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Mathematical problem-solving strategies and group dynamics of a small group of high school students

Sarah Richards

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Mathematical Problem-Solving Strategiesand Group Dynamics of a Small Group of High School Students

by

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Abstract

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I would like to thank the people who helped me put this project together and keep it going. All of you were busy people, and I truly appreciate the fact that you followed up on your promises to me. Many of you put in far more than what I had initially asked of you after others dropped the ball. Every one of you was essential to getting this done and keeping me sane.

Dr. Steven Blair: for putting up with my procrastination, rants, and semi-colons; you are extraordinarily patient. You gave me a lot of good ideas and a newfound respect for math education philosophy by challenging my ideas about how people learn math.

Mr. Joseph Ely: for being my inspiration to do mathematics from the start and for staying with me long after I was your responsibility. There is not enough room on my hard drive to list everything you have done for me.

Mr. J.P. Trías: for pretty much everything. Without you and the lab, I would not have gone on in mathematics after high school. I would have completely given up on this project in September as well. I may not have been admitted (with funding) to MSU. The list goes on and on.

The students, you know who you are: for being there every time I asked you and for getting excited about the problems. It would have been torture for me to come in and give you all a quiz you thought was boring. All of you are exceptionally bright (even those of you who may not think so), and you are all capable of much more than you think.

Mr. Brian Hart: for being a down-to-Earth teacher and a great person to talk to. After I had put so much time in, and the Tsunami of a let-down happened, you were one of the people who helped dig me out of the rut. I also appreciated having a place to work every once in a while, paper, scissors, and the like.

Mr. Dan Scott: for letting me do this project at your school and being thoroughly straightforward about it. I knew what was expected of me and what I could expect from you.

Contents:

Section 0:

This project started out as a study on peer tutoring. I have been tutoring college-level mathematics myself since shortly after arriving at college, and the ways people think about mathematics have always interested me. I credit tutoring for making more advanced concepts easier for me to understand, and I often wonder whether tutoring has the same effect on others. I decided to create a study to determine whether (and, if so, in what ways) being a tutor helped students to develop their skills for solving mathematical problems.

My goals were as follows:

- Give weekly quizzes to tutors, students who were receiving tutoring, and a control group comprised of students who were not involved in the tutoring project.
- Examine the results quantitatively to determine which groups were making the most progress.
- Collect surveys from all involved in the project to determine their attitudes about mathematics and problem-solving at the start and the end of the experiment.

After the project was started, working with the secondary school administration became more difficult; the intended scope of the project was not realized. I decided to focus on how well the students performed on the tasks themselves rather than how tutoring affected their abilities to solve the problems. I also decided to change to a qualitative, descriptive methodology with regard to how they solved the problems. I soon found that the students' responses to several of the tasks I had set them were engaging on their own without the tutoring aspect.

After seeing the results of the first few sessions, I began to wonder how the students would work together in a group. Their individual responses to some of the problems were immensely different, and their varying methods often led to alternate conclusions. For the few problems which none of the students answered correctly, it looked as though each student had a piece of the puzzle. I thought that it would be interesting to see how those ideas might join together.

The project has consequently progressed from a study on tutoring to a detailed description of the students' problem-solving methods and group dynamics.

My goals for the final version of the project were as follows:

- Determine by trial and error which types of problems the students had difficulty with by varying the content on the weekly problem sets. After finding difficult problems, give another like it on the next problem set to see whether or not the students have developed a strategy for solving such problems.
- Determine the types of problems for which the students have difficulty developing a successful strategy.
- Let the students work as a group on the last problem set made up of problems all of the students had difficulty solving, and describe how this affected their problem-solving ability.
- Observe the students tutoring. This was included mostly because I had agreed to participate in a tutoring project and did not want to go back on my word to the school. The students spent about an hour each week tutoring before spending half an hour on their problem set.

Section 1:

The study took place from October to December of 2008 at Annapolis High School in Dearborn Heights, Michigan. I asked teachers at the school to find volunteers who would be willing to tutor after school and take a brief quiz each week. One teacher responded, and I had three students to work with: Janey, Kent, and Jim (note, these are pseudonyms). The students were all juniors, and they had all finished geometry. Two of the students were in Algebra II at the time of the study. None of them had experience with tutoring. Although all of the students had volunteered for the study, some were more outgoing than others. Janey dived into tutoring but seemed to lack confidence in her mathematical skills at times. Kent was more shy about tutoring while Jim often walked up to students who looked like they were struggling whether they asked for help or not.

The school had an after-school tutoring program that allowed students to get help from a couple of teachers. The program was meant for students who were struggling; parents signed their students up for the program, and the student was expected to attend three times per week. This seemed like a good place to start a peer-tutoring study, and the student volunteers agreed to help there once per week. I had intended to procure a group of students who were receiving tutoring as part of the study, but that task quickly became unfeasible. The student volunteers worked at the tutoring program after that part of the study was abandoned, but they became more interested in the problems than tutoring toward the end of the study.

I wanted to create a set of tasks that measured problem-solving and adaptive reasoning rather than memorization as in the style of the Putnam Exam. The Putnam Exam is a competitive collegiate examination given each December to volunteer participants across North America. Students at any level after learning Single Variable Calculus have attained all of the prerequisite information needed to understand and to solve the problems, but solving them usually requires some sort of innovation. None of the problems on the exam are similar to what would typically be stressed in a classroom setting, so the test measures the student's capability to think on their own.

The book Problem Solving Strategies: Crossing the River with Dogs and Other Mathematical Adventures (Johnson & Herr, 2001) had a good selection of the types of problems I was looking for. They stressed several different types of mathematical reasoning skills including elimination, ability to represent a problem in a diagram, working backwards, spatial reasoning, and general logic. I invented most of my own spatial reasoning problems in order to allow some of the tasks to be more challenging than the level of the book. Crossing the River with Dogs is geared toward middle school level reasoning, but the more challenging problems were able to stump the high school students. The problems in the book were broken up into units, each of which stressed a different problem-solving strategy. I chose a number of problems from the various units.

