

Prospective Analysis Of The World Lithium Market Contribution To The Evaluation Of Supply Shortage Periods

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ABSTRACT

Despite the availability a priori of lithium geological resources and an assurance from producers to raise their production capacity in the event of an increase in lithium demand, the French government is not reassured because the lithium world market in general and the French market in particular are not still shielded from a supply shortage. Indeed, due to political or environmental reasons, a possible supply restriction policy could be applied by producers. The present work aims to contribute to assessing possible shortage periods, if such scenarios occur. Our approach consists in carrying out a prospective modelling and simulation of the lithium market. We use a Multi-Agent System (MAS) method so that the individual behaviour of the market players is considered. The results of our work enabled us to conclude in a time-limited risk of a shortage in France, even in the event of a restriction combined with an increase in demand. Regarding the rest of the world, the conclusion is more pessimistic. In that case, the work attempted to search possible alternative supply scenarios to compensate shortages resulting from restrictions. At a methodological level, this work provided a promising test regarding a first application of MAS on the modelling of an industrial mining market (a field of macroeconomics). This methodological choice was motivated by the increasing interest aroused by MAS in this field since the economic crisis, generated by individual behaviour, began in 2009; a crisis, where an important supply shortage may be a future occurrence.

Keywords: Lithium; Market; Supply Shortage; Simulation; Multi-Agents; Statistics

INTRODUCTION

Discovered in 1817 by the Swedish scientist Johan August Arfvedson, lithium is one of the metals stemming from mineral resources which, today, constitute one of the important elements in the manufacture of industrial products such as batteries and glass. Lithium products are generally presented in the market in the form of lithium compounds, in particular lithium carbonate (Li_2CO_3), lithium hydroxide (LiOH) or lithium chloride (LiCl).

Over the next few years, an increase in demand in the lithium market is expected, in particular because of the progressive arrival of electric vehicles. If we refer to the studies by Gruber et al., (2011), the quantity of available lithium resources at a world level, estimated at approximately 24 Mt, is *a priori* sufficient to face this increase, at least until 2100. Furthermore, according to Daw & Labbé (2012), all major producers of lithium have announced they would make increases in their production capacity in the event of an increase in lithium demand. *A priori*, the lithium market should not then be confronted with a supply shortage issue. However, these geological and economic situations do not completely reassure the lithium market and in particular fully importing (i.e. not producing any lithium) countries such as France. Indeed, environmental or political decisions from producing countries may still occur and may have consequences on this market. For example, sources of lithium such as the salt lakes of South America or China are generally protected ecological systems, the exploitation of which may affect the environmental equilibrium (Angerer et al., 2009). Moreover, difficulties could also arise in the event of

drastic changes to mining or commercial policies in some countries that currently dominate the world market, such as Chile (Daw & Labbé, 2012). Although such changes do not currently seem on the agenda, the weight of this country alone on the world export market (e.g. in 2008, approximately 42,000 t, i.e. 60 % market share) would generate important consequences if a restriction decision were to be taken. Finally, again according to Daw & Labbé (2012), the status of lithium as a nuclear material could also lead producers to eventually consider restrictions in the future.

The present work aims to contribute to the elaboration of a public policy support tool, by answering the following question: *Given the certainty of an increase in demand, coupled with uncertainty of supply, how long would a lithium supply shortage last (should the case arise) in the world market as well as in France?* Such a tool is expected by industrialists of vehicles of the future regarding the choices to be made towards their supplies. It is also expected by the Committee of Strategic Metals (set up by the French government) regarding future orientations they may undertake in the French industrial mining sector.

To try to answer the above question, the approach of the present work consists, via modelling and simulation, in creating prospective scenarios of supply shortage in the lithium market due to restrictions decided by a producing country, then (a) identifying the set of likely shortage periods that would correspond to these respective scenarios and (b) searching possible alternative supply scenarios to compensate the resulting shortages. The prospective period begins in 2012. The historic period is situated between 2005 and 2011, a period when the data necessary for this work are available.

STATE OF THE ART

Description

Various works have already been carried out on the economic prospective modelling of the lithium world market in order to deal with the supply shortage issue. These works were carried out either by the academic world (Yaksic & Tilton, 2009; Gruber et al., 2011), or by lithium consultants (Roskill, 2009), or by banks (McNulty and Khaykin, 2009, on behalf of the Credit Suisse), or by producing companies (De Solminihac, 2010, for the Chilean company SQM, the current leading lithium producer in the world). Table 1 shows samples of the results of these studies. The year of likely shortage (the last column) is estimated as the point where the supply can no longer keep pace with the increase in demand.

Table 1
Comparison of four studies giving hypotheses of the year of likely supply shortage on the lithium world market

	Estimated world demand (kt)	Estimated limit production capacity (kt)	Year of estimation	Evaluated year of a likely shortage
Roskill	225	150	2020	2016
Credit Suisse	200	140	2020	2017
SQM	190	290	2020	2026
Gruber and Medina	23,800	n/a	2100	2100+

Analysis

In all these studies, production and consumption were respectively extrapolated in an independent manner and the results next compared arithmetically. Thus, there was no mutual driving between the evolution of the supply and the demand values. Furthermore, all the works adopted a global scale as the level of their studies. Initiating this kind of analysis at a global level is necessary because mineral resources are spread out over the Earth as a whole. However, this is not sufficient. Indeed, knowledge of the quantity available at the global level does not automatically imply that of the distribution per country. Likewise, if the period of likely shortage is known at a global level (e.g.: in 2016 for Roskill), nothing says that for a given consuming country it will be the same, since the supply behaviour at a production side varies from one producing country to another, depending on its individual and collective interests. As a matter of fact, the global elements are important indications, but need to be refined.

PROPOSAL

Model, Method And Tools

To try to answer the question posed in the introduction while attempting to improve the above state of the art, our proposal consists in passing from the global scale, methodically adopted by the existing lithium market models, to a more detailed scale where it would be possible to start to explicitly represent the individual behaviour of producing and consuming countries, as well as their interaction. Indeed, it would be more interesting for the government of a consuming country to analyse the effects of the interaction between importation and exportation flow on its particular country, especially in a restriction context. Given this objective, we then use modelling by a multi-agent system approach (Wooldridge, 2009), in which the system is composed of semi-autonomous interacting countries and where the behaviour observed globally is the result *emerging* from these interactions. The software systems used for all our work are (a) Isatem (Andriamasinoro et al., 2010) for the simulation, and (b) the proprietary tool SAS® for the various statistical tests.

