CORE

# Using Student-Centered Projects To Teach Mathematics Content Standards To Middle School Teachers 

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#### Abstract

Teaching and learning standards across the country are becoming more content driven, especially in mathematics. As such, it is essential to develop substantive college level math courses for pre-service and in-service teachers. These courses should deliver mathematical content in a relevant way. Teachers must not only be proficient in their subject, but also recognize how to apply content meaningfully to student's lives. As part of a grant funded through the Department of Education in coordination with the Clark County School District, I have designed two college level courses that will deliver mathematical content standards to middle school teachers. These courses will help prepare teachers at the middle school level by focusing on mathematical content, making connections within the curriculum, using technology, and enhancing the role of mathematics in everyday life and problem solving.


The primary purpose of this paper is to share with the academic community the projects used to develop content understanding in mathematics topics such as: mathematical notation, proportion, computation, estimation, linear regression, reflections, translations, finding relationships, and analyzing data. This paper will present these projects, and tie them to mathematics standards outlined in the Clark County School District (and Nevada State) Standards.

## A. ESTABLISHING THE NEED: THE "CONNECTIONS" COURSES ARE BORNE

$\mathscr{I}$n 2003, Clark County Nevada had a population of over 1.5 million people, and was growing at a rate of 4,000 to 5,000 new people per month. Clark County School District (CCSD) is currently the fastest growing school district in the nation, and is the fifth largest. CCSD hired over 1,300 new teachers for the 2005-2006 school year, and anticipates a need for at least the same number next year. There are over 280 schools in the district, with more than ten new schools per year, and over 12,000 new students annually. It is projected the district will need over 88 new schools over the next ten years. Middle and high schools have a student to teacher ratio of thirty one to one [1].

The teaching ability of seventh grade mathematics instructors depends partly on the quality of their preservice learning [2]. One customary measure of the quality of pre-service learning is post-secondary education, in particular, whether the teacher has a college major in their primary subject area [3, 4]. Moreover, teachers are considered highly qualified when holding a masters degree in their primary teaching field. The percentage of CCSD seventh grade teachers holding a major degree in their teaching field is well below the national average. In fact, only four percent of math teachers in seventh grade classrooms hold a major in their specified field [5]; this is a full thirty one percent lower than the national average. Additionally, only three percent of CCSD teachers have a Masters degree in Mathematics [5].

Continued in-service teacher learning is also an important component in determining teacher quality [6]. The availability and content of professional development courses are crucial in helping in-service teachers perform at higher levels of excellence. Additionally, research has shown that the more hours teachers spend in professional
development activities, the more they feel their teaching improved in key areas, including new methods of teaching, state or district performance standards, integration of new technology, and addressing the need of special student groups [6].

The evaluation of CCSD professional development courses by grade groups [5] indicates a significant lack of math content and pedagogy courses designed specifically for middle school teachers. In particular:

- The majority of seventh grade mathematics teachers had less then 100 hours of mathematics specific professional development.
- $63 \%$ of teachers had less than 100 hours of professional development in mathematics teaching pedagogy, and $86 \%$ had less than 100 hours in mathematics content knowledge.
- Only $64 \%$ of teachers felt very confident when teaching seventh grade geometry and measurement standards to their seventh grade students.
- Only $25 \%$ of seventh grade mathematics teachers felt confident when teaching eighth and twelfth grade geometry and measurement standards to their seventh grade students.

To add further complexity to the problems above, CCSD has developed the Algebra Eight course curriculum initiative. A critical component of this initiative is that all eligible students will be enrolled in Algebra by the eighth grade. The initiative focuses on algebra, number sense, and data analysis standards, while integrating a few geometry and measurement standards. The heavy focus on Algebra has necessitated the teaching of many eighth grade standards in seventh grade. As a result, seventh grade mathematics instructors are required to teach additional, new, and unfamiliar standards. These new requirements call for a program that offers seventh grade mathematics instructor's support through pre-service learning and in-service development.

