Nov 7th, 3:30 PM - 3:45 PM

Using congruence in encoding musical partituras

Besim Shala
University for Business and Technology, besimi_77@yahoo.com

Besiana Mehmedi
State University in Tetovo, besiana.mehmedi@unite.edu.mk

Shkodran Tolaj
University of Audio and Visual Arts "Esra", shkodran.tolaj@hotmail.com

Azir Jusufi
State University in Tetovo, azir.jusufi@unite.edu.mk

Using congruence in encoding musical partituras

Besim Shala[1], Besiana Mehmedi[2], Shkodran Tolaj[3], Azir Jusufi [1,4]
[1]University for Business and Technology
[2]State University in Tetovo
[3]University of Audio and Visual Arts "Esra"
[4]State University of Tetovo
besimi_77@yahoo.com, besiana.mehmedi@unite.edu.mk, shkodran.tolaj@hotmail.com, azir.jusufi@unite.edu.mk

Abstract. Along with theoretical review of partituras and encryption systems, we have tried to conduct encryption of sheets by encoding all of its elements such as: encoding musical notes, encoding values of notes and intermissions, encoding accords, encoding tonalities and encoding rhythm whereby the original musical piece is transformed into an irregular and meaningless sheet. Information technology today has allowed for easier copying of authorial pieces; therefore, it is necessary to know encryption which allows protection of pieces from any misuse. Cryptology including knowledge of congruence deals with resolution of these insecurities. The significance of this paper lies in intertwining knowledge from music, math and computer sciences thus rendering our paper into an inter-disciplinary paper and we believe this will increase curiosity and the interest as well. In order to make our work more concrete, we have included encoding and decoding of a well-known melody from Shkodra"A'SAMAN TRËNDAFILI ÇELËS", whereby as encryption key we used a two-tact fragment from the song "O VENDI IM"

Keywords: pentagram, cryptosystem, encryption, decryption, music note, congruence.

1 Knowledge on congruence and cryptosystems

Definition 1: Let's have:$m \in \mathbb{N}.\forall a, b \in \mathbb{Z}$; Whereby$a$ is congruent with b based on module$m$ then and only if $m|(a-b)$. We note: $a \equiv b\ (mod m)$.

or

$$\mathrm{a} \equiv \mathrm{b}(\mathrm{mod}\,\mathrm{m}) \Leftrightarrow \exists \mathrm{k} \in \mathrm{Z} \mid \mathrm{a} - \mathrm{b} = \mathrm{km} \qquad (1)$$

The congruency relation is the relation of equivalence; therefore, the meaning of congruence is closely related with the meaning of residual classes.

Definition 2: Let's have:$m \in \mathbb{N}$. Classes of equivalence defined with the relation" ≡ " based on module *m(or as they are called in the theory of residual class numbers based on module m),* they represent the union of all the numbers, which when divided with *m* give the same residual value. Symbolically, residual classes based on module *m* are noted as in following:

$$K_m(a) = \{x \in \mathbb{Z} | x \equiv a(mod m)\} \subset \mathbb{Z} \qquad (2)$$

Every residual class based on module *m* is not empty since always $\exists a \in K_m(a)$whereby

$a \equiv a(mod m)$.

Note 1: In our paper we use smallest non-negative representatives of the equivalence class.

The following serves as reminder of some congruency features:

Theorem 1([1], p.52)**.** If $a, b, c \in \mathbb{Z}$ and$m \in \mathbb{N}$, where by $a \equiv b\ (modm)$, then:

1. $a + c \equiv b + c(modm)$
2. $a - c \equiv b - c(modm)$
3. $ac \equiv bc(modm)$.

Theorem 2([1], p.52)**:** If $a, b, c, d \in \mathbb{Z}$ and$m \in \mathbb{N}$, whereby $a \equiv b\ (modm)$and $c \equiv d(modm)$, then:

1. $a + c \equiv b + d(modm)$
2. $a - c \equiv b - d(modm)$
3. $ac \equiv bd(modm)$.