Data Sources

This work uses international trade data from GTIS (2012). The GTIS data presents flows between producing countries and transit countries (i.e. countries connecting producers and consumers) as well as between transit countries and consuming countries. It should be noted that for diverse reasons (administrative, geographical, etc.), a given consumer can be supplied by the same producer via several transits.

The chosen data are those situated between 2005 and 2011 and on a quarterly timescale. This scale was preferred to an annual timescale because it increases the number of our observations during the statistical tests.

Hypothesis Of The Model

The following hypotheses are adopted:

1. In order to better exploit the GTIS data, our market model will integrate not just producing and consuming countries obviously but also transit countries and will work "as if" consumers send their demands to transits even if, in reality, they directly address producers, which then send the product to the transits.
2. For the moment, we only focus on the market of lithium carbonate (Li_2CO_3) and not on the other lithium compound markets (LiOH , LiCl). This is not penalizing because, according to GTIS, France is currently supplied essentially with Li_2CO_3 . On the other hand, in future works where the research for alternative supplies will be delved into more deeply, extending the analysis to these other compounds would be interesting.
3. Even if we integrate producing countries, we do not use either export or production figures. Indeed, in the GTIS data, the figures for exports and imports at a given time are not always identical for reasons of transport delay, administrative procedures, etc. Thus, we only say that a producing country *supplies* the quantity effectively received by a consuming country at that time (after possibly having applied preliminary restrictions, during the scenarios).
4. In the model to be built, all chosen producing countries supply all chosen consuming countries, but with (obviously) different quantities, including 0.
5. The scenario of a supply without restriction, without increase in demand and without compensation will be called the *normal situation*.

Modelling

A *country* is modelled as an agent which is either a producer (pc), or a consumer (cc), or a transit (tc). The system contains η_{pc} producers, η_{cc} consumers and η_{tc} transits. A country may be in the following status¹: IsNormal, IsRestricting, IsMakingUp, IsWaiting and IsInRestriction. At the beginning of the simulation, each country is in

¹ In the agent literature, the term *state* is used instead of *status*. However, *state* is sometimes used to also indicate a *country*, a concept which also exists in this model. Thus, to avoid any confusion in the reading of this paper, the term *state* is simply changed.

IsNormal. The model also integrates agents called *ambassadors*. An ambassador $A(c1 \leftrightarrow c2)$ is a delegate agent which handles the flow exchanged between the countries $c1$ and $c2$. The concept of ambassadors has been introduced because, given the complexity of the internal and external behaviours of a country (as will be detailed later) and the topology of the system in general, it seems difficult for us to describe the exchanges between $c1$ and $c2$, in $c1$ and $c2$ at the same time; especially that, for a given $c1$, the mode of calculation of the exchanges changes from one $c2$ to another. A decentralisation (delegation) of the description of the exchanges consequently seemed more appropriate to us. Finally, communication between the agents is formally done via the exchange of events. They take the form $event(s, r, \langle q1, \dots, qn \rangle)$ where s is the sender, r the recipient and $\langle q1, \dots, qn \rangle$ a list of values to transfer from s to r .

In the simulation of the market model, the interaction between countries and ambassadors, at each time step, occurs by following the successive points below, in which the first three points concern the demand stage and the last three points concern the supply stage:

1. First, each consumer $cc_k, k \in \{1, \eta_{cc}\}$, will ask its ambassadors $A(tc_j \leftrightarrow cc_k), j \in \{1, \eta_{tc}\}$ to calculate the quantity $d(tc_j \leftarrow cc_k)$ to demand from all producers $pc_i, i \in \{1, \eta_{pc}\}$, the supply of which will next transit via the country tc_j . To calculate a demand over time, the approach, for now, consists in interpolating the time series $S_{d(tc_j \leftarrow cc_k)}$ of the GTIS data related to the demands from cc_k to tc_j , between 2005 and 2011, in order to obtain a regression line, which would describe and prolong that demand evolution. Let us note this line $DEq(tc_j \leftarrow cc_k, t)$. It may take a linear, logarithmic, exponential or average shape. The sum $cc_k \cdot \sigma_d$ of the quantities to be demanded by a cc_k is given by Equation 1.
2. Once each $d(tc_j \leftarrow cc_k)$ is calculated, each $A(tc_j \leftrightarrow cc_k), k \in \{1, \eta_{cc}\}$ sends that demand to tc_j .
3. Next, when tc_j has received the demands $\langle d(tc_j \leftarrow cc_1), \dots, d(tc_j \leftarrow cc_{\eta_{cc}}) \rangle$ from all the cc_k , it transfers them to each ambassador $A(pc_i \leftrightarrow tc_j), i \in \{1, \eta_{pc}\}$. The ambassador then calculates, from these demands, the part $d(pc_i \leftarrow tc_j)$ for which pc_i will have to answer. This part is here calculated as being a linear combination of all demands $d(tc_j \leftarrow cc_k), k \in \{1, \eta_{cc}\}$. It is formulated in Equation 2, in which K_{ij}^d and α_{ij}^d are the parameters of the linear equation, obtained by a linear regression on the GTIS data corresponding to respective variables described by Equation 2.
4. Next, from this demand part, the same $A(pc_i \leftrightarrow tc_j)$ calculates, in response, a supply $s(pc_i \rightarrow tc_j)$ that pc_i will provide all consumers via tc_j . The relation between the supply and the demand is shown by Equation 3. In the normal situation, supply and demand are theoretically identical except if pc_i decides, in its supplies, and at any time, to impose a restriction of rate $pc_i \cdot \rho_s$ (with $0 \leq pc_i \cdot \rho_s \leq 1$). In the normal situation, $pc_i \cdot \rho_s$ is 0.
5. Once a supply $s(pc_i \rightarrow tc_j)$ is calculated, $A(pc_i \leftrightarrow tc_j)$ sends it to tc_j .
6. Finally, when tc_j has received the supplies $\langle s(pc_1 \rightarrow tc_j), \dots, s(pc_{\eta_{pc}} \rightarrow tc_j) \rangle$ from all the pc_i , it transfers them to each ambassador $A(tc_j \leftrightarrow cc_k), k \in \{1, \eta_{cc}\}$, which then calculates, from these supplies, the part $s(tc_j \rightarrow cc_k)$, that will be provided to cc_k via tc_j . This part is calculated as being a linear combination of the supplies $s(pc_i \rightarrow tc_j), i \in \{1, \eta_{pc}\}$. It is formulated in Equation 4, in which K_{jk}^s and α_{ij}^s are the parameters of the linear equation, obtained by a linear regression on the GTIS data corresponding to respective variables described by Equation 4.
The sum $cc_k \cdot \sigma_s$ of the quantities supplying cc_k from different $tc_j, j \in \{1, \eta_{tc}\}$ is given by Equation 5.

