Thinking Outside the Box: An Introspective Look at the Use of Art in Teaching Geometry

Elizabeth Heskett

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Abstract

Geometry is typically thought of as a discipline in mathematics that is taught using formulas and basic shapes, but this idea is only the beginning. Geometry can be combined with art to enhance mathematical lessons for students. Educators must realize that visual representations of different mathematical concepts are a wonderful way to teach children geometry in a meaningful way. The "fundamental notion that integrating the arts into one's teaching can help facilitate learning in the mathematics classroom, as the arts can recapture the wonder of learning mathematics. The connection between instruction and imagination is bridged and learning becomes play, and play becomes learning" (Muller and Ward 22). This paper includes 7 lessons that integrate geometry with art, which will enhance the students' learning. Each lesson can be modified for any grade between kindergarten and 6th grade. These modifications will be explored in this paper along with the lesson idea. Finally, professional artwork of each type will be explored. Each piece investigated has significant mathematical ties and will be examined in the paper.

Degree Type

Open Access Senior Honors Thesis

Department

Mathematics

First Advisor

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Keywords

Geometry Study and teaching, Art Mathematics, Optical art, Origami in education

Subject Categories

Science and Mathematics Education

THINKING OUTSIDE THE BOX: AN INTROSPECTIVE LOOK AT THE USE OF ART IN TEACHING GEOMETRY

By

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A Senior Thesis Submitted to the

Eastern Michigan University

Honors College

in Partial Fulfillment of the Requirements for Graduation

with Honors in Mathematics

Approved at Ypsilanti, Michigan, on this date: May 8, 2007

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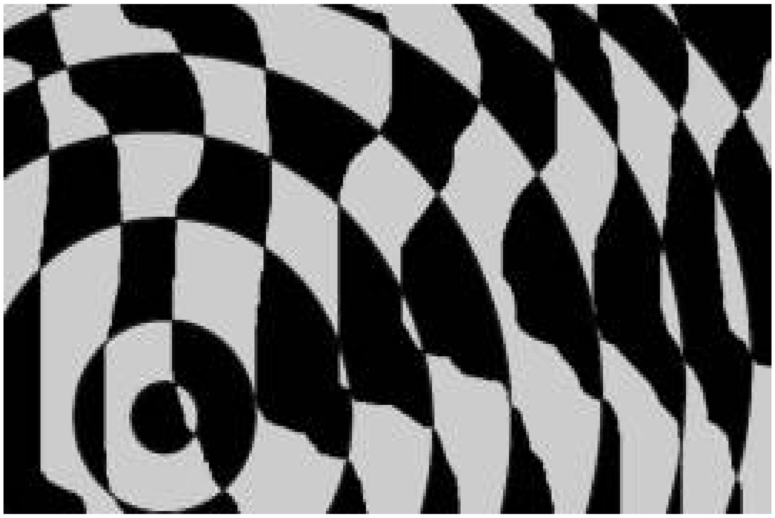
Geometry is typically thought of as a discipline in mathematics that is taught using formulas and basic shapes, but this idea is only the beginning. Geometry can be combined with art to enhance mathematical lessons for students. Educators must realize that visual representations of different mathematical concepts are a wonderful way to teach children geometry in a meaningful way. The "fundamental notion that integrating the arts into one's teaching can help facilitate learning in the mathematics classroom, as the arts can recapture the wonder of learning mathematics. The connection between instruction and imagination is bridged and learning becomes play, and play becomes learning" (Muller and Ward 22). This paper includes 7 lessons that integrate geometry with art, which will enhance the students' learning. Each lesson can be modified for any grade between kindergarten and 6th grade. These modifications will be explored in this paper along with the lesson idea. Finally, professional artwork of each type will be explored. Each piece investigated has significant mathematical ties and will be examined in the paper.

The first art style that can be used to further mathematical learning is optical art. Optical art deals with illusion, perception, and movement. It uses rhythm and a sense of movement to create motion or impossible structures. Optical art provides students with the opportunity to explore how geometric shapes are used in art. For example, students can use various shapes to create optical illusions like twisted cubes, which appear to never start or end.

Depending on the type of optical art your students make, students can discover multiple concepts like how to use concentric circles constructed with a compass to create art, how to work with symmetry, or how to create alternating patterns to create contrast in

the picture. An example of using concentric circles is in this piece by Victor Vasarely entitled *Blue/Red*. Students for many different reasons can explore this piece. Students can learn about symmetry by observing this piece. Reflection symmetry is evident in this piece. The line of symmetry is directly down the vertical middle of the piece. When the two sides are compared, the circles are exact copies of each other, just different colors. The colors are inverted as in the blue becomes black on the left and the black becomes red. This optical art creates the sense of depth on a two dimensional surface. When observing with students, ask them questions like "How does the artist use circles to create depth?" The students should realize that the basic properties of circles are manipulated to create depth. When the circles are squeezed to become ovals, they appear to be circles receding back in space in the picture rather than just ovals. For students to successfully view optical art and understand the images they are seeing, students must have basic knowledge of optical art vocabulary. The vocabulary includes words like perspective, symmetry, illusion, positive space, negative space, reversals, proximity, similarity, and progression. Students can create their own op-art by using their knowledge of geometric shapes and symmetry

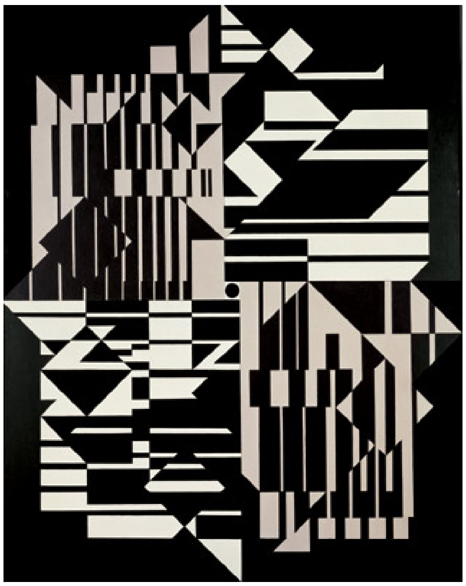


to create the illusion of depth. This project can be used for 5th and 6th grade students. Also, older students can describe their pictures. For example, when students learn the appropriate vocabulary for a project their "expressive language changes from 'It has a line on one side and another like just like it on the other side' to the mathematically oriented 'It had parallel lines that are vertical'" (Brewer 221). It allows students to explore geometric properties since manipulating the shapes creates the optical illusion.

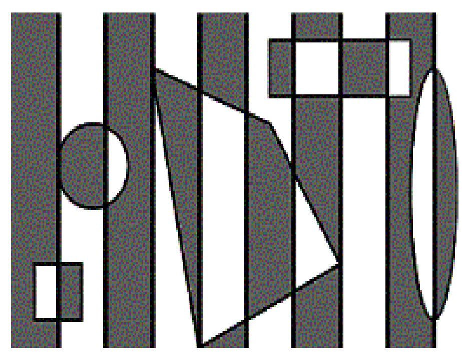
For the intermediate elementary grades, 3rd and 4th, teachers can introduce optical art through the "checkerboard" art. An example of "checkerboard" optical art is by Victor Vasarley. For this lesson, students are working with geometric shapes, patterning, and positive and negative space. Here are examples of students' work. First, students make concentric circles on a white piece of paper. Once the circles are drawn on

the paper, students will lightly draw curvy vertical lines, which will create the illusion of movement through the alternating patterns. After the circles and vertical lines are drawn, students alternate color in the "checkerboard squares" to create the checkerboard look.

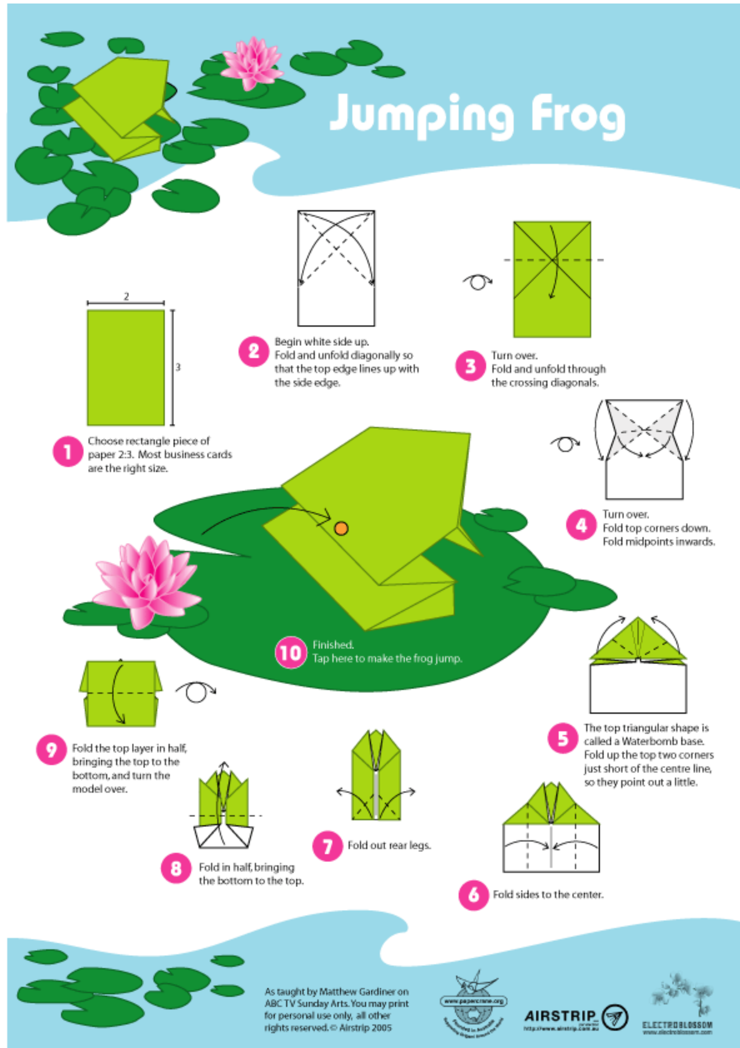
For 1st and 2nd grade students, they can create an optical illusion similar to the piece *Taimyr* by Victor Vasarely. By creating a simpler optical art design, students are



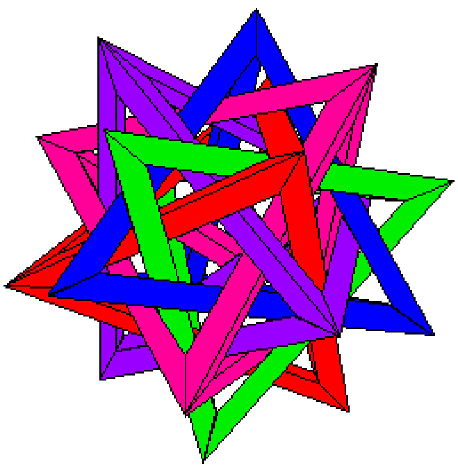
still gaining the crucial ideas behind optical art like symmetry, patterning, and positive and negative space. To create this type of piece, students must understand patterns to create the optical illusion since they have to alternate two colors for the background of the piece. Next, students investigate the properties of various geometric shapes. For example, if students want to create a square in their artwork, they must realize that two rectangles create a square. The students would then cut out a rectangle from one color and position that piece on the other color strip. This is shown in the student example below in the lower left corner of the picture. The use of optical art in the elementary class is a wonderful way to introduce students to shape characteristics, symmetry, and patterning. Optical art brings creativity and fun into the geometry lesson to engage students' mind and attention.