Following is a list of the tasks I gave to the students, and my rationale for choosing them. They are characterized by a number and a letter. The letter denotes which problem set it came from; A is the first week's set, B the second, and so on. The number denotes the order in which it appeared on the set.

Spatial reasoning problems:

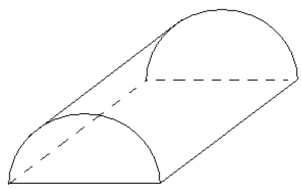
For a few of these types of problems, I decided to tell them about a three-dimensional object (a half-cylinder, a cone, and a prism of my own invention), and I instructed them to draw a net (i.e. a shape which would make the object if you cut it out and taped it together). I gave them the example of a Latin Cross which can be folded to make a cube.

I think that my directions were less clear with the cone; I thought that it was an easy problem, so I directed them to, "draw and label" the shapes. I gave them values for the height and radius of the cone, but after they became very confused about what they ought to have been labeling, I decided to drop that part of the exercise. I also asked them to perform the same task in reverse order in the last exercise set. I gave them a net and asked them what it would look like if they folded along the lines and taped the sides together.

The reason I kept using these types of problems is because they were the only types of problem which repeatedly stumped the students.

Following is a list of each problem, but I did not write the problems this way on the problem set. The spatial problems required more explanation than was acceptable to write on the page. The first time I gave one of these problems, I wrote, "I will give instructions for this problem," and I spent some time explaining what I expected them to do. I drew the three dimensional object on the board and gave them the Latin cross example. Once it was clear that they knew what was expected of them, I allowed them to attempt to answer the problem. I never used the word "net".

Problem 1A) Create a net for a half-cylinder like the one shown below.



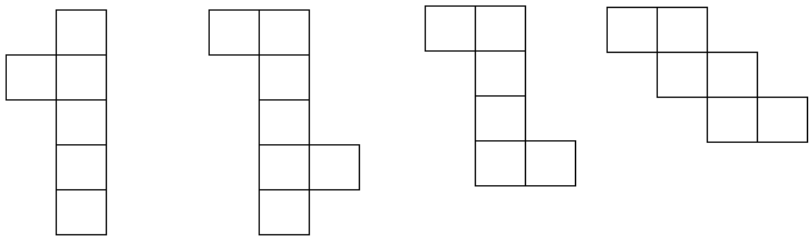
This problem was interesting because it had curved sides. I thought that the students could have difficulty figuring out the shape of the rectangle that is bent along the circumferences of the semi-circles.

Problem 1B) Original: Draw and label the shapes that, if cut out and taped together would make a cone with height 4 and radius 4.

Final: Draw and label two shapes that, if cut out and taped together would form a cone.

As I mentioned before, I changed this problem. I handed out the problem set, and the students were confused by what I meant by "label", even after a short explanation. Labeling the values on a 3-dimension object requires the ability to see how things fit together; a person can only realize on their own that the length of the rectangle in a cylinder is equal to the circumference of the circular bases by visualizing the rectangle wrapping around the circular base. By asking the students to label the values of the sides, I wanted to make sure that the students were not guessing what the shape may look like.

Problem 2C) which of the following can be folded into a closed box?



The second and fourth nets in this task are the best, since the other two are fairly straightforward. The second net has too many sides; but if you fold one side over another, you can still create a closed box. I wondered if the students would recognize that. The fourth net just does not look as obvious as the others. On first glance, you may say that it doesn't work; but actually it does.

Problem 5C) Make a half-cylinder out of paper.

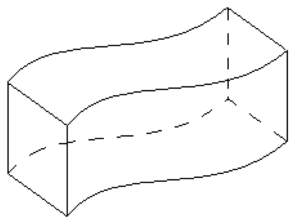
This was a follow-up to the first problem they did of this type. For the most part, they failed miserably the first time around; but I thought that they might do better if they had something physical in their hands to experiment with. It is one thing to draw a shape and experiment in your head with it; it is much easier to play with paper or something tangible to see if your idea works.

Problem 1D) which shapes can be made out of a cross-section of a cube?

This question was posed by my advisor. I had difficulty coming up with some of the shapes myself. It seems like you can make almost anything out of the cross-sections of a cube. But he told me that children in elementary school can figure it out a little easier when they had a clear plastic cube and a rubber band (or a closed plastic cube with water in it). Stretching the rubber band around the cube and looking at the shape the rubber band creates lets them see the cross-section. Moving the rubber band makes the students look at the problem in a more continuous manner than the more discrete the way they are taught to think about their math problems.

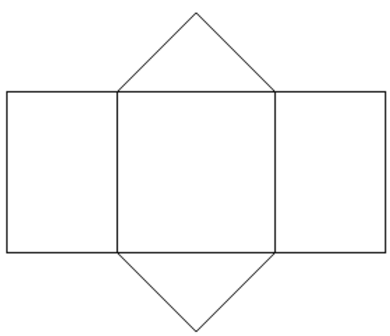
I gave the students the plastic cube at first and posed the question. They came up with a square and rectangle usually. When they started having trouble coming up with their own, I suggested shapes for them to consider. After they confirmed or rejected my ideas, I gave them the rubber band to see which cross-sections were possible.

Problem 1E) Draw the shape which, if cut out and taped together, would form the following figure.



This problem is even worse than the half-cylinder when it comes to the curves. I was thinking of problems I would consider doing for this project, and I just came up with this one. It was very challenging, and I didn't think the students would get it on their own. I decided to put it on the last problem set, where the students were allowed to work together.

Problem 3E) what 3-dimensional object would the following shape make if you cut it out and taped it together?