At the end of each time step, the available instantaneous stock $cc_k \cdot s$ of a consuming country cc_k is the difference between the initial demand sum $cc_k \cdot \sigma_d$ and the final supply sum $cc_k \cdot \sigma_s$. The process is shown in (Eq. 6). In a normal situation, this stock should be theoretically 0. In fact, this is not mathematically possible due to the residuals allowed by the different coefficients K and α during the calculus via the linear regression method.

$$cc_k \cdot \sigma_d = \sum_{j=1}^{\eta_{cc}} d(tc_j \leftarrow cc_k) \tag{1}$$

$$d(pc_i \leftarrow tc_j) = K_{ij}^d + \sum_{k=1}^{\eta_{cc}} \alpha_{ijk}^d * d(tc_j \leftarrow cc_k) \quad \forall (i \in \{1, \eta_{pc}\}, j \in \{1, \eta_{tc}\}) \tag{2}$$

$$s(p_{c_i} \rightarrow t_{c_j}) = d(p_{c_i} \leftarrow t_{c_j}) * (1 - p_{c_i} \cdot \rho_s) \tag{3}$$

$$s(t_{c_j} \rightarrow c_{c_k}) = K_{j,k}^s + \sum_{i=1}^{\eta_{cc}} \alpha_{i,j,k}^s * s(p_{c_i} \rightarrow t_{c_j}) \quad \forall (j \in \{1, \eta_{tc}\}, k \in \{1, \eta_{cc}\}) \tag{4}$$

$$c_{c_k} \cdot \sigma_s = \sum_{j=1}^{\eta_{tc}} s(t_{c_j} \rightarrow c_{c_k}) \tag{5}$$

$$c_{c_k} \cdot s(t) = c_{c_k} \cdot \sigma_s(t) - c_{c_k} \cdot \sigma_d(t) \tag{6}$$

The interpolation process used to obtain the $DEq(t_{c_j} \leftarrow c_{c_k}, t)$, $j \in \{1, \eta_{tc}\}$, $k \in \{1, \eta_{cc}\}$, takes place in two stages. First, finding all the $DEq(t_{c_j} \leftarrow c_{c_k}, t)$ turning around an average $\overline{d(t_{c_j} \leftarrow c_{c_k})}$. This is done by the analysis of a possible autocorrelation between the values in $S_{d(t_{c_j} \leftarrow c_{c_k})}$. Next, for all (j, k) for which an average cannot be found, determining experimentally the shape of $DEq(t_{c_j} \leftarrow c_{c_k}, t)$ so that:

- $d_{Eq}(t_{c_j} \leftarrow c_{c_k}, t)$, i.e. the value of $d(t_{c_j} \leftarrow c_{c_k})$ at a time t obtained via DEq , is as close to $s(t_{c_j} \rightarrow c_{c_k}, t)$ as possible, once equations 2, 3 and 4 are successively executed,
- and the curves formed respectively by $S_{d_{Eq}(t_{c_j} \leftarrow c_{c_k})}$ and $S_{s(t_{c_j} \rightarrow c_{c_k})}$ visually follow similar shapes.

Formalisation of a restriction scenario

Let us now assume that, as of an instant t_s , a producer p_{c_r} decides to restrict its supply by $p_{c_r} \cdot \rho_s$ points (with $0 < p_{c_r} \cdot \rho_s \leq 1$). In this case, it changes its status from $IsNormal$ to $IsRestricting$ and sends an event $restrict(p_{c_r}, c_{c_k}, < >)$ to each c_{c_k} , $k \in \{1, \eta_{cc}\}$. The c_{c_k} then switches from the status $IsNormal$ to $IsInRestriction$. Following this restriction, the stock $c_{c_k} \cdot s$ of a c_{c_k} will naturally decrease and finally be in shortage. Let us note $c_{c_k} \cdot \lambda$ the cumulated shortage over time. When $c_{c_k} \cdot s$ is less than a threshold $c_{c_k} \cdot \tau \leq 0$, it is added to $c_{c_k} \cdot \lambda$ (Eq. 7.a). The threshold $c_{c_k} \cdot \tau$ is necessary because due to the calculus, it may happen that, in the normal situation, the stock randomly or slightly adopts a negative value, which may be perceived by the c_{c_k} , $k \in \{1, \eta_{cc}\}$ as a restriction while this is not yet the case. Finally, Eq. 7.b indicates that the shortage to compensate cannot be, in a shortage period, greater than the demanded value. Let us here call this principle the *principle of the negative limit*, because the limit of the shortage is mathematically the negative value of the demand.

$$\begin{aligned}
 \text{a) } \quad c_{c_k} \cdot \lambda(t) &= \begin{cases} c_{c_k} \cdot \lambda(t-1) + c_{c_k} \cdot s(t) & \text{if } c_{c_k} \cdot s(t) < c_{c_k} \cdot \tau \leq 0 \\ c_{c_k} \cdot \lambda(t-1) & \text{otherwise} \end{cases} \\
 \text{b) } \quad c_{c_k} \cdot \lambda(t) &= \max(c_{c_k} \cdot \lambda(t), -c_{c_k} \cdot \sigma_d(t)) \quad \text{principle of the negative limit}
 \end{aligned} \tag{7}$$

Formalisation of a compensation scenario

On reception of the restriction imposed by p_{c_r} , each c_{c_k} , $k \in \{1, \eta_{cc}\}$ immediately sends an event $demandMakingUp(c_{c_k}, p_{c_i}, < c_{c_k} \cdot \lambda >)$ to all the p_{c_i} , $i \in \{1, \eta_{pc}\}$ and $i \neq r$. Each p_{c_i} that receives the message, either immediately responds and switches its status from $IsNormal$ to $IsMakingUp$, or waits for a delay $p_{c_i} \cdot \delta$ and, in that case, first switches its status from $IsNormal$ to $IsWaiting$ before switching from $IsWaiting$ to $IsMakingUp$, once this delay expires. This delay may be necessary for diverse reasons specific to p_{c_i} : inability to immediately respond, speculation, etc. At this stage of the work, the delay is determined by the user, not by the agent.