With the use of cryptography or cryptographic systems (also cryptosystem, code), we will understand the transformation of a message called open text through encoding function (or simply encoding) whereby only one authorized receiver can return the transformed message in the initial condition.

Definition 3: Cryptosystem is called a five (P, C, K, E, D) if it meets the criteria:

- P, is the final family of open texts
- C, is the final family of encoded texts;
- K, space of keys, is a final family of potential keys;
- Elements E andD are reflections respectively P into C, of C into P whereby every $k \in K$, has an encoding rule $e_k \in E$ and a decoding rule $d_k \in D$ for $\forall x \in P$ applies $\mathrm{d_k}(\mathrm{e_k}(\mathrm{x})) = \mathrm{x}$.

2 Encryption of music sheets

Taken in consider paper [2], [3], [4] and [5], we came to the following results:

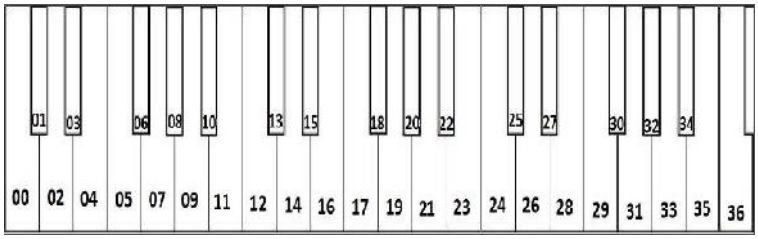
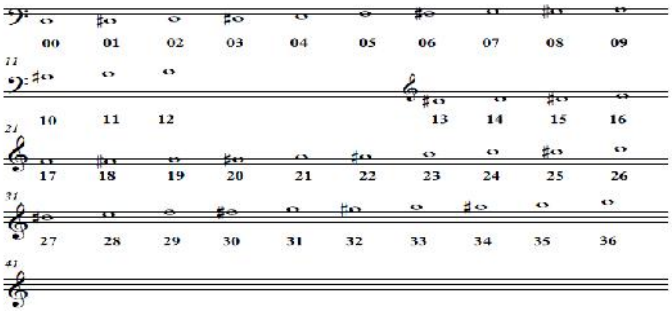


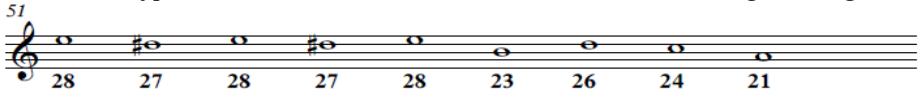
Fig. 1 Piano's keyboard

2.1 Encryption of notes in three octaves

(Keys in the small octave in the bass key, in the first and second octave in the violin key)

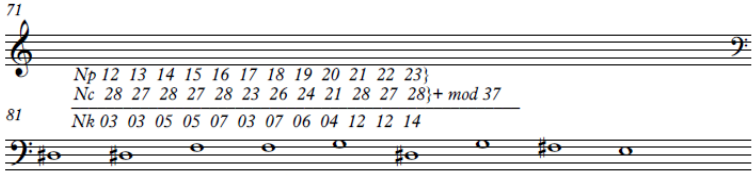


In order to encrypt notes into three octaves, we need another musical fragment, e.g. Elise



Note 2. Instead of this fragment, we can use any other musical fragment.
Encryption is as in following: Np + Nc = Nk (mod 37) (3)

Np means notes of the sheet we wish to encrypt; Nc means the notes of the key sheet (in our case, Elise [8]), while Nk means encrypted notes e.g. we encrypt the second octave with the help of key Elise



The second octave encrypted with the key Elise
Note 3. Decryption is done with

Nk – Nc=Np (mod 37) (4)

2.2 Value of notes and intermissions

(Apart from encryption of notes, values of notes and intermissions must also be encrypted)