To provide CCSD seventh grade teachers with additional content knowledge and pedagogy that they may not have received during their pre-service training, the Seventh Grade Grant Program [5] creates a seventh grade mathematics cohort learning community. This grant has been a collaborative effort between faculty at the University of Nevada Las Vegas (UNLV), the CCSD Mathematics Coordinator and the Grant Project Facilitator. Cohort participants are provided with a sixteen hour summer mathematics workshop, three eight hour professional development seminars, and financial support for nine semester hours of identified university pedagogical content course work. Thus, the university courses, connected to the CCSD school mathematics standards for grades five through eight, were borne - Mathematical Connections I and II.

## B. OVERVIEW OF COURSE

To be successful in today's world, one needs a comprehensive understanding of mathematics. Society needs individuals who have sound estimation skills and number and spatial sense, who are competent using and interpreting data, and who can use appropriate technology resources to solve problems and make informed decisions [7]. These skills are essential to becoming successful citizens, life-long learners, and competitive workers in a global market place [7].

Within the context of mathematics in every day life, students (and teachers) should continuously develop their ability to solve problems mathematically. Specifically, they should:

- Formulate and solve their own problems, develop and apply strategies to solve a wide variety of problems, and integrate mathematical reasoning, communication and connections (Nevada State Standard 6.0).
- Investigate significant mathematical ideas and construct their own learning in all content areas in order to justify their thinking; reinforce and extend their logical reasoning abilities; reflect on and clarify their own thinking; and ask questions to extend their thinking (Nevada State Standard 8.0).
- View mathematics as an integrated whole, identifying relationships between context strands, and integrate mathematics with other disciplines, allowing the flexibility to approach problems in a variety of ways within and beyond the field of mathematics (Nevada State Standard 9.0).

This course has been developed specifically with the premise that all individuals must have an awareness of how mathematics fits into every day life. The content topics have arisen within the framework of the CCSD Power Standards and the Nevada State Standards. Three out of the four topics below have originated from the CCSD Power Standards for School Mathematics (grades five through eight). The fourth topic, communication, has emerged specifically to address the need for further understanding of the rules of mathematics, which are often abused.

## 1. Communication

Mathematicians realize the importance of clear definitions, concise notation and following proper procedures. Rigor is characterized by the desire to be as clear as possible in things that are said and the things that are written. To accomplish rigor, mathematicians have extended natural language with precisely defined vocabulary and grammar for referring to mathematical objects, and stating certain common relations with accompanying notation [8].

As a result, anyone teaching or doing mathematics should have a fundamental understanding of its language. Perhaps, as with all languages, one will have to look up symbols and definitions from time to time, but appreciation for the simplicity and beauty of the notation and rigor should always be present. Further, all too often students (and teachers) misuse this notation, and it causes confusion for those who are attempting to follow their studies or instruction.

This topic will investigate mathematical notation, and assist in its identification, interpretation and use. Here, teachers will develop their ability to communicate mathematically, and translate between real world information and mathematical language. They will also be able to identify and correct common misuses in the language of mathematics encountered in the classroom.

## 2. Numbers, Number Sense, And Computation

Number sense refers to a pragmatic approach to using numbers. It does not necessarily imply that one has the ability to quickly compute, or use the computation facts in a particular algorithm. Rather, it implies adaptability in computation or estimation, and a desire to determine the reasonableness of a solution. Number sense has been operationally defined in such terms as the ability to use numbers flexibly when computing, estimating, judging number magnitude, or judging the reasonableness of results; the ability to move easily between different number representations; and the ability to relate numbers, symbols and operations [10]. Number sense is characterized by a desire to provide meaning of numerical situations, and is a way of thinking that must permeate all aspects of teaching and learning if mathematics is to make sense [9].

This topic will facilitate the assimilation of number sense and order of operations throughout the mathematics curriculum. Teachers will "accurately calculate and use estimation techniques, number relationships, operation rules, and algorithms; they will determine the reasonableness of answers and the accuracy of solutions" (Nevada State Standard 1).

## 3. Patterns, Functions, And Algebra

There is little question that algebra is a crucial foundation for all who aspire to have a mathematically based career. But why is it important for others? To begin, a lack of algebraic skills puts a glass ceiling over ones' head, limiting career options. Algebra is a means to empower individuals and provide them with goals, skills and opportunities [11]. The study of Algebra has been described as the fork in the road, a decision point where one direction leads to opportunities and the other direction leads to limited options for education and career choices [12]. Furthermore, over $75 \%$ of all jobs require proficiency in fundamental algebraic concepts, either as a prerequisite for advanced training, or as part of a licensure program [13].