The quantity $p_{c_i} \cdot v(t)$ to compensate by p_{c_i} at each instant t is then given by Equation 8, in which $p_{c_i} \cdot \rho_p$ is the compensation rate, $d(p_{c_i} \leftarrow t_{c_j}, t)$ is the value of $d(p_{c_i} \leftarrow t_{c_j})$ at the instant t, and $p_{c_i} \cdot \sigma_s(t)$ is the classical sum to supply by p_{c_i} during the normal situation. For a producing country that compensates (i.e. which does not restrict), the classical sum to supply is identical to the sum of the demands coming to this country (Eq. 8.a)).

$$\begin{aligned}
 \text{a)} \quad & pc_i \cdot \sigma_s(t) = \sum_{j=1}^{\eta_{pc}} d(pc_i \leftarrow tc_j, t) && \text{with } i \neq r \\
 \text{b)} \quad & pc_i \cdot v(t) = \begin{cases} 0 & \text{if } t < t_s + pc_i \cdot \delta \\ pc_i \cdot \sigma_s(t) * pc_i \cdot \rho_p * (t - (t_s + pc_i \cdot \delta)) & \text{otherwise} \end{cases} && 8
 \end{aligned}$$

The quantity $pc_i \cdot v(t)$ that has been compensated by pc_i is next distributed among the cc_k , $k \in \{1, \eta_{cc}\}$, proportionally to their respective shortage $cc_k \cdot \lambda$ and in absolute value (because for recall, $cc_k \cdot \lambda < cc_k \cdot \tau \leq 0$). Let us note $pc_i \cdot v_k(t)$ the proportion of $pc_i \cdot v(t)$ intended for cc_k (Eq. 9.a). The quantity that is finally compensated for cc_k is then the sum coming from each $pc_i \cdot v_k(t)$, ($i \in \{1, \eta_{pc}\}$ and $i \neq r$). This quantity naturally decreases the absolute value of the shortage $cc_k \cdot \lambda$ (Eq. 9.b).

In the end, for a cc_k , the part $cc_k \cdot \mu$ of its satisfied demand at a given instant is described in Equation 9.c.

$$\begin{aligned}
 \text{a)} \quad & pc_i \cdot v_k(t) = pc_i \cdot v(t) * \frac{|cc_k \cdot \lambda|}{\sum_{k=1}^{\eta_{cc}} |cc_k \cdot \lambda|} && \text{with } i \neq r \\
 \text{b)} \quad & cc_k \cdot \lambda = cc_k \cdot \lambda + \sum_{i=1}^{\eta_{pc}} pc_i \cdot v_k(t) && \text{with } i \neq r \\
 \text{c)} \quad & cc_k \cdot \mu(t) = \frac{cc_k \cdot \sigma_d(t) + cc_k \cdot \lambda}{cc_k \cdot \sigma_d(t)} && \text{with } cc_k \cdot \lambda < cc_k \cdot \tau \leq 0
 \end{aligned}$$

Formalisation of an increase in demand

When a demand increases, the additional quantity $cc_k \cdot a(t)$ to demand at each instant t , by cc_k , is given by Equation 10, in which $cc_k \cdot \sigma_d$ is the demand in the normal situation (recall Eq. 1), $cc_k \cdot \rho_a$ is the increased rate (with $0 < cc_k \cdot \rho_a \leq 1$) and $cc_k \cdot t_{a0}$ is the initial time when the increase starts.

$$cc_k \cdot a(t) = cc_k \cdot \sigma_d(t) * cc_k \cdot \rho_a * (t - cc_k \cdot t_{a0}) \tag{10}$$

Determination Of A Supply Shortage Period

For a country cc_k , the shortage period is that during which its cumulated shortage $cc_k \cdot \lambda$ is below 0 (the threshold $cc_k \cdot \tau \leq 0$ does not apply here, only during the normal situation). The process is formalised in Equation 11 where $cc_k \cdot t_{\lambda}$ is the date of the end of the shortage and then the return to an *equilibrium state*.

$$cc_k \cdot t_{\lambda} \text{ is } t > t_s \text{ such that } cc_k \cdot \lambda(t_{\lambda} - 1) < 0 \text{ and } cc_k \cdot \lambda(t_{\lambda}) \geq 0 \tag{11}$$

At this stage of our work, the model does not yet consider what happens after this return to an equilibrium state. As a thought process only, we may suggest that if pc_r decides to maintain its restriction, a solution would be for example to definitely reorganise the market by either (a) making permanent the quantities supplied by each current producing country at the time of the equilibrium state, and/or (b) the urgent introduction of new producers, etc. As the implementation of a reorganisation is not easy, the more in-depth elucidation of this stage will be the object of future work.

Simulation

Preamble: the simulation platform

While the statistical tests were performed with the proprietary tool SAS®, the simulation was implemented under the platform *Isatem* (Andriamasinoro et al., 2010). *Isatem* is constituted of a set of components in interaction.

Each component possesses:

- a set of *properties*,
- a set of *handlers* managing *input events*; for a received event eventX, this handler is formally written OnEventX (s, <pX1, ..., pXn>) where s is the component having sent the event and <pX1, ..., pXn > the event parameters. Only a handler OnTimeChange() is generic; it allows a component to react to the simulation timer.
- a set of *functions* managing the *output events*; for an event eventY to be sent, this function is formally written FireEventY(r, <pY1, ..., pYm >)) where r is the recipient component.
- a *behaviour*, which possesses the same OnEventX() and FireEventY() as a component to which it is associated. Actually, a component does not handle an event directly. It sends it to the handler of its current behaviour, having the same name. This mechanism allows a component, at any time, to change its behaviour, which is the way it handles the events, without changing the communication mode between it and its behaviour.

Each component possesses a default behaviour. It is then possible, by inheritance (as is defined by the object oriented concept, Booch et al., 2004), to *particularize* an OnEventX() or a FireEventY(). It is what allows two components to possibly have the same properties but totally different behaviours (e.g. a producing country and a consuming country). The body of functions in the default behaviour is either empty or groups the actions common to all the components of the same type (i.e. the *country* type and the *ambassador* type respectively).

