The Time Value of Money: A Clarifying and Simplifying Approach

Norman D. Gardner, (E-mail: gardneno@uvcs.edu), Utah Valley State College

Abstract

The concept of time value of money is critical for business students, financial managers, and anyone who deals with money. Financial managers must be able to employ TVM concepts competently in order to value both financial and real assets as they make decisions regarding capital budgeting, capital structure, and working capital management. Therefore business students must come to understand and apply correctly time value of money concepts.

This brief note suggests a clarification and a simplification of the pedagogy of TVM, which will result in greater understanding for the student. These suggestions include redefining the variable "n" in two of the TVM equations; dispensing with the pointless and purely semantic discussion of whether payments occur "at the beginning" or "at the end" of each period; and emphasizing the use of multiple-step problems.

Introduction

Time value of money (TVM) is a crucial component in understanding finance. Students must become comfortable and competent in the use of TVM techniques if they are to be successful in the business school. Financial managers make extensive use of TVM techniques in many areas, e.g., valuation of financial and real assets, capital budgeting and capital structure decisions, and the lease/buy analysis. A good understanding of TVM will also be valuable in the personal lives of students as they analyze major financial decisions, such as the purchase and financing of a home, and as they plan for retirement. Unfortunately, the way TVM is currently taught in the classroom and presented in textbooks often leaves students confused and uncertain as they attempt to understand the concepts and apply them correctly to meaningful and realistic problems.

This widespread difficulty experienced by students in achieving real understanding of TVM concepts has recently been discussed by Keasler (2003), Jalbert (2002), and Eddy & Swanson (1996). According to Jalbert, "Students nearly unanimously experience difficulty in identifying the appropriate technique to apply to TVM problems. While the TVM issue is complex, some of the difficulty can be attributed to the approach that finance texts take to the issue," Jalbert (2002). He analyzed seven popular finance textbooks in their coverage of TVM, and found them all lacking in clarity and teaching effectiveness. He then presents an exhaustive cataloging of the component characteristics of cash flow streams, including an annuities-due version of perpetuities with and without growing cash flows. Eddy & Swanson argue that instructors "do not sufficiently develop a frame of reference which begins with simple learning objectives focused on individual topics and progresses to higher levels of understanding." Eddy & Swanson (1996). They present a simplified step-by-step procedure in which students acquire TVM skills without initial reference to money or its value, but only a focus on the time variable. Keasler (2003) recommends the use of a new pedagogical aid he refers to as a "compounding period and payment completion table."

Each of these authors presents new procedures for teaching TVM concepts. Although their recommended methodologies differ widely, they all agree on the importance of TVM skills for the successful student of business.

The purpose of this teaching note is to present a more effective pedagogical approach to TVM by clarifying the definition of one of the TVM variables; by eliminating the needless confusion surrounding annuities due; and by emphasizing the use of multiple-step problems. This note provides no new theory. Its focus is more effective teaching through clarification and simplification. This note addresses pedagogical issues only, and does not suggest that theoretical changes should be made or that equations should be derived differently. Following are specific suggestions that reduce confusion and enhance understanding for the students.

Define "n" As The Number Of Payments, Not The Number Of Periods

The traditional textbook presents four standard TVM equations, each with its associated table of factors. For discussion purposes let us denote the present value of a single sum by PV(ss); the future value of a single sum by FV(ss); the present value of an annuity by PV(a); and the future value of an annuity by FV(a). Each equation contains four out of the five TVM variables: n, i%, PV, PMT, and FV.

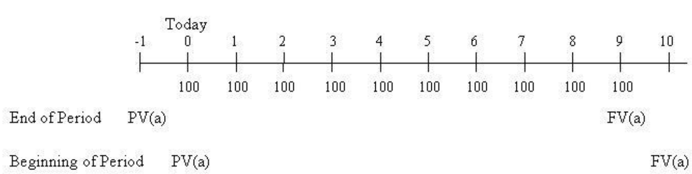
In the traditional textbook, the variable "n" is defined simply as the number of periods covered by the equation. This presents no ambiguity in the PV(ss) or in the FV(ss) equations, since we are only interested in the actual number of periods between the two-dollar amounts in the equation.

