

# A N-Assets Efficient Frontier Guideline For Investments Courses

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## Abstract

*This article provides directions that allow instructors and students to build an efficient frontier for investments courses. Our step-by-step approach intends to substantially reduce or eliminate the problems in combining the steps of downloading from the internet and use the data to build the efficient frontier and the capital market line, when short sales are present or not. In a less restricted theoretical framework, the approach can be applied to any subset of assets.*

## INTRODUCTION

The efficient frontier has long been taught by economic and finance professors as the envelope curve that contains the best risk-return combination for investors. According to Reilly and Norton (2003):

*“The efficient frontier represents that set of portfolios that has the maximum rate of return for every given level of risk or the minimum risk for every level of return.”*

Students are taught that a Markowitz’s investor should choose a point along the efficient frontier based on the investor’s utility function and attitude toward risk. They are also taught that the introduction of a zero variance asset into the Markowitz (1952) portfolio theory leads to the capital market theory. Noting that the risk-free asset would have zero correlation with all other risky assets, several authors have shown that any combination of this risk-free asset with any portfolio on that efficient frontier will be along a straight line between the points representing the risk-free asset and the market (or optimal) portfolio on the Markowitz efficient set (Sharpe, 1964; Lintner, 1965). As Reilly and Norton (2003) indicate: *“We can draw further lines from the point representing the risk-free asset to the efficient frontier at higher and higher points until we reach the point where the line is tangent to the frontier, which occurs at point M. The set of portfolio possibilities along the line RFR-M dominates all portfolios below point M.”*

This line RFR-M represents, therefore, a new efficient frontier, and is commonly referred as the Capital Market Line. Because point M is the point of tangency of the RFR-M line with the Markowitz efficient frontier, the risky portfolio at M has the highest return for its risk and every investor with such a risk tolerance want to invest in M. This portfolio which includes all risky assets is referred to as the market portfolio.

A major challenge faced by finance instructors is to lead students to situations where they can relate the theory to real life experiments. This challenge becomes quite substantial when teaching the modern portfolio theory. Our experience has shown, for instance, that going from the theory underlying the efficient frontier to its building and using the concept as a guide for the investment process is quite a challenge. At least four aspects have been identified as part of our problem in having students building and using the efficient frontier. First, they need to access data of the internet. Second, their computer skills are heterogeneous. We have sophomores, juniors and some seniors taking those classes. Third, textbooks have not provided a step-by-step guide which we could satisfactorily use to show how to combine the steps of downloading and use the data from the internet to build the efficient frontier and the capital market line.<sup>1</sup> Fourth, students face restrictions on how many asset classes they can use - most textbook examples show an efficient frontier with 2, 3 or at most 4 asset classes – and processes of optimization with PCs can be complex.

Our proposed guidelines intend to substantially reduce or eliminate the problems that students may have in combining the steps of downloading and use the data from the internet to build the efficient frontier and the capital market line, whether short sales are allowed or not. Our step-by-step approach has been very successful and popular among our students because it is simple and requires very little theory to be used. We believe it to be worthwhile passing on to others who also teach investments. Next we show, step by step, how to create the Markowitz efficient frontier with short sales restrictions and the inherent Capital Market Line, using an example of global strategic allocation product. Then, we conclude the paper by explaining how to relax the process to allow for short sales.

**RESTRICTED (NO SHORT SALES ALLOWED) EFFICIENT FRONTIER AND CML**

Our example stresses an efficient global asset allocation strategy. The step-by-step approach used for the example can be applied to any asset combination, though. To show the building process, the market index of 5 international regional blocks – European Monetary Union (EMU), the Asian Emerging Market (EMAS), the Latin American Emerging Market (EMLA), the East Europe and Middle East Emerging Market (EUMI), and the North American Market (NA) are considered.

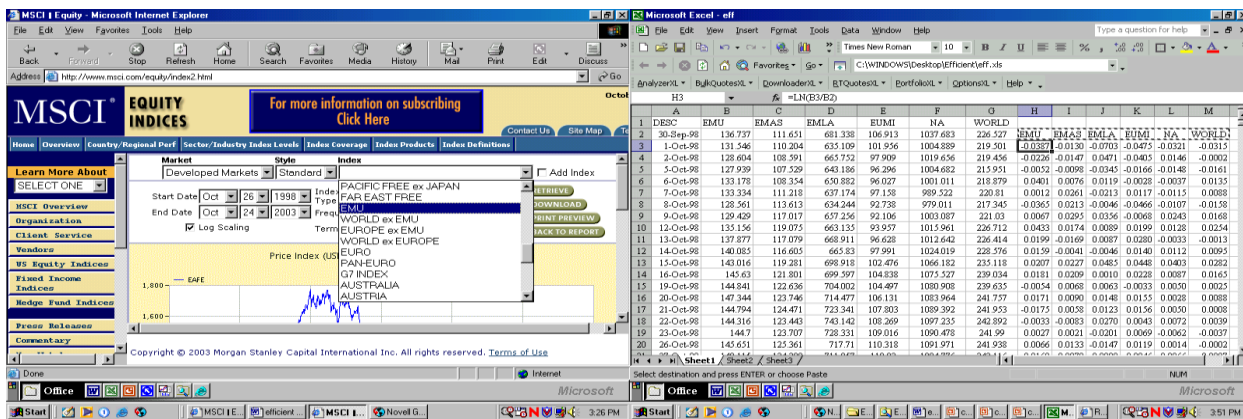
We create, first, a restricted (no short sales allowed) efficient set and, then, an unrestricted (short sales allowed) efficient set. In doing so, a five-step process is used to build an efficient set with *n* assets and design its inherent Capital Market Line. The first step consists of downloading historical time series on block indices from a particular website and transforming these “price series” in return series. The second step is to calculate the efficient frontier inputs (average, standard deviations, correlation matrix, and expected return). The third step sets up a calculation line for the optimization process (we show how to calculate the return and standard deviation of a portfolio). The fourth step consists of the actual construction of the efficient frontier using the calculation line created in the previous step. The fifth and last step is to build the inherent CML.

