Estimating Costs
With Curvilinear Analyses For Learning

Dennis F. Togo, (E-mail: togo@unm.edu), University of New Mexico

ABSTRACT

Even when the underlying mathematical relationship for the learning curve is a power function, most popular accounting textbooks perform a logarithmic transformation of the learning curve into a linear relationship. This transformation allows students to perform linear regression techniques; however, accounting students may be left with the misconception that all costs are linear. With the enhanced capabilities of spreadsheets, accounting students can now easily perform curvilinear analysis on historical learning data and then forecast costs where learning is present. The Smart Missile Company exercise illustrates this spreadsheet approach to teaching learning curves.

INTRODUCTION

The learning curve is important for cost estimation, yet, most managerial and cost accounting textbooks (Hilton *et al.*, 2000; Louderback *et al.*, 2000) do not perform an analysis of historical data in determining an appropriate learning model and rate to predict labor hours. Instead, most textbooks (Blocher *et al.*, 1999; Horngren *et al.*, 2000) introduce the log-linear learning curve model with its cumulative average-time and individual unit-time variations and then simply predict labor hours for specified learning rates. Furthermore, a logarithmic transformation of the underlying mathematical power function into a linear function is commonly performed when analysis or forecasting of learning is discussed. Such an approach to learning curves may leave students with the misconception that all cost estimations are inherently linear, even when more direct and appealing curvilinear data analysis techniques are readily available with spreadsheets.

The Smart Missile Company exercise illustrates curvilinear data analysis in estimating labor hours when learning is present. The students statistically support their use of either the cumulative average-time or individual unit-time learning model based on historical data, estimate the model's learning rate and predict labor hours for cost estimation. The students enhance their data analysis skills beyond linear regression, and recognize that non-linear cost behavior can be easily analyzed. This exercise is designed for undergraduate and graduate business students in managerial or cost accounting courses.

LEARNING CURVE OVERVIEW

The learning curve relationship is commonly modeled with a mathematical power function described as a log-linear or constant percentage model. The log-linear model below recognizes that labor hours decrease systematically by a constant percentage each time the volume of production increases geometrically (usually a doubling of units).

$(A, \text{ or } I_n) = aX^b$

The choice of a dependent variable is determined by whether the cumulative average-time learning model (A) or the individual unit-time learning model (I_n) is selected (Belkaoui, 1986). Hence, the left side of the relationship is either: A = the average cumulative labor hours for X number of units, or I_n = the number of labor hours required to produce the last nth unit. The independent variables are defined as follows: a = the number of labor hours required to produce the first unit, X = cumulative number of units produced, and b = learning exponent, which is always negative. The negative learning exponent b is (log r)/(log f), where r is the rate of learning represented by the constant

percentage decrease in hours, and f is the factor increase in output (usually in terms of 2). Hence, an 80% learning rate with a doubling of units has a learning exponent b equal to –0.3219, which is (log .80)/(log 2).

LEARNING CURVE EXERCISE: SMART MISSILE COMPANY

The students are to prepare a bid for the building of 15 XYZ missiles by Smart Missile Company. The company had built the XYZ prototype for the government because of its experience building the earlier ABC missiles. When the XYZ missile manufacturing specifications were released to other approved contractors, the government also released details of the $1,600,000 cost in building the prototype. Costs for direct materials, direct labor, and variable manufacturing overhead are listed below. Other manufacturing overhead costs were 10% of total variable manufacturing costs. The $170,000 equipment purchased by the government will be made available to the selected contractor.

Direct materials	$ 800,000
Direct labor (4,000 hours @ $100)	400,000
Variable manufacturing overhead (4,000 hours @ $25)	100,000
Other manufacturing overhead ($1,300,000 @ 10%)	130,000
Purchase of reusable government equipment	170,000
Total	$ 1,600,000

The Smart Missile Company will not assume 4,000 direct labor hours incurred by the prototype for each of the next fifteen XYZ missiles because of learning, which was noticeably present when building the ABC missiles. With no significant turnover in its highly skilled labor force, the company anticipates that the next fifteen XYZ missiles can be built with the same amount of learning found for the sixteen earlier ABC missiles. The direct labor hours for each of the sixteen ABC missiles follow:

1.	3,900	2.	2,740	3.	2,660	4.	1,980
5.	2,320	6.	2,000	7.	1,900	8.	1,700
9.	1,950	10.	1,950	11.	1,760	12.	1,540
13.	1,680	14.	1,320	15.	1,500	16.	1,100