00A Full note = 4

01B Half note = 2

02G Four note = 1

03D Eight note = 1/2

04E Sixteen note = 1/4

05F Full note with dot = 6

06G Half note with dot = 3

07H Four note with dot = 1.5

08I Eight note with dot

09L Full intermission = 4 10K Full intermission with dot = 2 11L Four intermission = 1

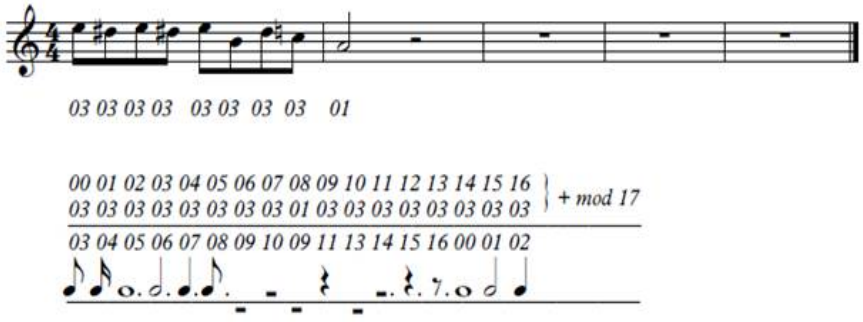
12M Eight intermission=1/2 3N Full intermission with dot=614O Half intermission with dot=2

15P Four intermission with dot = 1,5 16Q Sixteen intermission with dot = 0,75

Note 4*:* Dots extend the values of notes and intermissions by half their value.
Encryption of values is done based on module 17, e.g. one tact from Elise [8]

$$Vp + Vc = Vk \ (mod17) \qquad (5)$$

Piano

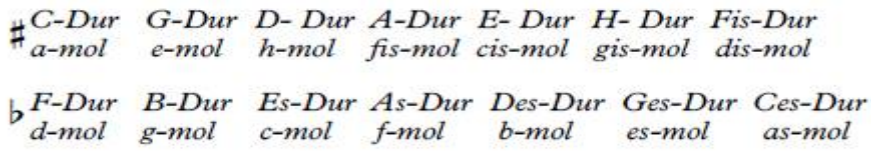


Note 5*:* Decryption of values will be done with

$$Vk - Vc = Vp \ (mod\ 17) \qquad (6)$$

Music partituras sheet, apart from notes, intermissions and their values that create the rhythm, it has the tonality, accords, the tact and a series of other composition elements. The following with introduce musical tonalities in order to encrypt the tonality.Musical tonality is the musical scale wherein the entire musical piece is developed.Tonalities are divided into Dur-Majeure or major or mol-minor or minor (with diezis and bemol) which differ greatly in sounds.

The musical tonalities are the following:



Encryption of the tonality

$$Tp + T\,c = Tk \ (mod.28) \qquad (7)$$

while decryption with

$$Tk - Tc = Tp \ (mod\ 28) \qquad (8)$$

2.3 Accords

Main accords in a musical piece are into three grades: first grade: tonics that represents the conclusion of the musical piece; fourth grade: sub-dominant, which represents the development of the musical piece and the fifth grade – dominant, which represents the culmination of the musical piece. We will stop at the main accords which are also divided into mol and dur, major and minor. Musical sheets contain a variety of accords such as quint accord 3/5 with rotations, sextaccord 6/3 and second accord, nonarord 9. Apart from these, it is important to emphasize the musical partituras sheet and intervals. Nevertheless, we will stop only at the main quit-accords.