**Step 1: Download Historical Time Series Indices, then Transform Them In Return Series**

Given the importance of international asset allocation, we first show students how to download Morgan Stanley Capital International block series.<sup>2</sup> The step-by-step approach that follows can be extended to any combination of assets, though. As indicated in figure 1, blocks are listed in the “index” box. In the “index type” box, a data type is selected—i.e., “price index”. The frequency is chosen in the “frequency” box. In the “term” box, the number of years of data to retrieve must be selected. In order to download into excel, click on “download” (top right-hand side). In our example, we download six series with a term of five years and in daily frequency—i.e., European Monetary Union market index (EMU), the Asian Emerging Markets index (EMAS), The Latin American Emerging market index (EMLA), the East Europe and Middle East Emerging market index (EUMI), the North American market index (NA), and the world index (WORLD).

Figure 1: Data downloading in MSCI

Figure 2: Block price and return series



Finally, we transform the “price series” listed on columns B-G of Figure 2 in return series. Columns H-M of Figure 2 show the equivalent returns to those six price series. Returns starting on row 3 are calculated as follows. Since field names are in row 1, returns are calculated with the function LN (designated cell for price in row 3)/designated cell for price in row 2), i.e., LN (B3/B2), and placed in row 3 of each column. Thus, the return -0.0387 in row 3 of Column H equals LN (B3/B2), where the prices 131.546 and 136.737 are in rows 2 and 3 of Column B, respectively. By copying and paste the function LN (B3/B2) on the other rows of Columns H-M, the other returns of Figure 2 are calculated by Excel.

**Step 2: Calculate the Efficient Frontier Inputs**

By the end of this step, you should have a worksheet that resembles Table 1—inputs for this step will be in he highlighted (gray-shaded) cells, and the numbers in these cells indicate the sub-steps described in this section. Inputs for each series are the historical *average* return, the *naïve forecast*, the historical *standard deviation*, and the *correlation matrix* between the series.

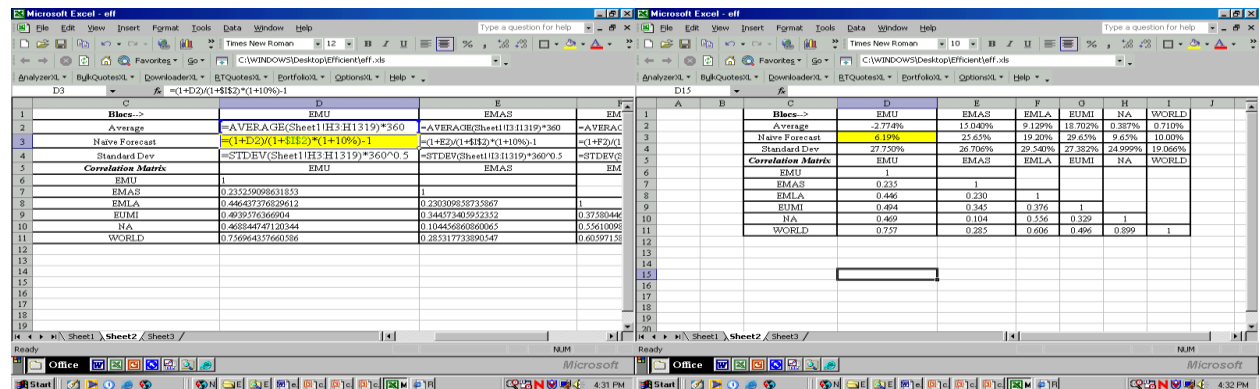
**Table 1: Desired Output from Step 2 (the numbers indicate the sub-steps described in step 2)**

	A	B	C	D	E	F	G	H	I	J
1			Blocks→	EMU	EMAS	EMLA	EUMI	NA	WORLD	
2			Average	1	1	1	1	1	1	
3			Naïve Forecast	4	4	4	4	4	4	
4			Standard Dev.	2	2	2	2	2	2	
5			Correlation Matrix	EMU	EMAS	EMLA	EUMI	NA	WORLD	
6			EMU	3						
7			EMAS	3	3					
8			EMLA	3	3	3				
9			EUMI	3	3	3	3			
10			NA	3	3	3	3	3		
11			WORLD	3	3	3	3	3	3	

As shown in Figures 3 and 4, the following computations need to be done. Firstly, compute the average “annualized” return for each daily return series —i.e., “Average (range)\*360”.<sup>3</sup> To do that, initially, select the cell where the average result should be placed. In our case, cell D2. Next, go to Insert, select Function, Select AVERAGE for function, and then in Number 1, place the cell references where the values to be used to compute the average are located. The other average values in cells E2:H2 may be determined by copying and pasting cell D2 value in these cells.