This topic will help teachers "analyze, illustrate, extend, and create numerous representations (words, numbers, tables, and graphs) of patterns, functions, and algebraic relations as modeled in practical situations" (Nevada State Standard 2).

## 4. Measurement And Data Analysis

Each and every day the public is constantly bombarded by data and data analysis. One cannot go far without running into sports statistics, the high and low temperature of the day, political polls, or advertisement comparisons; data analysis is on the news, billboards, radio and internet. Not only is it crucial to understand data and how to draw conclusions, but it is of paramount importance to realize that many of the statistics run across every day are presented in a way to sway the public one direction or another.

This topic will facilitate data analysis comprehension. In particular, teachers will learn how to read data critically and with discernment, collect data that provide unbiased answers to important questions, draw trustworthy conclusions, and decipher misleading data analysis methods.

## C. OVERVIEW OF PROJECTS

Each of the projects outlined below are presented in full online [14] and solutions can be provided upon request. They have been developed within the context of the above topics, and address many of the state and district standards for school mathematics in grades five through eight (See Section D).

## 1. Mathematical Notation And All It Implies

The purpose of this project is to understand how to read mathematical notation, interpret symbols, convert math language into English and vice versa, and appreciate the simplicity and beauty of mathematical notation.

## Project 1

Part A: The formal definition of the absolute value function is given by $|x|=\left\{\begin{array}{cc}x & x \geq 0 \\ -x & x<0\end{array}\right.$.

1. Write the above definition in words (no math symbols or numbers)
2. Using the definition, show $|x-a|= \begin{cases}x-a & x \geq a \\ a-x & x<a\end{cases}$
3. Explain (in words) what the statement $|x-a|>0 \Leftrightarrow x \neq a$ means

Part B: The formal definition of a limit is given by
$\lim _{x \rightarrow a} f(x)=L$ if $\forall \varepsilon>0 \quad \exists \delta>0$ such that. $0<|x-a|<\delta \Rightarrow|f(x)-L|<\varepsilon$

1. Write the above definition in words (no math symbols or numbers)
2. The informal definition of a limit is as follows: The limit of $f(x)$ as $x$ approaches $a$ equals $L$ if we can make the values of $f(x)$ arbitrarily close to $L$ by taking $x$ to be sufficiently close to $a$ on either side (but not equivalent to $a$ ). Match the formal to the informal...

- what part of the formal definition indicates " $f(x)$ arbitrarily close to $L$ "
- what part of the formal definition indicates " $x$ sufficiently close to $a$ on either side"
- how does the formal definition insure that $x$ is not equivalent to $a$ ?

Part C: A sequence of numbers is given by $a_{1}=1000, a_{n}=1.03\left(a_{n-1}\right)+100$

1. To find $a_{2}$ we use the formula $a_{2}=1.03\left(a_{2-1}\right)+100=1.03\left(a_{1}\right)+100=1.03(1,000)+100=1,130$. Find $a_{3}, a_{4}, a_{5} \ldots$ up to $a_{10}$.
2. What is the starting point for this sequence? Will it continue to get larger as $n$ gets larger?
3. Think of this in terms of an investment, where $a_{n}$ represents the amount of money in an investment at year $n$. What is the initial investment? What does the " $1.03\left(a_{n-1}\right)$ " represent? How about the " +100 "?

Part D: There are many mathematical symbols we have not covered. Provide 3 examples. Include what they mean and in what context they are usually used.

Part E: Provide an example of a mathematical statement, expression or formula that you have encountered and interpret it.

There are five parts to this project: In parts A, B and C, teachers are given a formal definition and are asked to interpret it and understand what it implies. In parts D and E they will show a few symbols not covered in class, and explain their meaning.

## 2. What Did You Say? Some Common Misuses Of Mathematical Notation

The purpose of this project is to identify common mistakes made when applying mathematical steps or replying to mathematical questions. The goal is to make teachers aware of such practices, so they can each set a great example for being mathematically concise.

