# Spare Parts Supply Chain Shipment Decision Making in a Deterministic Environment 

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#### Abstract

Determining the optimal lot size is a well discussed problem across many industries. This paper suggests a deterministic approach to shipment decision taking in a deterministic environment of spare parts supply chain. The model considered a deterministic forecast for the incoming faulty units. An objective function was modeled by taking in consideration three main elements: the number of malfunctioned units, the shipping cost and the storage cost. A deterministic forecast allowed calculating the cost effective schedule of shipments and its lot sizes. The purpose of this paper it to study the reverse logistics supply chain of a typical telecom devices company (that offers warranty/repairing services) and try to reduce its transportation and handling costs in order to bring added-value to the customer and by doing so, expand the competitive advantage of the company.


## Keywords

reverse logistics, linear programing, lot size, spare parts, supply chain management, planing

## 1. Introduction and Literature Review

Many industries nowadays are using expensive equipment for their manufacturing or services. This equipment at times takes a very large investment of the company and makes a large portion of its initial cost or capital. In many cases the equipment is custom build which increases the upkeep cost due to the lack of shelf spare parts. "Repairable items are referred to as components, which are expensive, critically important, and subject to infrequent failures. When they fail, they should be repaired and reused after repair since they are too expensive to be discarded" (Kim et al., 2007). A good example of such industry is the military industry; this industry requires various extraordinary demands due to the nature of its equipment (planes, war-ships, tanks, etc.), this is the reason why most early researches were military oriented (Mabini et al., 1992). "Inventory control of spare parts plays an increasingly important role in modern operations management. The tradeoff is clear: on one hand a large number of spare parts ties up a large amount of capital, while on the other hand too little inventory may result in poor customer service or extremely costly emergency actions" (Aronis et al., 2002).
This paper will focus on reverse logistics supply chain, the return process of defective units back to repair. In this process malfunctioned devices are gathered in a logistics center and wait to be shipped to a service center (technicians' lab for repair). Each malfunctioned device is being taken equivalent to a spare part needed to be held in ready-to-use spare parts inventory. The returned products will stay in the logistic center till enough units have been received to fulfill the optimal shipping quantity. The optimal quantity will vary due to many variables constantly changing for the nature of spare parts inventory. "The requirements for planning the logistics of spare parts differ from those of other materials in several ways: service requirements are higher as the effects of stock outs may be financially remarkable, the demand for parts may be extremely sporadic and difficult to forecast, and
the prices of individual parts may be very high" (Huiskonen, 2001). The main emphasis of this paper will be the specific challenge in the company's reverse logistics model. As shown in Table 1 the challenge would be to determine the optimal trade off point between the holding costs in the Logistics Center and the transportation cost from the LC to the repair center. This decision is being made by taking in consideration the forecast of receiving new faulty units and by that increasing the quantity shipped from the LC to the repair center taking advantage of the economic scale discounts. "Quantity discount is a common practice in retail sales and provides economic advantages for both the vendor and buyer. The vendor will be able to benefit from sales of larger quantities by reducing the unit order and setup costs. Similarly, as a buyer, one can reduce the per unit ordering cost and hold more inventory by paying a lower unit price" (Lin \& Ho, 2011).
Lot size problem is an extremely important and difficult problem. Since in fact, almost every business has some type of goods to transfer from one place to another this problem affects the vast majority of the markets and businesses. "Many authors have studied the single-item incapacitated lot sizing problem historically, the Economic Order Quantity (EOQ) presented by Harris" (Harris, 1990) predates this problem. EOQ is also known as the Wilson lot size formula since it was used in practice by Wilson (Wilson, 1934). "EOQ balances the setup cost and inventory holding cost" (Karimi et al., 2003). There are many extensions variations to lot Size Problem: "Single-Level Lot Sizing without Resource constraints (SLUR), Single-Level lot sizing with resource constraints (SLCR), Multi-Level lot sizing without resource constraints (MLUR), and multi-level lot sizing with resource constraints (MLCR), single-level dynamic lot sizing-the Capacitated Lot Sizing Problem (CLSP), Economic Lot Scheduling Problem (ELSP), Discrete Lot sizing and Scheduling Problem (DLSP), the Continuous Setup Lot sizing Problem (CSLP), the Proportional Lot sizing and Scheduling Problem (PLSP), and the General Lot sizing and Scheduling Problem (GLSP)" (Karimi et al., 2003). "It is one of the hardest problems to solve due the increased number constraints usually effecting the calculations. Medium-term planning often involves making decisions on Material Requirements Planning (MRP) and establishing production quantities or lot sizing over the planning period, so as to optimize some performance criteria such as minimizing overall costs, while meeting demand requirements and satisfying existing capacity restrictions" (Karimi et al., 2003). For some companies, especially ones that rely on operational profits to sustain themselves, minimizing the transportation and holding costs could be very important and even fatal, determining whether the company is profitable or not.
To simplify the problem, the deterministic approach the article suggests, considers shipments to each repair center separately. When looking at lot size problems, it is possible to compare the manufacturing and forward logistics end of view, to the repairable and revers logistics end of view. Instead of forecasting the demand for new products and derive from that forecast the lot-size and raw materials procuring; it is possible to treat the number of faulty devices as a deterministic demand, or furthermore treat the faulty devices in addition to future faulty units as stochastic demand. The manufacturer would be looked at as the manufacturer, and the repair process as the manufacturing process. Raw materials are the faulty units need repair and the spare parts to help fix those units. A perishable/expiration parameter will be defined to limit the time of the repair process can be added. This parameter can help measure customer satisfaction, for an instance if the item has an expiration date of 30 days; this could representation of the maximum length of the repair process. If the item is fixed within the time limit then the customer is satisfied.