**Figure 3: Efficient Frontier Inputs (formulas)**

**Figure 4: Efficient Frontier Inputs (values)**



Secondly, estimate “Naïve forecasts” to capture the historical thrust in each series using formula (1).<sup>4</sup> For this, assume that your benchmark has a long term run return of 10% (the number here does not matter as long as it is positive). Then, we calculate how well (or worse) each series performs as compared to the benchmark, which is the MSCI World index in our example. We define our naïve forecast as follows:

$$\text{Forecast} = \frac{(1 + \text{average for a series})}{(1 + \text{average for the benchmark})} \times (1 + \text{an estimated Long term return for the benchmark}) - 1 \quad (1)$$

Thirdly, compute the “annualized” standard deviation for each daily return series—i.e., “=STDEV (range)\*(360)^0.5”.<sup>5</sup> To accomplish this, do as follows: (1) select the cell where the standard deviation value will be placed. In our case, this cell is D4; (2) go to Insert, select Function, Select STDV for function, and then in Number 1, place the cell references where the values to be used to compute the standard deviation are located. The other standard deviation values in cells E4:H4 may be determined by copying and pasting cell D4 value in these cells. Fourthly, compute a correlation matrix for all “return data series,” as follows: select Tools, then Data Analysis, choose “Correlation” as the Analysis Tools in the Data Analysis dialog box, and click OK. When the Correlation dialog box appears, select the Intput Range for the return series (including labels), check Labels in First Row. In Output Options of the dialog box, click on Output Range, and define the cell(s) in your worksheet to place the correlation matrix. Click OK to validate. Values from these computations are shown in Figure 4.

**Step 3: Set up a Calculation Line for the Optimization Process**

By the end of this step, students should have a worksheet that resembles table 2—inputs for this step are highlighted and the numbers indicate the sub-steps described in this section. Row 14 is our *calculation line*. The role of this line will become clear soon. It is also crucial to proceed carefully during this step.

**Table 2: Desired Output from Step 3 ( the numbers indicate the sub-steps described in step 3)**

	A	B	C	D	E	F	G	H	I
5			Correlation Matrix	EMU	EMAS	EMLA	EUMI	NA	WORLD
6			EMU	1					
7			EMAS		1				
8			EMLA			1			
9			EUMI				1		
10			NA					1	
11			WORLD						
12	<b>WEIGHTS</b>								
13	<b>Portfolio Return</b>	<b>Portfolio Stdev</b>	<b>Sharpe Ratio</b>	EMU	EMAS	EMLA	EUMI	NA	SUM
14	2	3	4						5

(1) Downloading The Add-In To Compute Portfolio Returns And Standard Deviation

To compute the portfolio expected return and standard deviation, we use a Visual Basic Application (VBA) program. We believe this technique is more efficient than others that have been proposed elsewhere.<sup>6</sup>

Go to the proper website to retrieve the add-in, copy the file from the website and paste it on your PC desktop. Go back to your original Excel file to load your add-in. Select Tools, Add-Ins, and then choose Browse. The Browse dialog box will appear. At the top, in Look in choose the directory where your add-in has been saved. Double click on the add-in file (herein we named “efficient.xla”). Then, validate your choice as in figure 5. When you validate you will notice an ✓ on the left-hand side of the Solver Add-in included as one of the Add-Ins available. Click OK.

Figure 5: Load the “efficient.xla” addin into Excel      Figure 6: Portfolio risk, return and Sharpe ratio.

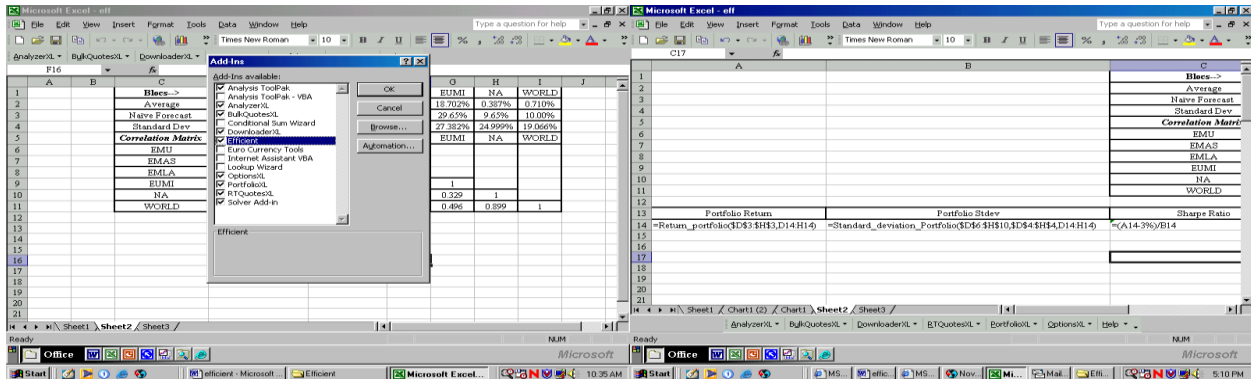
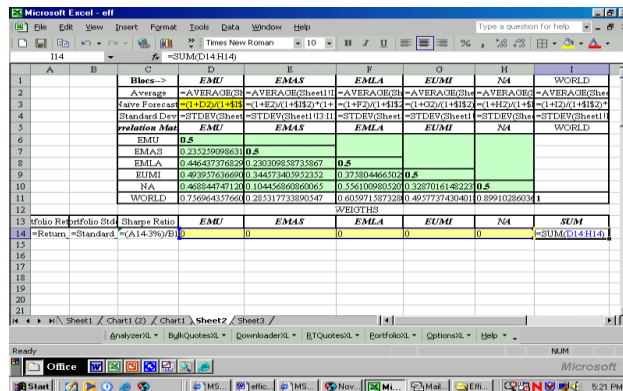


Figure 7: End of step 3 (Formula)



(2) Defining Cell Ranges And References For Different Functions

As suggested in Table 2, the setup for the optimization process consists of 5 sub-steps described in the next Step 4. Formulas and results are shown in figures 6 and 7.