There are seven parts to this project: Part A will help teachers understand the significance of similar notation having two distinct meanings depending on context. Parts B through F will help them understand some common mistakes made on student papers, why such mistakes are a problem and how to correct these mistakes. Part G will be an avenue to share other experiences with faulty notation.

## Project 2

Part A: Ordered pair $(a, b)$ vs. Interval $(a, b)$

1. Define what is meant by the ordered pair $(a, b)$
2. Define what is meant by the interval $(a, b)$
3. How can one tell the difference between the ordered pair and the interval?

Part B: The following question and two different student responses are given on a test... Evaluate $f(x)=2 x+7$ for $x=1$

1. $f(x)=9$
2. $f=9$
3. What is the problem with answer (1)? How would you correct this student response?
4. What is the problem with answer (2)? How would you correct this student response?

Part C: The following question and response is given on a test...

$$
\text { Compute } \begin{aligned}
& \frac{1}{2} \div\left(\frac{2}{3}+\frac{4}{5}\right) \\
&=\frac{10}{15}+\frac{12}{15}=\frac{22}{15} \\
&=\frac{1}{2} \times \frac{15}{22}=\frac{15}{44}
\end{aligned}
$$

1. The final answer given is correct, but what is wrong in the derivation of the solution?
2. How would you correct this student response to facilitate learning?

Part D: The following question and response is given on a test...
Find the exact solution of $\sin \left(45^{\circ}\right)$

$$
\sin =\frac{0 p p}{\text { hyp }}=\frac{1}{\sqrt{2}}
$$

1. What is the problem with this solution (even though it is indeed correct)?
2. How would you advise this student?
3. The solution given is not rationalized; does this mean it is incorrect?
4. What is the intention behind (or use for) rationalizing the denominator?
(Project 2, continued)
Part E: The following question and response are given on a test...
You see a stereo system you want to buy for $\$ 299.99$. The store says it is offering the system for
$15 \%$ off. You also can open a credit card account with the store for an additional $10 \%$ off (this
discount comes after the $15 \%$ is taken off). How much will you save just for opening the credit account?

$$
\begin{aligned}
& 299.99 \times 15=44.99 \\
& =255 \\
& 249.94 \times 10=29.99 \\
& 774.48
\end{aligned}
$$

1. There are multiple problems with this solution, including the fact that it is incorrect. What should the answer be?
2. Identify at least three "mistakes" this student has made.

Part F: The following question with two different student responses are given on a test...

$$
\begin{aligned}
& \text { 1. } 7 \frac{7}{4}+\frac{7}{7}{ }^{12}=\frac{19}{28} \\
& \text { 2. } 7\left(\frac{1}{4}\right)+\left(\frac{3}{7}\right) 4=\frac{7}{28}+\frac{13}{28}=\frac{19}{28}
\end{aligned}
$$

1. Although the solution is correct, what is wrong with the students' method in (1)? How would you correct this student?
2. Repeat this for response (2).

Part G: Identify a math-misconception in your own "world". This can be a mistake you commonly see on a test, something from the news, or product packaging.

## 3. Changing The Recipe: An Exercise In Proportion And Estimation

The purpose of this project is to facilitate teachers understanding of how to convert between measuring systems, and successfully use the concepts of ratio and proportion. They will also see how to mathematically evaluate a real-life situation.

This project has two parts: Part A involves using proportion to increase the serving size of a cookie recipe, and requires teachers to convert between Metric and English systems. Part B is an exercise in determining the most cost effective way to tile an area in their home, which involves estimation and problem solving.