Flow chart 1, describes the reveres logistics and the repair process. Firstly the products are being shipped by the end user back to one logistic centers spread around the world. Once the product arrives
to the logistic center it undergoes a conditional examination by a technician. The examination determines whether the product is in a fixable condition or not. Unfixable products are being scrapped. Products in fixable condition are sent to storage in the defectives area till the next scheduled shipment to one of the repair centers. Each product is assigned a repair center according to its malfunction and the expertise of the repair center it has been assigned to. The product remains in defective storage till the consolidate shipment to the repair center it has been assigned to reaches an optimal mass for shipment. This optimal mass is determined by the client's definition of the required service level and shipping costs. The goal is primarily to maximize service level considering the lowest costs possible. When the consolidated shipment reaches its critical mass it is being shipped out to the destined repair center. Repaired products are sent to the logistic enter and then back to the customer. Unsuccessful repairs are sent to scrap and are being replaced by other repaired units or by new ones.


Figure 1. Problem Formulation

Indices
k product group;
t period;
j logistics center node.

## Decision Variables

$\gamma \mathrm{kjt}$ Binary, setup shipment for product k from in logistic center j at period t ;
qkjt Lot size for product k in logistic center j at period t .

## Parameters

Mkjt Stock levels of product group k in logistic center j at period t ;
Skj Storage costs of product k at logistic center j ;
ekjx Transportation costs of product group k from logistic center j (depends on X , the amount of units shipped);
d Maximum storage time;
$\lambda$ kjt Stock entry from product group k , to logistic center j at period t ;
L A big number.
Objective Function
The objective function (1) calculates the sum of the holding ( Skj ) and transportation costs (ekjx) of every item all together. The function sums up the costs of all the logistic centers and through all of the time periods. The binary parameter ( $\gamma \mathrm{kjt}$ ) determines the type of cost to be calculated. A period of time that included a shipment will not include holding costs and vice versa, a period of time that did not include a shipment will not take transportation costs in account but only the inventory costs.
$\min Z=\sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{t=1}^{T}(\gamma \mathrm{kjt} \times \mathrm{qkjt} \times \mathrm{ekjx})+((1-\gamma \mathrm{kjt}) \times \mathrm{Mkjt} \times \mathrm{Skj}) ; \mathrm{X}=\mathrm{qkjt}$.