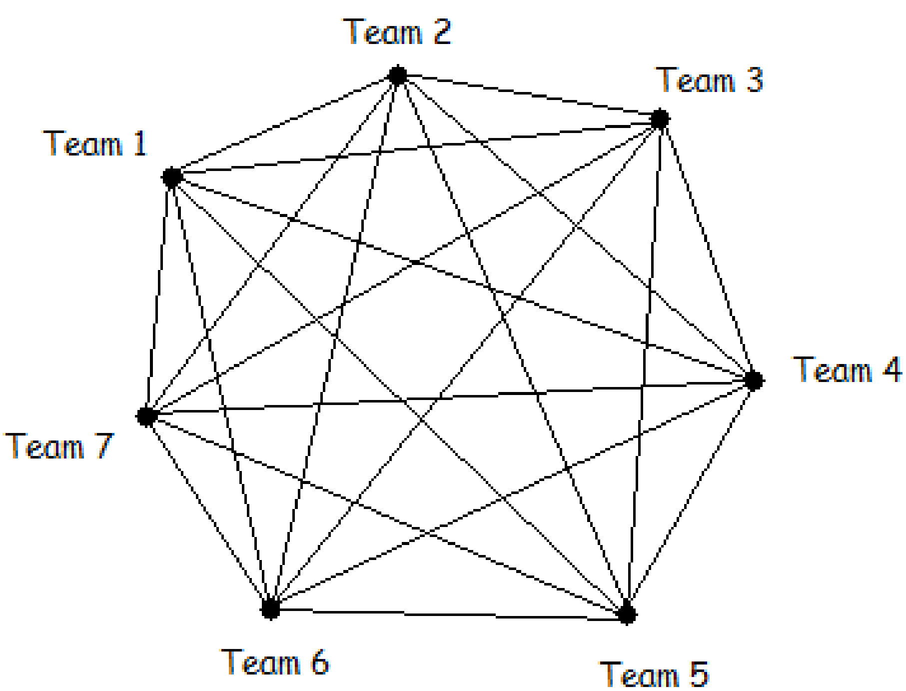


I thought it could be fun to do a net problem backwards. I assumed that the students would have an easier time with it than the other problems. In hindsight, I probably should have put it on an earlier problem set as a confidence booster for these problems.

I thought that the spatial reasoning problems were interesting to begin with, but the students had such difficulty with them that I felt justified using them again after the first problem set. They turned out to be exactly what I was looking for, an exercise that the students had not learned in school and that they had just enough difficulty with them that it frustrated the students a little, but was possible for them to get in the end.

<u>Problems which are easier with a diagram:</u>

Problem 3A/2E) A new basketball league was formed, and all of the teams in it have to play each other. If there are seven teams, how many games will be played in the league? This is a basic problem which is often presented to students who are beginning to use combinations. For students who have never seen the formula for combinations (or who do not know how to apply it), however, this problem provides a challenge. If the students could apply the formula for combinations, they could ask themselves, "How many ways can we get two teams to play each other if we have 7 teams?" They could then solve the problem by performing the calculation for 7 choose 2. If they did not know how to apply combinations to this problem the book gave the following solution: Draw a diagram with lines connecting each team, and count the lines. I did not think that this solution was any better than listing all of the games played, because counting errors are easy with this type of diagram. It would be best to count the lines as they were being drawn, but I was curious to see if the students could find a better solution.



Problem 4A) Find the function which created the table. For example, if I gave you the following table, you could determine that the function was 2*x.

x	y
1	2
2	4
3	6

y=2*x

x	y
1	4
2	7
3	12
4	19

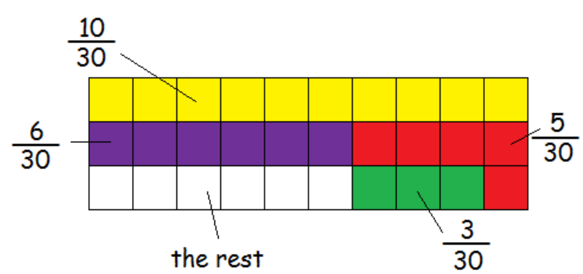
y=?

I found myself doing some of these types of problems in my second calculus course (when I was working on Taylor expansions), and I always found them a little challenging. I thought the students could graph the points and figure out what it looked like. Once they determined it was a parabola, they could use a standard equation and solve for co-efficients a, b, and c. I was unsure of whether they would be able to do that or find another way.

Problem 2B) A woman accidentally broke her necklace. One third of the pearls fell to the ground, one fifth stayed on the couch, one sixth were found by the woman, and one tenth were recovered by her husband; six pearls remained on the string. How many pearls were originally on the necklace?

This problem could be solved with an equation. Use n as the number of pearls on the necklace before it was broken. Then one third of the pearls would be n/3 and so on. Add 6 to the fractions of n, and set it equal to n. Solve for n. But I could see how the students might be overwhelmed by the fractions and may not see to set it up as an equation. I wanted to see how they would respond to it if they couldn't find the equation. I thought of a way to do it myself with a picture. My solution is as follows:

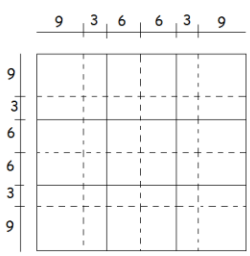
$$\frac{1}{3}=\frac{10}{30},\frac{1}{5}=\frac{6}{30},\frac{1}{6}=\frac{5}{30},\frac{1}{10}=\frac{3}{30}$$



We knew that there were 6 pearls left (the white part), so each cell is one pearl. There were 30 pearls.

Problem 3B) a model train is set up on a circular track with six telephone poles spaced evenly around it. If the train takes 10 seconds to go from the first pole to the third pole, how long does it take the train to go all the way around the track?
This was my favorite problem of the skill set. When I was attempting to pick out problems at the start, I realized that this one had a subtle trap. I thought that the students could fall into it. Let us define a "space" as the distance from any of these poles to the next. Some people may think that from the first to the third pole is three of the six spaces (since the first pole would be the starting point of the track), but it is actually only two.

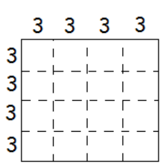
Problem 3C) How many 9-inch square floor tiles are needed to cover a rectangular floor that measures 12 feet by 15 feet?
This is a type of problem that many college students have trouble with. These were not easy for me to solve until I took physics and focused on unit conversions on the first day. It is sometimes difficult for us to visualize concepts like these. I wanted to see if the students could either visualize it or find some mathematical way of describing the situation. I solved it the following way:



I kept drawing on square feet until the 9 by 9 inch tiles filled the whole box. This was the least common denominator, and we can use it to find the answer.

$$16\ tiles = 9ft^2$$
$$12ft * 15ft = 180ft^2$$
$$180ft^2 = 20 * 9ft^2 = 20 * 16\ tiles = 320\ tiles$$

I then realized that the problem was worded in a funny way and that this was also a possible solution if you read it differently. I decided to accept either solution.
A 9-inch square floor tile could mean a 9 square inch floor tile (so a 3" by 3" tile).