## Project 3

Part A:

1. You found the best recipe for chocolate chip cookies below, which makes 24 servings. Absolutely the Best Chocolate Chip Cookies
```
INGREDIENTS:
1 cup butter flavored shortening
3/4 cup white sugar
3/4 cup brown sugar
2 eggs
2 teaspoons Mexican vanilla extract
21/4 cups all-purpose flour
1 teaspoon baking soda
1 teaspoon salt
2 cups milk chocolate chips
```

DIRECTIONS:

1. Preheat oven to 350 degrees $F$ ( 175 degrees C). Grease cookie sheets.
2. In a large bowl, cream together the butter flavored shortening, brown sugar and white sugar until light and fluffy. Add the eggs one
at a time, beating well with each addition, then stir in the vanilla .Combine the flour, baking soda and salt; gradually stir into the
creamed mixture. Finally, fold in the chocolate chips. Drop by rounded spoonfuls onto the prepared cookie sheets,
3. Bake for 8 to 10 minutes in the preheated oven, until light brown. Allow cookies to cool on baking sheet for 5 minutes before
removing to a wire rack to cool completely
4. You decide you want more! 36 servings. Convert the recipe accordingly.
5. You find yourself living in Europe, where they use different measurement systems and different ways of measuring. Cooks in the United States tend to measure all ingredients by volume, while it is common in Europe to measure dry ingredients by weight (e.g. ounces, pounds, grams, kilograms) and liquid ingredients by volume (e.g. tablespoons, cups, milliliters, liters).

Convert the recipe accordingly. This is not as easy as it looks, as a cup of brown sugar weighs more than a cup of powdered sugar. Help with converting these can be found at http://allrecipes.com/advice/ref/conv/conversions.asp

## Part B:

1. You want to tile your downstairs and are deciding between two different tile types. The schematic of your area is given below (you do not need to tile the gray area).

2. Type 1 is an $18 \times 18$ inch tile. The tiles come in a box of 15 which costs $\$ 20$.
3. Type 2 is a $12 \times 12$ inch tile. The tiles come in a box of 20 which costs $\$ 14$.
4. Both tiles cost $\$ 8$ per square foot to have installed (this includes the cost of grout and mortar). For each type of tile, find:

- Total square feet of area to be tiled
- Square footage covered by one box of tile
- Total number of boxes of tile needed
- Number of tiles left over
- Total price of tile
- Total price of job, including labor and $8 \%$ sales tax (you are taxed on labor and materials)


## 4. Why Did We Get Different Answers? A Study In Order Of Operations

The purpose of this project is to help teachers employ the proper order of operations, the 'ambiguity' that can sometimes arise when two operations have the same precedence, and common mistakes made in student responses.

In this project, several student responses are given, some incorrect. Teachers will investigate how students misuse the order of operations to arrive at incorrect solutions, and how to correct these responses to facilitate learning. They will also investigate how two students can use "different methods" and arrive at the same result.

## Project 4

Part A: The following problem was given on a test; there are 3 different student responses.

$$
\begin{aligned}
& \text { Compute } 12+5^{2}-2 \cdot(21+4) \\
& \text { 1. }=239 \\
& \text { 2. }=-11 \\
& \text { 3. }=12+25-2(25)=12
\end{aligned}
$$

1. For student responses (1) and (2) determine what error was made in order of operations, and specify how you would facilitate student understanding.
2. For student response (3), how would you explain that the 25 s do not cancel?

Part B: The following response was given on an algebra test...

$$
\frac{(x+y)-(2 x+5)}{3 x^{2}+(x+y)}=\frac{3 x+5}{3 x^{2}}=\frac{5}{3 x}
$$

1. Find the faults (there are a few)
2. Show how you would facilitate student understanding.

Part C: Show why the following student responses both lead to correct solutions, although they employ two "different" methods or orders.

$$
45-\left(2^{3}-4\right)
$$

$$
\text { 1. } 45-8+4=45-4=41
$$

$$
\text { 2. } 45-(8-4)=45-4=41
$$

## 5. Calculating Grades: An Exercise In Estimating And Projecting

The purpose of this project is to thoroughly understand the computation of a grade in the percentage system. Teachers will understand how to find the final grade, and answer common questions students have about their grade along the way, such as "what do I have to get on the final to get an A?" or "can I still get a B even though I failed the test?"

There are four parts to this project: In Part A teachers will calculate a students' final grade given all grades earned during the term. For Parts B and D they will answer some common student questions. For Part C they will determine how much the overall cumulative average is affected when dropping a particular item.