## 3. Constraints

The binary constraint (2) $\left(\gamma_{\mathrm{kjt}}\right)$ determines if there will be a shipment during the specific period or not ( t , if the variable equals to 1 , a shipment will take place at the mention period, otherwise there wouldn't be a shipment dispatch and the goods will remain in storage. As mentioned above, at time periods where a shipment occurs there are no holding $\operatorname{costs}\left(\mathrm{S}_{\mathrm{kj}}\right)$ and only the transportation costs are calculated. At periods where a shipment does not occur ( $\gamma_{\mathrm{kjt}}=0$ ), only the holding costs ( $\mathrm{S}_{\mathrm{kj}}$ ) will be taken in account.
The expiration (maximum process duration) constraint (3) defines the maximum length for a unit to be hold at the logistic center, this is by each company's service level agreement for the length of time which is required to fix the faulty part. Each repair process is bounded to a specific time length that is setup in advance. In order to understand if the shipment must leave the logistic center, the function sums up the binary parameter $\left(\gamma_{\mathrm{kjt}}\right)$. The sum of the binary parameter $\left(\gamma_{\mathrm{kjt}}\right)$ at each period represents the number of departing shipments.
Constraint (4) forces the binary parameter ( $\gamma_{\mathrm{kjt}}$ ) to match the size of the shipment. If a shipment did not leave the logistic center hence $\gamma_{\mathrm{kjt}}=0$, this forces the shipment size to equal zero. If a shipment did leave the logistic center hence $\gamma_{\mathrm{kjt}}=1$, this does not affect the outgoing shipment size.
Stock tracking constraint (5), this constraint takes the initial stock size in the previous time period $\left(\mathrm{M}_{\mathrm{kjt}}\right)$, the outgoing shipment size $\left(\mathrm{q}_{\mathrm{kjt}-1}\right)$ is being deducted from the initial stock size and the incoming products $\left(\lambda_{\mathrm{kjt}}\right)$ are being added.
Stock size and shipment size constraint (6), in case of an outgoing shipment, this constraint forces the shipment lot size $\left(\mathrm{q}_{\mathrm{kjt}}\right)$ to equal the stock size $\left(\mathrm{M}_{\mathrm{kjt}}\right)$. This means that in case of a leaving shipment all of the current inventory will be sent, leaving the new inventory level zero.
Natural constraints (7), (8), (9) the lot size $\left(\mathrm{q}_{\mathrm{kjt}}\right)$, the initial stock level $\left(\lambda_{\mathrm{kjt}}\right)$ are natural numbers.

| 1) | $\gamma_{\mathrm{kjt}} \in\{0,1\}$ | $;$ | $\forall \mathrm{k}, \mathrm{j}, \mathrm{t}$ |
| :--- | :--- | :--- | :--- |
| 2) | $\sum_{j=1}^{J} \sum_{t=1}^{T} \sum_{t=t}^{t+d-1} \gamma_{\mathrm{kjt}} \geq 1$ | $;$ | $\forall \mathrm{k}$ |
| 3) | $\mathrm{q}_{\mathrm{kjt}}-L \times \gamma_{\mathrm{kjt}} \leq 0$ | $;$ | $\forall \mathrm{k}, \mathrm{j}, \mathrm{t}$ |
| 4) | $\mathrm{M}_{\mathrm{kjt}}=\mathrm{M}_{\mathrm{kjt}-1}-\mathrm{q}_{\mathrm{kjt-1}}+\lambda_{\mathrm{kjt}}$ | $;$ | $\forall \mathrm{k}, \mathrm{j}, \mathrm{t}$ |

5) $\quad \gamma_{\mathrm{kjt}} \times \mathrm{q}_{\mathrm{kjt}}=\gamma_{\mathrm{kjt}} \times \mathrm{M}_{\mathrm{kjt}} \quad ; \quad \forall \mathrm{k}, \mathrm{j}, \mathrm{t}$
6) $\mathrm{q}_{\mathrm{kjt}} \geq 0 \quad$; $\quad \forall \mathrm{k}, \mathrm{j}, \mathrm{t}$
7) $\lambda_{\mathrm{kjt}} \geq 0 \quad$; $\quad \forall \mathrm{k}, \mathrm{j}, \mathrm{t}$
8) $\mathrm{q}_{\mathrm{kjt}} \lambda_{\mathrm{kjt}} \in \mathbb{N} \quad ; \quad \forall \mathrm{k}, \mathrm{j}, \mathrm{t}$