Here, you could say that there are 16 tiles in 1ft. Then you have

$$180ft^2 = 180 * 1ft^2 = 180 * 16\ tiles = 2880\ tiles$$

General Logic:

Problem 2A) A child has a set of 10 cubical blocks. The lengths of the edges are 1cm, 2cm, 3cm, ..., 10cm. Using all the cubes, can the child build two towers of the same height by stacking one cube upon another? Why or why not?
This problem is difficult more because of the "why" than anything else. As a high school student, I never really thought a lot about even and odd numbers; I was curious to see whether these students could explain how they discovered their result.

Problem 4B/4E) two friends decide to play a prank on Mr. Brown for two weeks. Jim lies to Mr. Brown on Fridays, Saturdays and Sundays. Fred lies to Mr. Brown on Tuesdays, Wednesdays, and Thursdays. F Mr. Brown calls both of them to the office after he figures out that each has lied to him, and they both tell him, "I lied yesterday," what day of the week is it?
This problem isn't so bad if you look at each day of the week and eliminate them one by one. But I wanted to see if the students could come up with their own strategy or employ elimination.

Problem 4C) Sam, Mamie, Ralph, and Gail are all good at Guitar Hero. Gail consistently scores higher than Ralph. Sam is better than all of them, and Mamie is better than Ralph. Is Mamie better than Gail?
I chose to use this problem because it doesn't give enough information to answer. I wanted to see if the students could recognize when information was relevant and sufficient.

Working backwards:

Problem 1C) A street vendor had a basket of apples. Feeling generous one day, he gave away one-half of his apples plus one to the first stranger he met, one-half of his remaining apples plus one to the next stranger he met, and one-half of his remaining apples plus one to a third stranger he met. If the vendor had one left for himself, how many did he have to start with?
I chose to use this problem because I got it wrong the first time I did it. I did not work backwards. I started with x amount of apples and kept whittling it down, and set what I got to the one apple that the street vendor had left. It was overly complicated, and I must have gotten bogged down with the arithmetic. I hoped that the students would catch on and solve the problem the way the book suggested after some struggle. It was the problem that I thought was most challenging toward their last problem set.

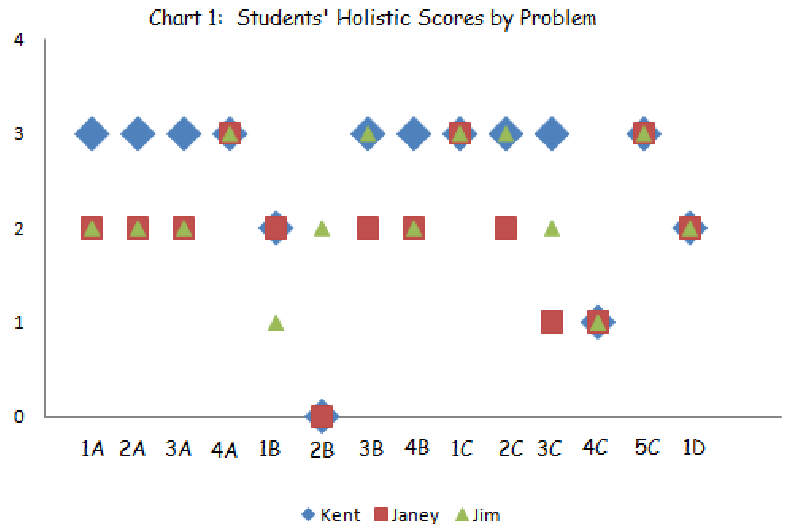
Section 2:

In general, the students seemed to have a difficult time with most of the problems; they spent between 30 and 45 minutes on each problem set (except the last, where they were allowed to work together). I scored their responses to the tasks on a four-point scale. A score of 0 was given for a response which had not made any progress toward solving the problem (irrelevant responses or no response). A score of 1 was given for a little progress toward the answer. A score of 2 was given if the problem was very nearly solved (they were missing one small step or they made a very small error somewhere). A score of 3 was given for a solved problem with adequate work.

The scores given for each problem from each student are shown in the table below:

Problem	Kent's Score	Janey's Score	Jim's Score
1A	3	2	2
2A	3	2	2
3A	3	2	2
4A	3	3	3
1B	2	2	1
2B	0	0	2
3B	3	2	3
4B	3	2	2
1C	3	3	3
2C	3	2	3
3C	3	1	2
4C	1	1	1
5C	3	3	3
1D	2	2	2

Table 1: Students' Holistic Scores by Problem



The students had the tendency to get similar scores on most of the problems. They were within 1 point of each other on all problems except 2B and 3C. Recall that 2B was the problem involving the broken pearl necklace and 3C was the problem involving floor tiles. The students seemed to have the most difficulty with the pearl necklace problem, since this was the only task for which I gave a score of 0. They all received a score of 1 on 4C, which was the Guitar Hero question. We could probably say that 4A, 1C, and 5C were the easiest, since all of the students got them right. Recall that 4A was the “Find the function” problem; 1C was the street vendor with apples problem; and 5C was the task where they were instructed to make a half-cylinder out of paper.

For the 14 problems done by the 3 students (42 problems total), I gave out the following amounts of each score.

