Testing For Unit Roots In The Presence Of Multiple Trend Breaks: An Application To Chilean Inflation Rate Series

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ABSTRACT

This paper is an application of the subject of testing for unit roots in a time series in the presence of structural change. The series used is that of the Chilean monthly inflation rate. A characteristic of this series is that, with the exception of the 1972-1977 period during which the inflation rate in Chile greatly accelerated then decelerated, the Chilean inflation rate over the 40-year period of the study stayed within a limited range. In spite of this relatively stable performance, the application of the standard unit root tests to the series results in the inability to reject the unit root. However, when a test that allows for three endogenously determined breaks in the slope of the trend function of the series is used to account for the structural change of the 1972-1977 episode; the result is a strong rejection of the unit root in favor of a, more reasonable, stationary characterization of the series.

INTRODUCTION

Chile has traditionally been known as a chronic inflation country. Chronic inflation has two characteristics: it lasts for long periods of time and is a type of inflation that has an intermediate intensity which is too high for the inflation to be considered moderate yet much lower than that of hyperinflations. An interesting feature of the Chilean monthly (annualized) inflation rate series presented in Figure 1 (in logarithmic scale) is that as much as it has fluctuated over a period of 40 years, it stayed for the most part in the 10 % to the 40 % range. It was only during the 1972 – 1977 years that the inflation rate was consistently above the 40 % mark. This relatively stable performance stands in contrast to the behavior of the inflation rate series of other high inflation countries like Brazil and Argentina that display accelerating inflation and persistence of shocks. It is therefore surprising that the application of standard univariate augmented Dickey Fuller tests to the Chilean monthly inflation rate series results in the failure to reject the unit root null. This result implies that the behavior of the Chilean monthly inflation rate series over 40 years is dominated by a 5-year episode.

The argument presented here is based on the findings of Perron (1989), who shows that the existence of structural change in a stationary time series biases the standard unit root tests toward nonrejection. A key assumption made by Perron (1989) is that the one-time break that he allows for is exogenously determined. This assumption was widely criticized. This resulted in subsequent papers such as Christiano (1992), Banerjee, Lumsdaine and Stock (1992), Zivot and Andrews (1992) and Perron and Vogelsang (1992) that introduce procedures to endogenize the determination of the break date by making it data dependent. Furthermore, Lumsdaine and Papell (1997) extend the endogenous break literature by allowing for a two break alternative.

The behavior of the Chilean monthly inflation rate series however, does not seem to be suitably modeled by any of the above mentioned procedures. This is because as shown in Figure 1, the Chilean inflation accelerated from 1972 to 1974, decelerated from 1974 to 1977 and then continued to fluctuate. As a result, tests that allow for one break or even two breaks will not be able to incorporate all these fluctuations.

Papell (2002) proposes a test that allows for three endogenously determined breaks in the slope of the trend function of a series. This test is very well suited to model the behavior of the Chilean monthly inflation rate series. The idea will be to treat the acceleration and deceleration of the inflation rate as exogenous events that are determined outside the data generating process. The first break will correspond to the start of the inflation rate's acceleration, the second will pick up the switch from acceleration to deceleration and the third will represent the end of the deceleration of the inflation rate.

This paper applies the test proposed by Papell (2002) to the Chilean monthly inflation rate series and determines whether this helps reach a more reasonable characterization of the series. In other words, the paper investigates whether the inability to reject the unit root in standard univariate ADF tests is an artifact caused by unaccounted for structural change in the form of one episode namely the acceleration and deceleration of the inflation rate between 1972 and 1977. The analysis further investigates whether, once this structural change is properly accounted for, the series can be described as a stationary process.

The data for this paper is obtained from the International Monetary Fund's International Financial Statistics. The Monthly data[1] for the Chilean inflation rate is used. The data covers a period of about 40 years; from January 1957 to March 1996. It can be seen in Figure 1 that the most dramatic movements in the Chilean inflation rate series are the acceleration of the inflation rate that started in 1972 and its subsequent deceleration in the period between 1974 and 1977.

UNIT ROOT TESTS

The first procedure used to test for unit roots in the Chilean inflation rate series is the univariate ADF test. This test involves regressing the first difference of a variable on a constant, the variable's lagged level and k-lagged first differences. The regression used here for the ADF test does not contain a time trend. Thus, the following equation is used:

$$\Delta\pi_t = \mu + \alpha\pi_{t-1} + \sum_{i=1}^{k} c_i \Delta\pi_{t-i} + \varepsilon_t . \qquad (1)$$

In this test, the null hypothesis of a unit root is rejected in favor of the alternative of level stationarity if α is significantly different from zero. The order of k is chosen using the recursive procedure[2].Maximum k (k_{max}) is set at 16 with the significance determined at the 10 % level of the asymptotic normal distribution. The critical values used are obtained from MacKinnon (1991) adjusted for 471 observations. The result of this test is reported in Table 1. Using the ADF test, the unit root null cannot be rejected at any considered level.

