Integration Of Fuzzy Logic Control Into Continuous Passive Motion Machines

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Abstract:

Thirty-five years ago, a method for approaching classes of problems having a continuum of grades of membership was developed by Zadeh (1965). Although Zadeh recognized the potential use of this fuzzy logic-based method within the medical profession, its current use in this field has been limited when compared to other scientific disciplines (Steimann, 1997). This paper builds on the existing literature in the medical discipline by applying fuzzy logic to a particular sub-discipline: physical therapy. More specifically, this paper presents a fuzzy logic inference engine that controls a continuous passive motion (CPM) device for use with Total Knee Arthroplasty (TKA) patients post surgery.

1. Introduction and Literature Review

The literature concerning diagnostic testing and procedures in the field of physical therapy is immense. For example, a simple query search under the heading of physical therapy in Medline (http://www.nlm.nih.gov/databases/freemedl.html) results in over seventy three thousand citations. However, the use of mathematical models within specific related segments of this profession are more limited, although some literature does exist. For example, multiple linear regression has been used to assess the association of pelvic inclination and the size of lumbar lordosis in a standing position with numerous demographic and physiologic variables (Youdas et. al., 1996). More recently, hierarchical linear modeling has assisted in establishing trends for the instruction and treatment of people with lower back pain. (Kerssens, Sluijs, Verhaak, Knibbe and Hermans, 1999). Neural network model results have predicted the place of discharge or discharge Functional Independence Measure score for stroke survivors with moderate disability (Oczkowski and Barreca, 1997).

The general medical literature also contains references to fuzzy logic. For example, (1) fuzzy logic has been associated with echocardiograms, electrocardiograms, coronary arteriograms, magnetic resonance imaging, and physiologic monitors as it concerns diagnostic testing, (2) fuzzy controllers have been applied to drug infusion devices, ventilators, artificial hearts, and pacemakers, and (3) fuzzy systems have been incorporated into expert systems which appear in both the diagnosis and treatment of cardiac disease, cancer, and antibiotic therapy (Vitez, Wada and Macario, 1996). For example, Suryanarayanan, Reddy, and Canilang (1995) develop a fuzzy logic diagnosis system for classification of patients with pharyngeal dysphagia. They conclude that there exists complete agreement between the fuzzy system classification into four categories of risk for aspiration and the clinician's classification in 18 of 22 patients.

Despite its appearance in the medical literature, the application of fuzzy logic to physical therapy is quite scarce, although some literature does exist. For example, Bell and Crumpton (1997) present a fuzzy linguistic model that predicts the risk of carpal tunnel syndrome in an occupational setting. Popovic (1993) develops a model that addresses the real time control of locomotion with functional electrical stimulation (FES) assistive systems. It is shown that a skill-based expert system containing basic production rules can be used in multi-joint FES systems where the synthesis of the production rules uses fuzzy logic in conjunction with artificial neural networks. Davoodi and Andrews (1998) use a computer model to assess the theoretical feasibility of FES assisted systems using a

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closed-loop self-adaptive fuzzy logic controller based on reinforcement machine learning. This controller accommodates simulated disturbances attributed to voluntary arm forces, FES induced muscle fatigue, and anthropometric differences between individuals.

The model presented here contributes to this basic body of literature by illustrating how fuzzy logic may be applied to a CPM device used for post TKA. The model provides recommendations for the torque applied to the knee joint. This recommendation may be based on factors including (1) “end feel,” (2) resistance encountered, and (2) subsequent excursion of range of motion (ROM).

Our proximate motivation is to present a methodology which may assist and perhaps improve CPM: The current literature indicates that considerable controversy exists regarding the benefits of CPM when compared to physical therapy as it concerns knee ROM (Worland et. al., 1998). As the general literature indicates that the use of fuzzy logic may improve performance when applied to technological processes (Stevens, 1993) such as commercial products for the home, automobile components, industrial equipment, and financial analysis tools (Munakata, T. and Jani, Y., 1994), it is hypothesized here that a similar implication may apply to CPM, i.e. CPM performance may improve when compared to physical therapy through the integration of fuzzy logic. Moreover, it has already been suggested that improved performance may result in cost reduction for TKA rehabilitation services overall (Worland et. al., 1998).

The paper proceeds as follows: Section 2 provides an overview of those components of fuzzy logic/sets that are used in the basic fuzzy logic controller proposed here and a description of the fuzzy inference engine. Section 3 provides a numerical example demonstrating how this fuzzy logic controller may be utilized and contrasts the numerical results from the fuzzy logic controller with a standard controller that does not utilize fuzzy logic. Section 4 is by way of conclusion and final comments.

2. The Fuzzy Inference Engine

In classical/binary logic the truth value of statements concerns the classification into one of two disjoint sets: “true” or “false.” However, there exist instances where the concept of true/false does not have distinct or crisp boundaries; in these cases situations are often better described with gradations or memberships within these two sets. The distinction between fuzzy and binary logic concerns the moderation of traditionally crisp concepts such as true/false, black/white, etc. Moreover, fuzzy logic also addresses the quantification of linguistic modifiers that are commonly encountered in clinical settings, such as *very* stiff or *rather* flexible. For example, a clinician would ordinarily choose to state that a knee joint is inflexible, rather than state that its inertia is 100 Newton-Meters ($N \cdot m$) of torque: we simply do not think or communicate in such crisp terms, despite the fact that such phenomena are measurable using crisp values.

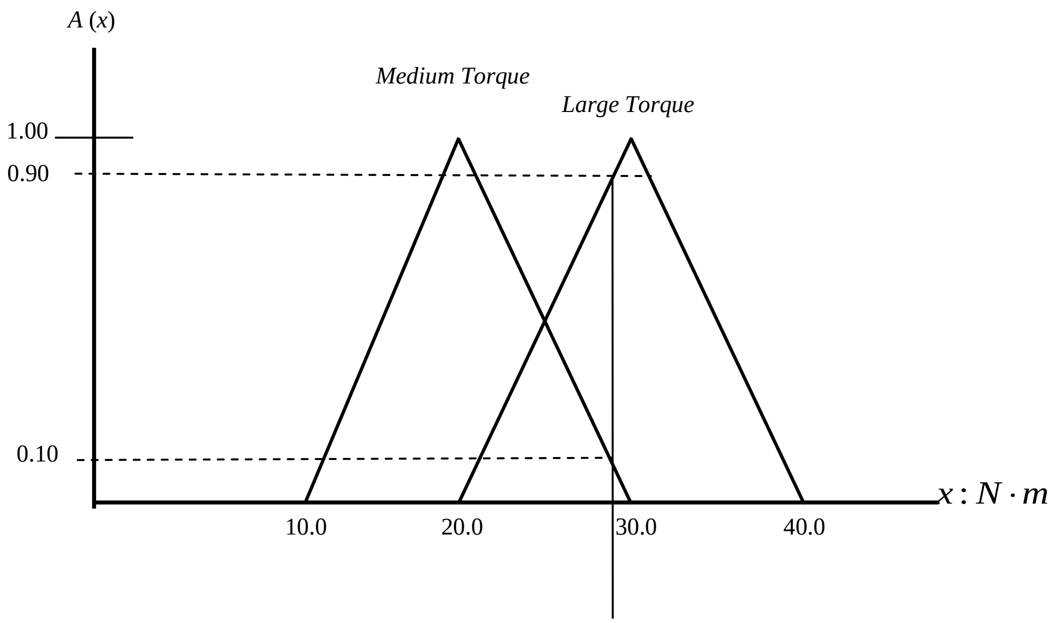
Fuzzy inference allows for expert judgment to be incorporated into the mapping of semantic inputs to numerical values with associated membership values. In the context of medical diagnosis, this translation can be accomplished by first developing a consensus from a panel of experts utilizing any of a number of well known principles, such as the delphi technique, among others (Cougar, 1995). The benefits of similar collaborative methodologies applied to physical therapy is not pursued further here, but rather, we consider this as an implication for future research. Note that the use of expert opinions and subjective evaluation has precedence in the medical literature: for example, Eddy (1982) addresses the application of Bayesian methodology to clinical diagnosis, and Tom and Schulman (1997) discuss the integration of subjective input into other common decision analytic methods applied to the health care industry, including decision trees, Markov models, and Monte Carlo simulation.