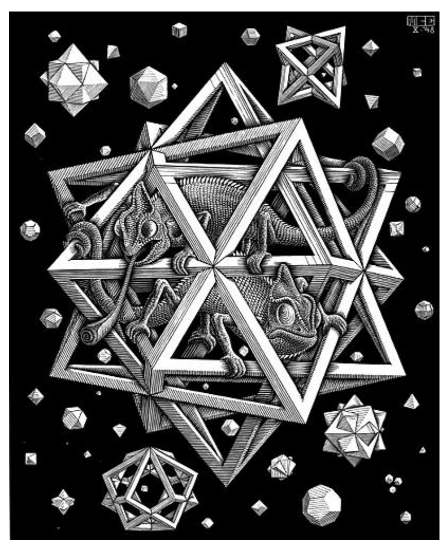
The second type of art that can be used in the geometry curriculum is origami, the ancient Japanese art of paper folding. Origami is more than simple paper folding; it is a tool to teach children about geometry. When working with paper, students encounter math concepts like lines of symmetry, congruency, and shape properties. Lines of symmetry are explored in origami when students fold the square paper exactly in half, since both sides are exactly the same. The first step to creating an origami frog, demonstrates the use of symmetry. The paper is folded exactly in half vertically. Origami demonstrates to students the various properties of shapes like when a square is folded in half it creates two rectangles. Once again, the first step to create a frog uses the properties of shapes to fold the paper. Working with origami also helps students discover congruent shapes and angles. Whenever one folds the paper on one side, the same process is repeated on the other so the sides are congruent. Steps 5 through 9 show that every fold is done on each side of the paper. Origami enables students to physically manipulate paper to explore these math concepts. Instead of memorizing mathematical definitions, students can create their own definitions



for symmetry, rectangle, and right triangle. Origami frogs are excellent for beginning math students in grades k-3. For upper elementary students, more challenging shapes can be created like cubes. The students can explore other geometric solids as well. When the students create these geometric solids, they can investigate the different properties of the shapes and gain a true understanding of the characteristics like angles and number of sides.



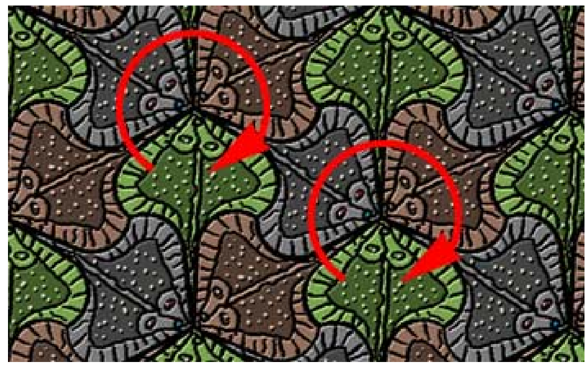
The final adaptation for origami is for 5th and 6th graders, which includes them making a shape that is five interlocking tetrahedral. A single tetrahedron has 4 corners and looks like a pyramid. The result of interlocking 5 tetrahedrons is a skeleton of a dodecahedron, which is a geometric solid with 20 corners. When students learn about geometric solids, 2-dimensional diagrams make the subject very abstract for students. Creating the objects will help students grasp the content and enable the students to have fun with their math lesson. Along with creating the object, students can observe geometric solids in professional artwork. M.C. Escher is a famous artist who used mathematical concepts in his artwork. An example of the use of geometric solids is in his piece entitled *Stars*. A wide variety of geometric solids can be examined in this piece of art. Students can observe



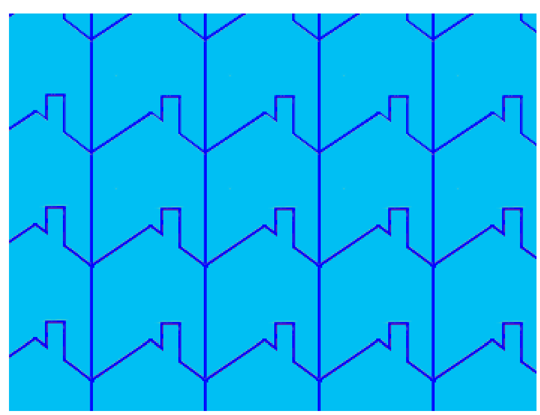
the relationships between the geometric solids and understand their similarities and differences. *Stars* includes geometric solids such as cubes and increase in difficulty up to the five interlocking tetrahedrons. Karen Baicker, author of *Origami Math*, stated, “The art of origami is truly hands-on-learning” (Baicker 42). Baicker summed up exactly why we have students create origami, because it is both fun and educational. As students progress through the grades, they loose their interest in math and harbor negative feelings toward the subject. By integrating art into the geometric curriculum, students can experience the creative side of math through geometry.

Third, tessellations are a wonderful way to integrate art into geometry class. Tessellations can be made from completing a simple translation or rotation of a particular shape. The manipulations of shapes help with the students’ understanding of the characteristics of different shapes. By designing a tessellation, students have a personal interest in their math lesson, which will enhance learning and retention. Many mathematical topics are learned and reinforced through tessellations. Tessellations help with symmetry, transformations, patterning, and angle and shape recognition. To help introduce children to tessellations, they can view the works of M.C. Escher. The tessellation by Escher on this page is entitled *Reptiles*. Students can view this piece and discuss patterning. The reptiles in the tessellation create patterns within the picture, since they repeat throughout the piece.

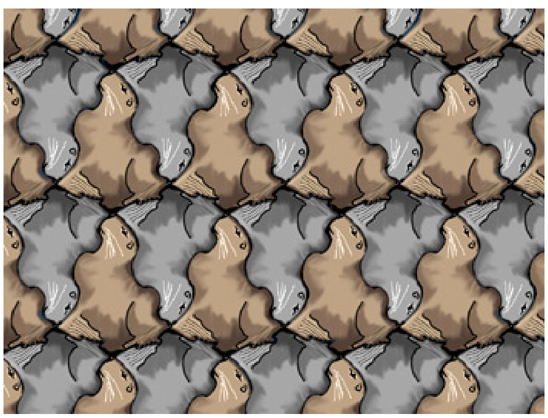
Along will patterning, symmetry can be explored through tessellations. Tessellations can include both rotational and reflection symmetry. The image to the right demonstrates rotational symmetry within a tessellation. Children can investigate the properties of both types of symmetry. *Reptiles* also exhibits rotational symmetry between the different reptiles and is a wonderful professional piece of art exhibiting symmetry. Reflection symmetry is also demonstrated in tessellations. Tessellations can be made using several different methods such as the slice method that cuts a section away or line method, which transforms straight lines into curves. The method the students use will depend on the grade level intended for the lesson.



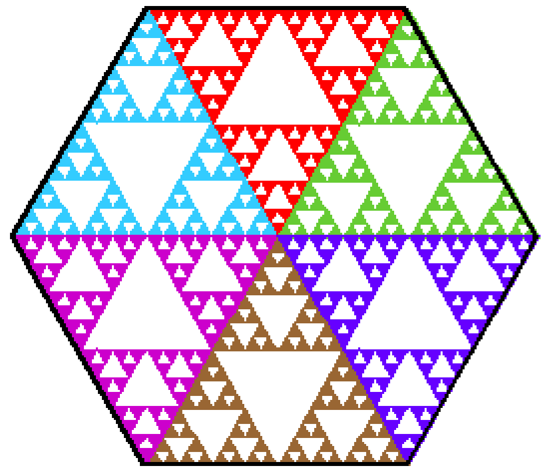
For kindergarten and 1st grade students, they should not create their own objects to tessellate but have different shapes provided for them to use. By creating patterns with different shapes, students can begin their journey with tessellations. Young children can observe professional pieces of art that use tessellations, like works by Escher. The teacher can discuss symmetry and patterning.



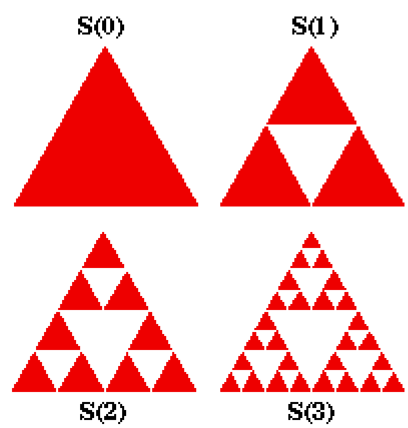
For the intermediate elementary students, 2nd and 3rd grade, the students can use the slice method to create a tessellation like the image on the right. The slice method requires the students to start with a square and then remove a part of one side of the square. Once the shape is removed, the slice is slid to the opposite side of the square. This transformation will allow the shape to tessellate with itself. The slice method can also help students learn about transformations. They can see how an object can slide or rotate depending on how they create their tessellation. Also, intermediate elementary students can also use original geometric shapes to investigate the angles of the shapes. They can learn about which shapes tessellate and which shapes do not. They can also manipulate shapes to see that a square with a right triangle on each side will create a straight line or 180º.



Upper elementary grades, 4th grade through 6th grade, can use the line method to develop their tessellation. The line method requires students to design their own shape by using a series of lines. The students start with a basic shape and manipulate the sides to make curved lines. An example of a tessellation constructed by the line method is the picture on the left. The seals are transformed kites fit together to tessellate. According to Tim Granger, a 5th grade teacher, his students "learned the attributes of different shapes and how those attributes affect a shape's ability to tessellation" (Granger 12). This is just one of the many topics students can learn through tessellations. Through tessellations, students can learn about symmetry, patterns, and shape characteristics throughout the grades.



Fractals are fascinating designs that can bring geometry alive for students. Fractals are “objects that appear to be broken into many pieces, each piece being a copy of the entire shape” (Naylor 360). They are inherently complex, yet simple designs. There are many famous fractals that students can reproduce and explore. They include the Sierpinski Gasket, Sierpinski Carpet, and Koch Snowflake. Once constructed, these fractals can also be used to explore perimeter and area. The topics of perimeter and area will come to life with the use of fractals. Instead of finding the perimeter and area of various shapes, students can use an object they created to explore those concepts.



First, students can investigate the fractals of the Polish mathematician Warclaw Sierpinksi “who is best known for his work with fractals and space-filling curves” (Naylor 363). Students can use the following steps to construct the Sierpinksi Gasket, also called the Sierpinksi Trinagle.