This *Isatem* architecture is largely inspired by the agent platform ADK that was next applied to the world of robots collecting pucks (Calderoni, 2002). The behaviour of the agents ADK is based on the principle of the triad perception-deliberation-action. The deliberation is made via what ADK calls the *controller*. If we compare ADK and Isatem, the perception corresponds to the OnEventX(), the controller corresponds to the behaviour and the action corresponds to FireEventY(). However, an ADK agent possesses only one type of controller for all the agents in a simulation. Only its states (here its statuses) change. In Isatem, two components of the same system may have its own “controller” thanks to the two mechanisms described above, which are (a) the dynamic association between a component and its behaviour and (b) the particularisation process.

INITIALISATION OF THE VALUES FOR THE SIMULATION

The Agents And The Ambassadors

The selected producing countries (pc_i) are Chile (cl), China (cn) and the United States (us). These countries are seen by GTIS as being those that regularly supply France with a high quantity (≥ 25 t/quarter) of Li_2CO_3 . We also add a (virtual) country called the *rest of the world* (rw). The quantity supplied by rw is the world quantity, decreased by that provided by cl, cn and us. Thus, $\eta_{pc}=4$.

The selected consuming countries (cc_k) are France (fr), the subject of our study and, again, the rest of the world (rw). The quantity consumed by rw is the world quantity, decreased by that of France. Thus, $\eta_{cc}=2$.

Finally, the transit countries (tc_j) are Belgium (be), Germany (de), United Kingdom (uk), Italy (it) and the Netherlands (nl). The analysis of the GTIS data shows that it is via these transit countries that cl, cn and us transfer their quantities of Li_2CO_3 to France. We also add, again, the rest of the world (rw). The quantity transiting by rw is the world quantity, decreased by that of be, de, uk, it, nl. Thus, $\eta_{tc}=6$.

All the ambassadors are next naturally created to connect all these countries (rw included) in keeping with the formalisms previously described in this paper.

Other Initialisations

To comply with the data we have chosen in GTIS, the simulation time step is 3 months. Let us note, for example, 2/2019 quarter 2 of year 2019. Furthermore, Table 2 provides samples of other initialisations used in the simulation. This table contains samples of linear regressions and interpolations we have realized. Regarding the interpolations, we have ascertained that most of the $DEq(tc_j \leftarrow cc_k, t)$ we have found turn around an average (i.e. an increase in the curve is generally followed by its decrease and vice-versa). Rare equations have a logarithmic trend but, even with these equations, the slope is rather weak. They are however left “as-is”. Finally, no curve having a linear or an exponential shape was found during the experimentation. Figure 1 and Figure 2 illustrate the results of these analyses. Figure 2 here shows the example of the supply part (in t) to France and to the rest of the world, coming from Chile (Figure 1) and transiting via Belgium. The GTIS data in the figures are set for reference purposes.

Table 2
Samples of initialisations of the system for the simulation

Initialisation	Comments
$s(uk \rightarrow rw) = 26.31 + 0.02 * s(cl \rightarrow uk) + 0.23 * s(us \rightarrow uk) + 0 * s(cn \rightarrow uk)$: Obtained by a linear regression (Cf. Eq. 4)
$DEq(be \leftarrow rw, t) = 1.25 * \ln(t) + 827$: Obtained by experimentation
$DEq(de \leftarrow fr) = 79$: Obtained by an absence of autocorrelation between the values of the demand time series
$fr.\tau = -1.9$ et $rw.\tau = 0$: Obtained by an observation of how the stocks evolve during the normal situation (Cf. Eq. 7.a)

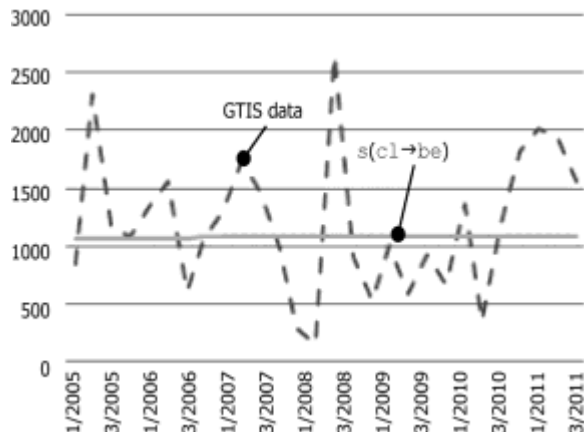


Figure 1: (Quarterly) world supply of Li_2CO_3 by Chile, via Belgium, according to GTIS and the model, respectively

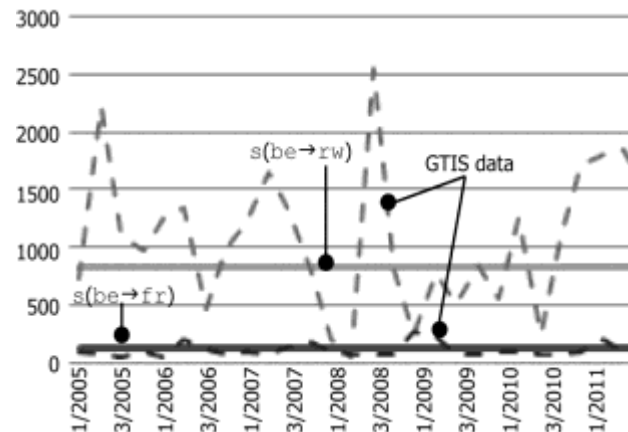


Figure 2: (Quarterly) supply of Li_2CO_3 to France and to the rest of the world, via Belgium, according to GTIS and the model, respectively

SIMULATION FOR VALIDATION (IN THE NORMAL SITUATION)

Figure 3 shows the result of the simulation of the stocks (cf. Eq. 6) of France and of the rest of the world respectively, between 2005 and 2030, and in a normal situation.