Fuzzy sets allow for partial membership in sets that are customarily considered to be disjoint. For example, an individual with knee joint inertia of 100 $N \cdot m$ could be classified as both stiff and *very* stiff, with a nonzero degree of membership in either set. Degree of membership expresses the extent of compatibility between the level of the attribute being evaluated and the concept represented by the fuzzy set **(**Klir, St. Clair, Yuan, 1997). This membership ranges, inclusively, between zero and one: zero implying no membership or compatibility in a set or class and one representing complete compatibility. More specifically, the membership for a crisp value x in fuzzy set

A, defined as $A(x)$, characterizes the degree of membership of that value in the fuzzy set (Klir, St. Clair, and Yuan, 1997).

Consider an example: 29 $N \cdot m$ of torque applied to the knee represents a crisp input. This numerical input may have membership values in both fuzzy sets A: *Medium Torque* and B: *Large Torque* of 0.1 and 0.9, respectively (The resulting sum of 1.0 is coincidental). The generalization of this process across all crisp input values (x) for torque results in a pictorial (and algebraic) representation of the corresponding fuzzy sets for both medium and large torque. A possible visual depiction of these resulting two fuzzy sets is shown below in Figure 1. The actual process of assigning fuzzy set memberships is customarily referred to as *fuzzification*.

Figure 1
Fuzzy Sets (Torque Applied To The Knee)



The determination of policy based on this fuzzification is pursued through the application of *inference rules* and *defuzzification*. Inference rules recommend actions/decisions based on the particular fuzzy membership values encountered. For example, a prototypical set of inference rules may include the following:

Fuzzy Inference Rule 1: If the previous torque applied (PTA) to the knee (at time $t-1$*) is medium and the change in resistance detected (CRD) from the knee (between times* $t-1$ *and t) is small positive then accelerate the torque applied to the knee by a small positive amount.*

Note that the descriptors *medium* and *small positive* are, as previously alluded to, fuzzy sets. The antecedents, such as medium torque applied and small positive change in resistance from the knee, result from the categorization of the particular crisp inputs witnessed for these phenomenon. In general, any crisp input may result in the simultaneous consideration of multiple inference rules, which, in turn, may each provide alternative or unique recommendations in the consequent. For example, we stated earlier (and illustrated in Figure 1) that a torque of 29 $N \cdot m$ also has positive membership in the fuzzy set *large torque*. Alternatively, consider the previously stated change in resistance from the knee of 3 $N \cdot m$ with membership in two fuzzy sets: small positive change and zero change. Based on this particular numerical crisp input, an alternative inference rule that would be considered simultaneously with Fuzzy Inference Rule 1 would be the following:

Fuzzy Inference Rule 2: If the PTA to the knee (at time $t-1$) is medium and the CRD from the knee (between times $t-1$ and t) is zero then accelerate the torque applied to the knee by a medium positive amount.

Note that for inference rule 2 above, the recommendation, or consequent, is to accelerate the torque applied to the knee by a medium positive amount, contrasted to the recommendation from inference rule 1, which was to accelerate the torque applied to the knee by a small positive amount. When crisp inputs result in the simultaneous consideration of multiple inference rules, we simply state that these multiple rules *fire* simultaneously.

In terms of continuous passive motion, the above rules may be applied in a context such as the following: A client with impaired ROM of the knee requires therapy to increase extension. Therapy involves the use of a CPM machine (CPMM) which will monitor both the torque applied to, and the change in resistance exerted by the knee. The CPMM then recommends changes in torque applied to the knee dynamically over time. For example, a medium torque is being applied to the knee. As the knee is extended a medium positive change in resistance occurs. The CPMM will use this information in conjunction with the current level of torque applied to the knee to modify the recommendation concerning torque applied to the knee. This recommendation then impacts on ROM. Note that for the purposes of the model presented in the following section, we limit our set of crisp inputs utilized to those specifically sited above: PTA to the knee and the CRD from the knee. Other possible inputs could include, but are not limited to: ROM, pain reports, relative position in the CPMM, etc.

Linguistic recommendations (or consequents) must be quantified through the process of defuzzification. Methods for defuzzification have evolved over time and are required in order to convert fuzzy recommendations into crisp numerical outputs **(**Klir, St. Clair and Yuan, 1997; Vitez, Wada, and Macario, 1996).

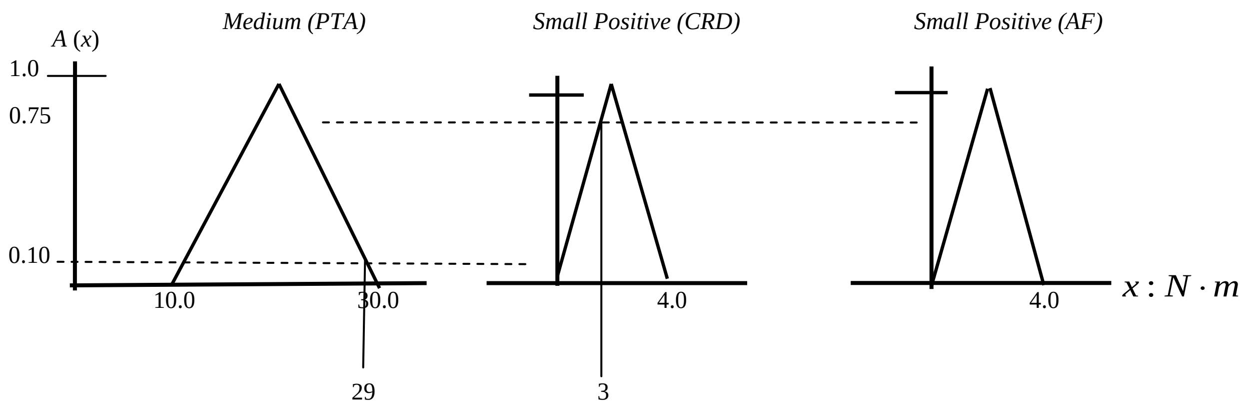
The process of defuzzification first involves the development and utilization of output membership functions for the consequents implied by the various inference rules that fire for any given set of crisp inputs. The membership for any categorical linguistic output fuzzy set, such as small positive acceleration in torque applied to the knee, is first determined by intersecting all fuzzy sets that apply to the antecedents for each of the firing rules. In the case of two fuzzy input sets *A* and *B*, each defined over some universal crisp input set X, containing values $x \in X$, the intersection of fuzzy sets *A* and *B* is defined as:

$$(A \cap B)(x) = \min[A(x), B(x)] \tag{1}$$

(Zadeh, 1965). This result may be extended to fuzzy sets that are measured in neither identical units nor scale. The reader is referred to Klir, St. Clair, Yuan (1997) for additional technical details which are further illustrated in Appendix 1, and visualized below, in Figure 2, where we illustrate how to determine the degree to which the consequent in fuzzy inference rule 1 fires by intersecting the two antecedents *Medium* and *Small Positive*:

The two fuzzy sets depicted at the left of Figure 2 represent antecedent fuzzy sets for PTA to and CRD from the knee. The leftmost fuzzy set indicates a crisp input of 29 $N \cdot m$ PTA to the knee with associated membership value .10 in the fuzzy set medium torque. Adjacent to this is the fuzzy antecedent set for small positive change in resistance exerted by the knee, with a crisp input of 3.0 $N \cdot m$ and associated membership value of .75 in the fuzzy set small positive. The rightmost fuzzy set represents the consequent fuzzy set for small positive torque applied to the knee. The membership within this fuzzy consequent set is derived from equation (1), namely, by taking the minimum membership from the two antecedent fuzzy sets, which results in a membership of .10. Note the consequent label "AF," representing acceleration factor. Further detail concerning this linguistic choice is provided in a subsequent section. A similar procedure would be followed and applied to all inference rules that fire for this given set of crisp inputs. These resulting fuzzy consequent sets (one per firing rule) must then be defuzzified so as to provide the recommended crisp output.