1. Start with an equilateral triangle S(0)
2. Connect the midpoints of each side
3. Remove the center triangle S(1)
4. Connect the midpoints of the three triangles remaining
5. Repeat steps 2 and 3 as many times as you want S(2) and S(3)

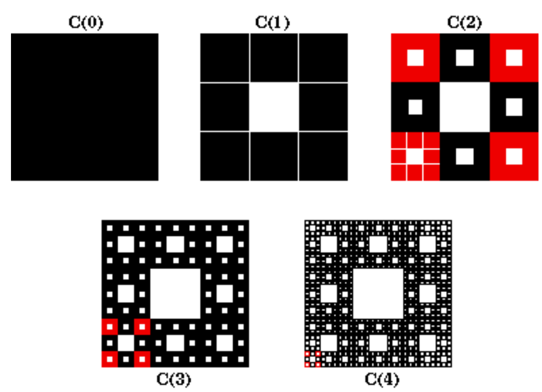
After the Sierpinski Gasket is constructed, students can be asked questions like "What happens to the area as 'n' approaches infinity?" and "How many points will never be removed?" (Naylor 363) The following are the formulas for the Sierpinski Gasket:

- Number of new holes = 3^{n-1} for $n \geq 1$
- Area of a new hole = $(1/4)^n$
- Area removed = $(1/3)(3/4)^n$ where $n \geq 1$
- Total remaining area = $(3/4)^n$

Students can explore these formulas using their copy of the fractal or students can discover these same formulas through their constructed fractal.

Another similar fractal students can create and investigate is the Sierpinski Carpet. Instead of using an equilateral triangle to begin the fractal, students use a square. To construct the Sierpinski Carpet, follow these simple directions:

1. Start with a square C(0)
2. Divide the square into 9 congruent squares
3. Remove the center square C(1)
4. Divide the remaining squares into 9 congruent squares
5. Remove the centers of the remaining squares C(2)
6. Repeat steps 2-5 as many times as you want C(3) and C(4)

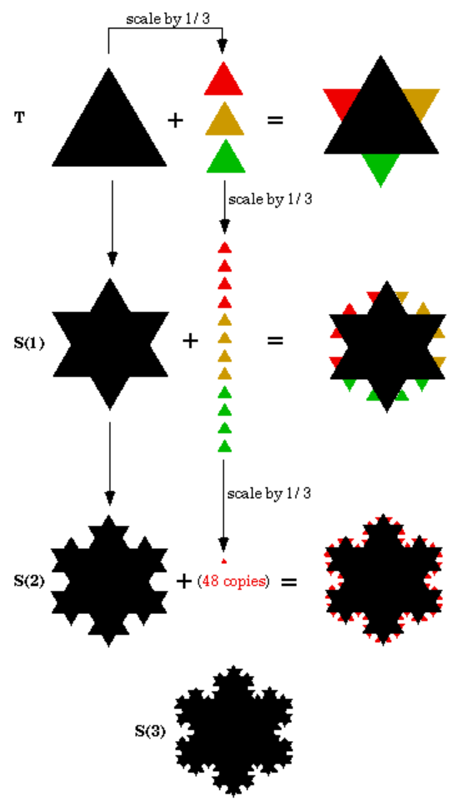


Once the Sierpinski Carpet is constructed, students can use the formulas below to investigate the area and perimeter. The formulas are:

- Side length of a new square = $(1/3)^n$ where $n \geq 1$
- Area of one square = $(1/9)^n$
- Number of new squares = 8^{n-1} for $n \geq 1$
- Area removed = $(8^{n-1})\ (1/9)^n$ where $n \geq 1$
- Total area remaining = $(8/9)^n$

After students have examined the formulas for area and perimeter, ask the students questions like "What is the area of this figure as 'n' approaches infinity?" and "Which parts will never be removed?" (Naylor 364) By asking these higher-level questions about the fractal, teachers can engage students in the lesson.

Also, students can examine the work of Niels Fabian Helge von Koch, a Swedish mathematician. His greatest contribution to the world of fractals is the Koch curve. To add difficulty to the Koch curve, the Koch Snowflake was created, which is three Koch curves together. The Koch Snowflake can be created using the following steps:



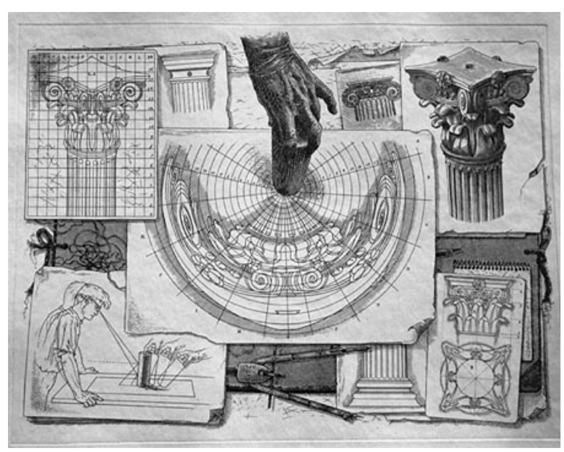
1. Start with an equilateral triangle (T)
2. Trisect each side of the triangle
3. Make smaller equilateral triangles on the exterior of the original side with two vertices on the trisection points S(1)
4. Repeat steps 2 and 3 as many times as you want S(2) and S(3)

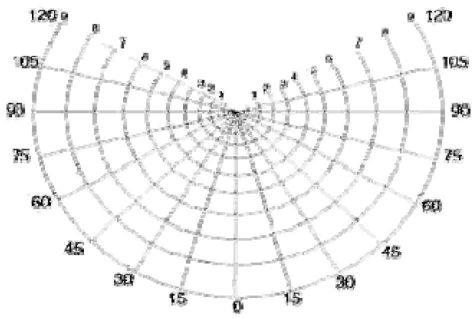
Like the previous fractals, the Koch Snowflake can be used to investigate perimeter and area. The general formulas for the Koch Snowflake are:

- Side length = $(1/3)^n$
- Number of sides = $3(4)^n$
- Perimeter = $3(4/3)^n$ where n = step number

A question like "What happens to the perimeter and area of this fractal as 'n' approaches infinity?" can be asked once the students explore the Koch Snowflake and it's characteristics (Naylor 362). By constructing fractals and investigating the characteristics of the Sierpinski Gasket, Sierpinski Carpet, and Koch Snowflake, students can learn about the perimeter and area of an object. Fractals present students with a unique and creative way to learn about and practice finding the perimeter and area.

Next, anamorphic art can be used to introduce students to many mathematical concepts. Anamorphic art involves taking an object and distorting it based on a curved grid or scaled grid. Through anamorphic art, students learn about reflections and graphing coordinate pairs. The picture to the right by Istvan Orosz actually includes the process of making an anamorphic design. The picture demonstrates the different steps to create anamorphic art like constructing a linear grid over the original picture and how the cylindrical mirror transforms the distorted image back to the original. To construct anamorphic art, students must take an original picture and transpose it onto another grid. Students can

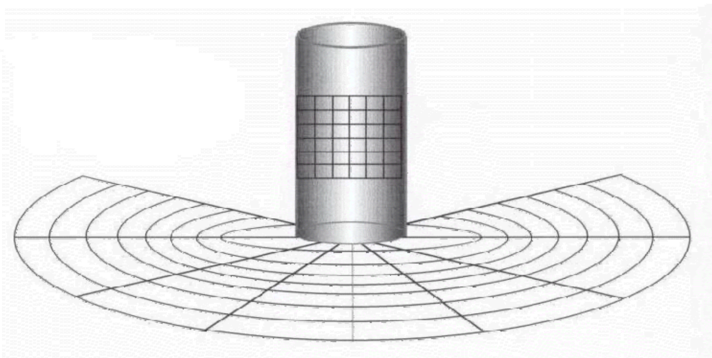




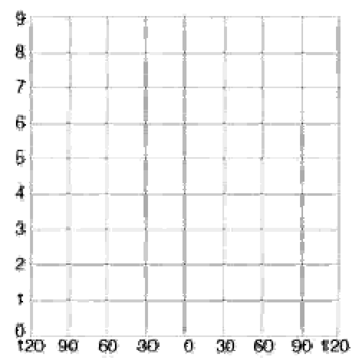
work with many different graphs like linear and non-linear graphs. When using a curved grid, students are able to work with a non-linear grid, which is rare in schools today. By transposing an image onto another graph, students are learning about and practicing working with coordinate pairs. Instead of providing students with a series of coordinate pairs to graph, students can create a piece of art by using the same skills. When transferring the image onto an alternative graph, the students are essentially graphing coordinate pairs.

Students not only learn about graphing coordinate pairs but reflections as well. The image of the anamorphic art is a distorted reflection of the original piece. For example, the piece *Cupola Anamophosis* by Istvan Orosz is an example of anamorphic art with the cylindrical mirror showing the reflection of the sketched image. “The anamorphic transformation produces a set of polar coordinates that return to their rectangular origins when viewed with a cylindrical mirror” (Houle). When a cylindrical mirror is used with the distorted image, students can see the reflection of the distorted picture.

The image below is a general template for anamorphic art. First, students create a regular grid over the original picture. Second, students move from grid section to grid section and recreate the designated section of the original picture onto the polar graph. By completing this process, students will eventually have a finished distorted image.

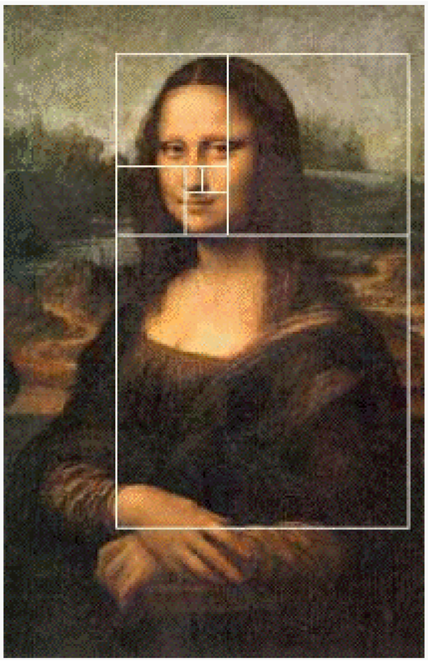


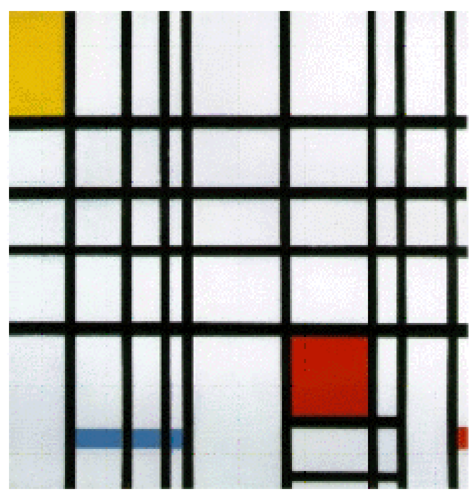
For younger students, 2nd and 3rd grade, they can take an original picture and transfer the image onto a larger or smaller regular grid. Instead of transferring an image onto a polar grid, students can stretch or shrink the original image. Going from one linear grid to another linear grid will provide students with the same experience as working with a non-linear grid, but easier.



