Original Papers

The Role of Ignorance about Keynes's Inexact, Approximation Approach to Measurement in the *A Treatise on Probability* in the Keynes-Tinbergen Exchanges of 1938-1940 in the *Economic Journal*

Michael E Brady[1*.]

[1] Department of Operations Management, College of Business Administration and Public Policy, California State University, Dominguez Hills, 1000 Victoria St, Carson, California, 90747, USA

[*] Michael E Brady, Department of Operations Management, College of Business Administration and Public Policy, California State University, Dominguez Hills, 1000 Victoria St, Carson, California, 90747, USA

Abstract

J.Tinbergen and J. M. Keynes held diametrically opposed positions on measurement. Tinbergen's physics background led him to deploy an exact approach to measurement based on the specification of probability distributions, like the normal and log normal, with exact and precisely probabilities that were linear, additive and definite. All probabilities for Tinbergen were assumed to be well defined, precise, exact, determinate, definite, additive, linear, independent single number answers, whether the field was physics or economics. Keynes's approach was an inexact one. Probabilities for Keynes were, with a few exceptions, partially defined, imprecise, inexact, indefinite, indeterminate ,non additive, non linear, and dependent. Probability estimates for Keynes required two numbers to specify the probability within a lower and upper bound (limit), and not one single number like Tinbergen's approach. Keynes called this approach Approximation. Keynesian probabilities are interval valued.

*The problem, from Keynes's perspective, was that Tinbergen was trying to apply to economic data techniques which were only sound when applied in physics ,where laboratory controlled environments with detailed experimental design could generate data and replicate/duplicate the experiments. Keynes had always argued that economics was **not** a physical or life science like physics, engineering, biology or chemistry and that it could never be like physics.*

Keywords

exact measurement, inexact measurement, precise probability, imprecise probability, approximation, interval valued probability

1. Introduction

The paper will be organized in the following fashion. Section Two will present an overview of Keynes's inexact approximation approach to measurement as presented by him in 1921 in the *A Treatise on Proba*bility (TP, 1921). All five parts of the TP will be examined in order to show that Keynes's concern with the inexact measurement of probability in statistics was one of the primary goals Keynes had in writing the book. Section Three will examine the mistaken assessments made by Tinbergen and other economists about Keynes's technical understandings of probability and statistics. It will be demonstrated that Tinbergen and other economists were advocating techniques for application in economics that worked only in physics. Section Four concludes the paper.

Keynes's unchanging lifetime views on inexact measurement versus exact measurement and imprecise probability versus precise probability explain the Keynes-Tinbergen debate of 1939-1940. Tinbergen's background in Physics meant that he was an advocate of the Limiting Frequency interpretation of probability. Tinbergen was thus an advocate of precise and definite probability and exact measurement. Tinbergen brought this view with him when he started working in economics. Keynes was the exact opposite.

Tinbergen was used to analyzing inanimate phenomenon, like atoms, protons, electrons, particles, cells, molecules, genes, chromosomes, fair decks of cards, dice, coins, etc. Keynes was used to analyzing animate human behavior. Humans, unlike inanimate phenomenon, like atoms, protons, electrons, particles, cells, molecules, genes, chromosomes, fair decks of cards, dice, coins, etc., can think and reason. Humans have brains, memories, emotions, minds, can reason and change their opinions. While exact calculation and precision can be applied in many areas of the physical and life science, it is doubtful in most social sciences, liberal arts, behavioral sciences and especially in most areas of economics, finance and business.

2. Method

It is impossible for technically trained reader of the TP not to recognize that Keynes's method is one of inexact measurement based on approximation unless the reader has not read Parts II and V. Mathematically confused readers of the TP would not be able to understand Keynes's use of Boole's concept of lower and upper ,interval valued, probability that was used by Keynes in parts I, III and IV, and heavily by Keynes in Parts II and V of the TP. Mathematical confusion may be the best explanation available that explains the incoherent, incomprehensible, contradictory Keynesian Fundamentalist claim that Keynes's method was one of ordinal measurement some of the time. Ordinal measurement appears nowhere in the TP. This section of the paper will be divided into seven subsections 2.1 to 2.7.

230

2.1 Keynes's Inexact Measurement and Approximation Method in Part I of the TP

Keynes refers to inexact measurement (intervals) a number of times in chapter III of the TP. His main target is the claim that all probabilities are numerical. In Part II, he shows that this would be the assertion that all probabilities are linear and additive. He warns the reader explicitly on page 37 that his discussions in chapter III are preliminary only and that it is in Part II that the issue of numerical measurement will be discussed in detail: "It will not be possible to explain in detail how and in what sense a meaning can sometimes be given to the numerical measurement of probabilities until Part II. is reached. But this chapter will be more complete if I indicate briefly the conclusions at which we shall arrive later…" (Keynes, 1921, p. 37; boldface added). Thus, Keynes's view of (pp. 38-40) is that they provide a "brief", but not "detailed", set of conclusions ahead of the main conclusions in Part II .It is in Part II ,and not in chapter III of Part I, where Keynes's provided the details concerning the question of measurement. Keynes's answer is that one needs to use approximation, using inexact measurement, involving Boolean interval valued probability. All Keynesian fund amentalists (Skidelsky, Carabelli, O'Donnell, Fitzgibbons, Runde, Meeks, etc.,), just Borel (1923), Edgeworth (1922), Ronald Fisher (1923), E. Wilson (1923; Pearl, 1922), and Ramsey skipped Part II of the TP. They thus arrived at a very incomplete answer as regards Keynes's theory of measurement. Their incomplete answer resulted in their reading into pp. 38-40 of the TP an ordinal theory of probability that does not exist in the TP. Nowhere in the TP does Keynes put forth an ordinal (ordinary language) theory of probability or even list the words "ordinal", "comparative", or "rank-order" in his index. The diagram that Keynes presents the reader on p. 39 (p. 42 of Volume 8 of the CWJMK) involving pages 38-40 is a brief introduction to his theory of Measurement as Keynes stated on pp. 37-38 of the TP. Chapter III is the alpha of Keynes's analysis of measurement. Chapters 15-17 are the omega of Keynes's analysis. The Keynesian "fundamentalists" (G. Meeks, R. Skidelsky, R. O'Donnell, A. Carabelli, A. Fitzgi-bbons, J. Davis, J. Runde, for example) chose to ignore Part II of the TP and base all of their analysis on the diagram on page 39 on chapter III of Part I of the TP alone. As Keynes stated on pp. 37-38, it will not be possible to understand his position on the measurement of probabilities without the important ,detailed analysis of Part II of the TP. Keynes is very clear that probabilities can be estimated using lower and upper bounds: "A relation of probability does not yield us, as a rule, information of much value, unless it invests the conclusion with a probability which lies between narrow numerical limits. In ordinary practice, therefore, we do not always regard ourselves as knowing the probability of a conclusion, unless we can estimate it numerically. We are apt, that is to say, to restrict the use of the expression probable to these numerical examples, and to allege in other cases that the probability is unknown. We might say, for example, that we do not know, when we go on a railway journey, the probability of death in a railway accident, unless we are told the statistics of accidents in former years; or that we do not know our chances in a lottery, unless we are told the number of the tickets. But it must be clear upon reflection that if we use the term in this sense,—which is no doubt a perfectly legitimate sense,—we ought to say that in the case of some arguments a relation of probability does not exist, and not that it is unknown.