Considerable ambiguity can arise, however, when dealing with the annuity equations, PV(a) and FV(a). Unfortunately, the traditional textbook also defines "n" as the number of periods in these two annuity equations and factor tables. Much confusion for the student will be eliminated if "n" is defined as the number of payments, not the number of periods, when dealing with PV(a) and FV(a).

While casual observation may lead to the conclusion that the number of payments is really the same as the number of periods, a simple example will demonstrate the difference. Suppose, for example, that we begin today making equal annual deposits of $100 into a savings account. Suppose further that we will make a total of ten such deposits into this account. When will our last deposit into the account be made? Ten years from today? No! Our tenth deposit (or payment) into the account will be made exactly nine years from today (see Figure 1). In other words, only nine years will have elapsed during which we will have made ten deposits.

FIGURE 1

Ten Savings Deposits



The present or future value of this, or any other annuity, may be correctly found using the standard TVM equations or factor tables, as long as "n" is defined as the number of payments (ten), not the number of periods (nine). This is not to suggest a flaw in the derivation of the TVM equations, rather that for pedagogical purposes we should carefully point out to students that when dealing with annuities, "n" equals the number of payments, not the number of periods. A survey of several basic finance textbooks reveals that, when referring to PV(a) and FV(a), none define "n" as the number of payments. Most textbooks surveyed define "n" as the number of periods, e.g., (Gitman, 2003), (Block & Hirt, 1994), (Brigham & Houston, 1996), (Hickman, Hunter & Byrd, 1996), and (Ross, Westerfield & Jordan, 2004). Some textbooks define "n" as the number of "years", or simply as "time", e.g., (Brealey, Myers and Marcus, 2004), (Van Horne, 1998), and (Lewellen, Halloran & Lanser, 2000). Still others give no clear definition at all for "n", e.g., (Keown, Martin, Petty & Scott, 2001), and (Lee, Finnerty & Norton, 1997). Clearly the current focus in finance textbooks is to define "n" in the two annuity equations and factor tables as the number of periods or the passage of time.

Of course if one adopts a point of reference one period before the first annuity payment (for ordinary annuities), or one period after the last payment (for annuities due), the argument can be made that the number of payments equals the number of periods. However, this artificial process of identifying appropriate points of reference in order that the number of payments of an annuity equals the number of periods, becomes confusing and arbitrary for students, especially when dealing with deferred annuities. On the other hand adopting the definition of "n" as the number of payments allows us to dispense with this artificial "point of reference" issue altogether, and to focus on the important elements of the stream of payments itself, i.e., the number of payments, their size, and when the first and last payments occur on the timeline. When dealing with the present or future value of annuities, defining "n" as the number of payments, allows for similar treatment of all annuities whether they be immediate, ordinary or deferred annuities.

Forget "Beginning" Or "End" Of Period

A related source of difficulty arises from inattention to the exact point in time corresponding to the answers obtained from the standard PV(a) and FV(a) equations. In the case of FV(a), the point in time that corresponds to the future value of the annuity, is the date of the last payment, or more precisely, the instant after the last payment of the annuity. In the case of the standard PV(a) equation, the point in time corresponding to the present value of the annuity is one period before the first payment of the annuity. This attribute of the standard PV(a) equation makes it very useful in dealing with the amortized loan situation, but can cause confusion when students attempt to apply it correctly in other situations.

