Optimal Portfolios For Different Holding Periods

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ABSTRACT

This study uses a block bootstrap method to construct random samples of returns of six major financial assets and identifies optimal portfolios for three different objectives relating to risk and return for short, medium, and long holding periods. Optimal portfolios minimizing risk consist solely of Treasury Bills and small company stocks for all periods, with an increasing allocation to small company stocks as the investment horizon lengthens. Optimal portfolios minimizing risk relative to return, as well as those maximizing the risk premium relative to risk, contain intermediate-term government bonds and stocks for all horizons, and the proportions of stocks in these portfolios increase with the investment horizon, small company stocks becoming the major component of the optimal portfolios for 10 years. These results indicate that, for investors optimizing any of these three objectives, the optimal portfolios contain increasing allocations of riskier assets, and decreasing allocations of safer assets, as the holding period increases.

Keywords: Optimal portfolios, risk and return, investment horizon, time diversification

INTRODUCTION

Supporters of time diversification suggest that, although stock returns are very volatile in short periods, they become less risky over long periods because they revert to the mean. Since enough historical data is not available for meaningful inferences about long-horizon returns, some researchers have bootstrapped returns from the available data. However, only Hansson and Persson (2000) have employed block bootstraps, which preserve cross-sectional and serial correlations of the sampled returns of different assets over time, to identify efficient portfolios. Their study only considers mean-variance efficient portfolios consisting solely of stocks and Treasury Bills. This study uses the block bootstrap method to generate returns for holding periods of 1, 5, and 10 years, and investigates the compositions and risk-return characteristics of optimal portfolios minimizing risk and risk-return tradeoffs, as well as those maximizing Sharpe ratios, considering six major financial assets. The results show that the optimal portfolios contain increasing allocations of risky assets, and decreasing allocations of safe assets, as the investment horizon increases.

LITERATURE REVIEW

Empirical evidence of mean reversion in stock returns provides a rationale for time diversification. Fama and French (1988) report that autocorrelations of stock returns are most negative for 3-5-years and return toward 0 for 6-10 years. Poterba and Summers (1988) also find negative autocorrelations of stock returns over long periods, and observe that "if stock price movements contain large transitory components, then for long-horizon investors the stock market may be less risky than it appears to be when the variance of single-period returns is extrapolated using the random-walk model." (p. 53) Hansson and Persson (2000) note that, if stock returns are mean-reverting and Treasury Bill returns have positive autocorrelations for some horizons, annualized risk should decrease for stocks and increase for Treasury Bills over long investment horizons.

Early researchers found support for time diversification. Lloyd and Modani (1983) find that although stocks have a higher standard deviation of returns than bonds and bills for holding periods up to 9 years, they have the lowest standard deviation of returns for periods exceeding 24 years. Lee (1990) observes that the proportion of the

optimal mean-variance efficient portfolio invested in stocks increases with the investment horizon up to three years.

Some studies provided evidence against the benefit of time diversification. McEnally (1985) argues that terminal total returns are more relevant for investors than average annual returns. For stock returns during 1926-1981, he shows that as the holding period rises from one year to ten years, risk measures such as standard deviation, variance, mean absolute deviation, range, and semistandard deviation decrease for annualized returns, but increase for total holding-period returns. Levy and Gunthorpe (1993) find that the standard deviation of the holding period return on stocks rises more than the mean return as the holding period increases, and indicate that mean-variance investors should invest mainly in intermediate government bonds.

Other researchers indicated that stocks become less risky over longer periods. Drawing random samples with replacement from historical real monthly returns, Butler and Domian (1991) report that the probability of common stocks underperforming Treasury bonds is 11% for 10 years, and only 5% over 20 years. Levy and Spector (1996) indicate that investors with a log-wealth utility function should invest only in equities, including more than 90% in small stocks, and reasonably risk-averse investors with myopic utility functions should invest mainly in equities. Hickman et al (2001) observe that most of the higher long-term volatility of riskier assets reflects uncertainty about the degree to which they will outperform safer assets. They find that long-term investors end up with much less terminal wealth if they do not invest in risky assets. Based on 60-month overlapping block bootstraps of real returns from 1900-97, Hansson and Persson (2000) report that, efficient portfolios of mean-variance investors have rising stock allocations and declining T-bill allocations as the investment horizon increases from 1 to 10 years.