For it is not this probability that we have discovered, when the accession of new evidence makes it possible to frame a numerical estimate. Possibly this theory of unknown probabilities may also gain strength from our practice of estimating arguments, which, as I maintain, have no numerical value, by reference to those that have. We frame two ideal arguments, that is to say, in which the general character of the evidence largely resembles what is actually within our knowledge, but which is so constituted as to yield a numerical value, and we judge that the probability of the actual argument lies *between* these two. Since our standards, therefore, are referred to numerical measures in many (author-note Keynes's emphasis in italics) two numerical measures, we come to believe that it must also, if only we knew it, possess such a measure itself". (Keynes, 1921, pp. 31-32; bold face added). This quotation is the foundation for F. Y. Edgeworth's conclusion in his 1922 review that Keynesian probabilities are intervals. Keynes also explicitly discusses intervals on pp. 22, 23, 24, 29, and 35 of chapter III. Keynes's diagram on page 39 (p. 42 of CWJMK version of the TP) presents no theory of ordinal probability just as chapter 3 of the GT contains no theory of expected aggregate demand and expected aggregate supply.

2.2 Keynes's Method of Inexact Measurement and Approximation in Part II of the TP

Keynes's characterization of his approach again emphasizes the importance of probabilities being "between" an upper bound and a lower bound: "It is evident that the cases in which exact numerical measurement is possible are a very limited class, generally dependent on evidence which warrants a judgment of equi-probability by an application of the Principle of Indifference. The fuller the evidence upon which we rely, the less likely is it to be perfectly symmetrical in its bearing on the various alternatives, and the more likely is it to contain some piece of relevant information favouring one of them. In actual reasoning, therefore, perfectly equal probabilities, and hence exact numerical measures, will occur comparatively seldom. The sphere of inexact numerical comparison is not, however, quite so limited. Many probabilities, which are incapable of numerical measurement, can be placed nevertheless between (Author-note Keynes's emphasis) numerical limits. And by taking particular non-numerical probabilities as standards a great number of comparisons or approximate measurements become possible. If we can place a probability in an order of magnitude with some standard probability, we can obtain its approximate measure by comparison. This method is frequently adopted in common discourse" (Keynes, 1921, p. 160).

Pp. 161-163 are of great importance as Keynes demonstrates his method of analysis using approximation, as he does also in chapter 17 on pp. 186-194, pp. 235-238, and pp. 253-255. Keynes's presentation of his method of inexact measurement and approximation in Part II can only lead to the complete and total rejection of the fundamentalist claim that purports to have discovered Keynes's "theory" on pages 38-40 of the TP in an analysis that Keynes calls "brief".

2.3 Keynes's Method of Inexact Measurement and Approximation in Part III of the TP

On page 192 of the TP, when applying his technique for solving Boolean interval probabilities Keynes used this technique in part III in order to set up an application of his concept of finite probability:

"There is one class of probabilities, however, which I called the numerical class, the ratio of each of whose members to certainty can be expressed by some number less than unity; and we can sometimes compare a non-numerical probability in respect of more and less with one of these numerical probabilities. This enables us to give a definition of "finite probability" which is capable of application to non-numerical as well as to numerical probabilities. I define a "finite probability" as one which exceeds some numerical probability, the ratio of which to certainty can be expressed by a finite number. The principal method, in which a probability can be proved finite by a process of argument, arises either when its conclusion can be shown to be one of a finite number of alternatives, which are between them exhaustive or, at any rate, have a finite probability, and to which the Principle of Indifference is applicable; or (more usually), when its conclusion is more probable than some hypothesis which satisfies this first condition" (Keynes, 1921, p. 237; bold face added).

Keynes repeats himself in chapter 22:   "There is a vagueness, it may be noticed, in the number of instances, which would be required on the above assumptions to establish a given numerical degree of probability, which corresponds to the vagueness in the degree of probability which we do actually attach to inductive conclusions. We assume that the necessary number of instances is finite, but we do not know what the number is. We know that the probability of a well-established induction is great, but, when we are asked to name its degree, we cannot. Common sense tells us that some inductive arguments are stronger than others, and that some are very strong. But how much stronger or how strong we cannot express. The probability of an induction is only numerically definite when we are able to make definite assumptions about the number of independent equi-probable influences at work. Otherwise, it is non-numerical, though bearing relations of greater and less to numerical probabilities according to the approximate limits within which our assumption as to the possible number of these causes lies" (Keynes, 1921, p. 259).

Of course, a reading of chapter 15 of the TP is absolutely required if a reader of Part III of the TP is to understand that non numerical probabilities are interval valued probabilities. There is no explicit evidence that the Keynesian fundamentalists understand this, given their over 40 years of claims that Keynes's non numerical probabilities were ordinals. Keynes's definition of finite probability, which is applicable to both numerical and non numerical, interval valued probabilities, makes no sense if Keynes's probabilities are ordinal. In fact, Part III of the TP is incomprehensible, inconsistent, and contradictory if based on a reading that argues that Keynes's theory of probability was an ordinal one. This would directly conflict with the Boolean foundations provided to Part III by Boole's Problem X on page 192 in chapter 17, which is interval. There are no ordinal probability problems in Boole's 1854 The Laws of Thought. Any interpretation of Keynesian probabilities as being ordinal probabilities renders any understanding of the Boolean foundation of Part III impossible to understand.

2.4 Keynes's Method of Inexact Measurement and Approximation in Part IV of the TP

Consider the necessity in Part IV of understanding Keynes's finite probabilities from Part III:

"For this purpose it is necessary to recur, briefly, to the analysis of Part III. It was argued there that the

233

methods of empirical proof, by which we strengthen the probability of our conclusions, are not at all dissimilar, when we apply them to the discovery of formal truth, and when we apply them to the discovery of the laws which relate material objects, and that they may possibly prove useful even in the case of metaphysics; but that the initial probability which we strengthen by these means is differently obtained in each class of problem. In logic it arises out of the postulate that apparent self-evidence invests what seems self-evident with some degree of probability; and in physical science, out of the postulate that there is a limitation to the amount of independent variety amongst the qualities of material objects. But both in logic and in physical science we may wish to consider hypotheses which it is not possible to invest with any à priori probability and which we entertain solely on account of the known truth of many of their consequences. An axiom which has no self-evidence, but which it seems necessary to combine with other axioms which are self-evident in order to deduce the generally accepted body of formal truth, stands in this category. A scientific entity, such as the ether or the electron, whose qualities have never been observed but whose existence we postulate for purposes of explanation, stands in it also. If the analysis of Part III. is correct, we can never attribute a finite probability to the truth of such axioms or to the existence of such scientific entities, however many of their consequences we find to be true. They may be convenient hypotheses, because, if we confine ourselves to certain classes of their consequences, we are not likely to be led into error; but they stand, nevertheless, in a position altogether different from that of such generalizations as we have reason to invest with an initial probability (Keynes,1921, pp. 299-300; bold face added).

Keynes's footnote one adds the following point that ordinal probability can't be applied:

"I am assuming that there is no argument, arising either from self-evidence or analogy, in addition to the argument arising from the truth of their consequences, in favour of the truth of such axioms or the existence of such objects; but I daresay that this may not certainly be the case. The reader may be reminded also that, when I deny a finite probability this is not the same thing as to affirm that 16 the probability is infinitely small. I mean simply that it is not greater than some numerically measurable probability" (Keynes, 1921, pp. 299-300; bold face added).