At this point, though, first, change the values of 1 on the correlation matrix diagonal to 0.5.<sup>7</sup> Second, define the cell ranges and references to be used to determine the portfolio return using the RETURN\_PORTFOLIO function: (1) Place the pointer in the cell where the portfolio return will be. For example, we choose A14 in our example. (2) Select Insert, and then Function. The Insert Function dialog box will appear. In “Or select category” scroll down until you find “User defined”. Select it. In “Select a function” choose RETURN\_PORTFOLIO. The Function Arguments dialog box will appear. In the return cell range, select the Naïve Forecast Range (\$D3:\$H\$3, in our example) which will include that forecast for EMU, EMAS, EMLA, EUMI and NA, and DO NOT include the forecast for the WORLD cell.<sup>8</sup> For the Weights Range, select the cells range (D14:H14, in our example) for the weights. For instance, notice that in Figure 10 we have introduced zero for the weights of EMU (cell D14), EMAS (cell E14), EMLA (cell F14), EMUI (cell G14), and NA (cell H14).

For the cells that include the naïve forecast we need to use absolute (\$D3:\$H\$3), for example, rather than relative (D3:H3) cell references. By doing so, when we copy the return portfolio figure that had been calculated by the proper formula, from one cell (in our case from cell D14) to another, the figures for the naïve forecast will not change. As a result, we should have for the return portfolio function the following representation =Return\_portfolio

(\$D\$3:\$H\$3, D14:F14) rather than =Return\_portfolio (D3:H3, D14:F14). This is necessary as we intend to copy line 14 different results to other lines and use the function over and over through out the worksheet.

Third, define the cell ranges and references to be used to calculate the portfolio standard deviation using the STANDARD\_DEVIATION\_PORTFOLIO function. (1) Place the pointer in the cell where the portfolio return will be computed. For example, we choose B14 in our example. (2) Select Insert, and then Function. The Insert Function dialog box will appear. In “Or select category” you should have “User defined”. Otherwise scroll down until you find “User defined” and, then you select it. In “Select a function” choose STANDARD\_DEVIATION\_PORTFOLIO. The Function Arguments dialog box will appear. For the “Cor” cell ranges select those for the correlation matrix (\$D\$6:\$H\$10, in our example). DO NOT include the cells in the correlation matrix values for the WORLD. For the StdDev, select the cells range where the standard deviation values are. For instance (\$D\$4:\$H\$4) for our example. For the Weights cells Range, select the same range we choose above for the RETURN\_PORTFOLIO function, i.e., (D14:H14) in our example.

Fourth, define the “reward to risk” ratio, that is, the Sharpe ratio. It equals the risk premium of the portfolio divided by the standard deviation of the portfolio. The portfolio risk premium is the return of the portfolio minus the risk free rate (in our example, we assume 3% for the T-bill rate). Hence, the function to be inserted in C14 is “=(A14-3%)/B14”.

Fifth, since in cell I14 we will have the sum of the weights from cells D14 through H14, we include the function SUM in cell I14. Thus, in our example we will have =Sum (D14:H14).

**Step 4: Build The Efficient Frontier Using The Calculation Line**

By the end of this step, you should have a worksheet that resembles table 3—inputs for this step are highlighted and the numbers indicate the sub-steps described in this section. Row 14, our *calculation line*, is used over and over in the execution of this step sub-steps.

**Table 3: Desired Output from Step4 ( the numbers indicate the sub-steps described in step 4)**

	A	B	C	D	E	F	G	H	I
12									
13	<b>Portfolio Return</b>	<b>Portfolio Stdev</b>	<b>Sharpe Ratio</b>	<b>EMU</b>	<b>EMAS</b>	<b>EMLA</b>	<b>EUMI</b>	<b>NA</b>	<b>SUM</b>
14	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4	1,2,3,4
15									
16	1	1	1	1	1	1	1	1	1
17 to 23	2	2	2	2	2	2	2	2	2
24	3	3	3	3	3	3	3	3	3
25 to 34	4	4	4	4	4	4	4	4	4

To load the solver that has been created in the previous step do as follows. Select Tools, and then Add-Ins. The Add-Ins dialog box will appear. Check “Solver Add-in” then validate it. When you validate you will see ✓ on the left-hand side of “Solver Add-in”. Click OK. At the bottom of the Add-Ins dialog box you will see the statement “tool for optimization and equation solving” appears. Now, we need to clarify some important issues regarding the efficient frontier computation. The solver function is a very inefficient tool, yet it is free. In this section we will show you how to “tame” the solver so that a global solution can be found during the each optimization process. The process is sequential and needs to be done very carefully in the following order.

Figure 8: Finding the Minimum risk Portfolio

Figure 9: Moving Along the Efficient Set

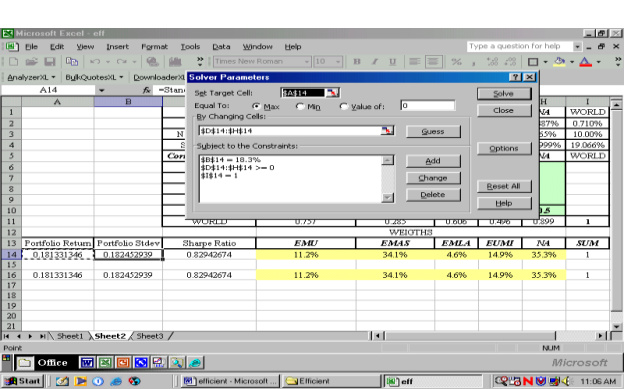
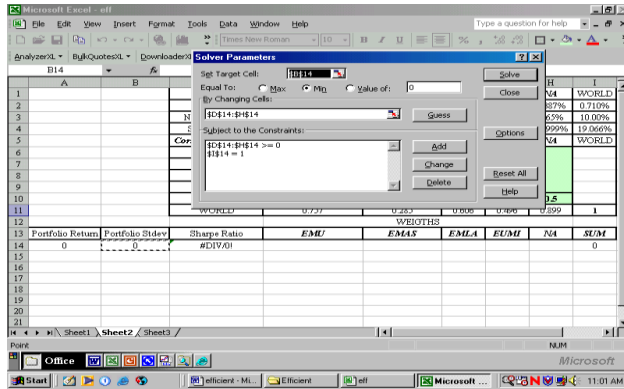