## Test Case

The following test case is taken from a real operational process for a company that markets fiber optics products for the communication industry. The defective units are being sent by the customer and on his expense to one of their two logistic centers $(\mathrm{j}=2)$. The logistic centers are located in the US and the Netherlands. The products are being repaired in three different repair centers, one in Israel, one in the US and one in Canada. Therefore the product groups are divided by 3. The periods are defined as "weeks". The examined length of time consists of four periods (equals to four weeks). The Test case assumes that there is a process continuity, which means that the operation started and continues before and after the examined time is over.
The storage costs are fixed per unit and remain unchanged throughout the test case duration. Storage costs are presented in Table 1 as follows:

Table 1. Storage Costs

|  | Logistic center 1 | Logistic center 2 |
| :--- | :--- | :--- |
| Product group 1 | 10 | 15 |
| Product group 2 | 20 | 17 |
| Product group 3 | 15 | 11 |

The shipping costs vary; as accustomed in the shipping industry there is a quantity discount on the shipping costs (the larger the lot size the bigger the discount). Table 2 presents the shipping tariffs for each product group and its potential lot size (for logistic center 1 whereas Table 3 presents the tariffs for logistic center 2).

Table 2. Shipping Tariffs

| Logistic center 1 |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Number of products | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Product group 1 | 0 | 18 | 34 | 48 | 61 | 71 | 79 | 85 | 89 | 92 | 93 |
| Product group 2 | 0 | 40 | 78 | 110 | 136 | 156 | 170 | 182 | 192 | 201 | 206 |
| Product group 3 | 0 | 27 | 50 | 72 | 82 | 90 | 90 | 90 | 90 | 90 | 90 |

Table 3. Tariffs for Logistic

| Logistic center 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: | :---: |
| Number of products | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |  |  |
| Product group 1 | 0 | 23 | 39 | 53 | 66 | 76 | 84 | 90 | 94 | 97 | 98 |  |  |
| Product group 2 | 0 | 37 | 75 | 107 | 133 | 153 | 167 | 179 | 189 | 198 | 203 |  |  |
| Product group 3 | 0 | 23 | 46 | 68 | 78 | 86 | 86 | 86 | 86 | 86 | 86 |  |  |

Incoming stock arrives to the logistic centers at a varied pace ranges from one to five units a week. Table 4 presents the forecast of faulty units over four weeks.

Table 4. Forecast of Faulty Units

|  | Logistic center 1 |  | Logistic center 2 |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Week 1 | Week 2 | Week 3 | Week 4 | Week 1 | Week 2 | Week 3 | Week 4 |
| Product <br> group 1 | 1 | 3 | 2 | 1 | 2 | 3 | 3 | 1 |
| Product <br> group 2 <br> Product <br> group 3 | 2 | 4 | 3 | 5 | 2 | 2 | 4 | 1 |

The Service level agreement requires that a unit will not be stored in each one of the logistic centers more than 3 weeks, therefore the maximum storage time, $\mathrm{d}=3$.
After running the model with the data presented above, the objective function reached a total costs of $1407 \$$ which is the optimal amount of combined storage and shipping costs. As for the decision variables, we can see that decision conclusion (ship or do not $-\gamma_{\mathrm{kji}} \in\{0,1\}$ ) is presented on the following Table:

Table 5. Decision Conclusion

|  | Logistic center 1 |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Week 1 | Week 2 | Week 3 | Week 4 | Week 1 | Week 2 | Week 3 | Week 4 |
| Product <br> group 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| Product <br> group 2 <br> Product <br> group 3 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 |

The lot size is determined by the stock available to ship. Constraint (6) forces the lot size shipped to equal stock levels. Once a shipment decision is made, the lot size is set to the available stock level. This occurs in order to ship the largest amount of units available to apply the maximum quantity discount in order to minimize shipping costs.
(6) $\gamma_{\mathrm{kjt}} \times \mathrm{q}_{\mathrm{kjt}}=\gamma_{\mathrm{kjt}} \times \mathrm{M}_{\mathrm{kjt}} \quad ; \forall \mathrm{k}, \mathrm{j}, \mathrm{t}$