Table 1: Augmented Dickey Fuller Test

α	t_{α}	k	Critical Values for ADF Tests		
			1%	5%	10%
-0.02	-2.35	12	-3.44	-2.87	-2.57

The argument made in this paper is that the Chilean inflation rate series is actually a stationary process but that the existence of structural change in the series biases the ADF test toward nonrejection. It is suggested that the behavior of the Chilean inflation rate series has been dominated by the acceleration and the subsequent deceleration of the inflation rate between 1972 and 1977. It is also argued that the acceleration and the deceleration of the inflation rate over this period are exogenous events that are not realizations of the usual data generating process. The solution of this problem requires the use of unit root tests that will isolate the effect of the rapid acceleration and deceleration of the inflation rate. It is obvious that the usual tests for unit roots in the presence of structural change that allow a one-time break in the intercept, slope, or both will not be able to account for these sharp fluctuations in the Chilean inflation rate.

[1] The possibility of the existence of seasonal unit roots in the data has been addressed.
[2] As shown by Campbell and Perron (1991) and Ng and Perron (1995), this procedure has better size and power properties than other methods that select k based on information criteria.

In a recent paper, Papell (2002) proposes a unit root test that extends Perron's changing growth model. This test allows for three endogenously determined changes in the slope of the trend function of a series. It is well suited to model the above mentioned swings in the Chilean inflation rate. The first break will pick up the beginning of the acceleration of the inflation rate in 1972, the second break will correspond to the switch from the acceleration of the inflation rate to its deceleration in 1974, and the third break will depict the end of the deceleration of the inflation rate in 1977. It is worth noting that since the break dates are endogenously determined, there is nothing that constrains them to be the same as, or even close to, the above-mentioned dates.

The proposed test therefore tests the null hypothesis of a unit root without breaks against the alternative of level stationarity with three breaks in (changes in the slope of) the deterministic trend of the series. Following Perron (1989, 1997) and Perron and Vogelsang (1992), the model used is of the additive outlier (AO) type since changes in inflation can occur rapidly[3]. This model is given by a two-step procedure. The first step detrends the series using the following regression:

$$\pi_t = \mu + \gamma_1 DT1_t + \gamma_2 DT2_t + \gamma_3 DT3_t + z_t , \qquad (2)$$

where the breaks occur at times TB1, TB2 and TB3, and the dummy variables DTi_t = (t-TBi) if t > TBi, 0 otherwise, i = 1,…,3. The fact that there is no time trend in equation (2) produces a constant mean prior to the first break.
The residuals from the above equation are then used in the following regression:

$$\Delta z_t = \alpha z_{t-1} + \sum_{i=1}^{k} c_i \Delta z_{t-i} + \varepsilon_t . \qquad (3)$$

The null hypothesis of a unit root is rejected if □ is significantly different from zero. The order of k is chosen using the recursive procedure with maximum k set at 16, and the 10% value of the asymptotic normal distribution used to determine significance. The coefficients on the dummy variables, which correspond to the breaks, are restricted to produce a constant mean following the third break by the addition of the restriction,

$$\gamma_1 + \gamma_2 + \gamma_3 = 0. \qquad (4)$$

As proposed by Perron and Vogelsang (1992), there are two possible methods for selecting the break dates endogenously. The first method chooses the breaks which minimize the t-statistic on □ in equation (3). The second chooses the breaks which maximize the joint F-statistic on DT1, DT2, and DT3 in equation (2). Because the goal here is to pick the episode that is possibly the reason for the nonrejection of the unit root null, the trend should fit the data as closely as possible, and therefore the second method for selecting the break dates is used.

Since the estimation of three globally chosen breaks is computationally intensive, especially with as many observations as there are in the data used, the following procedure is followed. The first break is chosen globally by picking the break that produces the best fit for the equation,

$$\pi_t = \mu + \gamma_1 DT1_t + z_t . \qquad (5)$$

Once a first break is obtained, it is fixed and a second break is chosen to produce the best fit for the equation,

$$\pi_t = \mu + \gamma_1 DT1_t + \gamma_2 DT2_t + z_t . \qquad (6)$$

After two breaks are obtained, these two breaks are fixed and a third break that produces the best fit for equation (2) is chosen, imposing the restriction given by equation (4). The next step is to set TB1 = TB2 and TB2 =

[3] Vogelsang and Perron (1998) find that there is not much loss in size and power if the AO model is incorrectly applied to IO data and vice versa which implies that the choice of one or the other should not affect the outcome of the test.

TB3 and choose a new TB3 that produces the best fit for equation (2). This process is repeated until convergence is obtained to three breaks that produce the best fit for equation (2) and that do not change with the repetition of the above-mentioned process[4]. The result of this procedure is presented in Figure 2. When the break dates are finally chosen, the unit root test statistic is calculated as in equation (3). The critical values for this test are obtained from Papell (2002). These critical values are –5.00 for the 1% level, -4.30 for the 5% level and –3.91 for the 10 % level[5].