The reader should note again that this passage makes absolutely no sense if Keynes's probabilities are ordinal, since finite probabilities must apply to both numerical and non numerical interval valued probabilities. Consider chapter 26, which was viewed as being very important by both Edgeworth and Russell, since the discussions in chapter 26 required a theory of inexact measurement to have been already presented in an earlier part of the book (Part II), according to Russell (1922):

"In Chapter III. of Part I. I have argued that only in a strictly limited class of cases are degrees of probability numerically measurable. It follows from this that the 'mathematical expectations of goods or advantages are not always numerically measurable; and hence, that even if a meaning can be given to the sum of a series of non-numerical 'mathematical expectations,' not every pair of such sums are numerically comparable in respect of more and less. Thus even if we know the degree of advantage which might be obtained from each of a series of alternative courses of actions and know also the

probability in each case of obtaining the advantage in question, it is not always possible by a mere process of arithmetic to determine which of the alternatives ought to be chosen. If, therefore, the question of right action is under all circumstances a determinate problem, it must be in virtue of an intuitive judgment directed to the situation as a whole, and not in virtue of an arithmetical deduction derived from a series of separate judgments directed to the individual alternatives each treated in isolation. We must accept the conclusion that, if one good is greater than another, but the probability of attaining the first less than that of attaining the second, the question of which it is our duty to pursue may be indeterminate, unless we suppose it to be within our power to make direct quantitative judgments of probability and goodness jointly. It may be remarked, further, that the difficulty exists, whether the numerical indeterminateness of the probability is in trinsicor whether its numerical value is, as it is according to the Frequency Theory and most other theories, simply unknown" (Keynes, 1921, p. 312).

Keynes has just ruled out ordinal probability due to intervals overlapping with one another meaning that "…not every pair of such sums are numerically comparable in respect of more and less." It has been known for centuries that it is mathematically impossible to add (sum), subtract, divide, or multiple ordinal probabilities. Keynes's comment can only be applied to overlapping interval valued probabilities. It is not possible to understand Part IV of the TP if Keynes's theory of probability is an ordinal one, since finite probabilities can't possibly be defined. Indeterminate probability means for Keynes, like it meant for Boole, that the addition of more relevant evidence is not going to result in a narrowing of the wide gap that exists between the lower and upper probabilities.

2.5 Keynes's Method of Inexact Measurement and Approximation in Part V of the TP

Keynes repeatedly refers to indirect and inexact measurement using intervals all through Part V of the TP: The "Stability of Statistical Frequencies" would be a much better name for it. The former suggests, as perhaps Poisson intended to suggest, but what is certainly false, that every class of event shows statistical regularity of occurrence if only one takes a sufficient number of instances of it. It also encourages the method of procedure, by which it is thought legitimate to take any observed degree of frequency or association, which is shown in a fairly numerous set of statistics, and to assume with insufficient investigation that, because the statistics are numerous, the observed degree of frequency is therefore stable. Observation shows that some statistical frequencies are, within narrower or wider limits, stable But stable frequencies are not very common, and cannot be assumed lightly. 19 The gradual discovery, that there are certain classes of phenomena, in which, though it is impossible to predict what will happen in each individual case, there is nevertheless a regularity of occurrence if the phenomena be considered together in successive sets" (Keynes, 1921, p. 336). I will simply list the great number of statements by Keynes where frequency probabilities are reinterpreted as interval probabilities that lie between upper and lower limits, so that approximation applies. One of the values of reading Part V of the TP is that it shows the reader what the real ,underlying conflict was that existed between Keynes and Tinbergen over measurement versus Tinbergen. It can be seen to revolve around

235

the fact that Tinbergen was committed to Exact measurement using precise probabilities while Keynes was committed to Inexact measurement using imprecise probability based on his approximation technique from chapters 15-17 of the TP or methods like those of the statistician Yule:

• Beginning with Bernoulli's Theorem, we will consider the various solutions of this problem which have been propounded and endeavour to determine the proper limits within which each method has validity (Keynes, 1921, pp. 337-338).

• …the value of this probability being calculable by a process of approximation… (Keynes, 1921, p. 338).

• For the second part of the theorem some method of approximation is required… (Keynes, 1921, p. 338).

• …by means of Stirling's Theorem, and obtain as its approximate value…(Keynes, 1921, p. 338).

• It is possible, of course, by more complicated formulae to obtain closer approximations than this.* But there is an objection, which can be raised to this approximation, quite distinct from the fact that it does not furnish a result correct to as many places of decimals as bit might. This is, that the approximation is independent of the sign of h, whereas the original expression is not thus independent. That is to say, the approximation implies a symmetrical distribution for different values of h about the value for h = 0; while the expression under approximation is unsymmetrical. It is easily seen that this want of symmetry is 20 appreciable unless mpq is large. We ought, therefore, to have laid it down as a condition of our approximation, not only that m must be large, but also that mpq must be large. Unlike most of my criticisms, this is a mathematical, rather than a logical, point (Keynes, 1921, pp. 338-339).

• This "fiction" will do no harm so long as it is remembered that we are now dealing with a particular kind of approximation…the probability* that the number of occurrences will lie between… (Keynes, 1921, p. 339).

• This same expression measures the probability that the proportion of occurrences will lie between… (Keynes, 1921, pp. 339-340).

• The probability that the proportion of occurrences will lie between given limits varies with the magnitude of the square root of 2pq/m, and this expression is sometimes used, therefore, to measure the 'precision' of the series. Given the à priori probabilities, the precision varies inversely with the square root of the number of instances.(Keynes, 1921, p. 340).

• Such a condition is very seldom fulfilled. If our initial probability is partly founded upon experience, it is clear that it is liable to modification in the light of further experience. It is, in fact, difficult to give a concrete instance of a case in which the conditions for the application of Bernoulli's Theorem are completely fulfilled. At the best we are dealing in practice with a good approximation, and can assert that no realised series of moderate length can much affect our initial probability… For this is an approximate formula which requires for its validity that the series should be long; whilst it is precisely in this event, as we have seen above, that the use of Bernoulli's Theorem is more than usually likely to be illegitimate (Keynes, 1921, p. 342).

• …the probability that the number of occurrences m of the event in the s trials will lie between the limits sp ± l is given by…The probability that the number of occurrences of the event will lie between sp ± γk (square root of s) is given by …(Keynes, 1921, p. 345).

• It seems in plain opposition to good sense that on such evidence we should be able with practical certainty…. to estimate the number of female births within such narrow limits. And we see that the conditions laid down in § 11 have been flagrantly neglected…" (Keynes, 1921, p. 352).

• Leibniz's reply goes to the root of the difficulty. The calculation of probabilities is of the utmost value, he says, but in statistical inquiries there is need not so much of mathematical subtlety as of a precise statement of all the circumstances. The possible contingencies are too numerous to be covered by a finite number of experiments, and exact calculation is, therefore, out of the question. Although naturehas her habits, due to the recurrence of causes, they are general, not invariable. Yet empirical calculation, although it is inexact, may be adequate in affairs of practice" (Keynes, 1921, p. 368).