Figure 10: Seek for the Optimal Sharpe Ratio

Figure 11: Computing the Optimal Sharpe Ratio

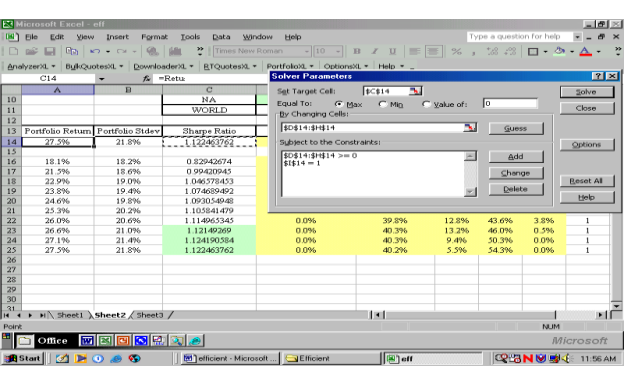
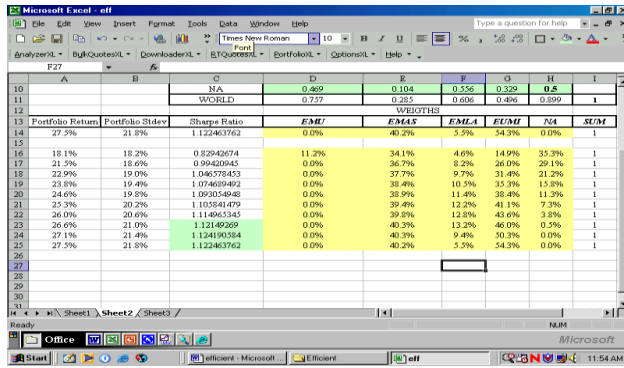
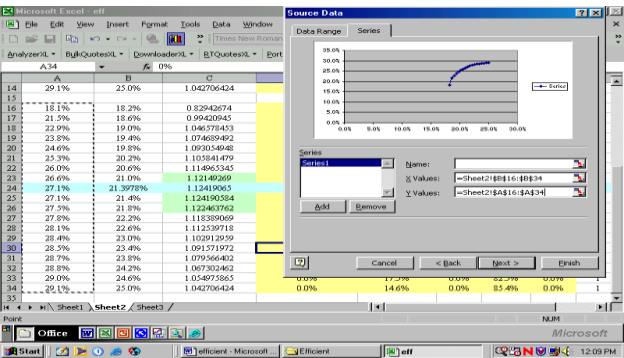
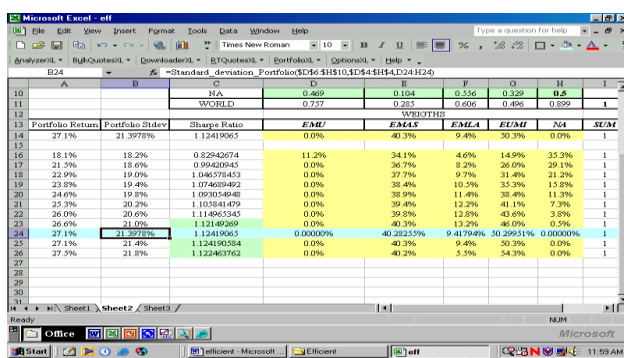


Figure 12: Inserting the Optimal Sharpe ratio Line

Figure 13: Charting the Efficient Set using a Scatter



(1) Seek For The Minimum Standard Deviation Portfolio And Inherent Weights - Done By Minimizing The Portfolio Standard Deviation

To calculate the minimum standard deviation portfolio composition, select Tools, then Solver. The “Solver Parameters” dialog box will appear. For the Set Target Cell pick the cell on the *calculation line* where the portfolio returns standard deviation will be placed. In our example that cell is B14 (see Figure 8). In “Equal to:” choose to

minimize by checking Min. For “By Changing Cells:” choose the cells range for the weights by typing  $\$D\$14:\$H\$14$ , as shown in Figure 8. In “Subject to the Constraints:” you insert the proper constraints for our calculation. First, choose Add to add the first constraint, which is the Sum of Weights should be 100%. In our example  $\$I\$14$  is set equal to 1, as indicated in Figure 8, because that is the previously selected cell reference for the sum value. Because short sales are not allowed at this point, the second constraint to be included must be that each weight is greater or equal to zero. Choose Add. The “Add Constraint” dialog box will appear. For Cell Reference choose the cells range where those weights are to be placed. In our example the range is  $\$D\$14:\$H\$14$ , as shown in Figure 8. For the middle box, where you see initially  $\leq$ , scroll down to select  $\geq$  instead. In “Constraint:” type the number 0. Click OK. The “Solver Results” dialog box will appear. Choose KeeP Solver Solution to validate your choice. In our example, the *calculation line* has values of 0.18133, 0.18245, 0.82943, respectively, for the Portfolio Returns, Standard Deviation, and Sharpe Ratio. The relative weights are also displayed in cells D4-H4. Next, you should copy the entire *calculation line* values (Row 14 in our example) and paste them a few rows below (row 16, in our example) as indicated in Figure 9. DO NOT delete the values you just moved from row 14, otherwise you will also delete the formulas in the target cell.