Figure 2
Intersection of Two Fuzzy Sets (Fuzzy Inference Rule 1)



Our method of choice for defuzzifying these fuzzy consequent sets follows the centroid method presented by Warburton (1992), although other methods, such as the center of area method (COA) **(**Klir, St. Clair, and Yuan, 1997), and the maximum defuzzification method (Adcock, 1993), have also been proposed. Note that the determination of the relative advantages and disadvantages for each of these methods lies beyond the scope of this analysis and is left as an implication for future research. Note that the defuzzification method utilized here incorporates all consequent fuzzy sets. Alternative methods, such as the COA method, neglect some output sets: a limitation that has been criticized in some of the existing literature (McNeill and Frieberger, 1993)**.** The determination of a crisp defuzzified output recommendation, as presented here using the centroid method, is reconciled using a weighted average. In general, the centroid is located for each fuzzy consequent set, and these centroids are then weighted by their associated membership values. A weighted average across all fuzzy consequent sets that fire is then determined. For the particular numerical inputs discussed earlier, the crisp output concerning the acceleration factor applied to the numerical torque to be administered to the knee equals 4.33 $N \cdot m$. The details concerning this calculation are provided in Appendix A.

3. Fuzzy Logic Control of CPM

CPM is an important part of a balanced and successful rehabilitation program for those who have undergone knee replacement. The current practice of CPM requires qualified staff to continually monitor patients (Booth, 1999) on a CPMM. These clinicians must frequently monitor the use of the CPMM to address changes over time in the physiological state of a patient's knee joint. Note that current CPMM designs incorporate neither artificial intelligence, expert systems, nor quantitatively driven decision analytic models, although rudimentary rule-based methods have been incorporated into the OptiFlex™ CPMM (whose rule-based methodology is not grounded in optimization). Vitez, Wada, and Macario (1996) provide insight concerning the benefits of classical crisp controllers versus heuristic techniques commonly used by clinicians, and the differences in fuzzy logic control versus three types of classical controllers, including the advantages of implementing a fuzzy controller rather than a proportional crisp/standard controller. Our model will illustrate how fuzzy logic may drive a CPMM to apply an ex-ante declared level of torque for flexion (or extension: these torques need not be identical) as prescribed by a physical therapist or physician to a knee joint, and demonstrate associated numerical advantages concerning mean time to achievement and lower variability in torque applied over time. Moreover, since fuzzy control error tends to diminish toward a goal (McNeill and Freiberger, 1993), it is anticipated that the fuzzy controller will provide a beneficial implication concerning overpressure to the knee joint, an undesirable condition that is possible and rather common when applying digital control or heuristics.

Normally, post surgery, a recommended ROM is determined through the prescription of both flexion and extension settings on a CPMM. This ROM is derived by a practitioner from “end-feel” (based on both resistance and change in resistance exerted by the knee through manual manipulation of the joint) in conjunction with visual input (usually with the assistance of a goniometer) of the joint’s excursion, although factors associated with resistance during this evaluation ordinarily supercede and limit all other ROM evaluations of joint rotation. Alternatively, the physician could make recommendations for maximum torque to be applied for both flexion and extension settings with the assistance of a dynamometer: This approach is taken here. The potential advantage of utilizing output measurements in terms of torque rather than angular ROM settings derives from the following: potential changes in the post operative knee over time with respect to changes in flexibility due to swelling, scarring, stretching etc., would create potential complications if a constant ROM is maintained. However, by utilizing a recommended torque setting, these potential changes would be accommodated through dynamic changes in ROM when applying constant torque. Alternatively stated, the “fuzzy” CPMM provides reasonable joint excursion resulting from the recommended level of torque applied to the knee. On the other hand, holding the terminal angles of flexion and extension constant (as opposed to constant torque) predisposes the joint to being subjected to over/under pressure if the state of the knee is changed with respect to volume, tension, or compressive constraints. This, in turn, can increase trauma to the knee in the case of an overpressure or conversely result in an inefficient excursion that will not effectively stretch the joint structure in the case of under pressure.

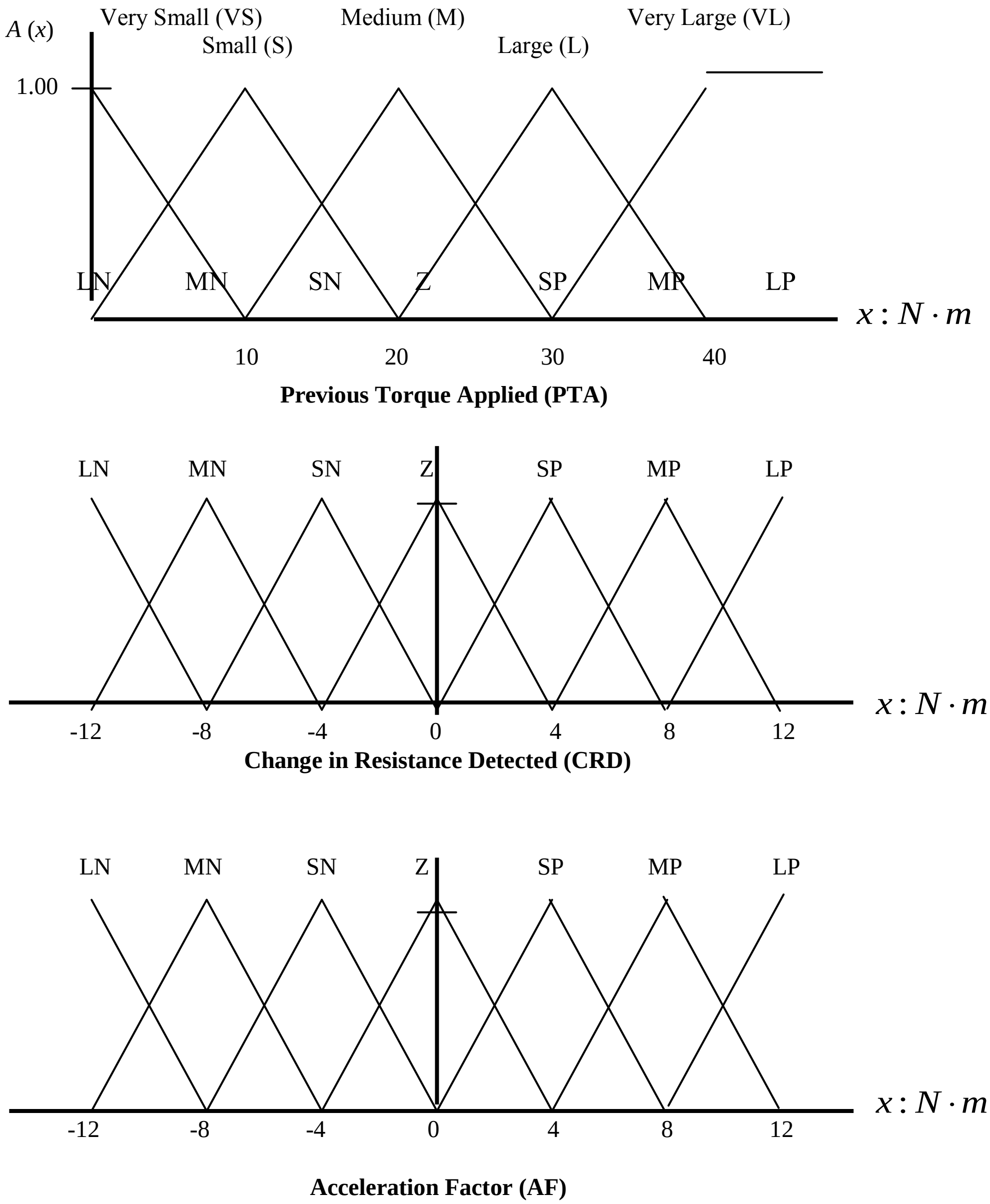
The fuzzy mathematical analysis presented here incorporates the three components of a fuzzy controller: fuzzification, fuzzy inference, and defuzzification, and contrasts numerical results from this resulting fuzzy controller with a standard controller. A proprietary simulation model was developed utilizing Excel^{TM} spreadsheet software and Visual BASIC to accommodate the model presented here. Results from the numerical analysis presented here support the claims discussed earlier, namely, that the fuzzy logic controller (1) consistently achieves and maintains an ex-ante declared goal, and (2) achieves this goal with more efficiency (i.e. smaller mean time to achievement coupled with lower variability).

To begin, a method of fuzzy set construction is chosen: we employ the common approach of utilizing triangular membership functions for inputs (PTA to the knee and CRD from the knee) and outputs (AF: acceleration in torque applied to the knee), although numerous techniques concerning the elicitation of membership values and the shape of membership functions have been discussed in the literature (Klir and Yuan, 1995). An example of two such membership functions were previously shown in Figure 1. The complete set of fuzzy sets for inputs and outputs utilized in this analysis is presented below, in Figure 3.