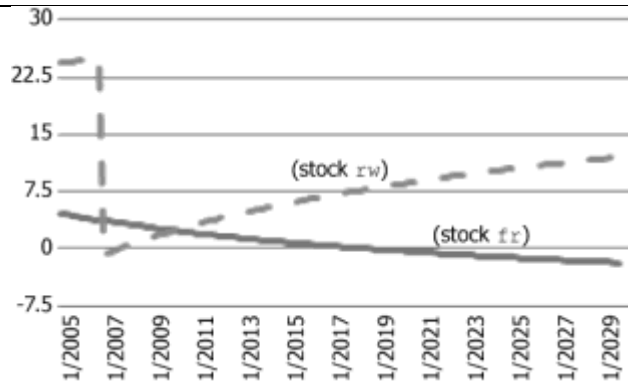


Figure 3: Simulation of the stocks of France and of the rest of the world in a normal situation

Regarding fr , Figure 3 shows that for an average demand of around 350 t/quarter (according to GTIS), the stock oscillates between 5 t and -1.8 t (whence $fr.\tau=-1.9$, cf. Table 2). This gives an error margin (i.e. a stock/demand ratio) of around 4%. As for rw , for an average demand of 13,080 t/quarter (according to GTIS), the stock oscillates between 25 t and 0 t (whence $rw.\tau=0$). This gives an error margin of around 0.1%.

The non-nullity of the stocks is due to the calculus by linear regression, the logarithmic effects and the experimental calibration of the curves corresponding to the initial demands.

The model is considered as valid under these conditions.

SIMULATION OF PROSPECTIVE SCENARIOS

General Principle

The pattern of the proposed (and currently fictitious) prospective scenario is the following: one assumes that as of 2014 (=t_s), Chile restricts its supply rate by $cl.\rho_s$ points. Following this situation, China accepts to assure compensation at a rate of $cn.\rho_p$ points, and does so immediately, i.e. $cn.\delta=0$. The United States also accepts, with a rate of $us.\rho_p$ points, but only as of 2016, i.e. $us.\delta=8$ (quarters). Finally, all increases in the demands $fr.\rho_a$ and $rw.\rho_a$, should the case arise, take place in 2013 (=fr.t_{a0} = rw.t_{a0}). The purpose of the simulation then consists in varying the value of these rates to find the shortage end date in France and in the rest of the world. A simulation will be formally written (example of France): $fr.t_{\lambda} = fr(-cl.\rho_s, +cn.\rho_p, +us.\rho_p, +fr.\rho_a)$. For each simulation performed, the result is read as follow: for each compensation $cn.\rho_p$ and $us.\rho_p$ and for each augmentation $fr.\rho_a$ and $rw.\rho_a$, the shortage end date for France and the rest of the world will be respectively $fr.t_{\lambda}$ and $rw.t_{\lambda}$. An inverse reading can also be performed: if one hopes that the shortage period does not extend beyond $fr.t_{\lambda}$ for France and $rw.t_{\lambda}$ for the rest of the world, the United States and China should increase their compensation rate by at least $cn.\rho_p$ and $us.\rho_p$ respectively and the increase in demand should not go beyond $fr.\rho_a$ and $rw.\rho_a$ respectively.

Results

Figure 4 shows examples of shortage end dates obtained for the rest of the world. Figure 5 is the “France equivalent” of Figure 4.

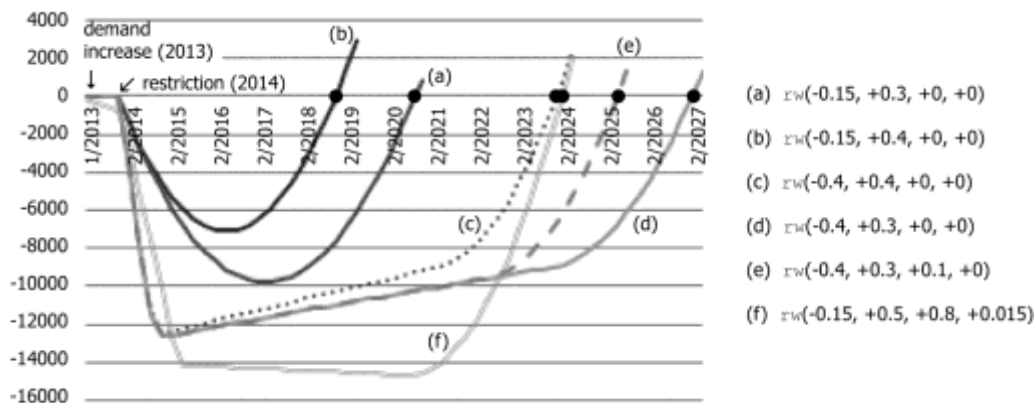


Figure 4: Prospective simulation of the evolution of stocks in the rest of the world after scenarios, combining (1) restriction and increase in demand with (2) compensation of supply

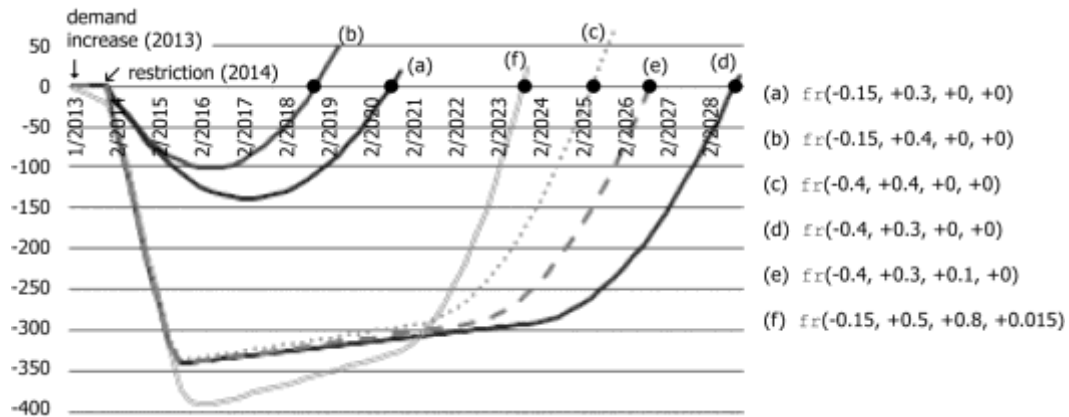


Figure 5: Prospective simulation of the evolution of stocks in France after scenarios, combining (1) restriction and increase in demand with (2) compensation of supply

In Figure 4, the curve (a) shows an end date $r_{w.t_{\lambda}}$ around 4/2020 if a restriction of -0.15 by Chile is immediately followed by a compensation of +0.3 by China. This date is brought forward to around 1/2019 if the compensation rate increases to +0.4 (curve (b)). But, with this last compensation, the date would be pushed back to around 2/2024 if the initial Chilean restriction was -0.40 (curve (c)) and around 2/2027 if, with this last restriction, the Chinese compensation was only +0.3 (curve (d)). Furthermore, this last date is brought forward from 2/2027 to around 4/2025 if, as of 2016, the United States also participates with a compensation of +0.1 (curve (e)). Finally, if the lithium demand increases by +0.015 (i.e. 1.5% per quarter) and Chile imposes a restriction of -0.15 in 2014, the date is situated around 3/2024 if, at the same time, there is a Chinese compensation of +0.5 associated with an American compensation of +0.8 (curve (f)). Besides, due to the principle of the negative limit (Eq. 7.b), the curves where $r_{w.p_a}$ is 0 immediately rise again, after reaching the threshold of the (around) -13,000 t per quarter. On the other hand, the curve (f), before rising again, first continues its descent due to the increase in lithium demand by +0.015 points.