• In dealing with the correspondence of Leibniz and Bernoulli, I have not been mainly influenced by the historical interest of it. The view of Leibniz, dwelling mainly on considerations of analogy, and demanding "not so much mathematical subtlety as a precise statement of all the circumstances", is, substantially, the view which will be supported in the following chapters. The desire of Bernoulli for an exact formula, which would derive from the numerical frequency of the experimental results a numerical measure of their probability, preludes the exact formulas of later and less cautious mathematicians, which will be examined immediately" (Keynes, 1921, p. 369).

• They showed, that is to say, that certain observed series of events would have been very improbable, if we had supposed independence between 22 some two factors or if some occurrence had been assumed to be as likely as not, and they inferred from this that there was in fact a measure of dependence or that the occurrence had probability in its favour. But they did not endeavour to pass from the observed frequency of occurrence to an exact measure of the probability. With the advent of Laplace more ambitious methods took the field" (Keynes, 1921, pp. 369-370).

• Thus, given the frequency of occurrence in μ trials, these writers infer the probability of occurrence at subsequent trials within certain limits, just as, given the à priori probability, Bernoulli's Theorem would enable them to predict the frequency of occurrence in μ trials within corresponding limits. If the number of trials is at all numerous, these limits are narrow and the purport of the inversion of Bernoulli's Theorem may therefore be put briefly as follows. By the direct theorem, if p measures the probability, p also measures the most probable value of the frequency; by the inversion of the theorem also measures the most probable value of the probability. The simplicity of the process has recommended it, since the time of Laplace, to a great number of writers. Czuber's argument, criticized on p. 399, with reference to the proportions of male and female births in Austria, is based upon an unqualified use of it. But examples abound throughout the literature of the subject, in which the theory is employed in circumstances of greater or less validity" (Keynes, 1921, pp. 370-371).

• What, in the first place, does Laplace mean by an unknown probability? He does not mean a

probability, whose value is in fact unknown to us, because we are unable to draw conclusions which could be drawn from the data; and he seems to apply the term to any probability whose value, according to the argument of Chapter III., is numerically indeterminate. Thus he assumes that every probability has a numerical value and that, in those cases where there seems to be no numerical value, this value is not non-existent but unknown; and he proceeds to argue that where the numerical value is unknown, or as I should say where there is no such value, every value between 0 and 1 is equally probable. With the possible interpretations of\the term "unknown probability", and with the theory that every probability can be measured by one of the real numbers between 0 and 1, I have dealt, as carefully as I can, in Chapter III. If the view taken there is correct, Laplace's theory breaks down immediately. But even if we were to answer these questions, not as they have been answered in Chapter III., but in a manner favourable to Laplace's theory, it remains doubtful whether we could legitimately attribute a value to the probability of an unknown probability's having such and such a value. If a probability is unknown, surely the probability, relative to the same evidence, that this probability has a given value, is also unknown; and we are involved in an infinite regress" (Keynes, 1921, p. 373).

Of course, Keynes's approximation approach uses two of the real numbers between 0 and 1. Chapter III is Keynes's introduction to his critique of exact and precise probability measurement with two exceptions (the POI is applicable; statistical data satisfies the Lexis—Q test for stability of frequency data). Chapters 10-17 of Part II contain the many details of his critique.

• Nobody supposes that we can measure exactly the probability of an induction. Yet many persons seem to believe that in the weaker and much more difficult type of argument, where the association under examination has been in our experience, not invariable, but merely in a certain proportion, we can attribute a definite measure to our future expectations and can claim practical certainty for the results of predictions which lie within relatively narrow limits. Coolly considered, this is a preposterous claim, which would have been universally rejected long ago, if those who made it had not so successfully concealed themselves from the eyes of common sense in amaze of mathematics" (Keynes, 1921, pp. 388-389).

It can be concluded from the extensive material covered in this section that Keynes's view of measurement was that ,in general, exact and precise probability measurement and assessments are not possible, in general, in macroeconomics. However, inexact and imprecise probability measurement and assessments are, in general, possible.

2.6 Keynes's Method of Inexact Measurement and Approximation in Chapters 3 and 4 of the GT

Keynes's views on measurement in the GT are identical to those expressed in the TP. Consider the following statements made by Keynes: "An entrepreneur, who has to reach a practical decision as to his scale of production, does not, of course, entertain a single undoubting expectation of what the sale-proceeds of a given output will be, but several hypothetical expectations held with varying degrees of probability and definiteness. By his expectation of proceeds I mean, therefore, that expectation of proceeds which, if it were held with certainty, would lead to the same behaviour as does the bundle of

238

vague and more various possibilities which actually makes up his state of expectation when he reaches his decision" (Keynes, 1936, ft.3, p. 24; boldface added).

"On this basis an attempt is made to erect a quantitative science. But it is a grave objection to this definition for such a purpose that the community's output of goods and services is a non-homogeneous complex which cannot be measured, strictly speaking, except in certain special cases, as for example when all the items of one output are included in the same proportions in another output…. The fact that two incommensurable collections of miscellaneous objects cannot in themselves provide the material for a quantitative analysis **need not, of course, prevent us from making _approximate_ statistical comparisons, depending on some broad element of judgment rather than of strict calculation, which may _possess significance and validity within certain limits_.** But the proper place for such things as net real output and the general level of prices **lies within the field of historical and statistical description, and their purpose should be to satisfy historical or social curiosity, a purpose for which perfect precision—such as our causal analysis requires, whether or not our knowledge of the actual values of the relevant quantities _is complete or exact_—is neither usual nor necessary**. (Keynes, 1936, pp. 37-39; italics, underline and boldface added).

Keynes states this again on p. 43: "It is my belief that much unnecessary perplexity can be avoided if we limit ourselves strictly to the two units, money and labour, when we are dealing with the behaviour of the economic system as a whole; reserving the use of units of particular outputs and equipments to the occasions when we are analysing the output of individual firms or industries in isolation; and the use of vague concepts, such as the quantity of output as a whole, the quantity of capital equipment as a whole and the general level of prices, to the occasions when we are attempting **some historical comparison which is _within certain (perhaps fairly wide) limits_ avowedly imprecise and _approximate_** (Keynes, 1936, p. 43; italics, underline and boldface added).

Keynes's emphasis above in the GT, which is on inexact method of measurement and approximation, is identical to his TP approach which he applied throughout Parts II, III, IV and V of the TP.

2.7 Conclusions

Keynes's unchanging lifetime views on inexact measurement versus exact measurement and imprecise probability versus precise probability explains the Keynes-Tinbergen debate of 1938-1940.Tinbergen's background in Physics meant that he was an advocate of the Limiting Frequency interpretation of probability. Tinbergen was thus an advocate of precise and definite probability calculations and exact, precise measurement. Tinbergen brought this view with him when he started working in economics. Keynes held the exact opposite, diametrically opposed position. Tinbergen was used to analyzing inanimate phenomenon, like atoms, protons, electrons, particles, cells, molecules, genes, chromosomes, fair decks of cards, dice, coins, etc. Keynes was used to analyzing animate human behavior. Humans, unlike inanimate phenomenon, like atoms ,protons, electrons, particles, cells, molecules, genes, chromosomes, fair decks of cards, dice, coins, can think and reason. Humans have memories, emotions, minds, and brains. While exact calculation and precision can be applied in many areas of the physical

239

and life science, it is doubtful in most social sciences, liberal arts, behavioral sciences, and especially in most areas of economics ,finance and business. The exception would be studies of consumer consumption spending and business inventory demand, which are highly stable over the short run (1-5 years). Irving Fisher's belief that there was an analogy between a particle and an individual ((Fisher, 1925, p. 85), so that a particle in mechanics corresponds to an individual in economics, only holds under the dubious claim that the sum of the parts is the whole, so that there is no interactions occurring among the parts as in a pizza that has been cut into 8 slices. The analogy breaks down completely once it is realized that a particle is an inanimate object while an individual is an animate object.