The fuzzy inference engine is then designed. As previously discussed, this engine relies upon the incorporation of fuzzy inference rules. The complete set of inference rules utilized in this numerical example are shown on page 122 in Table 1. These inference rules are designed with the objective of achieving a terminal torque applied to the knee of forty $N \cdot m$.

A brief denotation for acceleration, as utilized in this model, is presented here for purposes of clarity. First, note that it is anticipated that the torque applied to the knee should exceed resistance detected from the knee in order to continually achieve increasing range of motion. Should the resistance detected from the knee equal or exceed torque applied to the knee the CPMM would cease to achieve gains in ROM and the knee joint would remain static. Hence, for any CRD, the new torque applied will be increased or decreased from the previous torque by CRD $\pm$ an “acceleration” factor (AF). AF is positive when the recommended change in torque applied to the knee should exceed the CRD, and negative when the recommended change in torque applied to the knee is less than the CRD. When AF equals zero, changes in torque applied will simply offset CRD. Positive AF values are desirable for small ROM (i.e. the knee is extended) where low levels of resistance are ordinarily detected from the knee (McHugh, Kremenic, Fox, and Gleim, 1997). However, AF should typically approach zero and become negative when the torque applied to the knee approaches the ex-ante declared goal. This will result in the eventual convergence of torque applied and the ex-ante declared goal. The fuzzy inference rules in Table 1 were designed in order to achieve this ex-ante declared goal for torque applied to the knee.

Figure 3
Fuzzy Set Representation For Antecedents And Consequent



KEY: LN: Large Negative MN: Medium Negative SN: Small Negative Z: Zero
SP: Small Positive MP: Medium Positive LP: Large Positive

Table 1
Inference Rules Utilized For The Fuzzy Inference Engine

If the previous torque applied (PTA) to the knee (at time $t-1$) is:	And the change in resistance detected (CRD) from the knee (between times $t-1$ and *t*) is:	Then accelerate the torque applied to the knee by a ____________ amount.
Very Large (VL)	Large Negative (LN)	Large Positive
	Medium Negative (MN)	Medium Positive
	Small Negative (SN)	Small Positive
	Zero (Z)	Zero
	Small Positive (SP)	Small Negative
	Medium Positive (MP)	Medium Negative
	Large Positive (LP)	Large Negative
Large (L)	Large Negative (LN)	Large Positive
	Medium Negative (MN)	Large Positive
	Small Negative (SN)	Medium Positive
	Zero (Z)	Small Positive
	Small Positive (SP)	Small Positive
	Medium Positive (MP)	Zero
	Large Positive (LP)	Small Negative
Medium (M)	Large Negative (LN)	Large Positive
	Medium Negative (MN)	Large Positive
	Small Negative (SN)	Large Positive
	Zero (Z)	Medium Positive
	Small Positive (SP)	Small Positive
	Medium Positive (MP)	Small Positive
	Large Positive (LP)	Zero
Small (S)	Large Negative (LN)	Large Positive
	Medium Negative (MN)	Large Positive
	Small Negative (SN)	Large Positive
	Zero (Z)	Large Positive
	Small Positive (SP)	Medium Positive
	Medium Positive (MP)	Small Positive
	Large Positive (LP)	Small Positive
Very Small (VS)	Large Negative (LN)	Large Positive
	Medium Negative (MN)	Large Positive
	Small Negative (SN)	Large Positive
	Zero (Z)	Large Positive
	Small Positive (SP)	Large Positive
	Medium Positive (MP)	Medium Positive
	Large Positive (LP)	Small Positive

To run the inference engine, the Excel spreadsheet model generates random numbers concerning the crisp CRD input to the model for each iteration. Note that this method of determining input concerning changes in resistance detected from the knee was selected to facilitate required numerical calculations, however, prior to this model becoming operational in practice, further research concerning the relationship between torque applied and change in resistance detected must be considered. For example, information gathered from a torque/ROM curve (McHugh, Kremenic, Fox, and Gleim, 1997) would generate more realistic inputs related to joint status than random input alone. We also choose to bound the change in resistance detected to $\pm 12 \;\; N \cdot m$. Changes outside of this range are rarely encountered in practice, but can be accommodated by the model: this range increase would increase the degree of complexity as it concerns the mathematical calculations required here, without any substantive benefit. Finally, an initial condition of 12 $N \cdot m$ of torque is applied to the knee at time zero. This recommendation is consistent with standard practice in physical therapy, where an initial torque must be applied when the knee is at rest and expectations are that little resistance will be detected when torque is initially applied. It may be shown that this initial recommendation represents the crisp output that would be obtained from the fuzzy inference engine for initial crisp inputs of zero PTA and zero CRD (i.e. the knee is at rest and fully extended).

The fuzzy controller registers the accompanying CRD witnessed for any torque applied to the knee over a discrete time interval Δt. The torque applied together with the change in resistance witnessed form the crisp inputs which must then be fuzzified utilizing the previously elicited fuzzy input sets. Table 2, shown below, illustrates the Excel spreadsheet output concerning the fuzzification for the previously discussed crisp input values 29 $N \cdot m$ PTA and 3 $N \cdot m$ CRD.

Table 2
Excel Output For Fuzzification Of Crisp Inputs

Previous Torque Applied (PTA)						
Fuzzy Set		VS	SM	M	L	VL
Range		[0, 10)	[0, 20)	[10, 30)	[20, 40)	[30, ∞)
PTA	29*Nm*			29.00	29.00	
Membership:	*A* (*x*)	0	0	0.10	0.90	0

Change in Resistance Detected (CRD)								
Fuzzy Set		LN	MN	SN	Z	SP	MP	LP
Range		[-12, -8)	[-12,-4)	[-8,0)	[-4,4)	[0,8)	[4,12)	[8,12]
CRD	3 *Nm*				3.00	3.00		
Membership:	*A* (*x*)	0	0	0	0.25	0.75	0	0

The columns in Table 2 represent the fuzzy sets for inputs and the corresponding ranges. Crisp input values are shown adjacent to the headings PTA and CRD. Corresponding fuzzy set membership values for each crisp input are provided across the row labeled Membership. From Table 2, we see that the crisp value 3 $N \cdot m$ for CRD results in zero membership in all fuzzy sets with the exception of the two fuzzy sets, *zero* and *small positive*, whose membership values are .25 and .75, respectively. Similar results for 29 $N \cdot m$ PTA result in positive membership values of .10 and .90, respectively, in the two fuzzy sets *medium* and *large*, zero elsewhere. These values may also be visualized using the fuzzy sets previously shown in Figure 3. Note that the sum of membership values for each crisp input need not equal one: this coincidence results simply from the particular fuzzy sets and values utilized for this particular realization.

The CPM fuzzy model now converts these fuzzy inputs into fuzzy output utilizing the fuzzy inference rules previously shown in Table 1. We note that for the particular inputs presented here, four inference rules fire (see Appendix A). Each resulting consequent fuzzy set fires to a specified degree, as previously illustrated in Figure 2. The resulting recommended fuzzy outputs for the four rules that fire and their corresponding degrees of membership are illustrated in the Excel spreadsheet output shown on the following page, in Table 3:

From Table 3 note that four resulting cells are active (contain nonzero values). These cells correspond to the four fuzzy inference rules that fire as a result of the crisp inputs utilized. For example, from Fuzzy inference rule 1 (see section 2), recall that the resulting recommended output was to apply a small positive acceleration to the torque applied to the knee. As shown in Appendix 1 and Figure 2, this output fires with a degree of membership .10. This result corresponds to the cell in Figure 4 that is labeled *SP* and contains the value .10: this cell falls in row *M* and column *SP* which corresponds to the fuzzy input sets for fuzzy inference rule 1, medium PTA and small positive CRD. Similar reasoning may be utilized to account for the remaining three membership values that are nonzero in Table 3. We note that three of the four fuzzy inference rules that fire recommend applying a small positive acceleration in torque applied to the knee with varying degrees of membership. Table 3 also provides the overall AF recommendation for change in torque applied to the knee: 4.33 $N \cdot m$ (derived in Appendix A).