### (2) Seek For New Values Along The Efficient Set

We move along the efficient frontier by maximizing the return of the portfolio for a given standard deviation. We will choose a standard deviation that is slightly above the minimum standard deviation previously found. Indeed, the process uses the weights found in the first sub-step above as seeds to get the next optimum. In our example, we increase the amount of standard deviation by increments of 0.4%. If you use more assets, we advise you to choose the next standard deviation using increments of 0.1%. This is what we call “taming the solver.”<sup>9</sup>

Open the solver again. Select Tools, then Solver. The “Solver Parameters” dialog box will appear. In “Set Target Cell:” you should introduce now the cell reference for your portfolio return. In our example, it is  $\$A\$14$  as shown in Figure 9. Choose now Max in “Equal To:” and for “By Changing Cells” no entry is made, if you still have the same cells range reference we used as we selected “Min” in the previously sub-step; otherwise, you should make that entry for your cells range reference. As shown in Figure 9, you need to enter an additional constraint to those two previously considered. This new constraint is associated with the new value and cell reference of the standard deviation, which is in cell B14 for our case. Thus, proceed as follows.

Choose Add. The “Add Constraint” dialog box will appear. For Cell Reference we choose cell  $\$B\$14$ . For the middle box, where you see initially  $\leq$ , scroll down to select  $=$ . In “Constraint:” type a number for the new standard deviation you want to use to maximize the portfolio return. In the Solver Parameters dialog box, as shown in Figure 14, this constraint is set as follows:  $\$B\$14=18.3\%$ . We suggest you to choose a standard deviation that is slightly above the standard deviation you just found. In our example, we are increasing from 18.2 % to 18.3%, hence, by an increment of 0.1%. Also notice that we use higher increment values of 0.4% after that first computation. If you use a relative larger number of assets than we used, we advise you to select the next standard deviation using increments of 0.1%. Each time you add a new value for the standard deviation to the “Subject to the Constraints:” box you should validate it by clicking OK, then choose Solve, KeeP Solver Solution and then OK. Each time the solver place a new solution on the *calculation line* by following this procedure, you should copy it to a row below the calculation line (row 14). Keep repeating this procedure, as long the Sharpe Ratio is increasing.

As indicated in Figure 9, notice that two of the other three constraints in the “Subject to the Constraints:” box,  $\$D\$14:\$H\$14 \geq 0$  and  $\$I\$14 = 1$  had been added in the previous sub-step. The former includes the cells range where those weights are to be placed. The latter is for the sum of the weights. In our example is  $\$I\$14=1$ . Rows 16 through 25 of Figure 10 show the results of this sub-step.

### (3) Compute The Optimal Sharpe Ratio

As you are using the Solver and incrementally increasing the standard deviation as described above, new portfolio weights and returns results are determined. You will notice that the Sharpe ratio keeps increasing, then starts to taper off and finally decreases. At this point (when it starts decreasing), you are ready to find the optimal



Sharpe ratio. Understand that the “optimal portfolio” will have the best reward to risk—i.e., the highest Sharpe ratio. In our example (in the Sharpe ratio column), the best (highest) Sharpe ratio is included between row 23 and row 25.

To find the optimal Sharpe ratio, open the Solver (Select Tools, then Solver). Then as previously explained, chose  $C\$14$  (the Sharpe ratio cell for our example) for “Set Target Cell:”. Choose to maximize by checking “MAX” for “Equal To:” and for “By Changing Cells:” the weights range ( $D\$14: H\$14$ , in our example). In “Subject to the Constraints;” add the following two constraints: (1)  $I\$14 = 1$  for the Sum of Weights and, because short sales are not allowed, (2)  $D\$14: H\$14 \geq 0$ . Notice that no other constraint at this point is included, as shown in Figure 11. Validate your choice by checking on Solve and choose to Kee Solver Solution. As a result, the maximum Sharpe ratio value ( $=1.12419065$  in our example with a relative standard deviation of 21.3978) is found, as shown in Row 14 of Figure 12. Copy and paste these results onto the selected row (between rows 23 and 24 in our example).

(4) Moving Along The Efficient Set And Beyond The Optimal Sharpe Ratio

For this, repeat this step sub-step 2 (*Seek For New Values Along The Efficient Set*) as many times as you wish by increasing each time the previous standard deviation value you used in small increments (0.4% in our example). In order to do this you will need to add back the constraint for the new incrementally increased standard deviation value, which should start from the standard deviation of the optimal Sharp-ratio. The Target Cell will also be changed from  $C\$14$  to  $A\$14$  (Portfolio Return Cell). Hence, go to Tools, choose Solver, then Add in “Subject to the Constraints:”, and when “Cell Reference” dialog box appears introduce the new constraint to be added for the new incrementally increased standard deviation value ( $B\$14 = 21.4%$  in our case). Change also the Target Cell to  $A\$14$ , before you validate your two new choices. Keep moving along the efficient set by repeating the process with new incrementally values for the standard deviation constraint. In our example, we chose 21.8% for the next standard deviation value.

The question often asked by students is “when do we stop?” This is the moment to remind them that the portfolio return is always in between the lowest return and the highest return in the asset classes; so, the optimization process should stop for the standard deviation of the asset class with the highest return.