Keynes was never an ordinalist. Nowhere in the TP or GT does Keynes develop, apply, advocate or deploy an ordinal theory of probability. The Keynesian fundamentalists have misinterpreted pp. 38-40 of the TP by ignoring Keynes's clear statement on pp. 37-38 of the TP that his detailed analysis on measurement would take place in Part II, while only a brief introduction to measurement would take place on pp. 38-40 of chapter III of the TP. Keynes would subsume Tinbergen's approach as a special case of his more general case. There is no existing textual evidence that Tinbergen even understood what Keynes was talking about or what the original , Boolean approach to algebra-logic entailed.

3. Discussion

Consider the following exchange between Tinbergen and two interviewers in 1987:

"From Keynes's criticisms of your League of Nations first report, it seems fairly clear that he knew very little of the developments in econometrics over the 1920's and 1930's….(Magnus & Morgan, 1987, p. 129).

Tinbergen's response to the comment by Magnus and Morgan was that

"Indeed, I did feel that ,at least on certain points, he was badly informed. It was a bit strange to me because he had written the *Treatise on Probability*, so I thought he was somewhat familiar with statistics" (Magnus & Morgan, 1987, p. 129).

The assessment made by Tinbergen ,Magnus and Morgan of Keynes's knowledge of statistics is practically identical to the assessment made of Keynes's knowledge of statistics in the TP by Ronald Fisher in his 1923 book review of the TP, which was that Keynes was not knowledgeable about the developments in statistics over the period 1880-1920 about the Law of Large numbers, Central limit theorem ,and their support for the use of the Gaussian (Normal) probability distribution. Fisher never grasped that Keynes was using inexact, numerical approximation using intervals in part V of the TP .The far weaker data generated by economic phenomena over time, when compared with that in physics, chemistry, and biology, meant that the Normal distribution would not be the appropriate distribution to consider. Keynes's critical view of the assumption of Normality was examined by him in chapter 17 of the TP (1921) and was originally published in 1911 in the JRSS. Mandelbrot's repeated 50 years of demonstrations ,that exploratory data analysis in many areas of finance ,business, and economics does not support the use of a normal (log normal) distribution, only makes Keynes's critique

appear more prescient (See Brady, 2016).

Tinbergen appears to have made use of the Normal (lognormal) distribution throughout his career (See Brady's 2019 contribution at SSRN on Keynes and Tinbergen)

The problem that arises in the Tinbergen, Morgan, and Magnus assessments of Keynes's knowledge is that they are based on a shocking and stunning ignorance of Keynes's A Treatise on Probability. Neither Tinbergen, Morgan, or Magnus appear to understand Keynes's position that Economics is not Physics and the calculation of invariant regression constants over decades, based on unchanging parameters, is simply an impossibility. Tinbergen appears to have been trying to derive constants from economic statistical data that are like the constants of ,for example, light and speed in physics.

Tinbergen's decision to use a multivariate normal distribution, or log normal, in his regression analysis meant that he had to treat the problem of technological innovation, advance, change ,and obsolescence, which is the major factor causing changes in the composition of durable, physical, capital producer, investment goods over the business cycle over time, as well as being the main focal point of concern of businessmen in forward looking, long run business expectations formation, as being part of the residuals or as some lagged function of past, previous views. Given the highly irregular pattern of actual changes in technology over time and their non normally distributed bunched nature, Tinbergen's modeling strategy borders on the nonsensical. Keynes was very blunt in his assessment of Tinbergen's backward looking views on modelling the expectations formation by businessmen. If Tinbergen is placing these important variables in the residual (error) term of the regression equation, then very large errors should appear. This conflicts with Tinbergen's methodological goal of minimizing the size of the residuals (error terms).

There are a few econometricians who have grasped Keynes's point, even if they are not completely familiar with the inexact approach to measurement advocated by Keynes in both the TP and GT:

"The empirical content of economic theories is therefore systematically lower than those from physical theory. Testing an economic theory in quantitative form requires the introduction of all sorts of ad hoc statistical or econometric modelling assumptions so as to arrive at a fully specified empirical model (Blommestein, 1985). This ad hoc nature of economic model building generates a significant degree of specification uncertainty. For example, as noted above, the empirical pricing models for structured products such as CDOs and CDSs is hampered by a considerable degree of specification uncertainty. Semantically insufficient theories make it therefore very hard to formulate reliable empirical models. In other words, the big problem with economic theories is not that they are too simplistic or that so-called "unrealistic" assumptions are being used, but it is their semantical insufficiency (low degree of testability). By not taking the specification uncertainty caused by semantical insufficiency seriously enough, testing competing theories becomes a wasteful exercise. At the minimum one needs statistically reliable econometric models, although technical virtuosity can never substitute for increasing the information content of economic theories .(Blommestein, 2009, p. 2) and "Most of the original Cowles work was based on the additional assumptions that error terms are IID (Independent

241

and Identically Distributed) and that functions are parameters are LCC (Linear with Constant Coefficients). Tellingly, DF refer to them as "articles of faith" that had (and continue to have) a considerable influence on the applied literature" (Blommestein, 2009, p. 3). "However, economics and physics are qualitatively different enterprises, and always will be. Economics or finance as a social science differs in at least two important respects from the physical sciences. First, physical theories often formulate scientific predictions in the form of individual events, while predictions in economic theories (and other social sciences) are usually specified as patterns of a certain kind or type (Hayek, 1967; Blommestein, 1985, pp. 86-90). Second, theories of economic behaviour need to take into account reflexivity—the human capability for self-reference. Self-referential calculations have major (complicating) implications for economic explanations and predictions. For example, financial asset markets have a reflexive nature in the sense that prices are generated by the expectations of traders. But the latter expectations are based on the basis of the anticipation of others" expectations6 . This implication precludes the formation of expectations using deductive rules (Spear, 1989). In a fundamental way, economic issues (and social science topics in general) are harder and far more " (Blommestein, 2009, pp. 3-4).

And "Against the backdrop of this analysis, bringing together the insights from different parts of economics such as macroeconomics and financial economics is not necessarily the best response to the recent wave of criticism of academic finance. The end result [of this intended merger of theories] would still not go beyond disguised mathematics or sets of tautological structures. For example, the underlying behavioural paradigm of macroeconomic/macro finance models is that economic agents have rational expectations, meaning that the "representative" agent is at all times and everywhere completely informed, while fully understanding the complexities of the world (De Grauwe, 2009). This would imply that bankers and their clients at all times fully understand the complexities of the new financial landscape. It is difficult to reject the impression that these behavioural assumptions have been introduced to facilitate the underlying math and the formulation of quantitative models with closed-form solutions (Blommestein, 2009, pp. 5-6). Thus, Blommestein's work, along with that of D. Freedman(2009), E Leamer (1983), and H. Keuzenkamp (2000),if it became dominant in econometric practice ,would go along way toward satisfying Keynes's original objections about the use of econometrics in attempting to quantify business cycle research over time.