The results of the unit root test are tabulated in Table 2. The break dates obtained correspond very closely with the expected dates for changes in the slope of the series based on the history of the Chilean inflation. Also, the coefficient on the dummy variable depicting the first break is positive because this break corresponds to the beginning of the acceleration of the inflation rate. The coefficient on the second dummy variable is negative because this is when the inflation rate starts to decelerate; and the coefficient on the third break is positive, citing the end of the deceleration of the inflation rate. Finally, the t-statistic on □ is equal to –5.51. By comparing this t-statistic to the above-mentioned critical values, the null hypothesis of a unit root is strongly rejected at the 1 % level.

Table 2: Univariate Unit Root Test Allowing For Three Trend Breaks

□	$t_□$	k	**Critical Values**		
			1%	5%	10%
-0.35	-5.51	12	-5.00	-4.30	-3.91

Break Dates and Coefficients					
TB1	TB2	TB3	$□_1$	$□_2$	$□_3$
72:08	74:03	77:11	30.78	-44.19	13.41

CONCLUSION

This paper is an application of the subject of testing for unit roots in a time series in the presence of structural change. The series used is that of the monthly Chilean inflation rate. The use of the standard univariate augmented Dickey Fuller test resulted in the inability to reject the unit root null for this series.

The argument made here is in the spirit of the findings of Perron (1989). The paper investigates whether the presence of structural change in the series, as represented by the 1972 – 1977 episode of acceleration and deceleration of the inflation rate, biased the ADF test towards nonrejection of the unit root null.

To account for the structural change, a test developed by Papell (2002) which allows for three endogenously determined breaks in the slope of the trend function of the series is applied. This test is better suited to model the swings in the Chilean monthly inflation rate than the tests that allow for a single break in the intercept or slope of the series. Once the rise and fall of the Chilean inflation rate between 1972 and 1977 are taken into account, the unit root null can be strongly rejected in favor of the alternative of level stationarity with three breaks in the deterministic trend of the series.

The results of this paper show that the Chilean inflationary experience has been dominated by the 1972 – 1977 episode, which biased the standard ADF test toward nonrejection of the unit root null. However, the application of a unit root test that correctly accounts for this structural change results in a strong rejection of the unit root in favor of a stationary characterization of the series. This is a more reasonable outcome given the behavior of the Chilean inflation rate.

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[4] Papell (2002) finds that this procedure to determine the breaks approximated the global search procedure for the three breaks very well.

[5] Papell (2002) uses Monte Carlo methods with randomly generated data to calculate the critical values. This is done by fitting autoregressive (AR) models to the first differences of the data, using the BIC criterion to choose the optimal AR model. The optimal AR model is then treated as the true data generating process for the errors of the series. The optimal AR model is used with iid $N(0,\sigma^2)$ innovations to construct pseudo samples of size equal to the actual size of the series. This process is then repeated 5000 times to obtain the critical values for the finite sample distributions from the sorted vector of the replicated statistics.

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Figure 1. Inflation Rates - Chile (Jan 1957 - March 1996)

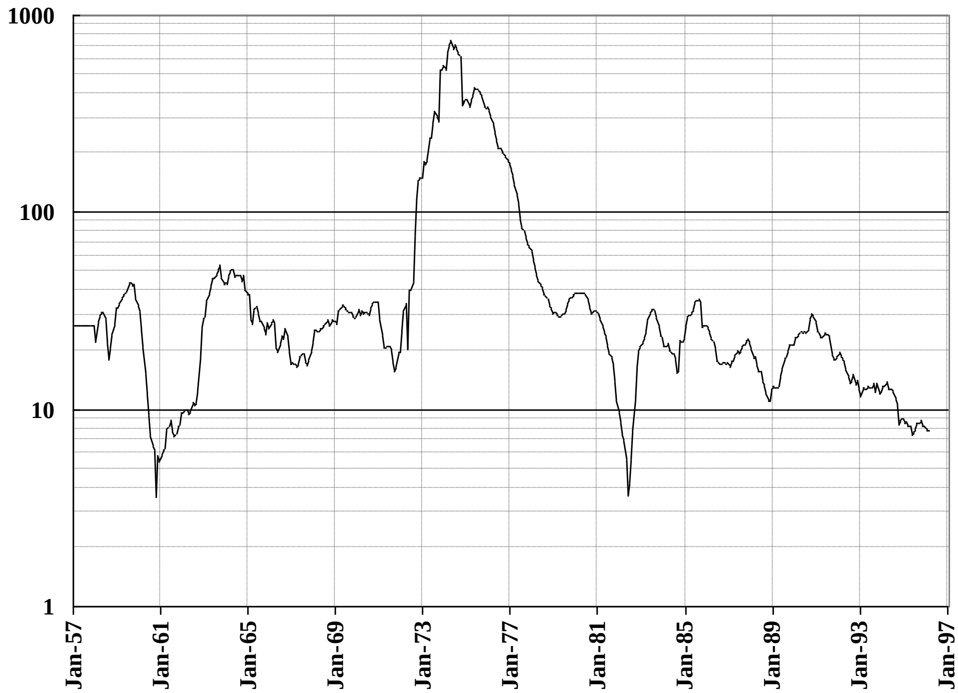

Figure 2. Inflation Rates and Fitted Curve
Chile (Jan 1957 - March 1996)