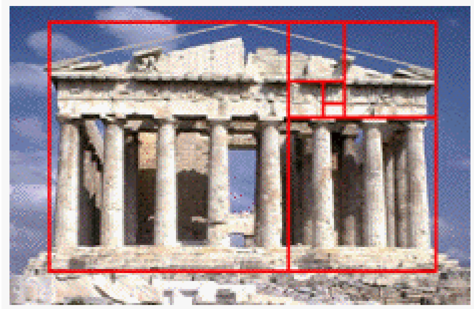
Students in the older elementary grades can use the polar grid. This will enable the students to work creatively with grids and coordinate pairs without the tedious practice of graphing coordinate pairs. For older students who have trouble transferring the image onto the non-linear grid, they can use the same regular grids of different sizes to stretch or shrink an image. By using a different grid, students will still learn the material and be able to participate in class with their classmates.

Another way to integrate art into the geometry curriculum is to introduce students to the Golden Rectangle. The Golden Rectangle is a rectangle whose two sides have a ratio of 1:1.618, the golden ratio. The Golden Rectangle is said to be "one of the most visually satisfying of all geometric forms" (Bergamini 94). This shape is found in many works of art and architecture. For example, *Mona Lisa* by Leonardo da Vinci and *Composition in Red, Yellow, and Blue* by Piet Mondrian both incorporate the Golden Rectangle. As seen to the left, the *Mona Lisa* can be sectioned off into different Golden Rectangles. The torso, head and face can all be enclosed by a rectangle with the golden ratio, as demonstrated in the picture. Another piece of art students can investigate is *Composition in Red, Yellow, and Blue* by Piet Mondrian, which is shown on the right. Instead of drawing Golden Rectangles on top of a picture, students can actually see the Golden Rectangles in the picture to the right. Several of the rectangles have the golden ratio. With this picture, the students can actually measure the sides of the rectangles and discover for themselves which ones are Golden Rectangles.

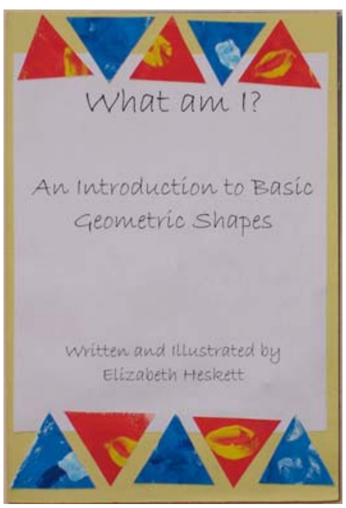




Finally, a Golden Rectangle can also be seen in architecture. The width and height of the Parthenon at Athens, once reconstructed, forms a Golden Rectangle. Phidias, a Greek builder built the Parthenon in Athens. "This early appearance of the Golden Ratio suggests that as long ago as the Fifth Century B.C., Greek builders were aware of its harmonious balances" (Bergamini 94). Using the Golden Rectangle is a wonderful way to help students appreciate art and discover the wonderful complexities of mathematics.



The last art project that can be integrated into the geometry classroom is book making. Using art projects can engage children in a creative project that will help reinforce and learn topics. Students can create a book about any topic in math, which can act as an assessment tool. To assess students' knowledge about shapes, if that was the topic being taught in class, have the students write and illustrate a children's book based on shapes. This will allow teachers to gain an insight into what concepts the students understand. While traditional tests include students writing down memorized definitions or drawing shapes, book making provides students with the opportunity to demonstrate their knowledge while still having fun and being creative. Making a book allows students to turn definitions into reality.



Students can create a math book during any elementary grade. For the younger students, kindergarten through 2nd grade, students can create a basic "Who Am I" book that requires very little time to assemble. This book style includes 4 flaps for the students to put information on. Older elementary students can create an accordion fold book, which can have as many pages as the student would like. The accordion fold book can be adjusted for 3rd grade up to 6th grade. Below is an example of an accordion fold book.

This book includes definitions of basic geometric shapes on the blue pages and the yellow pages include a picture of the shape in everyday life. No matter what grade, book making is always an excellent way to assess children on their mathematical knowledge.

Optical art, origami, tessellations, fractals, anamorphic art, Golden Rectangles, and book making are all wonderful ways to integrate art into the geometry curriculum. By incorporating art into the curriculum, the teacher is bringing fun back into math class. Many students never have the opportunity to experience the creative side of math, but these simple yet effective projects can change that. All of these projects can help teach children many valuable math concepts like symmetry, patterning, transformations, coordinates, and much more. Art is not just a fun and creative pastime activity; it is a valuable teaching tool for geometry.

Additional Optical Art Resources

Artist Biographical Information:

Victor Vasarely (Vásárhelyi Győző) (9 April 1906, Pécs - 15 March 1997, Paris) was a French Hungarian-born artist often acclaimed as the father of Op-art. Working as a graphic artist in the 1930s he created what is considered the first Op-art piece — *Zebra*, consisting of curving black and white stripes, indicating the direction his work would take. Over the next two decades, Vasarely developed his style of geometric abstract art. His work won his international renown and he received 4 prestigious prizes. He died in Paris in 1997.

From: http://en.wikipedia.org/wiki/Victor_Vasarely

Vocabulary Words:

Perspective: how objects appear to the eye.

Symmetry: where a center represents a focus point.

Illusion: a form of perspective that creates a feeling of movement. EX: lines seem to bend when diagonals are used to create an illusion of space.

Positive Space: the subject matter; the focus point(s) is dominant in an image.

Negative Space: the space that surrounds the subject matter; it is usually larger than the subject matter.

Reversals: when a starting point of a figure becomes an illusion; usually the negative and positive space has equal contrasts but provides no clues as to which is the subject or the ground (like wallpaper designs).....(link blue yellow)

Proximity: when the subject matter appears to form a group.

Similarity: when the subject matter relates in either size, color, value, or texture.

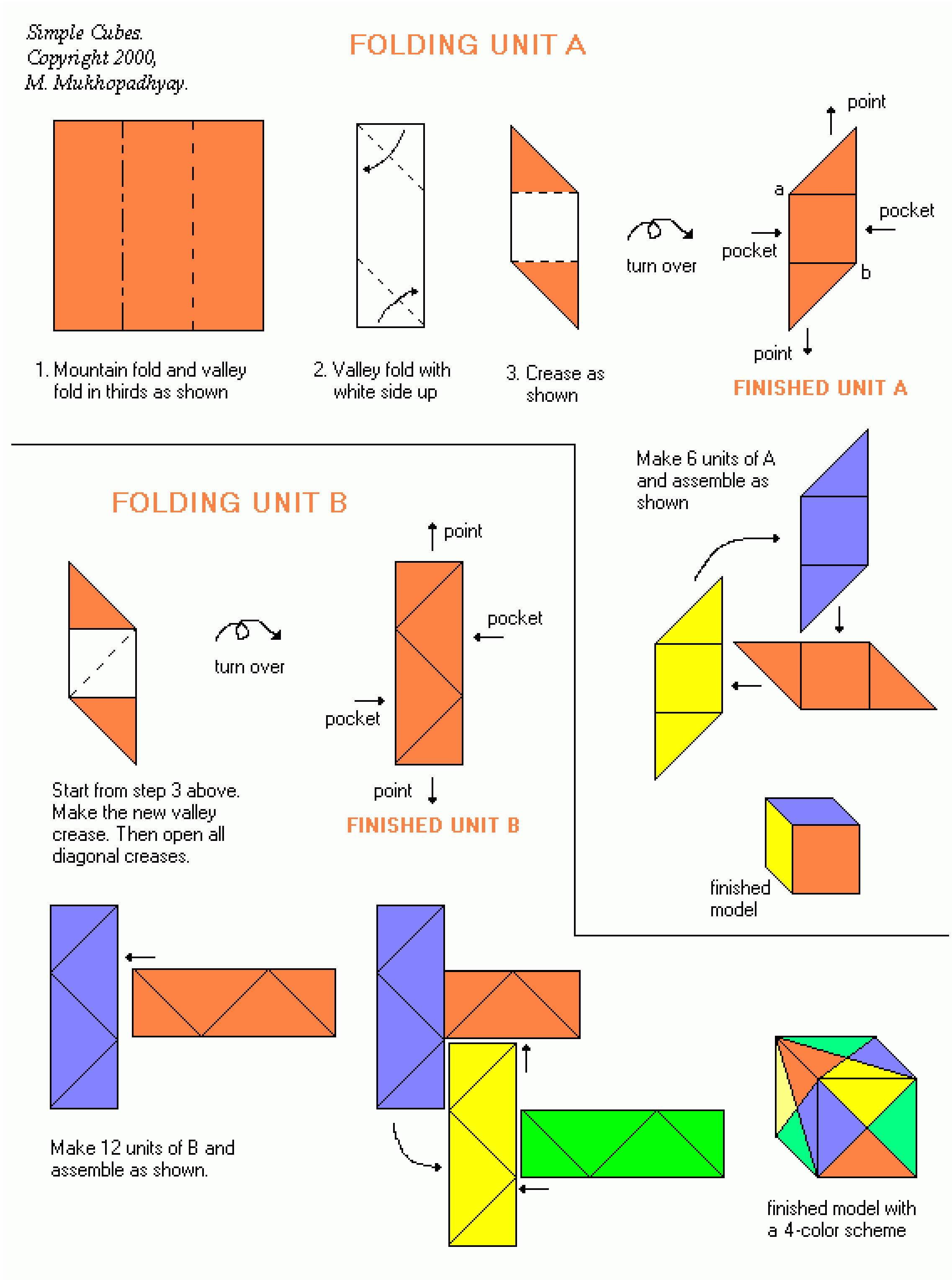
Moire: a pattern that includes intersecting angles (usually less than 30 degree angles) that are magnified and appear to fill space. EX: picket fences, sheer nylon curtains.

Progression: the subject matter increases in stages; includes both horizontal and vertical vanishing points.