4. Conclusions

The main reasons for the completely divergent views expressed by Keynes and Tinbergen in their 1938-1940 exchanges in the *Economic Journal* about measurement was due to their completely different technical backgrounds and different views about measurement being either exact or inexact. Keynes's background was in logic, mathematics and philosophy. Tinbergen's background was in physics. There simply was no common ground between the two academics.

Acknowledgement

I thank the referee for his suggestions.

References

———. (1922). *A Treatise on Probability* (pp. 72-85). https://doi.org/10.2307/2340524

---. (2004a). J. M. Keynes' Theory of Decision Making, Induction, and Analogy. *The Role of Interval Valued Probability in His Approach*. Philadelphia, Pennsylvania: Xlibris Corporation.

---. (2004b). *Essays on John Maynard Keynes and …. Philadelphia, Pennsylvania: Xlibris Corporation*.

_____. (1926). Truth and probability. In D. H. Mellor (Ed.), *Foundations: Essays in Philosophy, Logic, Mathematics, and Economics*. London: Routledge & Kegan Paul.

_____. (1986). *Boole's Logic and Probability* (2nd ed.). Amsterdam: North-Holland.

_____. (1996). *Sentential Probability Logic*. Bethlehem: Lehigh University Press.

Arthmar, R., & Brady, M. E. (2016). The Keynes-Knight and the de Finetti-Savage's Approaches to Probability: An Economic 1 Interpretation. *History of Economic Ideas*, *XXIV*(1), 105-124.

Arthmar, R., & Brady, M. E. (2018). Boole, Ramsey and the Keynes—Townshend exchange on subjective probability. *History of Economic Thought and Public Policy*, *14*(2), 55-74. https://doi.org/10.3280/SPE2018-002003

Backhouse, R., & Bateman, B. (2006). A cunning purchase: The life and work of Maynard Keynes. In Backhouse, & Bateman (Eds.), *The Cambridge Companion to Keynes*. Basingstoke: Palgrave Macmillan. https://doi.org/10.1017/CCOL0521840902

Bateman, B. (2003). The end of Keynes and philosophy? In J. Runde, & S. Mizuhara (Eds.), *The Philosophy of Keynes' Economics*. London: Routledge. https://doi.org/10.4324/9780203488973.ch6

Blommestein, H. J. (2009). Limits of Econometrics for Testing Economic Hypotheses. *International Econometric Review (IER)*, *1*(2), 1-11.

Blommestein, H. J. (2009). The financial crisis as a symbol of the failure of academic finance? (A methodological digression). *Journal of Financial Transformation*. https://doi.org/10.2139/ssrn.1477399

Boole, G. (1854). *An Investigation of the Laws of Thought on Which are Founded the Mathematical Theories of Logic and Probability*. New York: Dover Publications. https://doi.org/10.5962/bhl.title.29413

Brady, M. E. (1993). J. M. Keynes' theoretical approach to decision making under condition of risk and uncertainty. *The British Journal for the Philosophy of Science*, *44*, 357-376. https://doi.org/10.1093/bjps/44.2.357

Brady, M. E. (1994). On the application of J.M. Keynes's approach to decision making. *International Studies in the Philosophy of Science*, *8*(1), 99-112. https://doi.org/10.1080/02698599408573487

Brady, M. E. (2019). *Keynes (Inexact measurement, approximation, non linearity, non additivity, interdependence, imprecision) versus Tinbergen (Exact measurement, linearity, additivity, independence, precision) in 1939-1940: There was no middle ground between them.*" https://doi.org/10.2139/ssrn.3345430

Brady, M. E., & Arthmar, R. (2010). *A Road Map for Economists, Logicians, Philosophers, Mathematicians, Statisticians, Psychologists and Decision Theorists Seeking to Follow the Mathematical Structure of Keynes's Approach to Specifying Lower and Upper Bounds for Probabilities in the A Treatise on Probability*, *1921*(1973). https://doi.org/10.2139/ssrn.1618445

Brady, M. E., & Arthmar, R. (2012). Keynes, Boole, and the Interval approach to Probability. *History of Economic Ideas*, *20*(3), 65-84.

Broad, C. D. (1918). *The Relation between Induction and Probability*. https://doi.org/10.1093/mind/XXVII.4.389

Buitenhuis, M. (2015). Jan Tinbergen's Move from Physics to Economics, 1922-1930. *Research Master in History & Philosophy of Science* (1st of December 2015, pp. 1-72). Utrecht University.

Carabelli A. (1994). Keynes on menstruation and comparison. In K. Vaghned (Ed.), *Perspectives in the History of Economic Thought* (pp. 204-238). Aldershot, Edward Elgar.

Carabelli A. (2009). *Economic theory after Keynes: A new methodological approach*. University of Stirling, SCEME working papers.

Carabelli, A. (1988). Keynes on Cause, Chance, and Possibility. In Lawson, & Pesaran (Eds.), *Keynes' Economics: Methodological Issues* (pp. 151-180). New York: ME Sharpe.

Carabelli, A. (1988). *On Keynes' Method*. New York, St. Martins Press. https://doi.org/10.1007/978-1-349-19414-8

Carabelli, A. (1995). Uncertainty and measurement in Keynes: Probability and organisms. In D. Sheila, & H. Keynes (Eds.), *Knowledge and Uncertainty*. Aldershot: Edward Elgar.

Carabelli, A. (2003). Keynes: Economics as a branch of probable logic. In J. Runde, & S. Mizuhara (Eds.), *The Philosophy of Keynes' Economics*. London: Routledge. https://doi.org/10.4324/9780203488973.ch17

Cornelisse, P. A., & van Dijk, H. K. Jan Tinbergen. (1903-1994). *Jan Tinbergen Econometric Institute Report EI 2006-2009*. https://doi.org/10.1057/978-1-349-95121-5_1465-2

Damasio, A. (2005). *Descartes' Error: Emotion, Reason, and the Human Brain*. New York; Penguin Books.

Davis, J. (2003). The relationship between Keynes's early and later philosophical thinking. In J. Runde, & S. Mizuhara (Eds.), *The Philosophy of Keynes' Economics*. London: Routledge.

Dow, S. (2003). Probability, uncertainty and convention: Economists' knowledge and the knowledge of economic actors. In J. Runde, & S. Mizuhara (Eds.), *The Philosophy of Keynes' Economics*. London: Routledge. https://doi.org/10.4324/9780203488973.pt5

Dow, S. C. (2013). *Endogenising Uncertainty*. Annual Plenary Conference of INET, Hong Kong, April

4-7, 2013.

Dow, S. C. (2015). Addressing uncertainty in economics and the economy. *Cambridge Journal of Economics*, *39*, 33-47. https://doi.org/10.1093/cje/beu022

Edgeworth, F. Y. (1884). *Philosophy of Chance* (pp. 223-235). https://doi.org/10.1093/mind/os-IX.34.223

Edgeworth, F. Y. (1905). The Law of Error. *Transactions of the Cambridge Philosophical Society*, *20*, 36-65, 113-41.