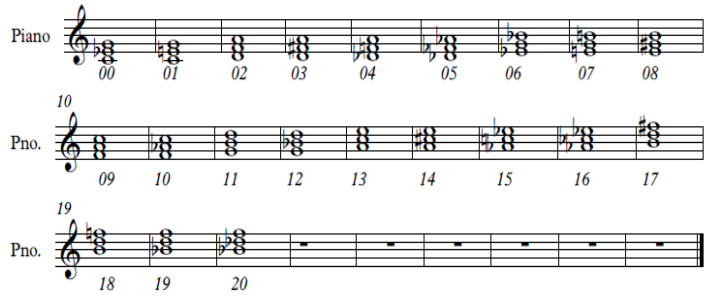
Encryption of accords follows

$$Ap + Ac = Ak \pmod{21} \tag{9}$$

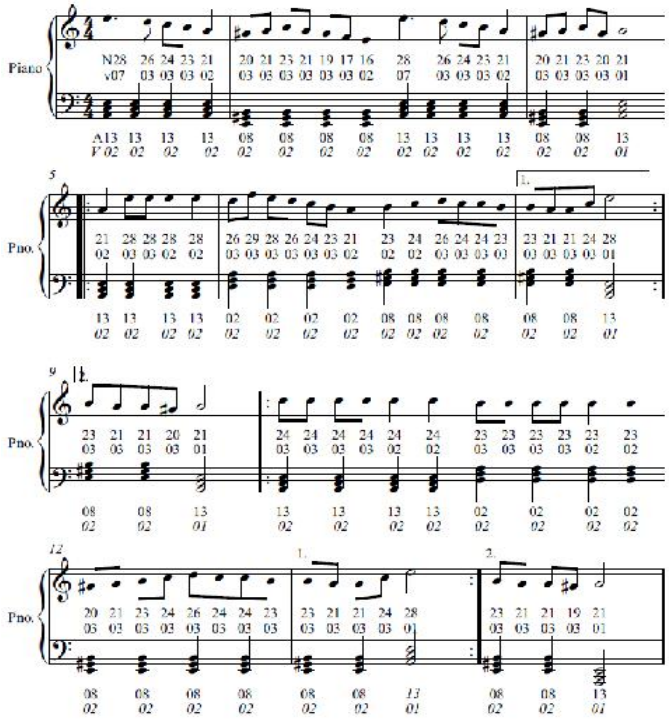
while decryption is done with

$$Ap - Ac = Ak \pmod{21} \tag{10}$$

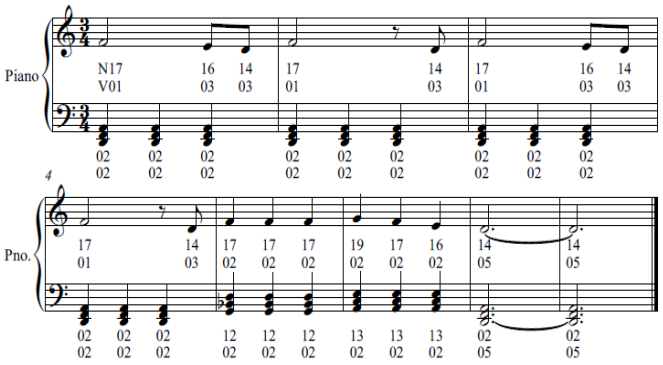
Note 6: Quint accord is a simultaneous sound of three sounds.



Example:
A'SAMAN TRËNDAFILI ÇELËS
(Melody from Shkodra [7], [9], [10])



Key – O vendi im ([7], [9], [10])



Tonality encryption

Since the musical sheet contains the tonality and the key, the encryption is simple.Main sheet is in a-mol (03) while the key in d-mol (21). Tonality encryption is done with module 28.

03+21=24 (mod 21)

The encrypted tonality is f-mol.

Rhythm

The same method applies with rhythm as well since the entire musical sheet is in one rhythm.Main rhythms are: 2/4 (00), 3/4 (01), 4/4 (02), 5/8 (03), 6/8 (04), 7/8 (05), 9/8 (06), 12/8 (07)

Rhythm "A'saman trendafil çeles" 02

Rhythm “O vendi im” 01

02 + 01 = 03 (mod 8)
Encrypted rhythm is (03) =5/8.