Project 5
For each of the questions we will be using the same grading scheme (below). All grades are provided, but some of the questions will assume only a portion of the grades are "in".

|  | Percentage Worth of Cumulative Grade | \% Grade Earned |
| :---: | :---: | :---: |
| Participation | 10\% | 96 |
| Project 1 | ) | 83 |
| Project 2 |  | 87 |
| Project 3 |  | 95 |
| Project 4 |  | 91 |
| Project 5 | Worth 40\% total | 86 |
| Project 6 |  | 81 |
| Project 7 |  | 94 |
| Project 8 |  | 95 |
| Project 9 | ) | 91 |
| Quiz 1 |  | 76 |
| Quiz 2 | Worth 40\% total | 81 |
| Quiz 3 |  | 87 |
| Quiz 4 | \} | 94 |
| Final Project | 10\% | 92 |

Part A: Assuming all grades are "in" determine the cumulative grade earned in this class by

1. Using a point system, where each project is assigned a worth of (40/9) \%, and each quiz is assigned a worth of (40/4)
2. Using an average system, where the average project grade and average quiz grade are computed and then multiplied by their respective weights.
3. Show mathematically how (1) and (2) yield the exact same result.
4. What if for part (1) you used each project counting 4\%. Are (1) and (2) still equivalent? Why or why not?

Part B: After taking Quiz 1, the student panics and is afraid they may not be able to get a B. They are thinking of dropping the class as a result. The grades that are in are: Participating, Projects 1 through 3, and Quiz 1.

1. Tell them their current "projected" grade. That is, use a weighted average to determine their cumulative with what they have right now.
2. Assuming their Project Average remains the same, and ignoring the grade for their final project, what does their quiz average need to be to get an $\mathrm{A}(90 \%)$ ?

Part C: You are feeling generous...

1. What difference would it make for this student if you were to drop the lowest Project grade, and take the average project grade from the best 8 scores?
2. What would be the most percentage points this would curve for any student?

Part D: All the grades are in but the final project.

1. This student has to leave town early and is considering not turning it in. What would be their cumulative grade if they earned a zero for the final project?
2. This student is worrying about their final grade. They want to know what they have to earn on the final project for a cumulative grade of $\mathrm{A}(90 \%)$.
3. Hmm, well maybe getting an $A$ is not realistic. How about a $B+(87 \%)$ ?

## 6. Finding Pi: An Exploratory Exercise In The Relationship Of Circumference To Diameter

The purpose of this project is to investigate the value of Pi . In particular, where does it come from, and how accurately can it be predicted using the measures of circles? Teachers will also be using technology to fit a linear equation to a set of data, using that equation to predict values, and analyzing error.

There are three parts to this project: Parts A and B teachers will measure and record data for circumference, radius and diameter of circles, and analyze this data by finding the best fit (linear regression) line. They will determine how the slope of this line relates to the value of Pi . For Part C they will use the knowledge they gained in Parts A and B to determine the circumference of a circle and the error in analysis.

## Project 6

## Part A

1. Measure the diameter and circumference of the following circles, and record in a table

2. Using MS Excel, graph the points $(d, C)$ on a scatter plot.
3. Using MS Excel, find the Linear Approximation (trend line) for the data, and display the equation on the chart.
4. Explain how (using the definition of slope) the slope of this line estimates the value of Pi.
5. For each data point, approximate $C / d$ to 5 decimal places.
6. Explain how this ratio estimates the value of Pi.
7. Find the relationship between the slope in part (4) to the ratio $C / d$ found in (5). Are they equal, if so, why? If not, how far are they off?

## Part B:

1. Find the radius of the circles, and record them (along with the circumference) in a table.
2. Using MS Excel, graph the points $(r, C)$ on a scatter plot.
3. Using MS Excel, find the Linear Approximation (trend line) for the data; display the equation on the chart.
4. Explain how (using the definition of slope) the slope of this line estimates the value of 2(Pi).
5. For each data point, approximate $C / r$ to 5 decimal places.
6. Explain how this ratio estimates the value of $2(\mathrm{Pi})$.