While existing studies have generally investigated the relative performance of risky and safe assets over different periods, risk-averse investors are expected to hold widely diversified portfolios optimized according to their risk-return objectives. This study uses a block bootstrap method to construct random samples of returns of six major financial assets for short, medium, and long periods, and identifies optimal portfolios for three different objectives relating to risk and return for each holding period.

DATA AND METHODOLOGY

Real monthly returns for 1926-2007 were obtained from Ibbotson Associates (2008) for six major U.S. financial assets: 30-day U.S. Treasury Bills (TB), intermediate-term government bonds (IGB), long-term government bonds (LGB), long-term corporate bonds (LCB), large company stocks (LCS), and small company stocks (SCS). The block bootstrap method can be used to approximate the distribution of long-horizon returns when limited historical data is available. One thousand blocks of 12, 60, and 120 months were drawn with replacement from the 984 months of available data, starting in random months. Logs of 1 + real returns were aggregated for each of the six assets to compute their continuously compounded monthly returns for 1, 5, and 10 years.

For each investment horizon, three different optimal portfolio allocations were identified, which minimized the standard deviation (SD) and coefficient of variation (CV) and maximized the Sharpe ratio (SR). These variables are defined below for asset i:

$$SD_i = \sqrt{\sum_{n=1}^{N}(R_n - R_i)^2/(N-1)} \quad (1)$$

$$CV_i = SD_i/R_i \quad (2)$$

$$SR_i = (R_i - R_f)/SD_i \quad (3)$$

where R_n is the continuously compounded monthly return in sample n, N is the total number of samples (1,000), R_i is the mean return of asset i in N samples, and R_f is the mean return on Treasury Bills in N samples.

For each optimization, the constraints were that no asset could have a negative allocation and the total portfolio weights would equal 100%, implying no short sales and full investment of the portfolio in one or more of the six assets.

RESULTS

Table 1 describes the log real returns and risk of different securities over various horizons. For all horizons, riskier securities generally have higher returns. SCS have the highest mean returns and standard deviations, followed by LCS, LCB, LGB, IGB, and TB, except that LCB have fractionally lower standard deviations than LGB over one and ten years. Means and standard deviations of returns consistently increase with the horizon for all securities, except that the standard deviation of SCS declines between 5 and 10 years, reflecting mean reversion in 5-year returns reported in earlier studies. For IGB, LCB, LCS, and SCS, means rise more than standard deviations, reducing the CV over longer horizons. For TB, the mean increases less than the standard deviation, raising the CV as the horizon lengthens. The CV of LGB decreases between 1 and 5 years but increases slightly over 10 years. TB are the safest asset, with the lowest standard deviation over all horizons. The best risk-return tradeoffs, indicated by the lowest CVs, are provided by IGB for 1 year, LCS for 5 years, and SCS over 10 years. The best risk premium-risk tradeoffs, denoted by the highest SRs, are offered by LCS for 1 and 5 years, and SCS over 10 years. These data indicate that riskier assets generally provide higher returns in each period, and both risk and return usually increase with the investment horizon for each asset. However, the returns of risky assets generally increase more than their risk, improving their tradeoffs of risk-return and risk premium-risk as the investment horizon lengthens.

Table 1
Log Real Returns and Risk of Different Securities over Various Horizons

	Treasury Bills	Intermediate Govt. Bonds	Long Govt. Bonds	Long Corp. Bonds	Large Coy. Stocks	Small Coy. Stocks
Panel A. 1-Year Returns						
Mean	0.84	2.26	2.38	2.70	6.32	7.95
Standard Deviation	4.03	6.45	9.25	9.16	20.72	29.35
Coefficient of Variation	4.82	2.86	3.89	3.40	3.28	3.69
Sharpe Ratio	0.00	0.22	0.17	0.20	0.26	0.24
Panel B. 5-Year Returns						
Mean	2.67	10.01	10.73	12.68	27.41	34.44
Standard Deviation	13.34	18.09	23.01	23.69	38.64	56.61
Coefficient of Variation	4.99	1.81	2.14	1.87	1.41	1.64
Sharpe Ratio	0.00	0.41	0.35	0.42	0.64	0.56
Panel B. 10-Year Returns						
Mean	4.10	18.77	18.25	22.33	71.12	93.57
Standard Deviation	22.22	31.01	40.91	40.81	51.68	52.01
Coefficient of Variation	5.42	1.65	2.24	1.83	0.73	0.56
Sharpe Ratio	0.00	0.47	0.35	0.45	1.30	1.72