Figure 5 illustrates that, for a given scenario, r_r and r_w easily have the same shortage end dates, except for the curve (f). This quasi-equality is globally normal because, according to Equation 9.a, the compensation is proportionally distributed among the consuming countries. The difference is situated at the depth of the curves because the shortage of France is “only” around 350 t/quarter, unlike around 13,000 t/quarter for the rest of the world. As for the curve (f), the difference can be explained by the high increase in the compensations rate (+0.5 and +0.8) by China and the United States respectively. This reduces the effect of the proportionality and accelerates the return of France to an equilibrium state, even though this is not immediately the case for the rest of the world. The latter continues its descent for several more quarters.

DISCUSSION

Thematic discussion

If we refer only to the results of these simulations, France should not worry about a possible prolonged shortage of its supply in lithium, even after a Chilean restriction. Indeed, Figure 5 can be seen as an extreme case (i.e. the most critical) for France: being imposed a proportional distribution of the compensations, and by Chile, the leading world producer. In fact, since the French proportion is small compared to that of the rest of the world, it is pragmatic to think that compensation would take place more quickly. Furthermore, even though remaining at only a calculation level, we saw in curve (f) that the compensation is faster for France as of a certain level of rates, despite the proportional distribution. Finally, we notice that for curves (a) and (b) in Figure 5, and for an average demand of about 350 t/quarter, the part of satisfied demand (recall Eq. 9.c) nevertheless turns around 57 % and around 71 % respectively, despite the restriction, which is acceptable in such a period. France would only need to decide which internal consumption sectors the imported quantity should first exceptionally supply during this period.

More generally, we can conclude that the results obtained here regarding France lead to optimism, whether the restriction is generated by Chile or by another country. Daw and Labbé (2012) already concluded that the risk of shortage is low for France, but they did not however in their study analyse the effects of a scenario of an effective restriction on a production side. The rest of the world would lead to a more pessimistic conclusion. Indeed, the best scenario (curve (b) of Figure 4) can reach a shortage of around 7,000 t/quarter. Nevertheless, what we could say to remain optimistic is that if the hypotheses chosen in the scenarios in this paper are approximately verified, the rest of the world would again be capable of satisfying 100 % of its demand before approximately 2030, even in the event of a supply restriction. Other optimistic results could also be sought via this model by introducing other rate values.

In any case, a return to an equilibrium state should be accompanied by a definitive reorganization of the market (the subject of one of our future works) by studying for example the new supply capacity of each producing country at the time of the equilibrium state. This reorganization should even be realized by anticipation, i.e. before 2030. Note that the value of 2030 mentioned here is not a time limit imposed by the model user, but an *emergent* result obtained during the simulation: it was observed that none of our periods of shortage exceeded this year.

Obviously, all the conclusions would have been different if we had simulated two or more restricting countries at the same time. But this scenario pattern, though currently methodologically feasible, has not yet been studied and consequently no results regarding this situation could be reported in this paper.

Limitations of the model

Although all of these conclusions seem interesting, they should however be interpreted with caution. Indeed, as rightly recalled by Feitosa et al., (2011), results obtained in any exercise of modelling complex systems do not represent either precise forecasts or deterministic answers. These results should mainly serve to feed the public debate concerning the subject. Our intention, by carrying out this work, goes in that direction. Firstly, the work tries to develop a methodology that would allow decision makers to analyse the sensitivity of the lithium market after a variation in important indicators (e.g. here, the diverse rates), and not to assert the precision of the values obtained during the simulation. In our (fictitious) scenarios, we took the example of Chile as being the country restricting its supply, but our work would be equally valid if we had taken China instead. Highlighting the values obtained in our scenarios are however important for illustration purposes. Secondly, the idea is to bring an additional thought process to the possible periods of a supply shortage of a particular country by trying to extend the state of the art (cf. Table 1) to a more detailed scale (of countries) and to observe the various effects of the individual countries' behaviour.

As regards the conceptual model itself, limitations exist. We think for example that the demand of a consuming country should not be a variable defined by an equation that is a function of only the *time* variable. The equation should also be dependent on other market variables such as the *geological reserves* available in the producing countries and also (and especially) the *price*. With the introduction of the price, China, for example, might decide to not immediately compensate the consuming countries' shortages but instead speculate before contributing. As Daw & Labbé (2012) specified, the risks of insufficiency of supply could appear only after a consequent decrease in the price and the production will be re-launched again after a price increase. We then observe the effect of the price in the behaviour of every producing country. An introduction of the price does not question the structure of our above scenario. On the contrary, it will rather help us to better specify the different producing countries' behaviour (for example, the value of the delays p_i, δ) and consequently the effects of these respective behaviours on the market.

Methodological discussion

In the industrial mining sector, which includes the lithium market, the modelling of raw materials markets to handle the supply shortage issue generally relies on purely mathematical or statistical models. Such is the case of the works presented in the "State of the art" section of this paper. Another example concerns a model called *Antag*, which analysed the supply shortage in aggregate resources (another product of industrial mining) on the French market: *Antag* was based on a dynamic systems approach (Rodriguez, 2010). We thus note that the MAS approach used in the present work was never applied in this field. An attempt to use MAS has already begun in the mining

field (Andriamasinoro and Angel, 2012) but it only concerned the artisanal mining network, i.e. at a micro scale, whereas the market of the industrial mining network is studied at a medium or macro scale.

