12	1.28	1.28	0.52	0.16	0.24
.97	1.00	0.25	4	1.25	1.43	1.43	0.60	0.16	0.24

Table 2
Ratio of sum total stock (TS_2 / TS_1) when cov = 0.30, M = 1.0
and combinations of SL, (dh1, dh2), (L1, L2) and (1LS, 2LS).
*** The ratio in this column is $TS_1/TS_1 = 1$.**

				location			
				1	1LS	1LS	1LS
				2	1LS	2LS	2LS
SL	dh1	dh2	L1 \ L2		1.0	0.1	0.5
.90	1.00	0.10	1		1.00*	0.94	1.00
.90	1.00	0.10	2		1.00	0.89	0.94
.90	1.00	0.10	3		1.00	0.86	0.90
.90	1.00	0.10	4		1.00	0.84	0.88
.90	1.00	0.25	1		1.00	1.01	1.07
.90	1.00	0.25	2		1.00	0.93	0.98
.90	1.00	0.25	3		1.00	0.89	0.93
.90	1.00	0.25	4		1.00	0.86	0.89
.95	1.00	0.10	1		1.00	0.94	1.00
.95	1.00	0.10	2		1.00	0.89	0.94
.95	1.00	0.10	3		1.00	0.88	0.93
.95	1.00	0.10	4		1.00	0.86	0.90
.95	1.00	0.25	1		1.00	0.94	1.00
.95	1.00	0.25	2		1.00	0.90	0.95
.95	1.00	0.25	3		1.00	0.87	0.91
.95	1.00	0.25	4		1.00	0.86	0.89
.97	1.00	0.10	1		1.00	0.94	1.00
.97	1.00	0.10	2		1.00	0.90	0.95
.97	1.00	0.10	3		1.00	0.87	0.91
.97	1.00	0.10	4		1.00	0.86	0.89
.97	1.00	0.25	1		1.00	0.98	1.06
.97	1.00	0.25	2		1.00	0.91	0.98
.97	1.00	0.25	3		1.00	0.88	0.93
.97	1.00	0.25	4		1.00	0.86	0.90