Table 3
Excel "Fuzzy Inference Engine"

then accelerate the torque applied to the knee (CAF) by a _________amount:

If Previous Torque Applied (PTA) to the knee at time *t* - 1 is:	LN	MN	SN	Z	SP	MP	LP
VL	LP 0	MP 0 0	SP 0 0	Z 0 0	SN 0 0	MN 0 0	LN 0
L	0 LP 0	0 0 LP 0 0	0 0 MP 0 0	0 0 SP 0 0.25	0 0 SP 0.75 0	0 0 Z 0 0	0 SN 0
M	0 LP 0	0 0 LP 0 0	0 0 LP 0 0	0 0.10 MP 0 0	0.10 0 SP 0 0	0 0 SP 0 0	0 Z 0
S	0 LP 0	0 0 LP 0 0	0 0 LP 0 0	0 0 LP 0 0	0 0 MP 0 0	0 0 SP 0 0	0 SP 0
VS	0 LP	0 0 LP	0 0 LP	0 0 LP	0 0 LP	0 0 MP	0 SP

And if the change in resistance detected (CRD) from the knee between time *t* - 1 and *t* is:

<u>PTA</u>
VL: very large
L: large
M: medium
S: small
VS: very small

<u>CRD</u>
LN: large negative
MN: medium negative
SN: small negative
Z: zero
SP: small positive
MP: medium positive
LP: large positive

<u>AF</u>
LN: large negative
MN: medium negative
SN: small negative
Z: zero
SP: small positive
MP: medium positive
LP: large positive

Following the determination of a recommended crisp output the appropriate acceleration in torque is applied and a subsequent iteration for the model is performed. For purposes of this analysis one run of the fuzzy CPM model constitutes thirty such iterations. For illustrative purposes, Figure 4 is a graphical representation of the output results for the CPM fuzzy controller for five such runs.

Figure 4
Output Of Five Fuzzy Controller Runs

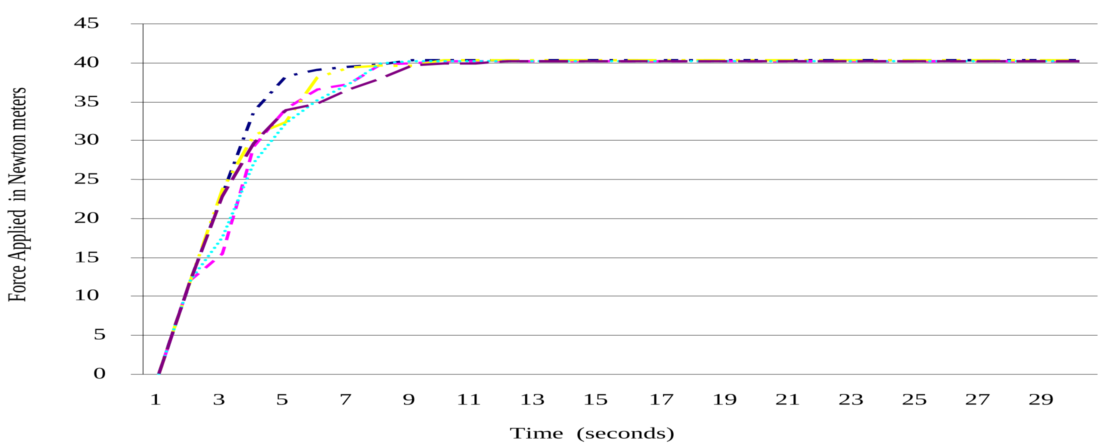


Figure 4 suggests that the torque applied to the knee increases monotonically from zero to the objective of forty $N \cdot m$ (although this is not a necessary condition of the fuzzy controller). We also note the relative consistency in the results between runs and the stability of the controller in applying the terminal torque once achieved. These findings may be contrasted with numerical results from a standard controller, which is now presented.

The standard controller for the CPMM utilizes crisp inference rules that do not incorporate fuzzy sets. A prototypical set of crisp inference rules, and the set of inference rules adhered to by the standard controller presented here, are shown below, in Table 4:

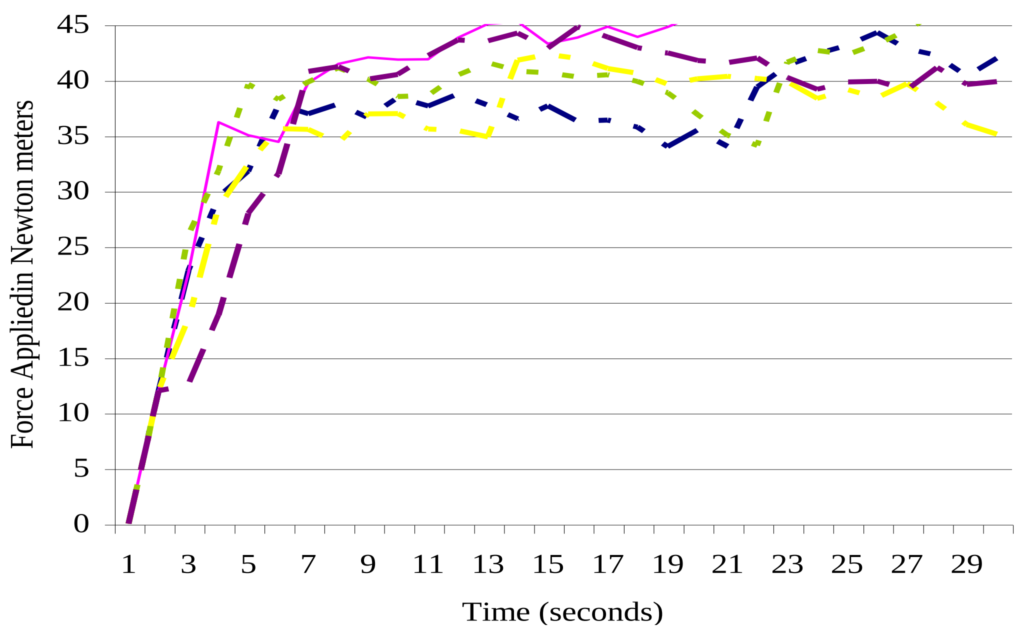
Table 4
Standard Control Inference Rules

If the previous torque applied (PTA) to the knee (at time *t*-1) lies in the range:	And the change in resistance detected (CRD) from the knee (between times *t*-1 and *t*) lies in the range:	Then accelerate the torque applied to the knee by __________ Nm/Δt.
[35, 40]	[-12, 10)	12
	[-10, -6)	8
	[-6, -2)	4
	[-2, 2)	0
	[2, 6)	-4
	[6, 10)	-8
	[10, 12]	-12
[25, 35)	[-12, 10)	12
	[-10, -6)	12
	[-6, -2)	8
	[-2, 2)	4
	[2, 6)	4
	[6, 10)	0
	[10, 12]	-4
[15,25)	[-12, 10)	12
	[-10, -6)	12
	[-6, -2)	12
	[-2, 2)	8
	[2, 6)	4
	[6, 10)	4
	[10, 12]	0
[5,15)	[-12, 10)	12
	[-10, -6)	12
	[-6, -2)	12
	[-2, 2)	12
	[2, 6)	8
	[6, 10)	4
	[10, 12]	4
[0,5)	[-12, 10)	12
	[-10, -6)	12
	[6, -2)	12
	[-2, 2)	12
	[2, 6)	12
	[6, 10)	8
	[10, 12]	4

Note that the inference rules above apply crisp output values for each crisp set of input values. Input sets follow the standard practice of being exhaustive and non-overlapping. In order to contrast the fuzzy and standard controllers, the standard controller's inference rules classify each crisp input value into the class that corresponds to the fuzzy set with the greatest membership for that crisp input value. For example, crisp inputs for PTA that are $\in [15,25)$ would be classified as medium PTA, since maximum membership values within this range of inputs, although nonzero for multiple fuzzy sets, are attained for the fuzzy class *medium*. Proceeding in this fashion, the inference rules in Table 4 are generated. Each fuzzy inference rule in Table 1 may be contrasted to a corresponding crisp inference rule in Table 4. For instance, the crisp analog for fuzzy inference rule 1 is the following: If $PTA \in [15,25)$ and $CRD \in [2,6)$ then accelerate the torque applied to the knee by 4 $N \cdot m$. The recommended output 4 $N \cdot m$ corresponds to the crisp value with maximum membership in the fuzzy output set *small positive*.