Encryption musical note done with the help of the formula (1)

Np 28 26 24 23 21 20 21 23 21 19 17 16 28 26 24 23 21 20 21 23 20 21
Nc + 17 16 14 17 14 17 16 14 17 14 17 17 17 19 17 16 14 17 16 14 17 14
Nk 08 05 01 03 35 00 00 00 01 33 34 33 08 08 04 02 35 00 00 00 00 35

21 28 28 28 28 26 29 28 26 24 23 21 23 24 26 24 24 23 23 21 21 24
+ 17 16 14 17 14 17 17 17 19 17 16 14 17 16 14 17 14 17 16 14 17 14
01 07 05 08 05 06 09 08 08 04 02 35 03 03 03 04 01 03 02 35 01 01

28 21 28 28 28 28 26 29 28 26 24 23 21 23 24 26 24 24 23 23 21 21
+ 17 17 17 19 17 16 14 17 16 14 17 14 17 16 14 17 14 17 17 17 19 17
08 01 08 10 08 07 03 09 07 03 04 00 01 02 01 06 01 04 03 03 03 01

20 21 24 24 24 24 24 24 23 23 23 23 23 23 20 21 23 24 26 24 24 24
+ 16 14 17 16 14 17 14 17 16 14 17 14 17 17 17 19 17 16 14 17 16 17
36 35 04 03 01 04 01 04 02 00 03 00 03 03 00 03 03 03 03 04 03 04

24 24 24 24 24 23 23 23 23 23 23 20 21 23 24 26 24 24 23 23 21 21 29 21
+ 14 17 17 17 19 17 16 14 17 16 14 17 14 17 16 14 17 14 17 17 17 19 17 16
01 04 04 04 06 03 02 00 03 02 00 00 01 03 03 03 04 01 03 03 01 03 36 00

Encryption of values and breaks done with the help of the formula (5)

Vp 07 03 03 03 02 03 03 03 03 03 02 07 03 03 03 02 03 03 03 01
Vc 01 03 03 01 12 03 01 03 03 01 12 03 02 02 02 02 02 05 01 03 03
08 06 06 04 14 06 04 06 06 04 15 05 09 05 05 05 04 05 08 04 06 04

02 03 03 02 02 03 03 03 03 03 02 02 02 03 03 03 03 03 03 03
+01 12 03 01 03 03 01 12 03 02 02 02 02 02 05 01 03 03 01 12 03
03 15 06 03 05 06 04 15 06 05 05 04 04 04 05 08 04 06 06 04 15 06

01 02 03 03 02 02 03 03 03 03 03 02 02 02 03 03 03 03 03 03
+01 03 03 01 12 03 02 02 02 02 02 02 05 01 03 03 01 02 03 01 03
02 05 06 04 14 05 05 05 05 05 05 07 03 05 06 04 15 06 04 06

03 03 01 03 03 03 03 02 02 03 03 03 03 02 02 03 03 03 03 03 03 03 03
+03 01 12 03 02 02 02 02 02 05 01 03 03 01 12 03 01 03 03 03 01 12 03
06 04 13 06 05 05 05 04 04 05 08 04 06 05 13 15 06 04 06 06 04 15 06

03 03 03 03 01 03 03 03 03 02 02 03 03 03 03 02 02 03 03 03 03 03 03 03
+02 02 02 02 02 02 05 01 03 03 01 12 03 01 03 03 01 12 03 02 02 02 02 02
05 05 05 05 03 05 08 04 06 05 03 15 06 04 06 05 03 15 06 05 05 05 05 05

03 03 03 03 03 01
+02 05 01 03 03 01
05 08 04 06 06 02

Encryption of accorddone with the help of the formula (9)