## Part C:

1. Using the information from Parts A and B, determine the formula for the Circumference ( $C$ ) for any given circle with diameter $(d)$ or radius $(r)$.
2. Fill out the table for the circles pictured in Part A.

| (RECA | $\%$ error $=$ | - estimate | $\times 100 \%)$ | $(\mathrm{Pi})^{\star} \mathrm{d}$ | $2^{*}(\mathrm{Pi})^{*} \mathrm{r}$ | \% Error |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | ctual |  |  |  |  |
| Circle | d = Diameter | Radius = d/2 | C = Circumference |  |  |  |
| A |  |  |  |  |  |  |
| B |  |  |  |  |  |  |
| C |  |  |  |  |  |  |
| D |  |  |  |  |  |  |
| E |  |  |  |  |  |  |
| F |  |  |  |  |  |  |
| G |  |  |  |  |  |  |

3. To what do you attribute your error? Is it what you expected? Why or why not?
4. If you used your value of Pi instead of the calculator value, how far off are you estimating the circumference of the earth?

## 7. Where's The Point? An Exercise In Translations, Rotations, And Reflections

The purpose of this project is to translate points on a "treasure" map, and use translations to determine where to find the next clue. Teachers will also be using their knowledge of translations and reflections to make their own treasure map.

This project consists of two parts: For Part A, a map of the UNLV campus has been put on a $10 \times 10$ coordinate axis. Starting with the ordered pair $(0,0)$ they will find the next point on the map by reflecting or translating. For Part B they will use geometry, equations of lines, perpendicular lines, etc, to make their own treasure map.

Project 7
Part A:

1. On the following page you will find a map of the UNLV campus, placed upon a coordinate axis (this map can be found online [14] ).
2. Starting from point $\mathrm{A}(0,0)$ perform the following translations

| Point | x-Value | $y$-Value | Found By Reflecting Previous Point Through the... |
| :---: | :---: | :---: | :---: |
| A | 0 | 0 |  |
| B |  |  | ...line $y=-4 / 3 x+2.08333$ |
| C |  |  | ...line $\mathrm{y}=2$ |
| D |  |  | ...line $\mathrm{y}=0$ |
| E |  |  | ...line $\mathrm{y}=4.5 \mathrm{x}-22.125$ |
| F |  |  | ...line $y=-17.6 x+33.21$ |
| G |  |  | ...line $\mathrm{y}=0$ |
| H |  |  | ....line $\mathrm{y}=-1.1333 \mathrm{x}-2.0333$ |
| I |  |  | ...origin |
| $J$ |  |  | ...line $x=3.85$ |

3. Don't forget to record each point in a table similar to above
4. Also record a dialogue of travels (building names) from start to finish.

## Part B:

1. Using a map of your school (or other applicable map) make your own lesson using from 3 to 5 separate points.
2. Realize this can be utilized as a "treasure map" of sorts, where students proceed to the next point to find the next clue.
3. Write up a lesson plan, and indicate where in the curriculum this can be presented.

## 8. Height And Weight, Finding The Relationship Between Weights Of A Cylinder Of Fixed Diameterfilled To Different Heights

The purpose of this project is to determine, using a cylindrical glass, the relationship between the weight of the water and the height of the water in the glass. Teachers will also learn how to develop an experiment, gather materials, collect data and analyze using technology.

This project has three parts: For Part A they will develop the method for collecting the data needed. For Part B they will gather the data, record methods, and make a table of values. For Part C they will analyze the data.

## 9. Finding the Relationship between an Individual's Forearm Length and Shoe Size

The purpose of this project is to determine if there is a relationship between the length of an individuals' forearm and their shoe size; and if there is a relationship, what it is. Teachers are also going to learn how to develop an experiment to answer a particular question, gather materials, collect data and analyze using technology.

There are three parts to this project: For Part A they will develop a method for collecting the data needed. For Part B they will gather the data, record methods, and make a table of values. For Part C they will analyze the data.

## Project 8

Now that we have seven other projects under our belt, you will have less written guidance on this project. It is your goal with this project to conduct your own experiment, gather data in your own fashion, analyze that data, and determine the validity of your experiment.

## Part A:

1. Determine the best way to gather the data.
2. Determine what materials will be needed.
3. Gather the materials needed.