Curvilinear Data Analysis

There are two related analyses of the ABC labor hour data: what learning curve model to adopt, and its percentage of learning. Using EXCEL, the students will generate a scatterplot of the direct labor hours incurred for the sixteen ABC missiles, and add to it a power function curve with its equation and r-squared value. This is performed for the individual unit-time and the cumulative average-time models. They then determine which model is best, and calculate its learning rate. In the next section, the impact of learning will be estimated for the next 15 XYZ missiles using the best model.
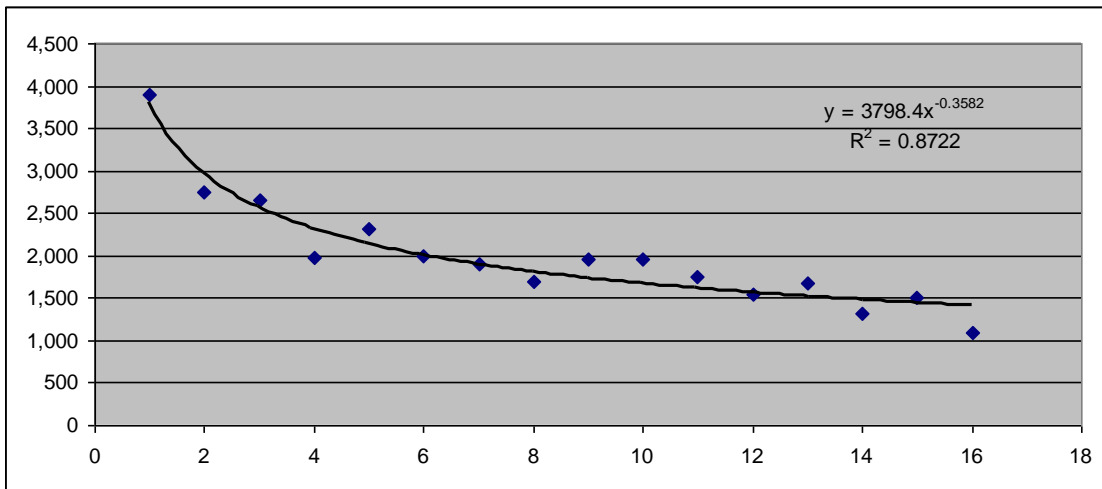
The data analysis begins with the students entering on a spreadsheet (see Exhibit 1) the data for the ABC missiles into two columns labeled as Unit and Individual. They then add columns for Total and Average. For the individual unit-time method, the students highlight the two columns Unit and Individual and use Chart Wizard to select XY (Scatter) as the chart type to generate the graph in Exhibit 1. The students right click on a data point within the graph, and then select Add Trendline and Power as the trend type from the drop down menus. Staying within Add Trendline, they select the Options tab and check Display equation on chart and Display R-squared value on chart.

Exhibit 1 displays the results for the individual unit-time model. The estimated learning curve is $Y = 3,798 X^{-0.3582}$ and its r-squared value is 0.8722. For the cumulative average-time model, the students repeat the above steps except substitute Individual with Average. The estimated learning curve is $Y = 3,944 X^{-0.2367}$ and its r-squared value is 0.9968. The data analysis supports the use of the cumulative average-time model because of its larger r-squared value. The EXCEL formula for computing the learning rate for the cumulative average-time model is $10^{\wedge}(-0.2367*Log(2))$ or rounded to 85%.

Exhibit 1: ABC Missiles Learning Curve Data and Analysis

Unit	Individual	Total	Average	Unit	Individual	Total	Average
1	3,900	3,900	3,900	9	1,950	21,150	2,350
2	2,740	6,640	3,320	10	1,950	23,100	2,310
3	2,660	9,300	3,100	11	1,760	24,860	2,260
4	1,980	11,280	2,820	12	1,540	26,400	2,200
5	2,320	13,600	2,720	13	1,680	28,080	2,160
6	2,000	15,600	2,600	14	1,320	29,400	2,100
7	1,900	17,500	2,500	15	1,500	30,900	2,060
8	1,700	19,200	2,400	16	1,100	32,000	2,000