From: http://www.geom.uiuc.edu/~demo5337/Group4/Opart.html

Additional Origami Resources

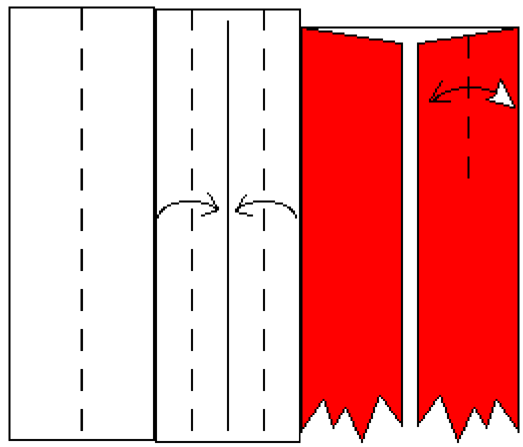
Directions to make a cube:



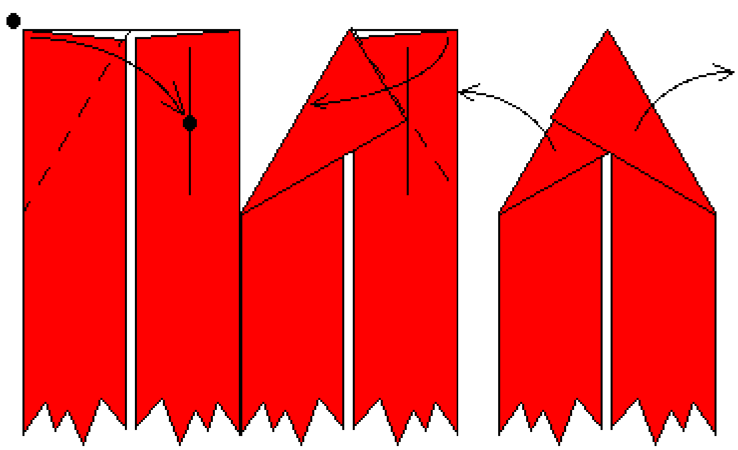
From: http://home.comcast.net/~meenaks/diagrams/

Directions to make five interlocking tetrahedrons:

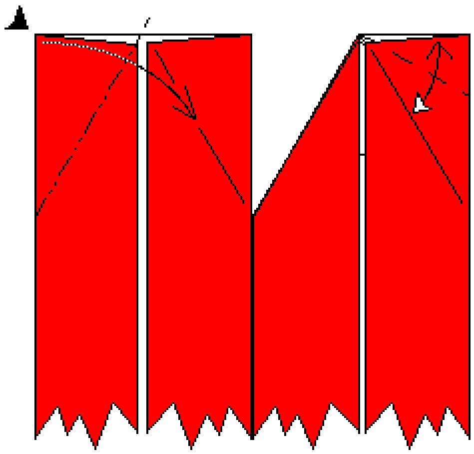
Making one tetrahedron frame requires six 1x3 pieces of paper. In other words, it will take two squares which then must be cut into 1x3 strips. To make the full 5 intersecting tetrahedra model you'll need to make 5 of these tetrahedra - that's a total of 10 squares of paper. To make each tetrahedron a different color, as in the picture above, you'll thus need 5 different colors and 2 square sheets per color.



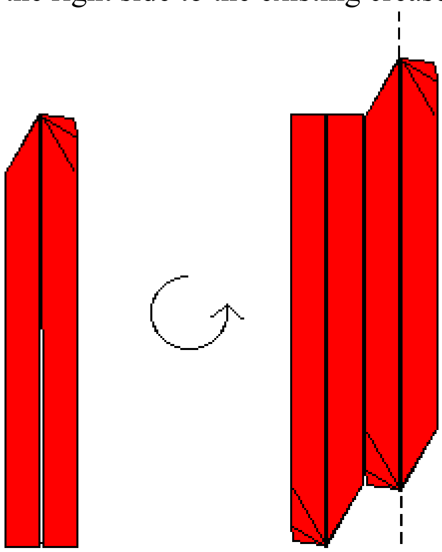
Take one of the 1x3 strips (white side up) and crease it down the middle. Then fold the sides to the center line. The right-most picture shows a close-up of the top end. Fold the right flap to the side, **only** making a pinch! This crease will be needed for the next step.



Then fold the upper-left corner to this crease line, **making sure** that the crease hits the midpoint of the top edge, as shown in the left-most picture. (Note that this is axiom (O5) in Huzita's axiom list (see Origami Geometric Constructions), and creates a 60 degree angle for us!) Then fold the upper-right corner over this flap, and unfold these flaps.

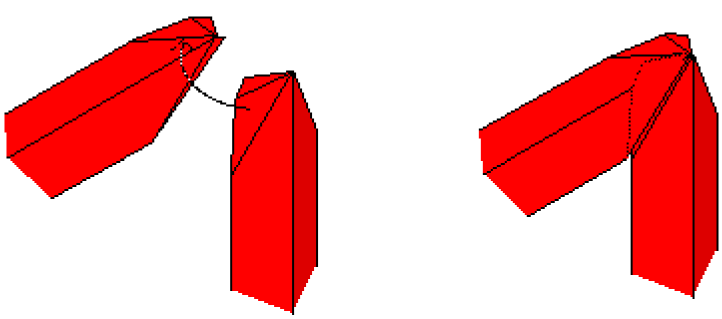


Now **reverse fold** the upper-left corner, using the crease that we just made. The reversed flap should go inside the model. Then (right-hand picture) fold and unfold the top edge of the right side to the existing crease line.

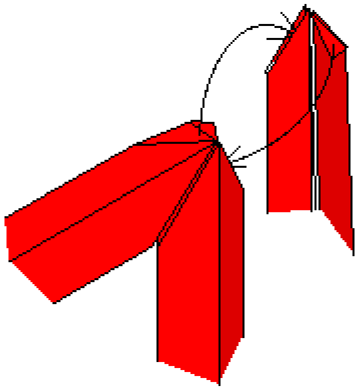


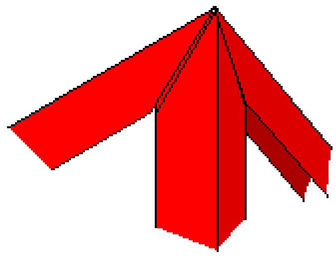
OK! We're done with one end, so rotate the model 180 degrees and repeat this process on the other end. (Note that the unit will have a left-handedness, like the Sonobe unit, and all of your units must have the same handedness in order to fit together properly.) Lastly, crease the unit down the middle, and you're done! You'll need 5 more to make one tetrahedron.

How to interlock the units



The end of each unit has a flap on one side and a pocket on the other. Insert the flap of one unit into the pocket of another as shown on the left. To the right is the result. Notice the nifty x-ray view effect, allowing you to see exactly how the flap **needs** to hook around the crease. This makes a strong lock.

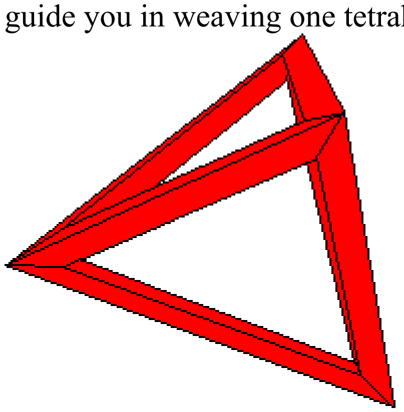


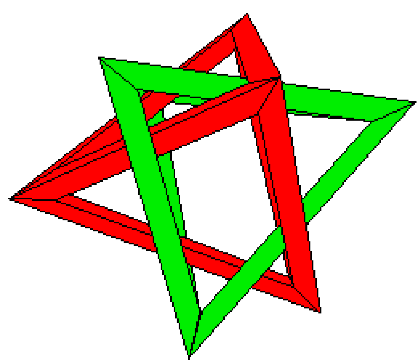


Now get ready to insert the third unit! This should complete one "joint" of the tetrahedron frame. Notice that each unit should form a "wedge" (in cross-section). However, when insertig the last one you might want to round-out the edges, so as to allow the last flap to hook around the other unit. Then pinch the sides to make everything stay in place. To build on this tripod you've just made, add two units to one of the tripod's legs to make another "joint". Then the last unit can be added to complete the tetrahedron.

Forming the object

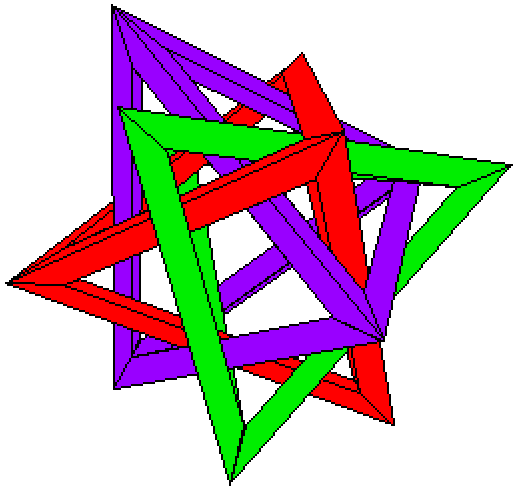
Unfortunately there's no easy way to describe how the tetrahedral frames need to weave around each other to create the 5 intersecting tetrahedra model. It really is a challenging puzzle to put it all together! I suggest that you use the following series of pictures to guide you in weaving one tetrahedron at a time.

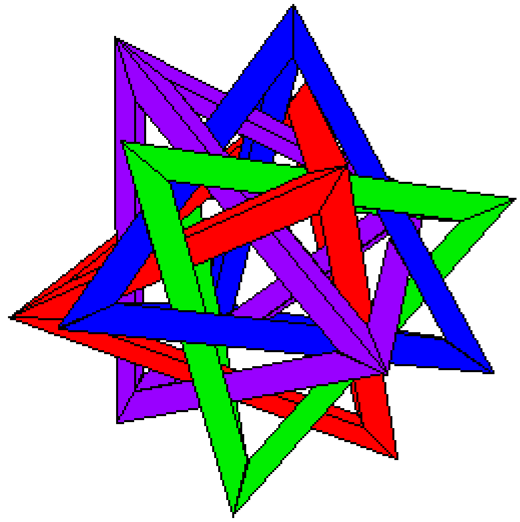




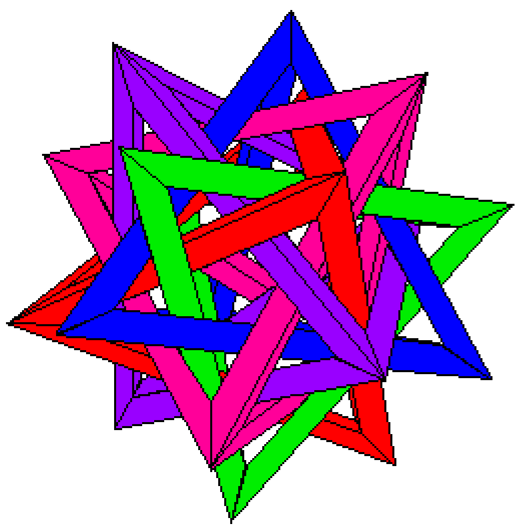
Notice how, in the right-hand picture, the left-most corner of the red tetrahedron is poking through a "hole" of the green one, and vice-versa, the right-most corner of the green tetrahedron is poking throught a "hole" of the red one. Further, this is done symmetrically. This observation is **key** to understanding how the tetrahedra fit together. Inspect the next pictures very carefully!

There is a very strong symmetry behind the formation of this structure, and understanding this symmetry can aid you in the construction. The finished object should have the following property: any two tetrahedra are interwoven with one corner poking through a hole of the other and vice versa, kind of like a 3-D Star of David but slightly twisted. (This is what we tried to describe above.)





The important part, though, is that **every** pair of tetrahedral frames in the finished model should have this property. I admit that this is a hard concept to grasp, but it can help in checking to see if you're "weaving" the frames properly.



From: http://www.merrimack.edu/~thull/fit.html

Additional Tessellation Resources

Artist Biographical Information:

Maurits Cornelis Escher, who was born in Leeuwarden, Holland in 1898, created unique and fascinating works of art that explore and exhibit a wide range of mathematical ideas.