Ap 13 13 13 13 08 08 08 08 13 13 13 13 08 08 13 13 13 13 02 02 02 02

Ac+ 02 02 02 02 02 02 02 02 02 02 02 02 02 12 12 12 13 13 13 02 02 02 02 02

15 15 15 15 10 10 10 10 15 15 15 15 20 20 04 05 05 05 15 04 04 04 04

08 08 08 08 08 08 13 13 13 13 13 02 02 02 02 08 08 08 08 08 08 13 13 13

+02 02 02 02 02 02 02 02 02 12 12 12 13 13 13 02 02 02 02 02 02 02 02 02

10 10 10 10 10 10 15 15 04 04 04 15 15 15 04 10 10 10 10 10 10 15 15 15

13 13 02 02 02 02 08 08 08 08 08 08 13 13 13 13 13 02 02 02 02 08 08 08

+02 02 02 12 12 12 13 13 13 02 02 02 02 02 02 02 02 02 02 02 02 02 12 12

15 15 04 14 14 14 00 00 00 00 10 10 10 15 15 15 15 15 04 04 04 04 10 20 20

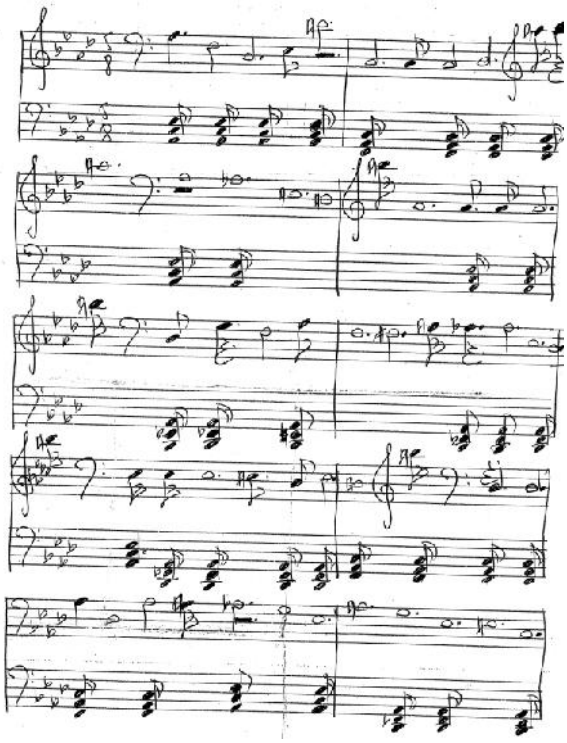
08 08 08 13

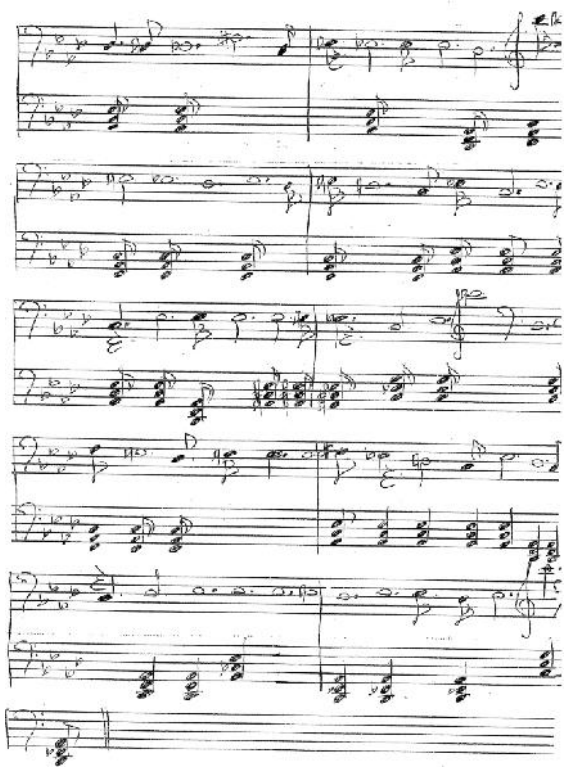
+12 13 13 13

20 00 00 05

Encrypted partiturasisan irregular combination and in fact it does not represent anything.

Encrypted Partituras





After having received the irregular sheet, the receiver deals with the following decoding whereby again as decoding key utilizing the two-tact fragment from the song "O VENDI IM"

Decryption musical notedone with the help of the formula (4)

08 05 01 03 35 00 00 00 01 33 34 33 08 08 04 02 35 .....

\- 17 16 14 17 14 17 16 14 17 14 17 17 17 19 17 16 14.....

28 26 24 23 21 20 21 23 21 19 17 16 28 26 24 23 21.....