Score	Count
0	2
1	5
2	17
3	18

Table 2: Score Frequency

Kent

Score	Count	Percent
0	1	7.14%
1	1	7.14%
2	2	14.29%
3	10	71.43%

Table 3:
Kent's Score Frequency

Janey

Score	Count	Percent
0	1	7.14%
1	2	14.29%
2	8	57.14%
3	3	21.43%

Table 4:
Janey's Score Frequency

Jim

Score	Count	Percent
0	0	0.00%
1	2	14.29%
2	7	50.00%
3	5	35.71%

Table 5:
Jim's Score Frequency

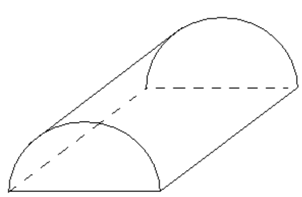
One of the interesting things about these scores is that Kent scored more than half of the perfect scores. Janey and Jim both had a high frequency of twos. While the students were working on the problem sets, I noticed that they tended to do two or three problems over the course of the first ten minutes, and then spend much of the time staring at their papers and briefly jotting things down every once in a while for the last problem or two. They seemed to enjoy solving the problems (although they sometimes became frustrated), and they were determined to give some response.

The following are summaries of how the students solved each of the problems.

<u>Spatial reasoning problems:</u>

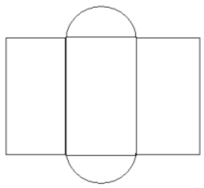
For the most part, the students had a terrible time with these although my directions seemed clear enough to demonstrate the objective; the students did catch on and get it right sometimes. These were the problems at which they spent the most time staring.

Problem 1A) Create a net for a half-cylinder like the one shown below.

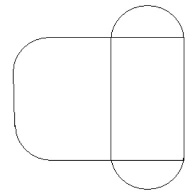


This problem was solved by Kent.

He drew this

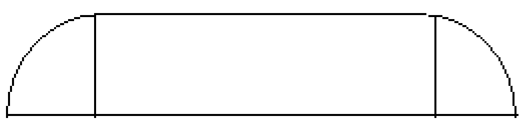


Jim drew this.

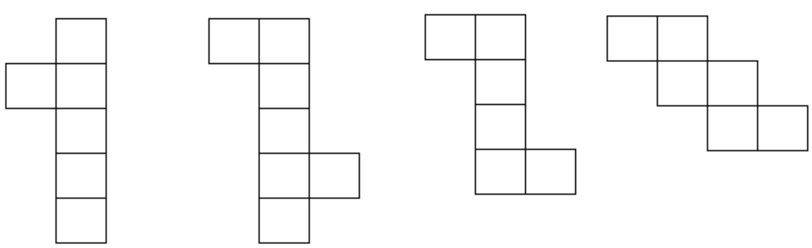


Jim figured out how the left flap attached to the rectangular base, but he didn't think that it was the same on the other side (even though he knew that this was a symmetric object).

Problem 1B) Original: Draw and label the shapes that, if cut out and taped together would make a cone with height 4 and radius 4.
Final: Draw and label two shapes that, if cut out and taped together would form a cone.
As I mentioned before, I changed this problem. I handed out the problem set, and the students were confused by what I meant by "label", even after a short explanation. I had three answers for this problem. Janey drew a circle and a cylinder (with height and radius 4). Jim wrote down "triangle, circle". Kent drew a circle and the following after folding his paper in various ways for a while.



Problem 2C) which of the following can be folded into a closed box?



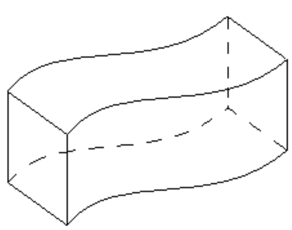
Kent and Jim thought that the last three could be folded into a closed box. I am not sure whether or not they recognized that there was an extra flap for the second net (see Section 1). Janey chose the middle two. None of the students mentioned anything special about the second one.

Problem 5C) Make a half-cylinder out of paper.
They all did very well with this. I gave them scissors, tape, a stapler, and a bunch of black paper, and they had no problem figuring out how to piece things together. Kent was a little more of a perfectionist about the semi-circle ends than the other two.

Problem 1D) which shapes can be made out of a cross-section of a cube?
I gave the students the plastic cube at first and posed the question. They came up with a square and rectangle usually. When they started having trouble coming up with their own cross-sections, I suggested shapes for them to consider. After they confirmed or rejected my ideas, I gave them the rubber band to see which ones worked.

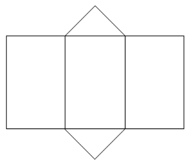
Janey came up with parallelogram, triangle, rectangle, square, and trapezoid. I asked her if you could make a pentagon. She said yes, but that it was “ugly-looking”.
Jim came up with pentagon, trapezoid, a triangle along the diagonal, and square if you cut the cube perfectly. I asked him about non-right parallelograms, and he said that they were impossible.
Kent came up with triangle, square, and rectangle on his own. I asked him about pentagons; he said they were possible but “diamond-looking” (like this ). He also said that parallelograms and hexagons were possible, but trapezoids (non-right) and decagons were impossible. He supported his claim that decagons were impossible by stating that a cube only has 6 sides and that it cannot create any object with more than six sides.

Problem 1E) Draw the shape which, if cut out and taped together, would form the following figure.



This problem is analyzed in detail in Section 3.

Problem 3E) what 3-dimensional object would the following shape make if you cut it out and taped it together?



This problem is also analyzed in detail in Section 3.

<u>Problems which are easier with a picture:</u>

Problem 3A/2E) A new basketball league was formed, and all of the teams in it have to play each other. If there are seven teams, how many games will be played in the league?
The students had a lot of trouble with this one. They all used a similar method; they focused on the first team, said they had to play all the other teams. Then they focused on the second team and said they had to play all the other teams minus the one they had already counted. Then they focused on the third team and so on until the seventh (which didn't count because they had already counted the seventh team's games when tallying up the other teams).
Despite the fact that they all used the same method, only Kent came up with the right answer. Jim counted the number of teams that Team#1 played as 7, Team#2 as 6, and so on so that he got an answer of 28 instead of 21. Janey just got 20, but that was most likely an error in the addition. How they solved this problem the second time they had it is discussed in detail in Section 3.

Problem 4A) Find the function which created the table. For example, if I gave you the following table, you could determine that the function was 2*x.

x	y
1	2
2	4
3	6

y=2*x

x	y
1	4
2	7
3	12
4	19

y=?

The students all came up with the right answer for this one, but it took them a long time. Kent tried a line first (he had mx+b written), but he found the differences between the y-values, found out that they were not equal and tried a quadratic next. Janey and Jim also found the differences in the y-values and tried a quadratic.