Individual Unit-Time Model

$y = 3798.4x^{-0.3582}$
$R^2 = 0.8722$

Cumulative Average-Time Model

$y = 3944.9x^{-0.2367}$
$R^2 = 0.9968$
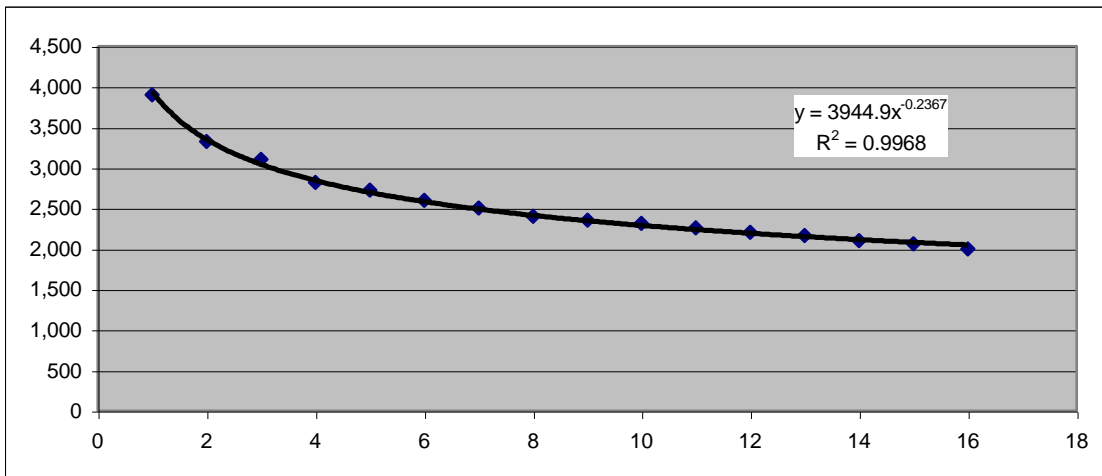
Cost Estimation With Learning

The students estimate the cost for the next 15 XYZ missiles with the cumulative average-time model and its learning rate of 85%. The formula ROUND(4000*16^(Log(0.85)/Log(2)),0) computes the 2,088 average direct labor hours to complete the 16 XYZ units, given 4,000 hours for the prototype. The total number of hours estimated to complete 16 units is 33,408 (2,088*16), therefore, the number of hours to complete the 15 XYZ missiles beyond the prototype is 29,408 (33,408-4,000).

The $17,243,600 bid is calculated for the next fifteen XYZ missiles based on 29,408 labor hours.

Direct materials ($800,000*15)	$ 12,000,000
Direct labor (29,408 hours @ $100)	2,940,800
Variable manufacturing overhead (29,408 hours @ $25)	735,200
Other manufacturing overhead ($15,676,000 @ 10%)	1,567,600
Total	$ 17,243,600

Supplementary Tasks

1. Solve for the logarithm of the cumulative average-time model. The solution is shown below. If $A = 3,944 X^{-0.2367}$, Log (A) = Log (3,944) + Log ($X^{-0.2367}$) = 3.5959 - 0.2367*Log (X)

2. Perform linear regression on ABC's equivalent logarithmic data. The result is the same as part 1. Proof: Assume X = 16, then Log (A) = 3.5959 – 0.2367*Log (16) = 3.5959 – 0.2367*1.2041 $\sim$ 3.3088. The antilog for Log (A) = 3.3088 is $A = 10^{+3.3088} = 2,088$.

3. Have the students contrast assumptions for the cumulative average-time model with the individual unit-time model. With the same 85% learning rate, the individual unit-time model estimates 37,534 (41,534 - 4,000) labor hours for the next 15 missiles beyond the prototype.

1.	4,000	2.	3,400	3.	3,092	4.	2,890
5.	2,743	6.	2,628	7.	2,535	8.	2,457
9.	2,390	10.	2,331	11.	2,280	12.	2,234
13.	2,192	14.	2,154	15.	2,120	16.	2,088

Exercise Effectiveness

The students enjoy this exercise based on course evaluations and in-class discussions. The spreadsheet modeling of learning curves broadens and enriches their study of cost estimation. Students recognize that any managerial decision where learning takes place can benefit from curvilinear data analysis of available historical data. Furthermore, they become aware of other nonlinear relationships such as polynomial, exponential, and moving average that can also be analyzed and modeled using the same approach.

OTHER FACILITATIVE USES OF COMPUTERS

As spreadsheets become more powerful and user-friendly, accounting educators can rejuvenate their teaching approaches for traditional accounting topics. For example, the teaching of topics such as master budgets (Togo, 1991), reciprocal service-department cost allocations (Savage and Wilburn, 1997) and capital budgeting (Togo and McNamee, 1996) are facilitated by spreadsheets. In addition, risk analysis with Monte Carlo simulation is conducive to accounting topics such as pensions (Kelliher *et al.*, 1997), flexible budgeting (Nordhauser and Chau, 1997), and capital budgeting and projecting cash flows or operating income (Togo, 2004).

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