## Part B:

1. Gather and record data in an organized way that facilitates analysis.
2. This section should include:
a. Materials used
b. A section describing your method for gathering data.
c. A table of data values (no less than 8 data points should be used).

Part C:

1. Analyze the data and draw conclusions.
2. This section should include:
a. An analysis of the data values in (2c), including a graphical representation and a description of the relationship (in words).
b. A comparison of the values you found with the known formulas for volume of a cylinder, weight of water, etc.
3. How accurate is your data? Perform an analysis of the error.
4. Are you satisfied with your experiment? If not, what would you change?

## Project 9

Again as in project 8 , you will have less guidance on this project. It is also your goal with this project to conduct your own experiment, gather data in your own fashion, analyze that data, and determine the validity of your experiment.

## Part A:

1. Determine the best way to gather the data.

Determine what materials will be needed.
3. Gather the materials needed.
4. Consider data that could be skewed, such as choosing only one gender, age group, etc. You want to make the data collection as random as possible.

## Part B:

Gather and record data in an organized way that facilitates analysis.
This section should include:
a. Materials used
b. A section describing your method for gathering data
c. A table of data values (no less than 10 data points should be used)

Part C:

1. Analyze the data and draw conclusions.
2. This section should include:
a. An analysis of the data values in (2c), including a graphical representation and a description of the relationship (in words).
b. A best fit line (least squares line) of the data.
c. A decision about whether there indeed is a correlation between the two variables.
3. How accurate is your data? Using the best fit line predict and test a data value (you will need an additional data point for this exercise). What is the error?
4. Are you satisfied with your experiment? If not, what would you change?

## D. DISTRICT AND STATE STANDARDS COVERED

The CCSD Power standards for grades five through eight are broken up into the following six categories:

1. Numbers, number sense and computation
2. Patterns, functions and algebra
3. Measurement
4. Spatial relationships and geometry
5. Data analysis
6. Mathematical problem solving, communication, reasoning, and connections.

This course specifically addresses topics in each category with the exception of (4), Spatial Relationships. The next course in this sequence continues further with all topics, and stresses additional topics in (4).

Table 1: Standards Covered in Projects

|  | No. of Standards <br> Addressed | Total No. of <br> Standards | Percentage |
| :--- | :---: | :---: | :---: |
| (1) Number Sense | 15 | 30 | $50 \%$ |
| (2) Patterns, Functions and Algebra | 15 | 26 | $58 \%$ |
| (3) Measurement | 10 | 16 | $63 \%$ |
| (4) Spatial Relationships and Geometry | 8 | 30 | $27 \%$ |
| (5) Data Analysis | 7 | 17 | $41 \%$ |
| (6) Problem Solving, etc |  | 48 | 52 |

Table 1 is an outline of the number of standards covered with all the projects combined (not including class discussions). A list of these standards can be found at [15] and [16]. As can be seen from Table 1, the projects are heavy in the area of "Problem Solving, Communication, Reasoning and Connections" (6), with more than $92 \%$ of all standards addressed. Over half of all standards are covered in the categories of "Number Sense" (1), "Patterns" (2) and "Measurement" (3). In total, over sixty percent of all CCSD Power Standards (grades five through eight) are covered within the context of these projects.

Figure 1: Number of Standards Addressed by Each Project


Figure 1 displays the number of standards addressed by each project alone. On average, each project addresses almost a third of all 171 CCSD Power Standards (grades five through eight) in all categories.

## E. CONCLUSION

As the teaching and learning standards in mathematics across the country become more content driven, college level math courses for teachers must focus specifically on delivering mathematical content in a meaningful way. The nine projects presented in this paper not only assist in developing content understanding, but show teachers how to apply the content to student's lives.

Development is underway for the second course in this two part series. Again, it will be heavy in problem solving and reasoning, but the projects will focus on topics such as: geometry, the Pythagorean Theorem, unit conversions (money and time), and probability.

Mathematics is everywhere, but is not usually recognized because it doesn't look like the algebra struggled through in school. It is the job of educators to demonstrate, to their communities, the relevance and importance of mathematics in every day life. It is time to embrace the learning and understanding of mathematics, and the projects presented here are a small step in achieving that goal.

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