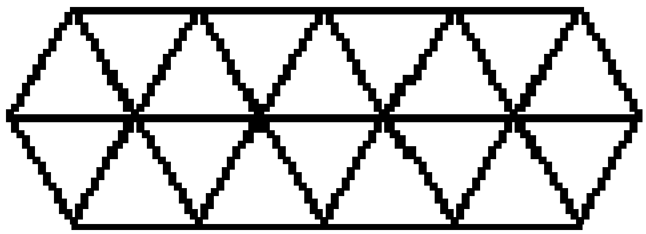
While he was still in school his family planned for him to follow his father's career of architecture, but poor grades and an aptitude for drawing and design eventually led him to a career in the graphic arts. His work went almost unnoticed until the 1950's, but by 1956 he had given his first important exhibition, was written up in Time magazine, and acquired a world-wide reputation. Among his greatest admirers were mathematicians, who recognized in his work an extraordinary visualization of mathematical principles. This was the more remarkable in that Escher had no formal mathematics training beyond secondary school.

As his work developed, he drew great inspiration from the mathematical ideas he read about, often working directly from structures in plane and projective geometry, and eventually capturing the essence of non-Euclidean geometries. He was also fascinated with paradox and "impossible" figures, and used an idea of Roger Penrose's to develop many intriguing works of art. Thus, for the student of mathematics, Escher's work encompasses two broad areas: the geometry of space, and what we may call the *logic* of space.

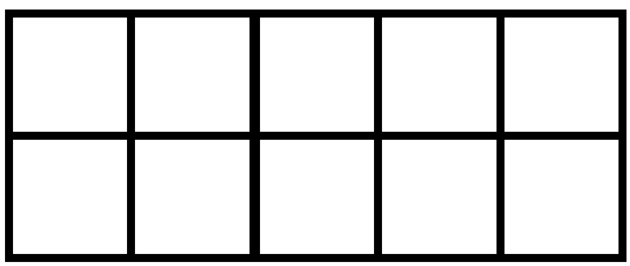
From: http://www.mathacademy.com/pr/minitext/escher/

Shapes that will tessellate:

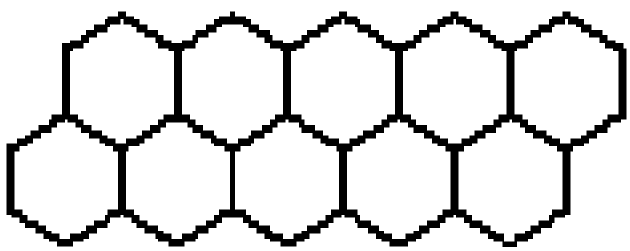
Triangle:



Square:



Hexagon:

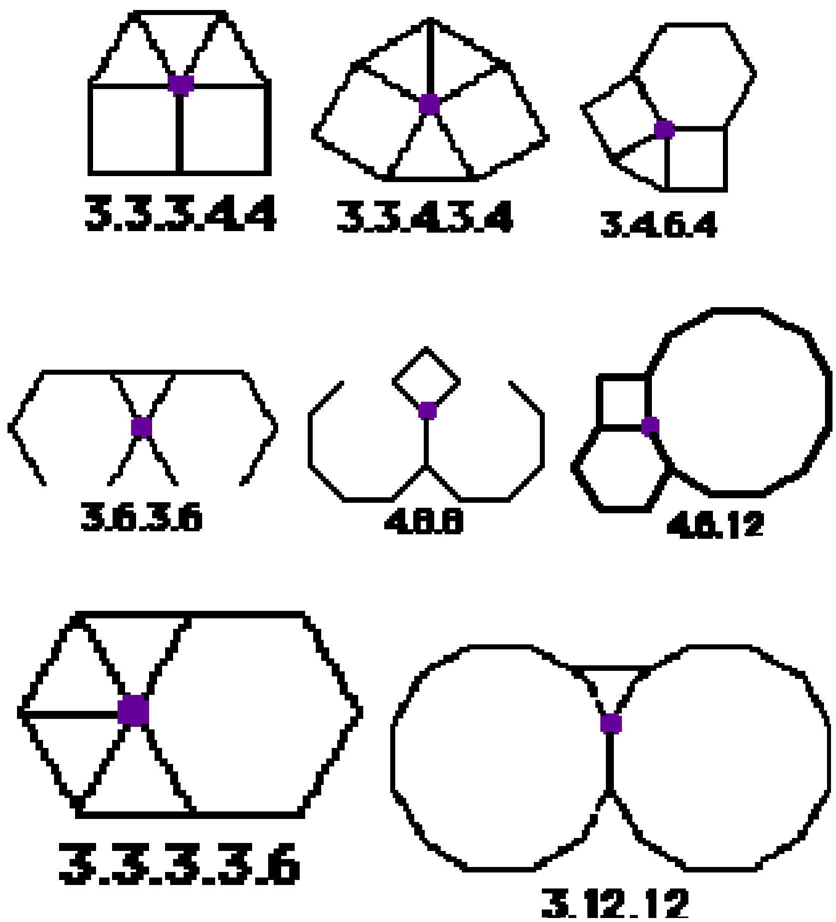


Shapes that will tessellate with each other:

3 = triangle

4 = square

6 = hexagon



From: http://mathforum.org/sum95/suzanne/whattess.html

Additional Fractal Resources

Mathematician Biographical Information:

Wacław Franciszek Sierpiński (March 14, 1882 — October 21, 1969), a Polish mathematician, was born and died in Warsaw. He was known for outstanding contributions to set theory (research on the axiom of choice and the continuum hypothesis), number theory, theory of functions and topology. He published over 700 papers and 50 books.

Three well-known fractals are named after him (the Sierpinski triangle, the Sierpinski carpet and the Sierpinski curve), as are Sierpinski numbers and the associated Sierpiński problem.

From: http://en.wikipedia.org/wiki/Sierpinski

Niels Fabian Helge von Koch (January 25, 1870 - March 11, 1924) was a Swedish mathematician, who gave his name to the famous fractal known as the Koch snowflake, which was one of the earliest fractal curves to have been described.

He was born into a family of Swedish nobility. His grandfather, Nils Samuel von Koch (1801-1881), was the Attorney-General ("Justitiekansler") of Sweden. His father, Richert Vogt von Koch (1838-1913) was a Lieutenant-Colonel in the Royal Horse Guards of Sweden.

von Koch wrote several papers on number theory. One of his results was a 1901 theorem proving that the Riemann hypothesis is equivalent to a strengthened form of the prime number theorem.

He described the Koch curve in a 1904 paper entitled "Sur une courbe continue sans tangente, obtenue par une construction géométrique élémentaire."

From: http://en.wikipedia.org/wiki/Helge_von_Koch

Additional Anamorphic Art Resources

Artist Biographical Information:

Istvan Orosz studied at the Hungarian University of Arts and Design (now Moholy-Nagy University of Art and Design) in Budapest as pupil of István Balogh and Ernő Rubik. After graduating in 1975 he began to deal with theatre as stage designer and animated film as animator and film director. He is known as painter, printmaker, poster designer,and illustrator as well. He likes to use visual paradox, double meaning images and illusionistic approaches while following traditional printing techniques such as woodcutting and etching. He also tries to renew the technique of anamorphosis.

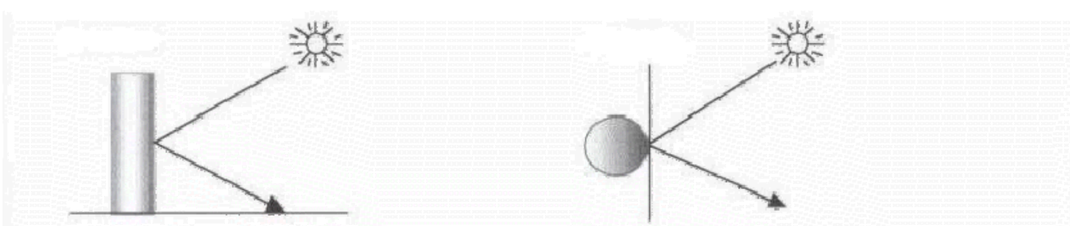
From: http://en.wikipedia.org/wiki/Istv%C3%A1n_Orosz

How Cylindrical Mirror Anamorphosis Works

The Law of Reflection

A cylindrical mirror distorts information in two different directions. To see why this is true, look at the cylinder from two different points of view...

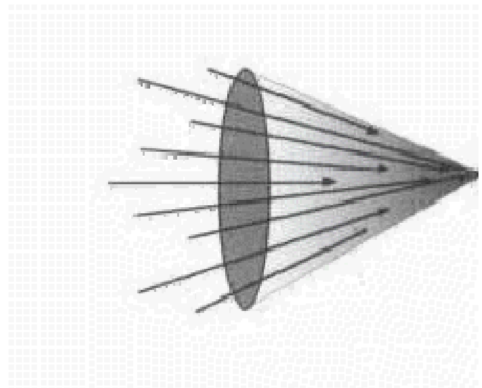
The side of the mirror is straight,like the surface of a flat mirror.... but its edges are rounded, like the surface of a curved mirror.



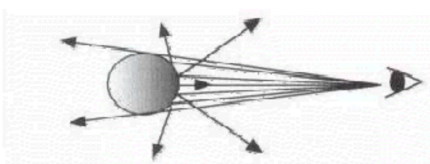
In both situations, the angle of incidence is equal to the angle of reflection. In the case of a curved surface, these angles are measured from a line tangent to the curve at a specific point.

The Cone of Vision

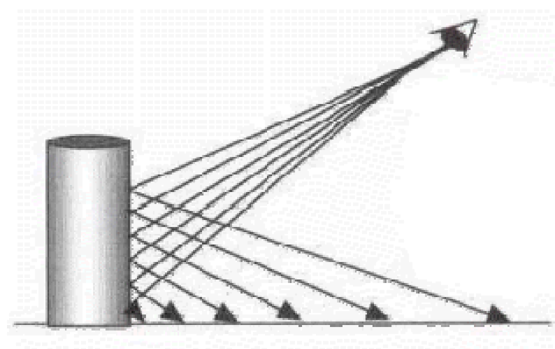
Light rays travel to our eyes in straight lines from all directions. The area of the pupil is small compared to the area from which light may travel. This causes an effect called the "cone of vision".



The cone of vision causes some interesting patterns when combined with the way light is reflected in curved mirrors. Rays (shown as traveling from the pupil) strike the surface of the mirror at various angles.



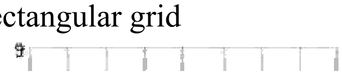
This pattern shows the radial reflection of light due to the law of reflection for curved mirrors and the cone of vision.



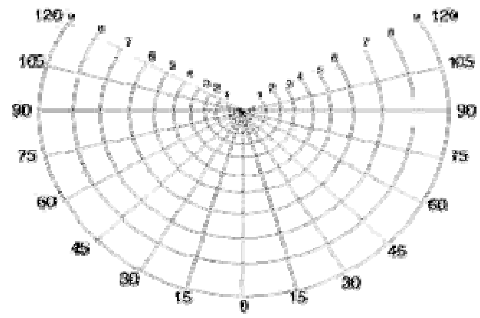
This pattern shows how light rays spread out as they strike farther and farther from the mirror, due to the variation in the angle of incidence.