Referring to the savings deposit example above (see Figure 1), where the first deposit is made today, the point in time that corresponds to the future value of this ten payment series, using the standard FV(a) equation or factor table, is the date of the last payment, i.e., exactly nine years from today. Assuming this account pays 8% per year, and solving for the FV(a) using a financial calculator with "n" set to ten payments gives a future value of $1,448.66. On the other hand, the point in time that corresponds to the present value of this ten-deposit annuity is exactly one year ago today (see Figure 1). This will be the case if we use the standard PV(a) equation and factor table. Applying this equation with "n" still set to ten payments gives a present value of $671.01.

If we desire to find the value of this ten-deposit annuity as of today (the date of the first payment), there are two possible approaches. The most commonly used approach leads to the greatest confusion. It involves switching modes on the calculator to "beginning of period" mode, and then solving for the present value. While this procedure gives the correct answer with a single calculation, I nevertheless strongly recommend against it because I have found that it contributes to more confusion for the students than it is worth.

The problem is that in switching modes on the calculator in this fashion, we in essence introduce two additional TVM equations and factor tables for the "annuities due" situation, which assumes that payments occur at the "beginning" of each period. These two additional equations are variations of the standard (or ordinary annuity) equations, and for the most part, neither these "beginning of period" equations nor their factor tables are found in the traditional textbook. In contrast to the standard (or ordinary annuity) equations, these "beginning of period" (or annuity due) variations of the PV(a) and FV(a) equations find the present value on the date of the first payment, and the future value one period after the last payment (see Figure 1.) By switching to "beginning of period" mode on the calculator, and thus subtly introducing these two variations of the standard annuity equations, we only add needless complication to the understanding of the subject.

Aside from this needless complication, there is no real substance to the argument that a given series of payments occurs at the beginning of each period, while another series of payments occurs at the end of each period. Clearly, any series at all may simply be defined as occurring either at the beginning or at the end of each period. In the case of the savings deposit example above, we may say that the deposits occur either at the beginning, or at the end of each year. We could correctly say, for instance, that the first deposit occurs at the beginning of the year that begins today and ends one year from today. But it would also be completely correct to say that the first deposit occurs at the end of the year that began one year ago today, and that the second deposit occurs at the end of the year that begins today. Therefore I recommend that we completely dispense with the pointless and purely semantic discussion of whether payments occur "at the beginning" or "at the end" of the period, and focus instead on a correct and complete understanding of each of the four standard TVM equations. The additional TVM equations and factor tables that apply to annuities due are not needed to solve TVM problems, no matter how difficult. They only serve to complicate the subject for students and prolong the effort needed to really understand it.

Use More Multiple-Step Problems To Foster Real Understanding

Perhaps the chief motivation for introducing the concept of "beginning of period" payments, as opposed to "end of period" payments is that in so doing, the problem of finding the present value of an annuity on the date of the first payment can be accomplished with only one calculation. This preoccupation with single-step problems fosters shallowness of understanding of TVM.

Admittedly, using only the standard TVM equations will necessarily require two steps to compute the present value of a series of payments or deposits, if the desire is to know the lump sum value of the annuity on the date of the first payment. As noted above, applying the standard PV(a) equation to any series of payments results in finding the present value of the series at a point one period before the first payment. In the example above, this would be at a point exactly one year ago today. This presents no great challenge for students who understand TVM. If they desire to know the equivalent value of an annuity on the date of the first payment, or at any other point along the time line, once they have computed the present value of the annuity using the standard PV(a) equation, they simply perform a second calculation. This second calculation will utilize either PV(ss) or FV(ss), depending on the point in time where they wish to know the equivalent lump sum value of the annuity.

In the example we have been referring to above, the first step in finding the value of the ten deposits as of today, is to apply the standard PV(a) equation for the present value of an annuity by setting "n" equal to ten payments. As noted above, the answer is $671.01. This value corresponds to a point one year ago today. Therefore, to find the equivalent value today, a second calculation is required. This second step is to apply the FV(ss) equation to the lump sum value found in step one, setting "n" equal to one this time. The answer is $724.69--the value of this ten-deposit annuity as of today.