Edgeworth, F. Y. (1922a). *The Philosophy of Chance*, *31*, 257-283. https://doi.org/10.1093/mind/XXXI.123.257

Edgeworth, F. Y. (1922b). A Treatise on Probability, by John Maynard Keynes. *Journal of the Royal Statistical Society*, *85*, 107-113. https://doi.org/10.2307/2340524

Ellsberg, D. (2001). *Risk, Ambiguity and Decision* (Studies in Philosophy). England; Routledge.

Fioretti, G. (2003). No faith, no conversion: The evolution of Keynes's ideas on uncertainty under the influence of Johannes von Kries. In J. Runde, & S. Mizuhara (Eds.), *The Philosophy of Keynes' Economics*. London: Routledge. https://doi.org/10.4324/9780203488973.ch10

Fisher, R. (1922). On the Mathematical Foundations of Theoretical Statistics. *Philosophical Transactions of the Royal Society*, *222*, 309-368. https://doi.org/10.1098/rsta.1922.0009

Fisher, R. (1923). Review of J. M. Keynes's Treatise on Probability. *Eugenics Review*, *14*, 46-50.

Fitzgibbons, A. (2003). Keynes's epistemology. In J. Runde, & S. Mizuhara (Eds., 2003). *The Philosophy of Keynes' Economics*. London: Routledge. https://doi.org/10.4324/9780203488973.ch5

Freedman, D. A. (2009). Limits of Econometrics. *International Econometric Review*, *1*(1), 1-13.

Fuchs, E. P. (2015). Reconciling Keynes and Tinbergen? *Lawrence R. Klein and the Making of Macroeconometrics. Conference: The History of Economics Society Conference*. Michigan State University, East Lansing (June).

Fuchs, E. P. (2017). PHD THESIS SUMMARY: Economics as a "tooled" discipline: Lawrence R. Klein and the making of macro econometric modeling, 1939-1959. *Erasmus Journal for Philosophy and Economics*, *10*(1), 133-136. https://doi.org/10.23941/ejpe.v10i1.253

Gardner, M. (2009). Is Reuben Hersh "Out there"? *EMS Newsletter*, *June*, 23-24.

Gerrard, B. (2003). Keynesian uncertainty: What do we know? In J. Runde, & S. Mizuhara (Eds., 2003). *The Philosophy of Keynes' Economics*. London: Routledge. https://doi.org/10.4324/9780203488973.ch19

Gigerenzer, G. (2008). *Gut Feelings: The Intelligence of the Unconscious*. New York; Penguin Books.

Gillies, D. (2003). Probability and uncertainty in Keynes's The General Theory. In J. Runde, & S. Mizuhara (Eds.), *The Philosophy of Keynes' Economics*. London: Routledge. https://doi.org/10.4324/9780203488973.ch9

Good, I. J. (1950). *Probability and the Weighing of Evidence*. London: Charles Griffin.

Good, I. J. (1956). WHICH COMES FIRST, PROBABILITY OR STATISTICS? *JIA*, *82*, 249-255. https://doi.org/10.1017/S0020268100046448

Greenspan, A. (2004). Risk and Uncertainty in Monetary Policy. *American Economic Review*, *94*(2), 33-40. https://doi.org/10.1257/0002828041301551

Hailperin, T. (1965). Best possible inequalities for the probability of a logical function of events. *American Mathematical Monthly*, *72*, 343-359. https://doi.org/10.1080/00029890.1965.11970533

Hansen, P. et al. (2000). Probabilistic satisfiability with imprecise probabilities. *International Journal of Approximate Reasoning*, *24*, 171-189. https://doi.org/10.1016/S0888-613X(00)00033-5

Harrell, E. (2010). *A Bold Opening for Chess Player Magnus Carlsen*. Retrieved from http://content.time.com/time/magazine/article/0,9171,1950939,00.html .

Holyoak, K. I., & Kyunghee, K. (1987). *Surface and Structural similarity in analogical transfer, Memory and Cognition*, *15*(4), 332-340. https://doi.org/10.3758/BF03197035

Hunt, M. (1982). *The Universe Within*. New York ,Simon and Schuster.

Keuzenkamp, H. (2000). *Probability, Econometrics and Truth: The Methodology of Econometrics*. England; Cambridge University Press. https://doi.org/10.1017/CBO9780511493300

Keynes, J. M. (1921). *A Treatise on Probability*. London: Macmillan.

Keynes, J. M. (1921). A Treatise on Probability. *The Collected Writings of John Maynard Keynes (CWJMK)*, *VIII*. London: Macmillan.

Keynes, J. M. (1931). F. P. Ramsey. In *Essays in Biography, CWJMK* (Vol. X). London: Macmillan for the Royal Economic Society (reprinted from The New Statesman and Nation, 3 October).

Keynes, J. M. (1937). The General Theory of Employment. *Quarterly Journal of Economics*, *51*, 209-223. https://doi.org/10.2307/1882087

Keynes, J. M. (1964). *The General Theory of Employment, Interest and Money*. New York: Harcourt, Brace and World.

Keynes, J. M. (1972). *Essays in Biography* (Vol. X). The Collected Writings of John Maynard Keynes (CWJMK). London: Macmillan.

Klein, G. (2003). *The Power of Intuition*. New York; Doubleday.

Koopmans, B. O. (1940). The axioms and algebra of intuitive probability. *Annals of Mathematics*, *41*, 269-292. https://doi.org/10.2307/1969003

Kyburg, H. E. Jr. (1970). *Probability and Inductive Logic*. New York: Macmillan.

Kyburg, H. E. Jr. (1995). Keynes as a philosopher. In A. F. Cottrell, & M. S. Lawlor (Eds.), *New Perspectives on Keynes*. Durham: Duke University Press. https://doi.org/10.1215/00182702-27-Supplement-7

Kyburg, H. E. Jr., & Smokler, H. E. (1964). Introduction. In H. E. Jr. Kyburg, & H. E. Smokler (Eds.), *Studies in Subjective Probability*. New York: Wiley.

Kyburg, H. E. Jr.. (2003). Are there degrees of belief? *Journal of Applied Logic*, *1*, 139-149. https://doi.org/10.1016/S1570-8683(03)00010-7

Kyburg, H. E. Jr.. (2006). Belief, evidence, and conditioning. *Philosophy of Science*, *73*, 42-65. https://doi.org/10.1086/510174

Lawson, T. (2003). Keynes's realist orientation. In J. Runde, & S. Mizuhara (Eds.), *The Philosophy of Keynes' Economics*. London: Routledge. https://doi.org/10.4324/9780203488973.ch12

Leamer, E. (1983). Let's Take the Con Out of Econometrics. *American Economic Review*, *73*(1), 31-43.

Lehrer, J. (2010). *The Frontal Cortex-Chess Intuition*. Retrieved from http://scienceblogs.com/cortex/2010/01/18/chess-intuition/

Levi, I. (1967). *Gambling with Truth*. New York: Knopf.

Levi, I. (1974). On indeterminate probabilities. *Journal of Philosophy*, *71*, 391-418. https://doi.org/10.2307/2025161

Levi, I. (2001). Introduction. In D. Ellsberg (Ed.), *Risk, Ambiguity and Decision*. New York: Routledge.

Mazur, B. (2008). Mathematical Platonism and its opposites. *European Mathematical Society Newletter*, 1-5.