(5) Graph Your Findings

Select Insert, then Chart, and “Scatter” for Chart type. In the Chart Wizard dialog box, choose Next at the bottom, then “Series”. In Series, select Add. Input the portfolio standard deviations in the “X values” ( $B16: B34$  in our example) and portfolio returns in the “Y series” ( $A16: A34$  in our example), click Next twice and in “Place Chart” choose “As new sheet:” or “As object in:” as you wish. Now you have an efficient frontier and an optimal portfolio, for our example) that suggests an allocation of 40.3% in EMAS, 9.4% in EMLA and 50.3% in EUMI. The process is shown in figure 13.

**Table 4: Desired Output from Step5 ( the numbers indicate the sub-steps described in step 5)**

	A	B	C	D	E	F	G	H
36	<b>Targeted Return</b>	<b>Targeted Risk</b>	<b>Cash Weight</b>	<b>EMU</b>	<b>EMAS</b>	<b>EMLA</b>	<b>EUMI</b>	<b>NA</b>
37 to 46	3	0% to 27% (by increments of 3%)	1	2	2	2	2	2

**Step 5: Build the Capital Market Line (CML)**

This step will allow you to combine the optimal portfolio – the one with the optimal Sharpe’s Ratio we just found in the previous step - with a risk-free asset (herein it is assumed to have a 3% return and 0% standard deviation). Several targeted risk levels are also considered to show how different portfolios that can be chosen along

the CML, depending on the investor risk tolerance. At this point, it is always a good time to remind students how personal risk preferences are expressed in terms of utility functions, since risk level tolerances may differ among individuals. Hence, each individual should choose portfolios reflecting his/her risk tolerance, which is recommended to be defined as part of the student’s investment plan policy statement. By the end of this step, you should have a worksheet that resembles Table 3—inputs for this step are highlighted, and the numbers indicate the sub-steps described in this section.

Figure 14: Portfolio Return and Targeted Risk

Figure 15: Charting the CML with the Efficient Set

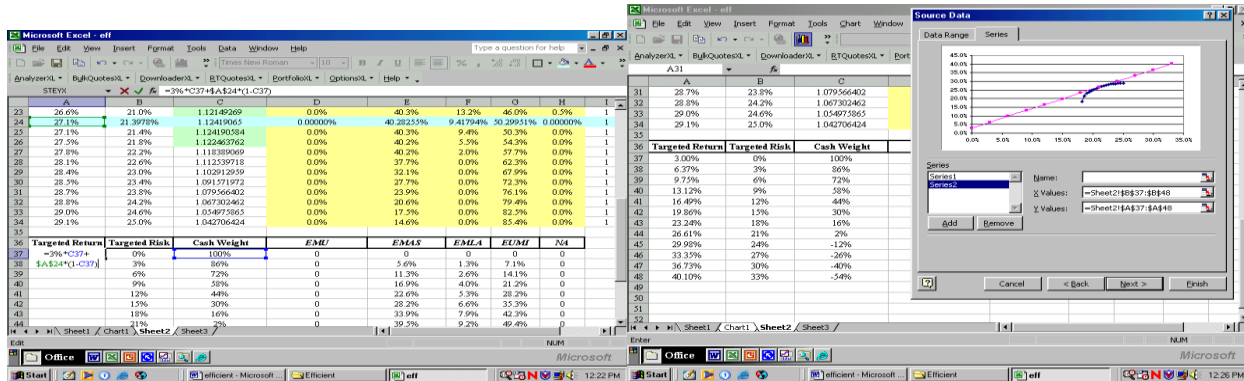
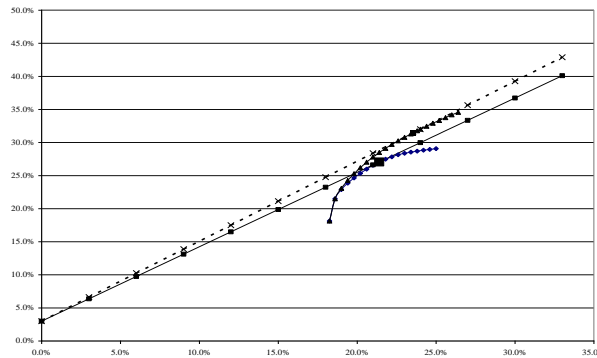


Figure 16: Efficient Set and Inherent CML With And Without Short Sales Restrictions



Hence, starting on row 37, the titles Targeted Returns, Targeted Risk, and the Cash Weights – proportion to be invested in the risk-free asset- are placed on columns A, B, and C, respectively. The proportions to be invested in the risky assets, EMU, EMASA, EMLA, EUMI, and NA, will be calculated in this step and placed on columns D, E, F, G, and H.

(1) Calculate The Weight In T-bills For Each Targeted Risk

Select a set of targeted risks (0% to 33% in our example) and calculate the T-bill weights for each targeted standard deviations using the following formula.<sup>10</sup> As shown, the optimal portfolio standard deviation (21.3987%- cell B24- in our example) is used in the calculation.

$$\omega_{Rf} = \left(1 - \frac{\sigma_p}{\sigma_m}\right) \quad (2),$$

where  $\sigma_p$  is the targeted risk;  $\sigma_m$  is the risk of the optimal (Sharpe) portfolio from the efficient frontier and  $\omega_{Rf}$  is the proportion of money to be put in T-Bills in order to get  $\sigma_p$ .

In excel format, i.e.,  $= 1 - B37/\$B\$24$ , formula (2) is placed in cell C37. Once this done, copy and paste the formula (in our case) in cells C38 through C48, to extend the computations to other targeted risk levels.