The standard inference engine provides results similar in nature to those generated by the fuzzy inference engine. Specifically, for any set of inputs (PTA and CRD), the model provides a recommendation for an acceleration factor concerning torque to be applied to the knee. Again utilizing thirty iterations per run, the graph of five such runs for the standard controller is shown below, in Figure 5:

Figure 5
Output Of Five Standard Controller Runs



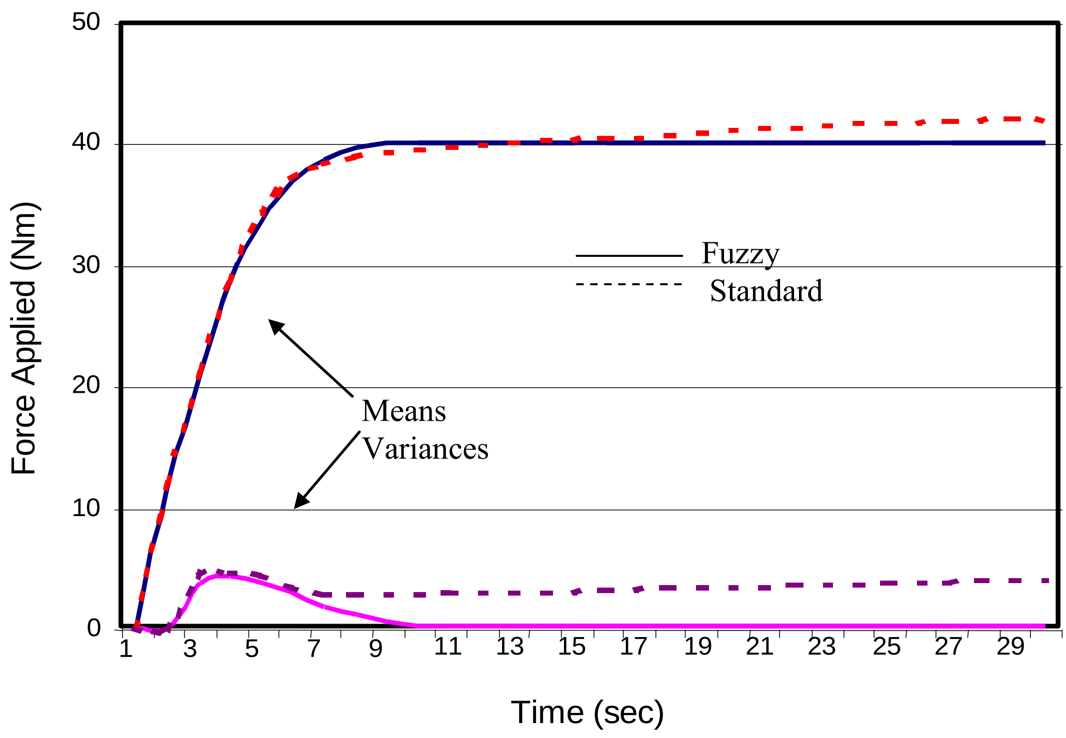
For comparative purposes, note that Figures 4 and 5 provide graphical support for three claims. Specifically, the fuzzy inference model (1) achieves the ex-ante declared goal of forty $N \cdot m$ of torque applied to the knee in less time than the standard inference model, (2) demonstrates significantly lower variability as torque applied increases, and (3) demonstrates greater stability once achieving forty $N \cdot m$ torque applied to the knee.

As it concerns (1), mean time to completion is a commonly used measure of efficiency when comparing fuzzy and standard controllers, and the slight decrease in time for objective achievement illustrated by the fuzzy controller presented here is consistent with findings elsewhere. The benefits of (2) involve provision of smooth transitions from lower torques to higher torques and vice versa. This prevents unnecessary moments of stress on knee joint structures that may occur with more abrupt transitions, and in turn provide additional comfort for the patient. As the CPMM approaches the ex-ante declared goals, fuzzy control prevents "over/undershooting" the goal which provides the advantages as stated earlier, and also provides additional patient comfort.

Concerning (3), once the goal of 40 $N \cdot m$ of torque applied is achieved, we may wish to maintain this degree of torque over a period of time. As seen from Figures 4 and 5, the crisp/standard controller shows a higher degree of variability around the goal relative to the fuzzy controller. This is significant in that trauma to the knee when overshooting and inefficient stretch when undershooting are critical drawbacks in optimizing knee range-of-motion. While some proportional, derivative and integrative controllers do address the problems of over/undershooting, the extent of programming and degree of elaboration required in the development of these models in order to achieve the same level of variability as a fuzzy controller is sometimes prohibitive (Vitez, Wada, and Macario; 1996).

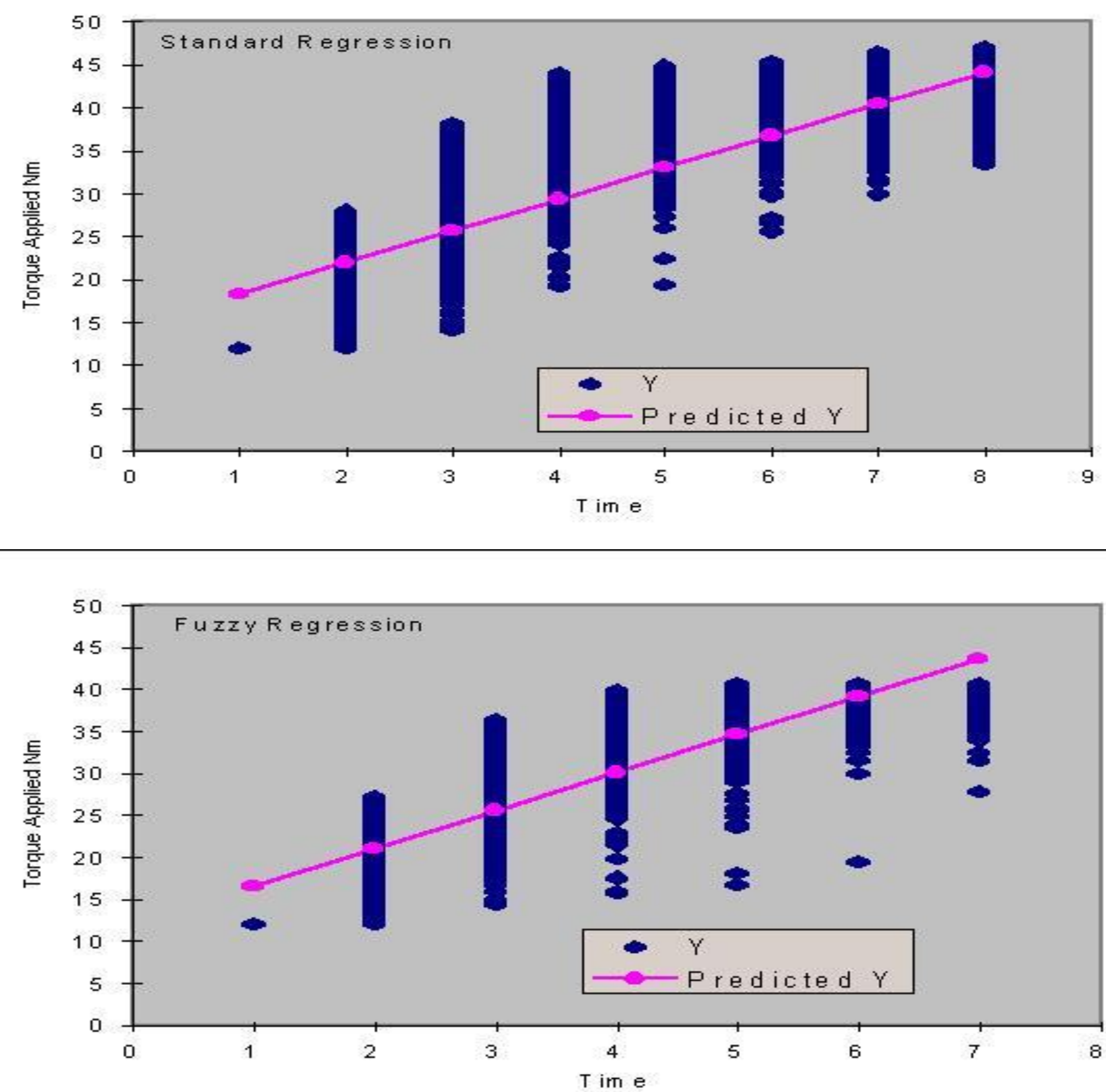
Further support for claims (1)-(3) may be generated through the examination of multiple runs, depicted graphically in Figure 6. This figure illustrates results based on five hundred runs for each controller: fuzzy and standard. Upper contours illustrate the mean torque applied for both the fuzzy (solid) and standard (dotted) controllers at each point in time. As expected, these mean contours suggest that mean torque applied over time for the fuzzy controller achieves the ex-ante declared goal of forty $N \cdot m$ in less time than the standard controller. The two lower curves suggest that the variance (over five hundred runs) at each point in time, for torque applied, is also lower for the fuzzy controller. Finally, it is clear from the figure that the fuzzy inference engines shows much greater stability when the goal of 40 $N \cdot m$ is attained. As noted earlier, similar advantages have been noted for fuzzy controllers in other contexts.