This process may seem needlessly complex, since the same answer can be obtained in a single step by simply switching to "beginning of period" mode on the calculator, or by using factor tables designed for annuities due. On the other hand, once students have understood the principles involved in the two-step process suggested here, they are then prepared, without confusion or further explanation, to find the equivalent value of any annuity at any point on the timeline. This insight will greatly facilitate real understanding of TVM, and enable the student to easily solve more complex and more realistic, multiple-step problems involving all annuities, whether they be immediate, ordinary or deferred annuities.

The vast majority of end-of-chapter TVM problems in textbooks designed to teach this material are single-step problems, e.g., (Gitman, 2003), (Keown, Martin, Petty & Scott, 2001), (Brealey, Myers and Marcus, 2004), (Brigham & Houston, 1996), (Hickman, Hunter & Byrd, 1996), and (Ross, Westerfield & Jordan, 2004).

Authors of textbooks should provide more end-of-chapter problems that require multiple-step solutions. Such problems will accelerate students' understanding of TVM, and will likely be more interesting and almost certainly more relevant to the real world.

Conclusion

In order to help students become more comfortable and competent in the use of TVM, I call on authors of finance textbooks to revise their presentation of the TVM material such that the variable "n" in the PV(a) and FV(a) equations and factor tables be defined as the number of payments, not the number of periods. This will allow for all annuities, whether immediate, ordinary or deferred, to be analyzed with the same, straightforward process. I also call for more challenging end-of-chapter problems that require multiple-step solutions, and for the complete elimination of the purely semantic discussion of whether a series of payments occurs "at the beginning" or "at the end" of each period.

References

1. Block, Stanley and Geoffrey Hirt (1994). *Foundations of Financial Management*, 7th ed. USA: Irwin.
2. Brealey, Richard, Stewart Myers, Alan Marcus (2004). *Fundamentals of Corporate Finance*, 4th ed. New York: McGraw-Hill.
3. Brigham, Eugene and Joel Houston (1996). *Fundamentals of Financial Management*, Florida: Harcourt Brace & Co.
4. Eddy, Albert and Gene Swanson (1996). A Hierarchy of Skills Approach to Teaching Accounting Present Value, *Journal of Accounting Education*, vol. 14, 1. Great Britain: Elsevier Science Ltd.
5. Gitman, Lawrence (2003). *Principles of Managerial Finance*, 10th ed. Boston: Gitman.
6. Hickman, Kent, Hugh Hunter, John Byrd (1996). Foundations of Corporate Finance, Minnesota: West.
7. Jalbert, Terrance (2002). A New Method for Teaching the Time Value of Money, *Journal of American Academy of Business*, Cambridge, vol. 2, 1.
8. Keasler, Terrill (2003). Abstract: The Compounding Period and Payment Completion Table as a Guide for Learning Multiple Step Time Line Problems, *International Journal of Services Technology and Management*, vol. 4, 2.
9. Keown, Arthur, John Martin, J. William Petty, David Scott, Jr. (2001). *Foundations of Finance*, 3rd ed. New Jersey: Prentice Hall.
10. Lee, Cheng, Joseph Finnerty, Edgar Norton (1997). *Foundations of Financial Management*, Minnesota: West.
11. Lewellen, Wilbur, John Halloran, Howard Lanser (2000). Financial Management: An Introduction to Principles and Practice. Ohio: South-Western College
12. Milburn, J. Alex (1992). Book Review*: Incorporating the Time Value of Money Within Financial Accounting*, Contemporary Accounting Research, Spring, vol. 8, 2. (Toronto: Canadian Institute of Chartered Accountants, 1983)
13. Ross, Stephen, Randolph Westerfield, Bradford Jordan (2004). *Essentials of Corporate Finance*, 4th ed. New York: McGraw-Hill.
14. Van Horne, James (1998). *Financial Management and Policy*, 11th ed. New Jersey: Prentice Hall.

Notes