Table 6. Presents the out Bound Lot Sizes for each Product Yype in each One of the Logistic Centers at each Time Period

|  | Logistic center 1 |  |  | Logistic center 2 |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | Week 1 | Week 2 | Week 3 | Week 4 | Week 1 | Week 2 | Week 3 | Week 4 |
| Product <br> group 1 | 1 | 0 | 5 | 0 | 2 | 3 | 3 | 0 |


| Product <br> group 2 | 2 | 0 | 7 | 0 | 2 | 0 | 6 | 0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Product <br> group 3 | 0 | 3 | 0 | 6 | 0 | 6 | 0 | 0 |

Results show the model suggests to ship on the first period even though the amount of units is small. The reason for that is due to forecasted arrival of faulty unit and their shipping expenses.
The dynamic variable $\mathrm{M}_{\mathrm{kjt}}$ which represents the stock amount for each one of the product groups in the logistic centers during the periods, it is based on the results given on the lot size and the variable $\mathrm{M}_{\mathrm{kjt}}$ is presented on the following Table:

Table 7. Stock Amount

|  | Logistic center 1 |  |  |  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: |
|  | Week 1 | Week 2 | Week 3 | Week 4 | Week 1 | Week 2 | Week 3 | Week 4 |  |  |  |  |  |
| Product <br> group 1 | 1 | 3 | 5 | 1 | 2 | 3 | 3 | 1 |  |  |  |  |  |
| Product <br> group 2 <br> Product <br> group 3 | 2 | 4 | 7 | 5 | 2 | 2 | 6 | 1 |  |  |  |  |  |

Analyzing both storage expenses and shipping expenses, it is possible to simplify the objective function into two main sections, the first one is-the shipping costs and the second is-the storage cost. The shipping cost is given by the following equation:
Shipping costs $=\sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{t=1}^{T}\left(\gamma_{\mathrm{kjt}} \times \mathrm{q}_{\mathrm{kjt}} \times \mathrm{e}_{\mathrm{kjx}}\right) \quad ; \quad \mathrm{X}=\mathrm{q}_{\mathrm{kjt}}$
The shipping costs of this test case totaled in $\$ 984$ which represents about $70 \%$ of the total amount spending that was given on the result of the objective function.
The storage costs can be calculated in two ways. The first takes the total output of the objective function and then deducts the shipping costs. The second uses the formula below:

$$
\text { Storage costs }=\sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{t}^{T}\left(1-\gamma_{\mathrm{kjt}}\right) \times\left(\mathrm{M}_{\mathrm{kjt}} \times \mathrm{S}_{\mathrm{kj}}\right)
$$

Since there are no storage costs in a period of time with an outbound shipment, the formula will only consider periods with storage costs, hence $\left(1-\gamma_{\mathrm{kjt}}\right)$. Then it will multiply the storage cost for product k at logistic center j times the stock level of product group k in logistic center j at period t . The storage costs of this test case totaled in $423 \$$ which represent about $30 \%$ of all expenses.
Graph 1, presents the distribution of the shipping and storage expenses during the examined period for both logistic centers. It is easy to notice that the shipping expenses are more significant than the storage costs.


Figure 2. Expenses Distribtion

## 4. Conclusions

Using the suggested model presented in this article, the organization can plan its operation of returning faulty units to repair in the most cost effective way through maintaining the required service level for the client. The output of the objective function is shipping versus storage decision; the function calculates the most cost effective array of shipping schedule and the lot size being shipped. In order to give optimal results, It takes in consideration all of the related elements such a shipping, storage and the forecast for faulty units. Combining this decision tool with other popular existing decision making tools can be a very helpful instrument in the hands of mangers in their mission to reduces costs and maximize profits in the operations field of work. In addition, the given model can easily fit itself to other constrains and can be customize to the organization needs. Other variables relate to the process can easily fit in under either the storage section or the shipping section and can even be defined as a separate one with other types of related expenses. Also, in most cases the demand (the faulty units forecast) is not known in advanced. This model is limited in this aspect because it does not operate in a stochastic environment. An opportunity for future research is to adjust the suggested model to operate in a stochastic environment.

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