Problem 3B) a model train is set up on a circular track with six telephone poles spaced evenly around it. If the train takes 10 seconds to go from the first pole to the third pole, how long does it take the train to go all the way around the track?
All of the students drew a picture for this one (correctly). But Janey miscalculated when performing the number-crunching part of it. None of the students fell for the trick in the wording that I anticipated (see Section 1).

Problem 2B) A woman accidentally broke her necklace. One third of the pearls fell to the ground, one fifth stayed on the couch, one sixth were found by the woman, and one tenth were recovered by her husband; six pearls remained on the string. How many pearls were originally on the necklace?
I got 5400, 24, and 11 as the answers for this problem. Jim nearly had it. He found that the fraction of pearls which had been found off of the string was 4/5. All he had to do (but unfortunately did not) was say that 1/5 of the pearls were on the string, and that there were 6 on the string. So n/5=6. n=5*6=30. I have a more detailed explanation of the responses in Section 3.

Problem 3C) How many 9-inch square floor tiles are needed to cover a rectangular floor that measures 12 feet by 15 feet?
Janey made a mistake in her arithmetic early on, but didn't really have a useful strategy even after that point. She wrote this vertically, but basically had 12*15=60+12 instead of 60+120 (she forgot to add the zero on the end of 12). Kent converted everything into inches. He got 12ft=144" and 15ft=180" Then he divided them by 9" to get a width of 16 tile lengths and length of 20 tile lengths. He then multiplied them to get 320 tiles (the same answer I had). None of the students used the method I had thought of for this (see Section 1).

<u>General Logic:</u>

Problem 2A) A child has a set of 10 cubical blocks. The lengths of the edges are 1cm, 2cm, 3cm, ..., 10cm. Using all the cubes, can the child build two towers of the same height by stacking one cube upon another? Why or why not?
The students could not find a way to put the blocks together, and they all said that the answer was "no", but only Kent said that it was because the sum of the lengths of the bricks totaled an odd number which could not be split evenly in two. Jim was close; he added up the lengths of the blocks and got an odd number, but didn't really pay attention to the fact that he could use this result to justify his answer.

Problem 4B/4E) two friends decide to play a prank on Mr. Brown for two weeks. Jim lies to Mr. Brown on Fridays, Saturdays and Sundays. Fred lies to Mr. Brown on Tuesdays, Wednesdays, and Thursdays. If Mr. Brown calls both of them to the office after he figures out that each has lied to him, and they both tell him, "I lied yesterday," what day of the week is it?
I got a different answer from each student, but they all listed the days of the week. Kent eliminated Monday (since Fred doesn't lie on that day), and then moved on to find Friday as the answer. Jim listed the days and which student lied on each day but didn't put much thought into it and said Thursday. Perhaps he thought the question was asking what day "yesterday" was. How the students solved this problem the second time is discussed in detail in Section 3.

Problem 4C) Sam, Mamie, Ralph, and Gail are all good at Guitar Hero. Gail consistently scores higher than Ralph. Sam is better than all of them, and Mamie is better than Ralph. Is Mamie better than Gail?
As I stated in Section 1, I wanted to see if the students could recognize when information was relevant and sufficient. They did not. I got "yes", and I got "no", but none of the students recognized that there was insufficient evidence. This would also have been a good problem for the last set if I had had more room.

Working backwards:

Problem 1C) A street vendor had a basket of apples. Feeling generous one day, he gave away one-half of his apples plus one to the first stranger he met, one-half of his remaining apples plus one to the next stranger he met, and one-half of his remaining apples plus one to a third stranger he met. If the vendor had on left for himself, how many did he have to start with?
Kent did work backwards. He has 1+1*2+1*2+1*2 and 2 4 5 10 11 22 written on his paper with 22 circled. Janey wrote 22 apples, but has no work. I think that was the day when she couldn't get one of the problems and someone just told her the answer because she got frustrated with it.

Section 3:

I was able to locate several problems which were difficult for the students to solve and invoked interesting responses. I put some of these tasks again on the last problem set, allowed the students to work in a group, and observed them working on the problems.

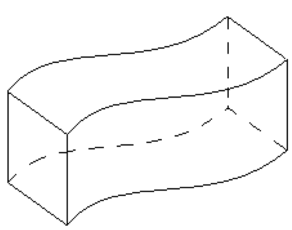
The tasks I chose to use again on the last problem set were the following:

A new basketball league was formed, and all of the teams in it have to play each other. If there are seven teams, how many games will be played in the league?

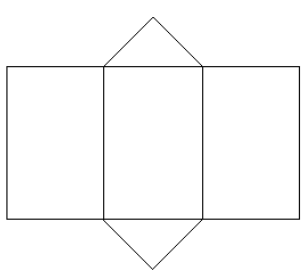
Two friends decide to play a prank on Mr. Brown for two weeks. Jim lies to Mr. Brown on Fridays, Saturdays, and Sundays. Fred lies to Mr. Brown on Tuesdays, Wednesdays, and Thursdays. If Mr. Brown calls both of them to the office after he figures out that each has lied to him, and they both tell him, "I lied yesterday," what day of the week is it?

I also invented some new spatial reasoning problems, because I was sure they would remember the answers to the old ones.

Draw the shape which, if cut out and taped together, would form the following figure.



What 3-dimensional object would the following shape make if you cut it out and taped it together?



The students solved the problems in ten minutes or so for this set; compared to the 30-40 minutes it took them to do a problem set during every other session. The last problem on this set was the most time-consuming for them since they spent a good portion of their time (4-5 minutes) arguing about it. In general, Kent worked more independently, while Janey and Jim worked things out together. After the first few minutes, Kent thought he had solved everything. He passed his work on to the other two, who had finished about half of the problem set. Janey passed back his work and told him he was wrong on the last one. As Janey and Jim worked together, Jim tended to produce ideas (some of which were not ideal), while Janey gave feedback. She was very good at spotting flaws, but she seemed either unable to come up with ideas or too shy to share them. I do not think that my facial expressions gave her clues, since she did not look toward me while she was working on the problems. In the end, they got all of the answers to the tasks correct.