(2) Calculate The Relative Weights For Each Asset Class In M For Each Targeted Risk

The formula  $(1 - w_{Rf}) \times w_{series}$  is used in these calculations. For our example, the values for  $w_{series}$  are 0.00%, 40.28%, 9.42%, 50.29%, and 0.00%, respectively, for EMU, EMAS, EMLA, EUMI, and NA, the risky assets in M. Thus, in excel format, i.e.,  $(1 - C37) * D\$24$ , formula (3) is introduced in cell D37. Also, in our case, this formula will be copied and pasted in the cell ranges D38-D47, E37-E47, F37-F47, G37-G47, and H37-H47, respectively, for EMU, EMAS, EMLA, EUMI, and NA, which are the risky assets in M. Once this done, you will have the computation resulting figures in those cells, as shown in Figure 14 lower portion.

(3) Calculate The Portfolio Return For Given Targeted Risk Level

The return of the portfolio is equal to the weight in the risk free asset,  $w_{Rf}$ , times the return of the risk free asset,  $Rf$ , that is  $w_{Rf} \times Rf$ , where  $Rf$  is 3%, plus  $(1 - w_{Rf}) \times Rm$ , where  $Rm$  is the return on the previously-step-determined-optimal portfolio. In our case,  $Rm$  was found to be 27.1%. Thus, as shown in Figure 14, the formula in excel format, i.e.,  $= C37 * 3\% + (1 - C37) * \$A\$24$ , is inserted in cell A37.

Next, copy and paste the formula also in cells A38-A48 to get the results of Figure 15. At this point, students are ready to graph their results following similar process as in sub-step 5 of step 4. Figure 15 results show the graph if short-sales are not allowed.

### CONCLUDING REMARKS

We have been successfully using this technique in the classroom for almost five years. Our students have access to this “recipe” and, with the help of graduate assistants; it takes usually around one and half hour to go through the process in a computer lab with less than 25 students. In this paper, we show the efficient frontier in the context of global block allocation strategies. We utilize several variants of the process using stocks, sector indices and even mutual funds.

The process described is restricted to no short sales. However, it is quite easy to relax this assumption. Indeed, assume that no more than 20 percent of the amount allocable in each asset can be shorted. In this case, we start over the optimization process in step 4 and, in the “subject to constraint” box of the solver, and then we change the positive weight constraint  $\$D\$14:\$H\$14 \geq 0$  into a negative weight constraint, such as  $\$D\$14:\$H\$14 \geq -20\%$ .<sup>11</sup> We illustrate our findings in figure 16. Notice the optimal Sharpe ratio is now much higher and that the efficient envelope is shifting up indicating more potential return for the same level of risk tolerance. This is what text books mention, and our students can understand and actually do it.

### ENDNOTES

<sup>1</sup> An excellent textbook on financial modeling is Simon Benninga’s (2000). However, our step-by-step approach is easier to follow and considerably decreases the execution time.

<sup>2</sup> In our class, we use mostly two sources of free historical data—i.e., Yahoo (<http://chart.yahoo.com/d>) and MSCI ([http://www.ms\\_cidata.com/mstool/mschart.wsx/cty](http://www.ms_cidata.com/mstool/mschart.wsx/cty)). In Yahoo, students can select a quote for an index, a mutual

fund or stock by using the ticker symbol (the symbol of a particular series can be found in “Symbol Lookup”). After selecting a date range and frequency, series are copied and pasted into a new Excel worksheet. Then the data needs to be cleaned up, so that each series’ date matches. MSCI series are already clean.

<sup>3</sup> For weekly data: “=Average (range)\*52”; for monthly data: “=Average (range)\*12”.

<sup>4</sup> At this point, we assume that students have not yet been introduced to asset pricing models. An alternative to the naïve forecast could be to use historical return or a “guesstimate” of what the long run returns in each block could be. Indeed, it is important to emphasize that expected returns should be used.

<sup>5</sup> For weekly data: “=STDEV(range)\* (52)^0.5”; for Monthly data: “=STDEV(range)\*(12)^0.5”

<sup>6</sup> You can go to [http://misnt.indstate.edu/egirard/fin434\\_534/fin434\\_534.html](http://misnt.indstate.edu/egirard/fin434_534/fin434_534.html) to download the add-in and the spreadsheet used here.

<sup>7</sup> This is to be done as a result of the VBA program for the standard deviation of a portfolio—i.e. we need a full correlation matrix, so we added to the current matrix its transpose. If we don’t change the trace to 0.5, the resulting trace would be 2.

<sup>8</sup> The purpose of the world index is to compute the naïve forecast; it does not need to be included in the construction of the efficient frontier.

<sup>9</sup> It is very important to proceed this way as the optimization process will lead to a global (Vs. local) solution. In essence we are using the weights from the previous optimization as seed values for the next optimization.

<sup>10</sup> At this point, students can be reminded that a portfolio that combine the optimal portfolio M and the risk free asset has a standard deviation of  $\sigma_p = (1 - \omega_{Rf}) \sigma_m$  and a return of  $R_p = Rf + (Rm - Rf) / \sigma_m \times \sigma_p$ .

<sup>11</sup> One can use an infinity of variants to the short sales restriction by adding, changing or deleting constraints in the “subject to the constraints” box of the solver: (1) If the weight constraint is deleted (delete “D14:H14>=0”), then unlimited short sales are allowed; (2) If you have only access to index futures in North American Markets, short sales can be restricted in the other blocks (“E14:H14>=0”); (3) if you have different short sales restriction in each block (say a maximum of -30% in NA, -20% in EMU, -10% in EMLA and EMAS, and 0% in EUMI), then add as many constraints in the “subject to constraints” box (“D14>=-20%”, “E14:F14>=-10%”, “G14>=0”, and “H14>=-30%”).

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