Table 2 shows the correlations of log real returns of the six securities for different investment periods. Over all horizons, IGB, LGB, and LCB returns are very strongly positively correlated with each other. Returns on the three bonds also have strong positive correlations with TB returns, and their correlations with TB returns increase with the investment period. LCS returns are strongly positively correlated with SCS returns, but their correlation decreases as the horizon lengthens. LCS returns are moderately positively correlated with returns on TB and the three bonds over 10 years, while SCS returns have moderate negative correlations with returns on TB and the three bonds for 5 and 10 years. These data indicate that combining SCS with TB or one of the three bonds will provide the largest diversification benefit, and including more than one of the three bonds in the portfolio will yield the smallest benefit.

The asset allocations and risk-return profiles of optimal portfolios minimizing the standard deviation over different periods are presented in Table 3. The optimal portfolio consists solely of TB and SCS for all periods, and the SCS allocation increases while the TB weight decreases as the horizon lengthens. The mean and SD of the optimal portfolio both rise with the investment horizon, but the mean increases more than the SD, lowering the CV and raising the SR as the horizon lengthens. Owing to the negative correlations between SCS and TB, observed in Table 2, combining increasing proportions of SCS with TB results in the optimal portfolios having lower SD than the SD of TB alone, shown in Table 1, for all horizons. In addition, the optimal portfolios have much lower CV and higher SR than TB alone, especially over longer horizons.

Table 2
Correlations of Log Real Returns of Different Securities over Various Horizons

	Treasury Bills	Intermediate Govt. Bonds	Long Govt. Bonds	Long Corp. Bonds	Large Coy. Stocks
Panel A. 1-Year Returns					
Intermediate Govt. Bonds	0.72				
Long Govt. Bonds	0.56	0.93			
Long Corporate Bonds	0.54	0.93	0.95		
Large Company Stocks	-0.05	0.04	0.11	0.18	
Small Company Stocks	-0.19	-0.09	-0.02	0.05	0.82
Panel B. 5-Year Returns					
Intermediate Govt. Bonds	0.85				
Long Govt. Bonds	0.68	0.93			
Long Corporate Bonds	0.70	0.95	0.98		
Large Company Stocks	-0.03	0.03	0.13	0.13	
Small Company Stocks	-0.37	-0.28	-0.19	-0.17	0.63
Panel C. 10-Year Returns					
Intermediate Govt. Bonds	0.88				
Long Govt. Bonds	0.70	0.94			
Long Corporate Bonds	0.74	0.96	0.99		
Large Company Stocks	0.21	0.23	0.33	0.30	
Small Company Stocks	-0.18	-0.23	-0.14	-0.19	0.56

Table 3
Allocations and Risk-Return Profiles of Optimal Portfolios Minimizing Standard Deviations

	1-Year Returns	5-Year Returns	10-Year Returns
Panel A. Percentage Weights in Optimal Portfolios			
Treasury Bills	95.76	88.42	80.56
Intermediate Govt. Bonds	0.00	0.00	0.00
Long Govt. Bonds	0.00	0.00	0.00
Long Corporate Bonds	0.00	0.00	0.00
Large Company Stocks	0.00	0.00	0.00
Small Company Stocks	4.24	11.58	19.44
Panel B. Real Returns and Risk of Optimal Portfolios			
Mean	1.14	6.35	21.49
Standard Deviation	3.82	11.19	18.89
Coefficient of Variation	3.36	1.76	0.88
Sharpe Ratio	0.08	0.33	0.92

Table 4 shows that for optimal portfolios minimizing the CV, IGB constitutes 81% of the optimal portfolio for 1 year and 68% for 5 years, with the remainder in stocks, mainly small company stocks. Over 10 years, however, the optimal portfolio comprises 53% small stocks and 47% IGB. Therefore, combinations of IGB and stocks minimize the risk relative to return for all horizons, and the proportions of risky assets in these portfolios increase with the investment horizon, the riskiest asset becoming the major component of the optimal portfolio for 10 years. The rising allocations to stocks, especially small company stocks, increase the mean return much more than the standard deviation, sharply reducing the CV as the investment horizon lengthens. The CVs of 2.13, 1.01, and 0.48, of the optimal portfolios for 1, 5, and 10 years, respectively, are 14% to 28% lower than the lowest CVs of 2.86, 1.41, and 0.56 of individual assets shown in Table 1. The optimal portfolios, therefore, improve the risk-return tradeoffs offered by individual assets.