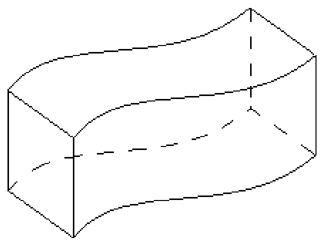
Two friends decide to play a prank on Mr. Brown for two weeks. Jim lies to Mr. Brown on Fridays, Saturdays, and Sundays. Fred lies to Mr. Brown on Tuesdays, Wednesdays, and Thursdays. If Mr. Brown calls both of them to the office after he figures out that each has lied to him, and they both tell him, “I lied yesterday,” what day of the week is it?

Recall (from section 2) that each student gave a different answer for this problem. Only Kent had it right. While the students were working together, it looked like the only person who attempted this problem was Kent. He just scribbled the initials of the students, and the first letter of the days they lied, and wrote “Friday” underneath. Jim and Janey trusted Kent’s answer, or they quickly checked his. Neither of them wrote anything on paper, and I did not hear any discussion. In retrospect, I could have picked a better problem, but I wanted to see if this problem would create interaction because all of their answers were different.

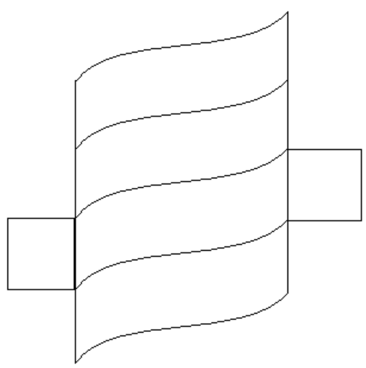
A new basketball league was formed, and all of the teams in it have to play each other. If there are seven teams, how many games will be played in the league?

This time, all of the students came up with 21 as their answer, and they all did it the same way, by listing out every game. They had all done this in a similar manner the first time (see Section 2), but each had had some slight flaw in their assumptions or counting.

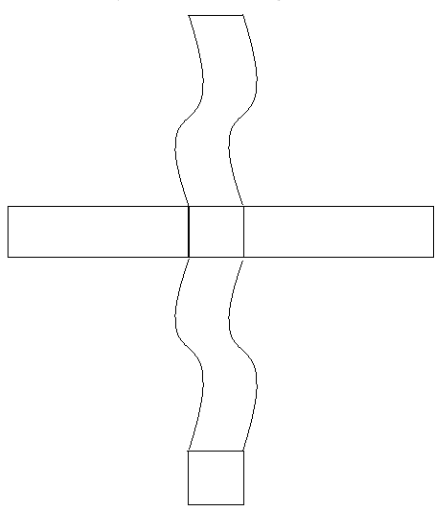
Draw the shape which, if cut out and taped together, would form the following figure.



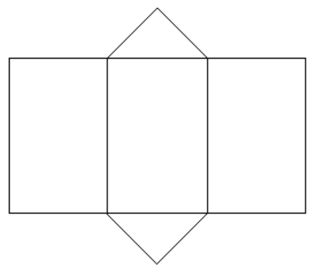
Kent and Jim both drew something like this.



Janey recognized that you would not be able to fold along the curved lines, so she told them that some of the sides would have to be straight. After hearing her criticism, Kent fixed it and came up with the right answer, as shown below.



What 3-dimensional object would the following shape make if you cut it out and taped it together?



Kent recognized that this would form a triangular prism, and there was not much discussion on the subject. Neither of the other two students have anything written down for this problem, but I assume that they looked at his answer and agreed due to the fact that this was a brand new problem; I do not think that Jim or Janey would have trusted his answer to a new problem without looking at it.

A problem that I did not choose to include on the last set of tasks was Problem 2B, the pearl necklace problem. Although none of the students were successful at answering the question, their responses are interesting because their strategies varied so greatly from one another. Therefore, I will discuss in further detail how each student attempted to solve the problem.

A woman accidentally broke her necklace. One third of the pearls fell to the ground, one fifth stayed on the couch, one sixth were found by the woman, and one tenth were recovered by her husband; six pearls remained on the string. How many pearls were originally on the necklace?

Jim found a least common denominator for all of the fractions. He then added them all together to get $\frac{48}{60} = \frac{8}{10} = \frac{4}{5}$. He then wrote 24 as his answer. He was very close. If he had just said that 1/5 of the pearls (because 4/5 of the pearls were the ones found elsewhere) were the six left on the string, then 6*5=30.

Kent has four variables and the fractions given in the problem. He then multiplies various large numbers by the fractions and comes up with 5400 pearls. There is some erased work, but none of it really makes a lot of sense to me.

Janey multiplied all of the fractions by 6, got $2, 1\frac{1}{5}, 1, and \frac{6}{10}$ as her answers, and wrote down 11. Again, it doesn't make a lot of sense. Perhaps she had been trying to tie this to what she had been taught in class. She knows how to multiply fractions and numbers, so maybe she was trying to apply that, but did not know how to set up the problem.

If I had been solving this problem and been as confused as it looks like these students were, I would have started guessing at values, particularly values the denominators of the fractions were factors of. I know that the answer to the question has to be a whole number, and that these people were probably not finding fractions of pearls all over the place. Since the denominators are 10, 3, 5, and 6, the lowest number they all go into would be 30. If I guess 30, that is the right answer, and I could check by finding one third of the pearls=30/3=10 and so on. Guessing is the way that most students start finding x when they are first introduced to simple algebraic equations; I wonder why the students would not employ the same method for this.

I think that the students just got very confused when fractions were put in front of them. They did not seem to think about the problem; they just did a bunch of arithmetic. I would like to give them the equation I had mentioned in Section 1 to see if they could solve that. It would have been interesting to put the equation and this question on the same problem set and ask if they noticed anything about the two to see if they could find the parallel between them.