Figure 6
Comparative Analysis – Fuzzy And Standard Controller (500 Runs)



Statistical calculations further support the claims made above. For example, the mean time to goal achievement of 40 $N \cdot m$ (measured in number of time intervals required) for the fuzzy controller over five hundred runs was $\bar{x}_f = 8.89$ units of time with an associated variance of $s_f^2 = 3.21$. In contrast, the standard controller required a mean time of $\bar{x}_f = 10.32$, with a variance of $s_f^2 = 32.9$. As suggested earlier, the fuzzy controller achieves the target objective in less time, on average ($H_0 : \mu_f = \mu_s$ rejected in favor of $H_A : \mu_f < \mu_s$, $p < .01$), and with more consistency ($H_0 : \sigma_f = \sigma_s$ rejected in favor of $H_A : \sigma_f < \sigma_s$, $p < .01$). To check for uniformity in torque increases over time as the goal of 40 $N \cdot m$ was approached, a simple regression model was run and utilized for fit. Desirable uniform increases would imply linearity in the torque curve over time, hence, the linear model is appropriate due to its constant slope. Resulting regression curves for both controllers are shown below, in Figure 7:

Figure 7
Regression Comparison: Fuzzy And Standard Controllers



The resulting adjusted r^2 values are 74.1% and 82.0% for the fuzzy and standard controller, respectively, with associated standard errors of 5.14 and 4.22. Hence, the fuzzy controller's torque curve does appear to exhibit greater linearity, and hence a more uniform increase, when contrasted to the standard controller. Note that we do not claim that the particular tests utilized here are unique in supporting the results previously suggested: we present these tests simply for the purposes of indicating that simple statistical analysis on the data does suggest that results conform to the general tenets commonly observed when comparing controllers utilizing fuzzy and standard controllers. The primary purpose of this manuscript is to introduce the reader to the concept of a fuzzy inference engine within an area previously not imbued with much analysis of this nature, specifically, the area of physical therapy, as opposed to the uncontestable verification of hypotheses based on the numerical facts presented herein. This analysis is left as an implication for future research.

4. Implications for Future Research and Conclusions

This manuscript builds on the existing literature by presenting a fuzzy logic inference engine that controls a continuous passive motion device for use with Total Knee Arthroplasty patients post surgery. The paper addresses an area that is not well represented in the literature, specifically, the integration of fuzzy logic into the field of physical therapy. Physical therapists are often faced with the dilemma of being constrained from using their expertise to develop policy: fuzzy logic provides an alternative whereby subjective inputs are embodied within a formal optimization methodology. We suggest that fuzzy logic and fuzzy decision making may therefore serve as an attractive and viable alternative decision analysis tool within physical therapy, as it offers a reasonable level of interaction and control as it concerns inputs from physicians and physical therapists while simultaneously adhering to mathematically attractive optimization principles. Given the wealth of literature suggesting that benefits are attainable through the use of fuzzy logic controllers in many technological areas, it is speculated here that similar benefits may apply within the medical field, specifically physical therapy.

Preliminary results from the model presented here support this contention. Specifically, numerical results for both a standard and fuzzy controller are contrasted for inference rule based systems that attempt to achieve an ex-ante declared goal for torque applied to the knee. These results suggest that the fuzzy controller (1) achieves the ex-ante declared goal of forty $N \cdot m$ of torque applied to the knee in less time than the standard inference model, (2) demonstrates lower variability and greater uniformity as torque applied increases, and (3) demonstrates greater stability once achieving the terminal value of forty $N \cdot m$ torque applied to the knee.

Considering (1) the current practice utilized, which involves continuous monitoring of continuous passive motion machines visually and adjusting range of motion manually based on imprecise measurements obtained by "end feel" and visual inspection, (2) the rudimentary mathematical rule-based systems currently incorporated into continuous passive motion devices, (3) the resulting expected cost benefits obtained in Total Knee Arthroplasty rehabilitation services overall through increased performance and efficiency, and (4) the debilitating effects of overpressure or inexcursion of the knee joint as a result of high levels of variability in torque applied both prior to and following the achievement of a predetermined goal concerning terminal torque applied or range of motion, the results presented here are encouraging and consistent with benefits achieved in other contexts when implementing fuzzy controllers.

It should be noted that prior to becoming operational in practice, much work remains to be done. For example, refinements in the inference engine and the appropriate choice of antecedents requires a more thorough investigation, and the incorporation of empirically derived torque/ROM curves would provide greater realism. However, the model, as presented here, provides the opportunity for those within the health care industry to visualize how fuzzy inference and the use of inference engines may be of potential benefit to patients requiring health care services in physical therapy: a discipline which has not traditionally utilized analytic optimization models. The numerous open questions that remain, potential implications to future research, and extensions of the work presented here are indications that this research may serve as a beneficial and fertile area of further investigation. Hopefully this introductory piece will initiate a more thorough investigation of the application of fuzzy inference engines within the field of physical therapy. 🕮

References:

1. Adcock, B., “Implementation of Fuzzy Logic: Selected Application,” Texas Instruments Publication Docket SPRA028, 1993.
2. Bell, P. M. and Crumpton, L., “A fuzzy linguistic model for the prediction of carpal tunnel syndrome risks in an occupational environment,” *Ergonomics*, Vol. 40, No. 8, pp. 790-09, 1997.
3. Booth, R. E., “Continuous passive motion enhances recovery and flexibility,” *Joint Line* (a publication of the University of Pennsylvania Health System), Winter, Vol. 4, 1999.
4. Cougar, J. D., *Creative Problem Solving and Opportunity Finding*, Boyd and Fraser: New York, 1995.
5. Davoodi, R. and Andrews, B. J., “Computer simulation of FES standing up in paraplegia: a self-adaptive fuzzy controller with reinforcement learning,” *IEEE Transactions in Rehabilitation Engineering*, Vol. 6, No. 2, 151-161, 1998.
6. Eddy, D. M., “Probabilistic reasoning in clinical medicine: Problems and opportunities,” in *Judgment Under Uncertainty: Heuristics and Biases, Kahneman*, D., Slovic, P. and Tversky, A. (editors), pp. 249-67, Cambridge University Press: New York, 1982.
7. Kerssens, J. J., Sluijs, E. M., Verhaak, P. F., Knibbe, H. J. and Hermans, I. M., “Back care instructions in physical therapy: A trend analysis of individualized back care programs,” *Physical Therapy*, Vol. 79, No. 3, pp. 286-95, 1999.
8. Klir, G. J., St. Clair, U. H. and Yuan, B., *Fuzzy Set Theory: Foundations and Applications*, Prentice Hall: New Jersey, 1997.
9. Klir, G. J. and Yuan, B., *Fuzzy Sets and Fuzzy Logic*, Prentice Hall: New Jersey, 1995.
10. McHugh, M. P., Kreminic, I. J., Fox, M. B. and Gleim, G. W., “The role of mechanical and neural restraints to joint range of motion during passive stretch,” *Medical Science and Sports Exercise*, Vol. 30, No. 6, pp. 928-32, 1998.
11. McNeil, D. and Freiberger, P., *Fuzzy Logic. The revolutionary computer technology that is changing our World*, Touchstone Publishers: New York, 1993.
12. Munakata, T. and Jani, Y., “Fuzzy systems: An overview,” *Communications of the Association of Computing Machinery*, Vol. 37, No. 3, pp. 68+, 1994.
13. Oczkowski, W. J. and Barreca, S., “Neural network modeling accurately predicts the functional outcome of stroke survivors with moderate disabilities,” *Archives of Physical Medical Rehabilitation*, Vol. 78, No. 4, pp. 340-45, 1997.
14. Popovic, D. B., “Finite state model of locomotion for functional electrical stimulation systems,” *Progress in Brain Research*, Vol. 97, pp. 397-407, 1993.
15. Stevens, T., “Fuzzy logic makes sense,” *Industry Week*, Vol. 24, No. 5, 34-42, 1993.
16. Steimann, F., “Fuzzy set theory in medicine,” *Artificial Intelligence in Medicine*, Vol. 11, No. 1, pp. 1-7, 1997.
17. Suryanarayanan, S., Reddy, N.P. and Canilang, E.P., “A fuzzy logic diagnosis system for classification of pharyngeal dysphasia,” *International Journal of Bio-medical Computing*, Vol. 38, No. 3, pp. 207-15, 1995.
18. Tom, E. and Schulman, K. A., “Mathematical Models in Decision Analysis,” *Infection Control and Hospital Epidemiology*, Vol. 18, No. 1, pp. 65-72, 1997.
19. Vitez, T. S., Wada, R. and Macario, A. (1996) “Fuzzy logic: Theory and applications,” *Journal of Cardiothorac Vascular Anesthesia* 10 (6), 800-08.
20. Warburton, D., “How to design fuzzy logic controllers,” *Machine Design*, November, Vol. 26, pp. 92-94, 1992.
21. Worland, R. L., Arredondo, J., Angles, F., Lopez-Jimenez, F. and Jessup, D. E., “Home continuous passive motion machines versus professional physical therapy following total knee replacement,” *Journal of Arthroplasty*, Vol. 13, No. 7, pp. 784-87. Fuzzy systems: An overview, 1998.
22. Youdas, J. W., Garrett, T. R., Harmsen, S., Suman, V. J. and Carey, J. R., “Lumbar lordosis and pelvic inclination of asymptomatic adults,” *Physical Therapy*, Vol. 76, No. 10, pp. 1066-81, 1996.
23. Zadeh, L. A., “Fuzzy sets,” *Information and Control*, Vol. 8, pp. 338-53, 1965.