This absence of MAS in the modelling of the industrial mining markets insofar can be explained by at least three reasons: (1) the method is not known by the studied field, (2) the method is known (because MAS has existed since the 1990s) but its use has always been considered as not indispensable or (3) its use has been considered as indispensable but its development is too difficult. That being said, the observed fact is that currently a methodological lack exists in the modelling of the industrial mining market, which means, for example regarding *Antag*, that the analysis contented itself with observing the global market flow in France while it has been known for a long time that the opening of production zones is decided at a regional (i.e. more detailed) subdivision even at a departmental subdivision, levels where MAS would have been suitable to complete this national level study.

To overcome the misreading of the MAS method (consequences of reasons 1 and 3) and favouring its acceptance at the same time, Hamill (2010) suggests that it is necessary (a) to demonstrate the value of MAS modelling by showing-by-doing and offering training projects and (b) to prove to analysts that this method provides something better than they already have. This is, we think, the case of the present work: it is an additional MAS application (even if, we agree, not yet sufficient to convince) and it attempts to be a proposal to overcome the limit of the models mentioned in the state of the art. Yet, regarding MAS acceptance, Andriamasinoro and Angel (2012) suggest favouring the coupling between MAS models and mathematical or statistical models, instead of sticking to a solely MAS approach. In this work, we have integrated statistical approaches to describe, not a global phenomenon, but the behaviour of certain agents to which the approach is suitable. It particularly concerns the *ambassadors*. Our linear regression tests (cf. Eq. 2 and 4) were used with this objective. As a result, we have a system where certain agents behave in a linear manner (*ambassadors* and *transit countries*) while others (*producing* and *consuming countries*) adopt more complex and more discrete behaviours, which depend on their status.

As regards the necessity or not to use the MAS approach in the industrial mining market field, i.e. at a medium-macroeconomic scale (reason 2), let us first recognize that the world markets have always traditionally preferred to rely on conventional economic equilibrium models such as DSGE (Fernández-Villaverde, 2010). However, the economic crisis in 2008-2009, resulting from interactions between local players (individual banks, households, traders, etc.) and which then had repercussions on the world market, has made economists reconsider modelling at a macroeconomics level. Indeed, they observed that models such as DSGE were no longer sufficient to anticipate a crisis situation and that MAS could be a solution (Farmer and Foley, 2009; Fagiolo and Roventini, 2012). The Economist (2010) review presents this insufficiency as follows: if conventional models perform well enough in a business-as-usual economy, based only on the existence of an ideal state of equilibrium, there is no equilibrium during crashes. Agent-based models may be more suitable because they make no assumptions about the existence of efficient markets or general equilibriums. Instead, they are focused on the assignation of particular behavioural rules to each agent and large fluctuations and even crashes are inherent to the system. Supply shortage, the subject of the present work, may be a future market crisis occurrence. Indeed, it includes an important period of disturbances, resulting from individual decisions, but being able to trigger major consequences on the global market and on importing countries. And, as we have stated in the “thematic discussion” section, the year 2030, the approximate date of return to an equilibrium state in the worst scenario, is an emergent result not an imposed initial date. Consequently, we believe that the choice of MAS is here appropriate. More generally, it may now be time to gradually consider MAS in the macroeconomics field in general, and the industrial mining field in particular, since the method is clearly appropriate regarding the issues to be solved. In return, however, those who work on MAS also need to make efforts to progress in the coupling of the approach with conventional market models acting at a global level.

CONCLUSIONS

Summary

Within forthcoming years, an increase in demand for lithium is expected, in particular because of the progressive arrival of electric vehicles on the market. Despite the availability, *a priori*, of lithium geological resources and the assurance by producers to increase their production capacity to keep pace with this demand

increase, the lithium market in general and fully importing (i.e. those without any lithium production) countries such as France, in particular, are not reassured. Indeed, environmental or political decisions from producing countries may still occur and may have consequences on this market. The present work aims to contribute to the elaboration of a public policy support tool to help French industrialists as well as the French government, through the Committee of Strategic Metals, to ensure that in forthcoming decades there will always be a continuity of supply on the world market and in France. Failing this, at least if the shortage is temporarily unavoidable, the best compensation scenario should be found so that the shortage period is as brief as possible. The approach we have proposed here consists in developing a model and simulating prospective scenarios, combining restriction of supply, increase in demand and compensation of the resulting shortages. The state of the art showed that existing models that have dealt with the subject have limited their studies to a purely global level without modelling and simulating the behaviour of the countries which, nevertheless, are heterogeneous. Thus, the present work suggests extending these purely global models to models directly representing the interactions between the market players (producing, consuming and transit countries). In order to deal with country heterogeneity, we have used the multi-agent system (MAS) modelling approach.

Our results are situated at two levels: thematic and methodological.

Regarding the thematic level, the model allowed us to strengthen the idea that in the event of a supply restriction, the risk of shortage in France remains very low or, at least, very limited in time. It completes the study previously carried out by Daw and Labbé (2012) who, however, did not measure the impact on France of a possible supply restriction. As for the rest of the world (i.e. the “sum” of the worldwide countries other than France), the shortage periods are more consequent. Regarding that case, what we have done is to attempt to search possible alternative supply scenarios to compensate the shortages resulting from restrictions.

We believe the modelling results obtained are very interesting but they need to be interpreted with caution, as in any exercise of modelling complex systems. Indeed, these models do not represent either precise forecasts or deterministic answers. The results could mainly be used to feed public debates on the subject.

Regarding the methodological level, this work was a promising test on a first application of MAS in the modelling of an industrial mining market, thus strengthening the use of MAS in the field of macroeconomics. This methodological choice was motivated by the increasing interest aroused by MAS in this field since the onset in 2009 of the economic crisis and the fact that an important supply shortage may be a future occurrence.

Future works

Since this work is only at its initial stages, the perspectives are numerous. In the first place will be the introduction of other variables to possibly explain supply and demand. We are thinking in particular about *geological reserves* and *price*. Secondly, it will be necessary to make the system more complex by introducing the other producing and consuming countries (which will involve, in the model, their “withdrawal” from the virtual country “rest of the world”). Thirdly, the model will integrate the management of what will take place when the market again returns to an equilibrium state, at the end of a restriction consequence. In particular, it is planned to add a mechanism of a market reorganization, in the event where a restriction initially decided is definitely maintained over time. Fourthly, a methodological transposition of all the work on other metals will be carried out. There are naturally strategic metals such as rare earths, but one should not completely forget major metals, with iron and steel to the forefront, but also aluminium or copper, which have an economic importance infinitely greater than lithium.

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