Section 4:

The students took on distinct roles when they worked in a group, and some of those roles clashed with what I initially saw of their personalities. When I first met the students, I had what turned out to be fairly accurate first impressions. I thought that Janey was somewhat unsure of herself; for example, when she tutored, she often asked me to make sure she was doing the problem right. She was outgoing in a way, because she always asked people if they needed help. While she was good at doing the math for the most part, she never committed to an answer right away. She always checked herself. While working on the problem sets, she took a long time and sometimes confused herself by constantly checking to make sure things made sense. She never just tried something to see if it worked. Problem 4A is the best example of this; she never just tried a random quadratic formula and guessed it was x^2 or $x^2 +1$. She found it out eventually, but she could have come up with the answer much sooner if she had simply guessed something like a quadratic after she'd had that idea.

I thought that Kent was just quiet. He was more confident about his math skills when he was asked a question, but he didn't walk up to people and ask if they needed help like Janey. He tended to sit back until someone raised their hand. When he did the problems, he worked through them steadily. I never saw him get stuck while answering someone's question. When he got stuck on one of my problem sets, it looks like he just tried to see the problem a different way. He eventually would find something that worked. He was never afraid to guess at something if he couldn't figure it out at first.

Jim was definitely the most hyper of the three. He raced through everything and was very impatient. I think this is the reason he got so many 2's on the problem sets; he never really thought things through long enough to see if it made sense. The necklace problem was a prime example of this. He was so close to having the answer, but he just did something silly in his head. He didn't write whatever he did down. I think that this was because he was in a hurry to get to the next problem. When he tutored, he walked up to students who looked like they may have been having trouble and just started helping them. He hardly ever asked whether or not they needed help. One time, I saw him walk up to a student, look at his work, say, "You need help," and start explaining the problem.

As the students worked in a group, Jim and Kent generated solutions while Janey was the critic. Kent worked more independently; he only wanted feedback after he thought he had found a solution, and if he was challenged, he went back to working on his own. Jim tended to look for feedback while he was in the middle of solving things; he did not mind being interrupted when necessary.

I thought that something was odd about some of their roles within the group. Jim, the most impatient of the three was the one that didn't mind being interrupted as he was working. Janey, the least confident of the three, was telling the other two when they were wrong. Kent was the only one that really did the same thing in a group that he did on his own. He worked steadily on the problems and didn't let anything interrupt his train of thought.

The main conclusion I drew personally from this is that individual performance on tasks involving mathematics unfamiliar to the student is strongly dependent on confidence. The skill levels of these students were very similar. When I watched them tutor, they all were able to answer the same questions at about the same level of knowledge and skill. Despite the similar skill levels, the student who was least confident scored at or below the other students' scores on all but one task when they worked individually (see Table 1 in Section 2). Further supporting my claim is the fact that she repeatedly corrected them as they worked in a group.

In addition to the above conclusions, I would like to assess how successful I was in terms of my individual goals.

- Goal 1:

  Determine by trial and error which types of problems the students had difficulty with by varying the content on the weekly problem sets. After finding difficult problems, give another like it on the next problem set to see whether or not the students have developed a strategy for solving such problems.

  - Assessment of goal 1:

    I kept the students engaged with the spatial reasoning problems, and I also found other problems that were challenging for the students. As Table 1 indicates (see Section 2), the students were unable to solve most of the spatial reasoning problems. The pearl necklace problem (Problem 2B) was a particularly difficult problem for the students, but I did not use a similar problem in the subsequent problem sets. It is plausible that the results for this problem were senseless because of the students' aversion to fractions. Guessing would have lead to an answer to the problem, but the students did not attempt this method.

- Goal 2:

  Determine which types of problems the students have difficulty developing a successful strategy for.

  - Assessment of goal 2:

    The spatial reasoning problems were definitely the group that fulfilled this goal. None of the students developed a repeatedly successful strategy for solving the problems. Perhaps this skill is difficult for the students to develop without intervention. I could have done more with fractions like the pearl necklace problem.

- Goal 3:

  Let the students work as a group on the last problem set made up of problems all of the students had difficulty solving, and describe how this affected their problem-solving ability.

  - Assessment of goal 3:

    When the students worked in a group, I was able to obtain more information about the students' personalities but not as much about their individual problem-solving methods. The students worked quickly and wrote very little as they worked in a group. As a result, I was unable to write quickly enough to obtain a complete picture of how the students worked. I was also unable to determine from their written work how they thought about the problem individually. I wish I had anticipated the students taking less time while working together and given more problems on the last set. The pearl necklace problem (2B) and the guitar hero problem (4C) would have been excellent candidates for the last problem set.

- Goal 4:

  Observe the students tutoring. This was included mostly because I had agreed to participate in a tutoring project and did not want to go back on my word to the school. The students spent about an hour each week tutoring before spending half an hour on their problem set.

  - Assessment of goal 4:

    This gave some insight into the personalities of the students taking part in the project that I could not have gained otherwise. Tutoring behavior was one of the major indicators of how outgoing, confident, and mathematically capable the students were. Since they had to explain what they were doing while they tutored, it gave me the opportunity to see what they were thinking a little more clearly.

Due to the small number of students involved in the study, the results may not necessarily reflect the behavior or abilities of the population of similar students. This is not a random sample; all of the students came from the same high school and had the same teacher.

This study suggests that there is more work to do to determine for which unfamiliar problems students have trouble developing successful strategies. The students in this study were exposed to problems that they had not seen in a classroom setting. They did well on some types of problems and poorly on others. The fractions involved in the pearl necklace problem (problem 2B) seem to have confused two of the students to the point where they started performing calculations which were irrelevant to the problem. The manner in which students, especially those who are confused, explain their strategies may also be of interest.

Next year, I will start teaching college-level mathematics classes. I enjoyed having motivated students who were challenged by the problems I set them. It is difficult to engage students who are already motivated without leaving the rest of the class behind, but I intend to supply a reasonable challenge for all of my students. By using the types of problems described in Section 2, a teacher can challenge their best students while not requiring a large amount of previously learned techniques. I view mathematics as a problem-solving discipline. While the memorization of computational methods can be important for efficiency, the ability to develop a strategy to solve a problem is far more important.

Reference

Johnson, K., & Herr, T. (2001). Problem Solving Strategies: Crossing the River with Dogs and Other Mathematical Adventures