McCann, Jr., C. (2003). On the nature of Keynesian probability. In J. Runde, & S. Mizuhara (Eds.), *The Philosophy of Keynes' Economics*. London: Routledge. https://doi.org/10.4324/9780203488973.ch3

Meeks, G. T. (2003). Keynes on the rationality of decision procedures under uncertainty: The investment decision. In J. Runde, & S. Mizuhara (Eds.), *The Philosophy of Keynes' Economics*. London: Routledge.

Mellor, D. H. (1995). *E.P. Ramsey, Philosophy*, *70*, 243-257. https://doi.org/10.1017/S0031819100065396

Moll, A. (2010). *Jonah Lehrer on Carlsen and chess intuition*. Chessvibes. Retrieved from http://www.chessvibes.com/reports/jonah-lehrer-on-carlsen-and-chess-intuition

Monk, R. (1991). Russell and Ramsey, London Reviews of Books, August, 11-13. *Reprinted in Russell Society News*, *72*, 25-26.

Moore, G. E. (1903). *Principia Ethica*. Amherst, New York: Prometheus Books.

Mumford, D. (2008). Why I am a Platonist. *EMS Newsletter*, 27-30.

Myers, D. G. (2002). *Intuition*. New Haven, Yale University Press.

O'Donnell, R. (2003). The thick and the thin of controversy. In J. Runde, & S. Mizuhara (Eds.), *The Philosophy of Keynes' Economics*. London: Routledge.

O'Donnell, R. (2012). Treatise on Probability. In J E King (Ed., pp. 359-365), *The Elgar Companion to POST KEYNESIAN ECONOMICS*. England ; Edward Elgar.

O'Donnell, R. (2014). Two Post-Keynesian Approaches to Uncertainty and Irreducible Uncertainty. In G. C. Harcourt, & P. Kriesler (Eds.), *The Oxford Handbook of Post Keynesian Economics* (Vol. 4, pp. 124-142). Critiques and Methodology. Oxford University Press, New York.

Raffaelli, T. (2006). Keynes and philosophers. In R. E. Backhouse & B. W. Bateman (Eds.), *The Cambridge Companion to Keynes*. Publisher: Cambridge University Press,

247

England.https://doi.org/10.1017/CCOL0521840902.009

Ramsey, F. P. (1922). Mr. Keynes on Probability, Cambridge Magazine, XI, 1, (Jan) 3-5. *Reprinted in the British Journal of the Philosophy of Science* (pp. 219-222). https://doi.org/10.1093/bjps/40.2.219

Ramsey, F. P. (1926). Truth and Probability. In R*amsey, 1931, The Foundations of Mathematics and other Logical Essays* (pp. 156-198). London: Kegan, Paul, Trench, Trubner & Co., New York: Harcourt, Brace and Company.

Rehmeyer, J. (2008). Still debating with Plato: Where do mathematical objects live? *Science News*, 1-3.

Runde, J., & Mizuhara, S. (2003). Introduction. In J. Runde, & S. Mizuhara (Eds.), *The Philosophy of Keynes' Economics*. London: Routledge. https://doi.org/10.4324/9780203488973

Runde, J., & Mizuhara, S. (Eds). (2003). *The Philosophy of Keynes' Economics*. London: Routledge. https://doi.org/10.4324/9780203488973

Russell, B. (1922). Review of A Treatise on Probability by J. M. Keynes. *Mathematical Gazette*, *11*(159), 119-125. https://doi.org/10.2307/3603283

Russell, B. (1930). Probability and Fact. *Atlantic Monthly*, *146*, 163-170.

Russell, B. (1931). Review of Ramsey. *Mind*, 40, 476-482. Reprinted in Russell, 1996. https://doi.org/10.1093/mind/XL.160.476

Russell, B. (1932). Review of Ramsey. *Philosophy*, *7*, 84-86. Reprinted in Russell, 1996. https://doi.org/10.1017/S0031819100053705

Russell, B. (1959). *My Philosophical Development*. Unwin Book: London.

Russell, B. (1996). *Collected Papers of Bertrand Russell*. Unwin Book: London.

Shackle, G. L. S. (1939). Expectations and Employment. *Economic Journal*, *49*(195), 442-452. https://doi.org/10.2307/2224801

Shackle, G. L. S. (1949). A Non-Additive Measure of Uncertainty. *Review of Economic Studies*, *17*(1), 70-74. https://doi.org/10.2307/2295790

Shackle, G. L. S. (1983). The romantic mountain and the classic lake: Alan Coddington's Keynesian Economics. *Journal of Post Keynesian Economics*, *6*(2), 241-246. https://doi.org/10.1080/01603477.1983.11489433

Shackle, G. L. S. (1984). Comment on the papers by Randall Bausor and Malcolm Rutherford. *Journal of Post Keynesian Economics*, *6*(3), 388-394. https://doi.org/10.1080/01603477.1984.11489448

Skidelsky, R. (1983). *John Maynard Keynes: Hopes Betrayed (1883-1920)*, *I*. London: Macmillan.

Skidelsky, R. (1992). *John Maynard Keynes*, *II*. The Economist as Saviour 1920-1937. London: Macmillan.

Skidelsky, R. (2010). The Social Market Economy Revisited. In *The Social Market Foundation*. London, England.

Smith, A. (1776). *The Wealth of Nations. Modern (Cannan) Library Edition with the foreward by Max Lerner*. N.Y., N.Y.

Squartini, T., & Garlaschelli, D. (2014). Jan Tinbergen's Legacy for Economic Networks: From the Gravity Model to Quantum Statistics (Chapter 9). In F. Abergel et al. (Eds.), *Econophysics of Agent-Based models, New Economic Windows*. https://doi.org/10.1007/978-3-319-00023-7_9

Suppes, P. (2006). Ramsey's Psychological Theory of Belief. In M. C. Galavotti (Ed.), *Cambridge and Vienna: Frank P. Ramsey and the Vienna Circle*. Dordrecht: Springer Netherlands.

Tinbergen, J. (1938). Statistical Evidence on the Acceleration Principle. *Economica, New Series*, *5*(18), 164-176. https://doi.org/10.2307/2549019

Tinbergen, J. (1940). Econometric Business Cycle Research. *The Review of Economic Studies*, *7*(2), 73-90. https://doi.org/10.2307/2967472

Tinbergen, J. (1970). A POSITIVE AND A NORMATIVE THEORY OF INCOME DISTRIBUTION. *Review of Income and Wealth*, *16*(3), 221-234. https://doi.org/10.1111/j.1475-4991.1970.tb00699.x

Wakker, P. (2006). Uncertainty. In L. Blume, & S. N. Darlauf (Eds.), *The New Palgrave: A Dictionary of Economics* (2nd ed.). London: Palgrave Macmillan.

Wilson, E. (1934). Boole's Challenge Problem. *JASA*, *29*(187), 301-306. https://doi.org/10.1080/01621459.1934.10502722

Winslow, E. (2003). The foundations of Keynes's economics. In J. Runde, & S. Mizuhara (Eds.), *The Philosophy of Keynes' Economics*. London: Routledge. https://doi.org/10.4324/9780203488973.pt3

Zambelli, S. (2007). A Rocking Horse That Never Rocked: Frisch's "Propagation Problems and Impulse Problems". *History of Political Economy*, *39*(1), 145-166. https://doi.org/10.1215/00182702-2006-027