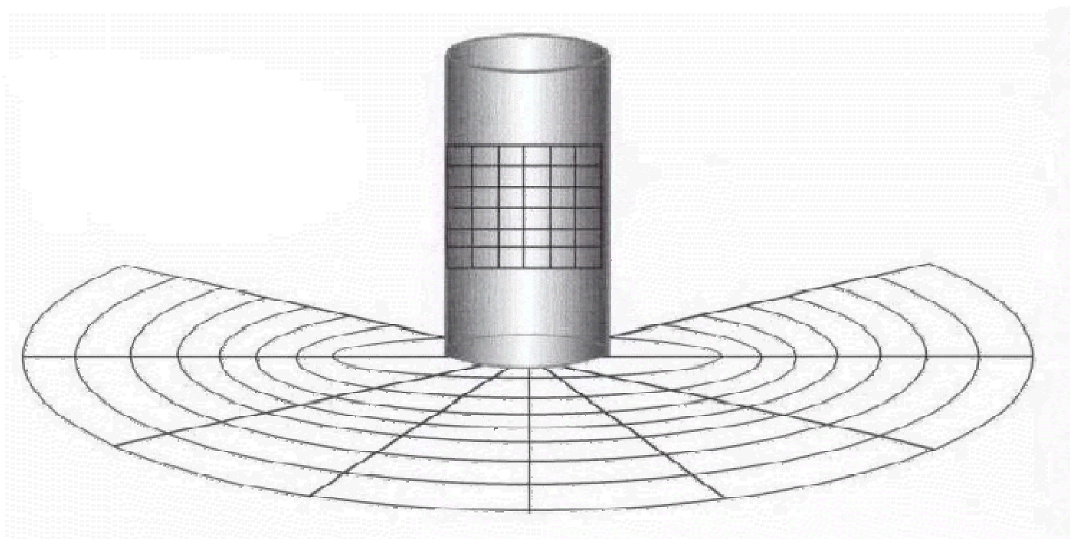
The anamorphic transformation produces a set of polar coordinates that return to their rectangular origins when viewed with a cylindrical mirror.

rectangular grid

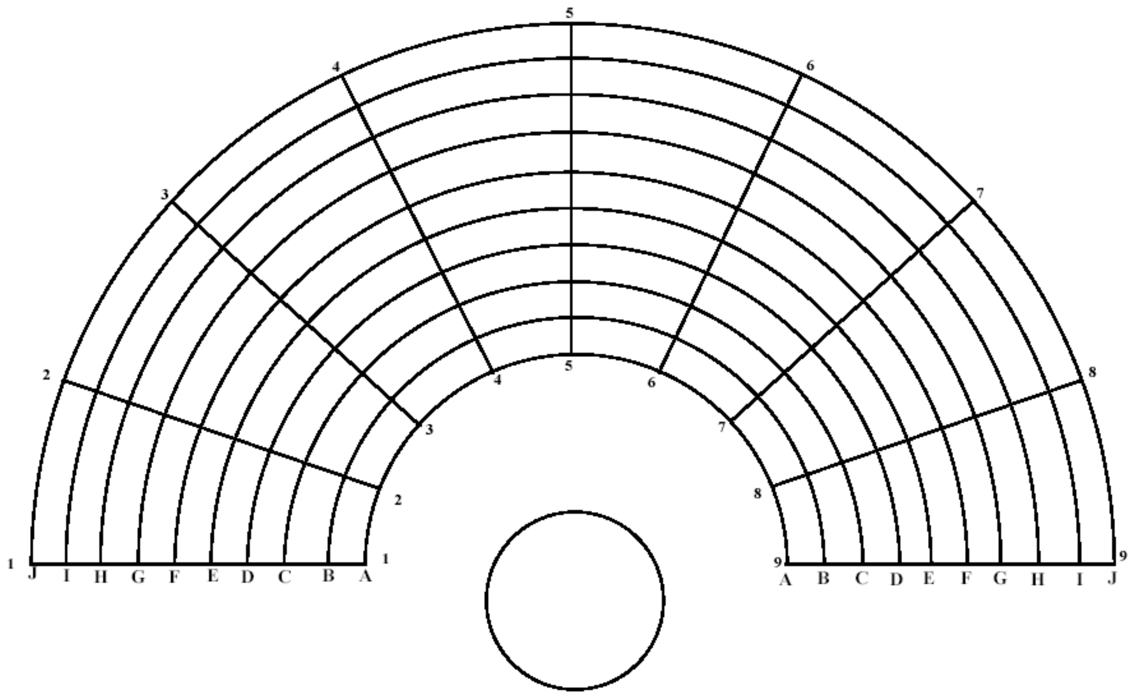


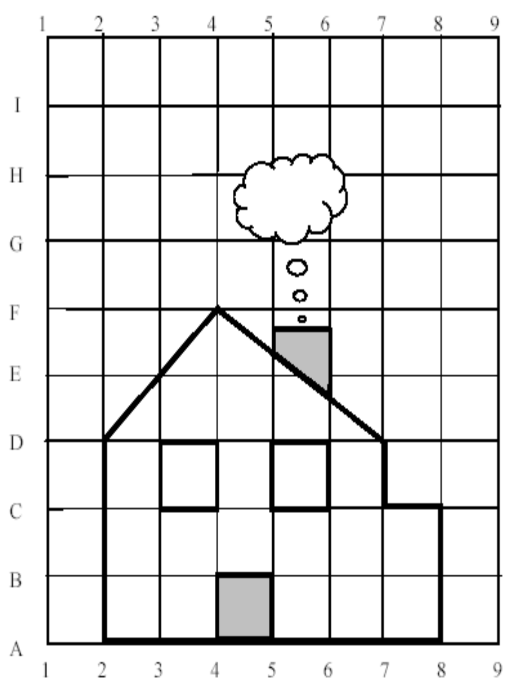
polar grid

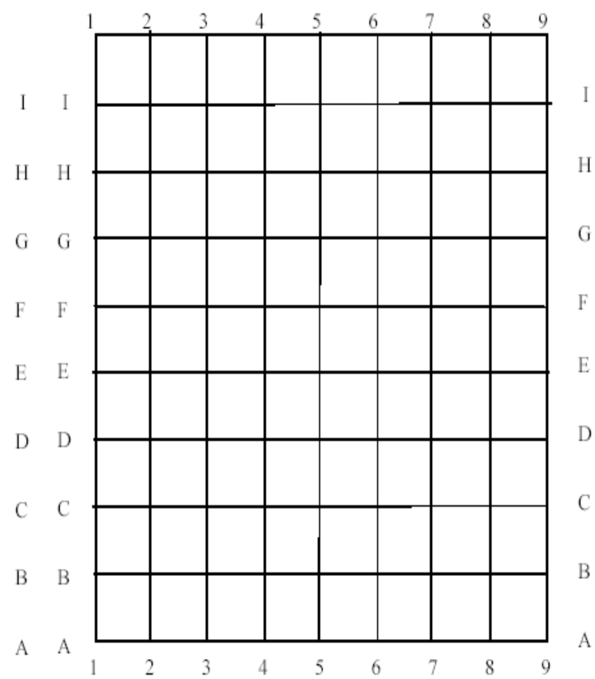




From: http://www.kellymhoule.com/about_anamorphosis_frameset.htm







Grid templates from http://www.raft.net/ideas/Anamorphic%20Art.pdf.

Additional Golden Rectangle Resources

Definition:

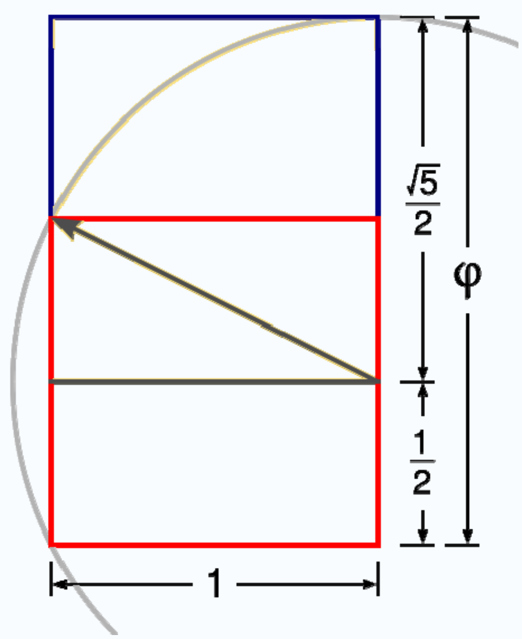
A **golden rectangle** is a rectangle whose side lengths are in the golden ratio, 1:φ, that is, approximately 1:1.618.

A distinctive feature of this shape is that when a square section is removed, the remainder is another golden rectangle, that is, with the same proportions as the first. Square removal can be repeated infinitely, which leads to an approximation of the golden spiral.

According to astrophysicist and math popularizer Mario Livio, since the publication of Luca Pacioli's *Divina Proportione* in 1509, when "with Pacioli's book, the Golden Ratio started to become available to artists in theoretical treatises that were not overly mathematical, that they could actually use,"many artists and architects have proportioned their works to approximate the form of the golden rectangle, which has been considered aesthetically pleasing. The proportions of the golden rectangle have been observed in works predating Pacioli's publication.

Construct a Golden Rectangle:

1. Construct a simple square
2. Draw a line from the midpoint of one side of the square to an opposite corner
3. Use that line as the radius to draw an arc that defines the height of the rectangle
4. Complete the golden rectangle



From: http://en.wikipedia.org/wiki/Golden_rectangle

Artist Biographical Information:

Pieter Cornelis (Piet) Mondriaan, after 1912 **Mondrian**, (b. Amersfoort, Netherlands, March 7, 1872 — d. New York City, February 1, 1944) was a Dutch painter.

He was an important contributor to the De Stijl art movement and group, which was founded by Theo van Doesburg. Despite being well-known, often-parodied and even trivialized, Mondrian's paintings exhibit a complexity that belies their apparent simplicity. He is best known for his non-representational paintings that he called "compositions", consisting of rectangular forms of red, yellow, blue, white or black, separated by thick, black rectilinear lines. They are the result of a stylistic evolution that occurred over the course of nearly 30 years and continued beyond that point to the end of his life.

From: http://en.wikipedia.org/wiki/Piet_Mondrian

Leonardo di ser Piero da Vinci (April 15, 1452 – May 2, 1519) was an Italian polymath: scientist, mathematician, engineer, inventor, anatomist, painter, sculptor, architect, musician, and writer.

He was born and raised near Vinci, Italy, the illegitimate son of a notary, Messer Piero, and a peasant woman, Caterina. He had no surname in the modern sense, "*da Vinci*" simply meaning "of Vinci". His full birth name was "Leonardo di ser Piero da Vinci", meaning "Leonardo, son of (Mes)ser Piero from Vinci."

Leonardo has often been described as the archetype of the "Renaissance man", a man whose seemingly infinite curiosity was equalled only by his powers of invention. He is widely considered to be one of the greatest painters of all time and perhaps the most diversely talented person ever to have lived.