Appendix

Inputs to the system are: 29 $N \cdot m$ PTA and 3 $N \cdot m$ CRD. From Figure-3 it is straightforward to show algebraically that the following membership values apply: PTA membership is .10 in the fuzzy set Medium and .90 in the fuzzy set large, and CRD membership is .25 in the fuzzy set Zero and .75 in the fuzzy set Small Positive. Hence, the following four fuzzy inference rules from Table 1 will fire:

If the previous torque applied (PTA) to the knee (at time $t-1$) is:	And the change in resistance detected (CRD) from the knee (between times $t-1$ and t) is:	Fuzzy Consequent: Then accelerate the torque applied to the knee by a ____________ amount.
Medium	Zero	Medium Positive
Medium	Small Positive	Small Positive
Large	Zero	Small Positive
Large	Small Positive	Small Positive

Resulting memberships for the fuzzy consequents are determined from equation 1, and illustrated on the following page.

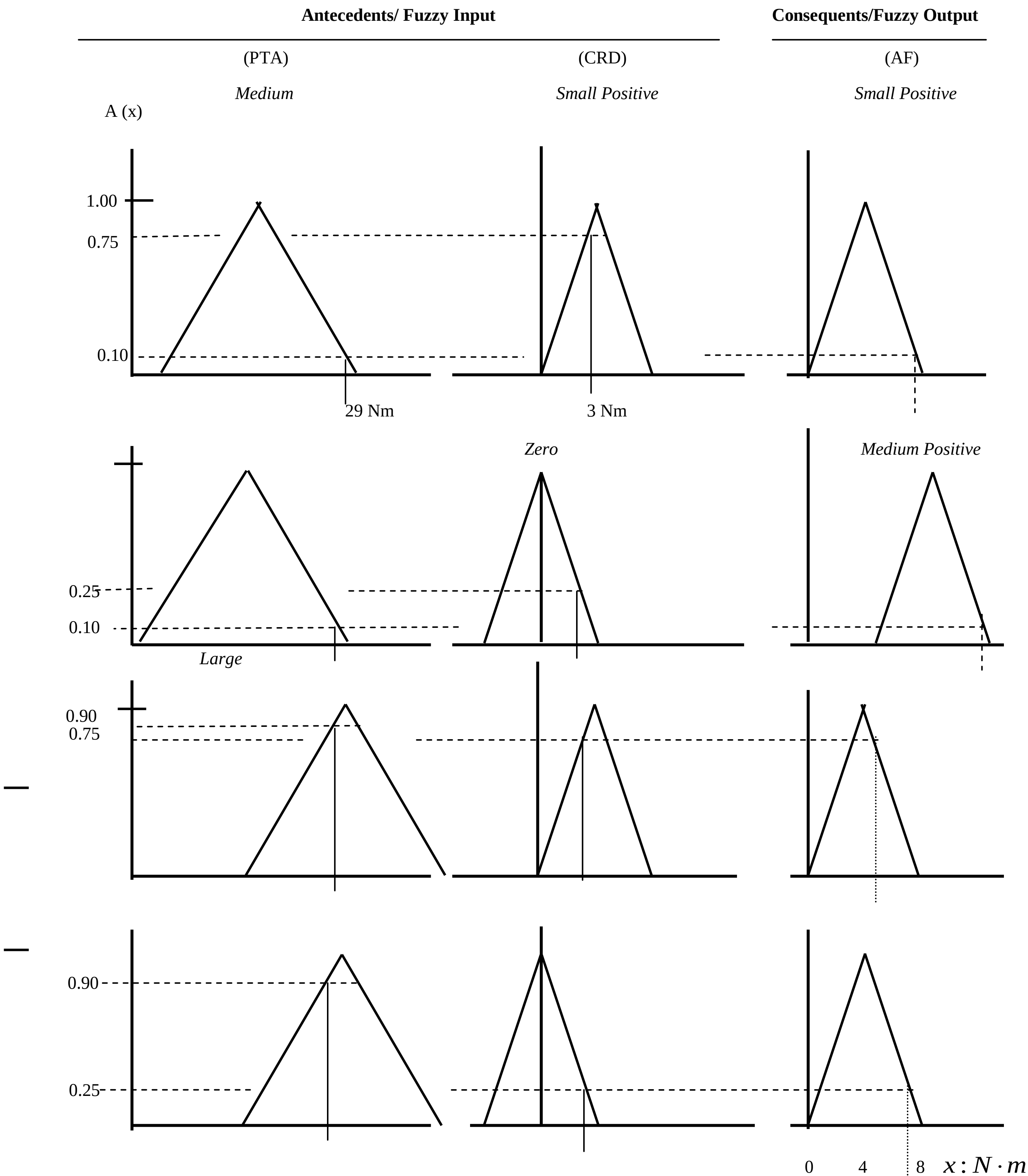
Deffuzification for the output (AF : acceleration factor) requires taking a weighted average of the consequents' centroids.. These centroids will coincide with the peaks of the triangles, for this realization, since all fuzzy sets are symmetric. The defuzzified weighted average is found as follows:

$$AF = \sum_{i=1}^{n} \left(A_i(x) \cdot x_i \right) / \sum_{i=1}^{n} A_i(x) = (.10 \cdot 4 + .25 \cdot 4 + .75 \cdot 4 + .10 \cdot 8) / (.10 + .25 + .75 + .10) = 4.33\, N \cdot m$$

where x_i and $A_i(x)$ correspond to the centroid and membership value for the associated fuzzy set i. These values are summed across all inference rules $i = 1,\ldots,n$ that fire. Thus, 4.33 $N \cdot m$ is the recommended acceleration factor. As stated in the body of the paper: the new torque applied will be increased or decreased from the previous torque by CRD $\pm$ an "acceleration" factor (AF). Therefore, the model then proceeds to determine the final output value for new torque applied to the knee by utilizing the following equation:

$$Torque_t = Torque_{t-1} + CRD_{t-1,t} + AF_t$$

Hence, the new value for torque applied to the knee for a previous torque of 29 $N \cdot m$ and change in resistance of 3 $N \cdot m$ would be: 29 + 3 + 4.33 = 36.33 $N \cdot m$.


Antecedents/ Fuzzy Input
Consequents/Fuzzy Output
(PTA)
(CRD)
(AF)
Medium
Small Positive
Small Positive
A (x)
1.00
0.75
0.10
29 Nm
3 Nm
Zero
Medium Positive
0.25
0.10
Large
0.90
0.75
0.90
0.25
0
4
8
$x : N \cdot m$