Decryption of values done with the help of the formula (6)

Vk 08 06 06 04 14 06 04 06 06 04 15 05 09 05 05 05 04 05 08 04 06 04 .....

Vc 01 03 03 01 12 03 01 03 03 01 12 03 02 02 02 02 02 02 05 01 03 03 .....

Vp 07 03 03 03 02 03 03 03 03 03 03 02 07 03 03 03 02 03 03 03 03 01 .....

Decryption of accord done with the help of the formula (10)

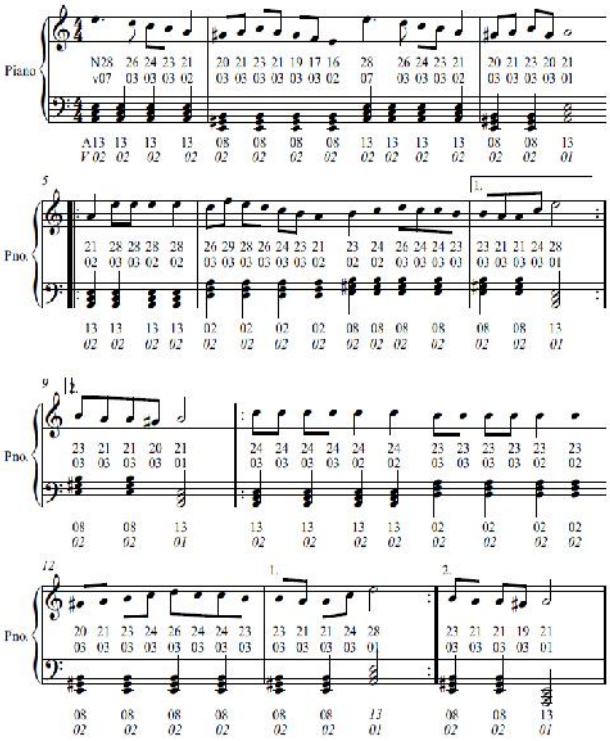
15 15 15 15 10 10 10 10 15 15 15 15 20 20 04.....

-02 02 02 02 02 02 02 02 02 02 02 02 12 12 12.....

13 13 13 13 08 08 08 08 13 13 13 13 08 08 08.....

Decrypted rhythm is 03- 01 = 02 (mod 8), (02) is 4/4.

Decrypted element placing in the partitures and we receive Shkodran melody "A'SAMAN TRËNDAFILI ÇELËS",



References

[1] WissamRaji, An Introductory Course in ElementaryNumber Theory, https://www.artofproblemsolving.com/.../intr..., 2014
[2] M. Yamuna, Krishna Pandey, Nikhil Choudhary , CRYPTOGRAPHY USING MUSIC NOTES, Journal of Global Research in Computer Science, Volume 4, No. 4, April 2013
[3] Sandip Dutta, Chandan Kumar, Soubhik Chakraborty, A Symmetric Key Algorithm for Cryptography using Music, International Journal of Engineering and Technology, Vol 5 No 3 Jun-Jul 2013,
[4] Sandip Dutta Soubhik Chakraborty N.C.Mahanti, A Novel Method of Hiding Message Using Musical Notes, International Journal of Computer Applications, Volume 1 – No. 16, 2010
[5] Bruce Schneier, Applied cryptography, Wiley-India, 2007
[6] Rexhep Munishi, IDENTITETI MUZIKOR, , Prishtinë GME, 2001
[7] Rrustem Berisha, Diemensione të kenges popullore , shtypshkronja KGT, Instituti Albanologjik Prishtinë, 2011
[8] Tomor Berisha, Përmbledhje e veprave më të njohura nga Bethoven, Shtepia Botuese Rilindja, 1999
[9] Përmbledhje Këngesh Qytetare nga kompozitorë të ndryshëm, Toena Tirane, 2004
[10] Ferit Bala, Kënga dhe Këngetaret Shkodran, Shtepia Botuese , Tirane, 2014