From: http://en.wikipedia.org/wiki/Leonardo_da_Vinci

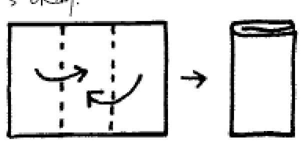
Additional Book Making Resources

This is a fun form. I've used it a lot with second and third grades. Each of the four folded pages has one or two facts. When they are all opened, the center page reveals the identity. I've used them for biographies in history, countries in geography, animals in science, and numbers in math- I am half of 8, I am 2 + 2, etc. You can use velcro instead of the tie.

You Need:

- 1 piece 12" x 18" paper
- 1 piece 6" x 18" cover stock or oaktag
- 1 piece 24" yarn
- Scissors
- Glue stick and scrap paper
- Hole punch
- Pony beads (a fun extra for decoration)

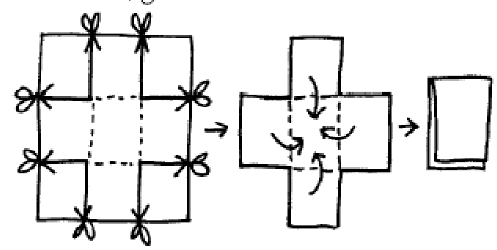
1. Place the large paper in front of you so that it is a sideways rectangle and fold it into thirds. I find the best way is to roll it into a three part tube and then flatten it. This fold can be tricky. If it's not exact, that's okay.



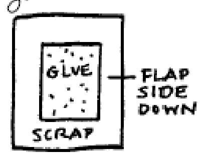
2. Keep the paper folded and fold it into thirds the other way. Because the paper is smaller, it will be easier this time.

3. Open the paper and cut out the four corner pieces to make a large plus sign.

4. Fold the pages in. The order doesn't matter.

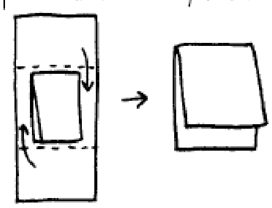


5. Place the folded pages flap side down on a piece of scrap paper. Cover the top surface with glue.

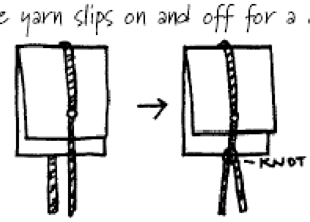


6. Glue the folded pages to the center of the cover.

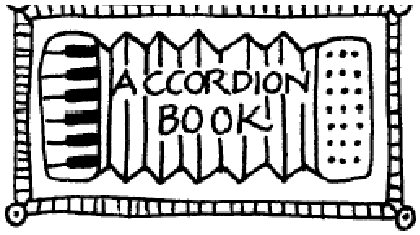
7. Fold the bottom cover up over the pages and the top down. Be careful not to fold the pages.



8. Punch a hole in the center of the top flap. Thread the yarn through the hole. Pull it over the top of the cover so that there is yarn in the front and back of the book. Make the ends even. Tie the yarn at the bottom with a double knot. Don't make it too tight. The yarn slips on and off for a closure.



More bookmaking ideas are available at Susan's website, makingbooks.com. Ebooks with thorough directions for specific projects are available for purchase at the Bookstore at makingbooks.com.

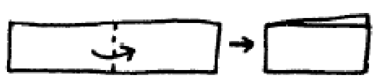


I like accordion books because you can stand them up and view all the pages at once. This makes them great for displays and exhibits. Accordion books have a rich history around the world. They are made in many parts of Asia, including China, Japan, Korea, Thailand, India, and Burma. They were also the book form of the Aztecs and Maya in Mexican and Central America. Book artists frequently experiment with the form.

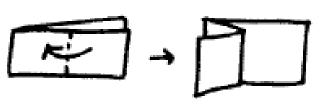
You Need:

- 1 piece 6" x 18" paper (I chose this size assuming you'd start with 12" x 18" drawing or construction paper, but any long piece of paper will do. The cover should be 1/4" taller and 1/4" wider than the folded accordion.)
- 2 pieces 6 1/4" x 4 3/4" posterboard, oaktag, or cardboard
- 1 piece 24' long ribbon or yarn
- Glue stick and scrap paper

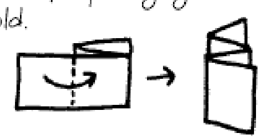
1. Place the paper in front of you horizontally and fold it in half.



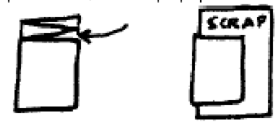
2. Fold the top page in half by folding it back to meet the fold.



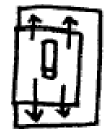
3. Turn the paper over and fold the top page in half by bringing it back to meet the fold.



4. Open the first page of the accordion and slip a piece of scrap paper inside.

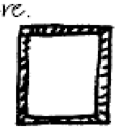


5. Cover the entire surface with a thin coat of glue. I start in the middle and then go in stripes up and then down the page. Go over the edges and onto the scrap paper.



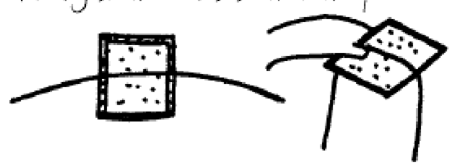
6. Remove the scrap paper and fold it in half with the glue on the inside.

7. Center the accordion on one of the cover pieces and press. Smooth to help the glue adhere.

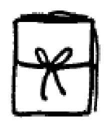


8. Place the accordion with the cover side down, insert the scrap paper under the top page, and cover it with glue. Remove the scrap paper and fold it in half.

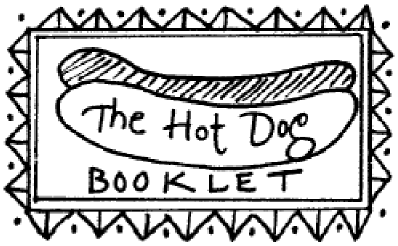
9. Lay the ribbon across the back of the book. Adjust the ends so that they are even.



10. Wrap the ribbon around the front of the book and tie it in a bow. When you write in your book, place the ribbon side down and stretch the pages out to the left.

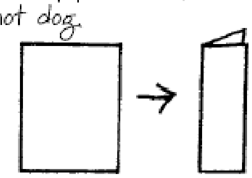


Susan's Multicultural Books To Make And Share, which contains sixteen bookmaking projects from around the world including several projects using the accordion book form, is available only from the Bookstore at makingbooks.com.

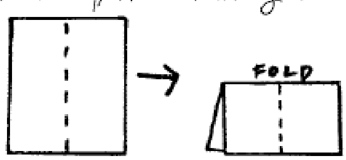


The Hot Dog Booklet has a front and back cover and six pages inside. You can use any size paper. Standard copy paper makes a cute little one; bigger paper makes a bigger book. I like this form because it's easy to make multiple copies. Make a blank book following the directions. Write and illustrate your book with black marker or pen. Open the sheet and lay it face down on a copier. Make as many copies as you want. Fold and cut each one according to the directions.

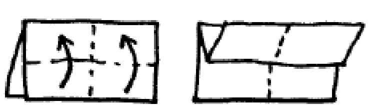
1. Fold the paper in half the long way, like a hot dog.



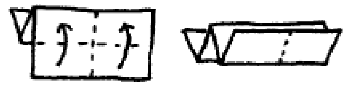
2. Open the paper and fold it in half the short way, like a hamburger.



3. Take one layer of paper and fold the edge back to meet the fold.

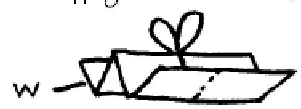


4. Turn the paper over and fold the edge of the paper back to meet the fold.

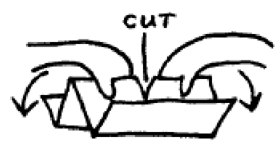


5. Place the paper on the table so that you see a W when you look at the end. You can also think of it as a hot dog in a roll.

6. Cut the hot dog in half along the center fold. You'll be cutting through two layers of paper and stopping at the cross fold.



7. With your wrists above your fingers, hold the two halves of the hot dog from the top.



8. Turn your wrists to the sides. You will have an open book with four sections.

9. Bring three of the sections together. Fold the last section on top of the other three so that you have a flat book.

Susan's Multicultural Books To Make And Share, which contains two projects using the Hot Dog Booklet, a Comic Book and a Newsbook based on the forerunners of newspapers in Europe, is available only from the Bookstore at makingbooks.com.

All instructions from http://www.makingbooks.com/freeprojects.shtml

Bibliography

About Anamorphic Art. 2005. Kelly M. Houle. 14 Apr. 2007 <http://www.kellymhoule.com/about_anamorphosis_frameset.htm>.

Anamorphic Art. 2003. RAFT Education Department. 19 Mar. 2007 <http://www.raft.net/ideas/Anamorphic%20Art.pdf>.

Artlex. 22 Mar. 2007. Michael Delahunt, MFA. 24 Mar. 2007 <http://www.artlex.com>.

Baicker, Karen. “Origami Math.” *Instructor* Apr. 2004: 42-45.

Brewer, Evelyn. “Geometry and Op Art.” *Teaching Children Mathematics* Dec. 1999: 220-224.

Classic Iterated Function System. 2007. Agnes Scott College. 14 Apr. 2007 <http://ecademy.agnesscott.edu/~lriddle/ifs/ksnow/ksnow.htm>.

Five Intersecting Tetrahedra. 1997. Thomas Hull. 19 Mar. 2007 <http://www.merrimack.edu/~thull/fit.html>.

Golden Ratio in Art and Architecture. 2005. University of Georgia. 13 Apr. 2007 <http://jwilson.coe.uga.edu/EMT668/EMAT6680.2000/Obara/Emat6690/Golden%20Ratio/golden.html>.

Granger, Tim. “Math Is Art.” *Teaching Children Mathematics* Sep. 2000: 10-13.

Increidble Art Department. 2007. Princton Online. 19 Mar. 2007 <http://www.princetonol.com/groups/iad/lessons/elem/patti-opart.htm>.

Istvan Orosz. 2006. Marsha Child Contemporary. 24 Mar. 2007 <http://www.mchildcontemporary.com/artists/Orosz.html>.

Making Books with Children. 2007. Susan Kapuscinski Gaylord. 14 Apr. 2007 <http://www.makingbooks.com/freeprojects.shtml>.

Optical Art Designs. 24 Jul. 1996. LaToniya Jones-McKinnon. 14 Apr. 2007 <http://www.geom.uiuc.edu/~demo5337/Group4/Opart.html>.

Origami- Folding Instructions. 2006. Meenakshi Mukerji. 19 Mar. 2007 <http://home.comcast.net/~meenaks/diagrams/>.

Papercrane. 2007. Origami Australia. 19 Mar. 2007 http://www.papercrane.org/articles/abcTV/jumping-frog.gif>.

Platonic Realms. 2006. Math Academy Online. 19 Mar. 2007 <http://www.mathacademy.com/pr/minitext/escher/>.

Tessellations. 2007. David Annal. 19 Mar. 2007 <http://www.tessellations.org>.

What is a Tessellation? 2007. Drexel University. 14 Apr. 2007 <http://mathforum.org/sum95/suzanne/whattess.html>.

Wikipedia. 16 Mar. 2007. Wikimedia Foundation, Inc. 25 Mar. 2007 <http://en.wikipedia.org/>.