Table 5 indicates similar patterns as Table 4 for optimal portfolios maximizing the Sharpe Ratio. The optimal portfolios contain only IGB and stocks for all horizons, with IGB being the primary component for 1 and 5 years, and small company stocks having the major allocation for 10 years. However, compared to the optimal portfolios minimizing CV, those maximizing SR have slightly lower allocations to IGB and higher allocations to both large and small company stocks over all horizons. The Sharpe Ratio of the optimal portfolio rises with the investment horizon. These ratios of 0.35, 0.84, and 1.94 for 1, 5, and 10 years, respectively, are 13% to 35% higher than the highest SRs of 0.26, 0.64, and 1.72 for individual assets shown in Table 1.

Table 4
Allocations and Risk-Return Profiles of Optimal Portfolios Minimizing Coefficient of Variation

	1-Year Returns	5-Year Returns	10-Year Returns
Panel A. Percentage Weights in Optimal Portfolios			
Treasury Bills	0.00	0.00	0.00
Intermediate Govt. Bonds	81.41	68.02	46.74
Long Govt. Bonds	0.00	0.00	0.00
Long Corporate Bonds	0.00	0.00	0.00
Large Company Stocks	8.45	13.19	0.00
Small Company Stocks	10.14	18.79	53.26
Panel B. Real Returns and Risk of Optimal Portfolios			
Mean	3.18	16.89	58.61
Standard Deviation	6.77	17.01	28.20
Coefficient of Variation	2.13	1.01	0.48
Sharpe Ratio	0.35	0.79	1.93

Table 5
Percentage Weights and Risk-Return Profiles of Optimal Portfolios Maximizing Sharpe Ratios

	1-Year Returns	5-Year Returns	10-Year Returns
Panel A. Percentage Weights in Optimal Portfolios			
Treasury Bills	0.00	0.00	0.00
Intermediate Govt. Bonds	75.81	64.09	43.68
Long Govt. Bonds	0.00	0.00	0.00
Long Corporate Bonds	0.00	0.00	0.00
Large Company Stocks	12.08	16.01	1.02
Small Company Stocks	12.11	19.90	55.30
Panel B. Real Returns and Risk of Optimal Portfolios			
Mean	3.44	17.66	60.67
Standard Deviation	7.42	17.85	29.23
Coefficient of Variation	2.16	1.01	0.48
Sharpe Ratio	0.35	0.84	1.94

CONCLUSION

While existing studies have generally investigated the relative performance of risky and safe assets over different periods, risk-averse investors are expected to hold widely diversified portfolios optimized according to their risk-return objectives. This study uses a block bootstrap method to construct random samples of returns of six major financial assets and identifies optimal portfolios for three different objectives relating to risk and return for short, medium, and long investment horizons.

The risk-return characteristics of the individual assets indicate that riskier assets generally provide higher returns in each period, and both risk and return usually increase with the investment horizon for each asset. However, the returns of risky assets generally increase more than their risk, improving their tradeoffs of risk-return and risk premium-risk as the investment horizon lengthens. Correlations between the returns suggest that combining small company stocks with Treasury Bills or one of the three bonds will provide the largest diversification benefit, and including more than one of the three bonds in the portfolio will yield the smallest benefit.

Optimal portfolios minimizing the standard deviation consist solely of Treasury Bills and small company stocks for all periods, and the allocation to small company stocks increases while the weight of Treasury Bills decreases as the horizon lengthens. Optimal portfolios minimizing the coefficient of variation, as well as those maximizing the Sharpe ratio, contain intermediate-term government bonds and stocks for all horizons, and the proportions of stocks in these portfolios increase with the investment horizon, small company stocks becoming the major component of the optimal portfolios for 10 years. These results indicate that, regardless of whether investors seek to minimize risk, minimize risk relative to return, or maximize the risk premium relative to risk, the allocations of riskier assets in the optimal portfolios increase, while the allocations of safer assets decrease, as the holding period increases.

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