A WAVELET-BASED VARIABLE CONTROL PROCEDURE FOR DETECTING PROCESS MEAN SHIFT

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This paper develops a wavelet-based approach for a variable control chart, and adopts the data decomposition and linear combination techniques to detect process mean shifts. The Shewhart, exponentially weighted moving average (EWMA), and cumulative sum (CUSUM) control charts, the most popular monitoring process graph tools, were developed for different process situations. If a user chooses an inappropriate control chart to monitor a process, the correct control result will not be obtained. This study used the wavelet transform to develop a novel variable control procedure. First, the Haar function was used as the basis for data decomposition. Next, the linear combination technique was used to combine different resolution data through wavelet transform decomposition. Simulation techniques and real data were adopted to evaluate performance. The analysis shows that the detection ability of the wavelet-based variable control chart is superior to the EWMA control chart in a comparison of average run length (ARL) results.

1. INTRODUCTION

In practice, the control chart is the most popular tool for monitoring and detecting process variation. Controlling process variations prevents the manufacture of poor products, the need to rework products, and waste. The Shewhart control chart detects process variations using a graph technique for quality characteristic data. Further, it can continuously record the process data and allows engineers to use the graphic information to make process changes by parameter adjustment. Engineers can use control chart pattern changes to explore and evaluate the phenomena of process variations. In addition, the control chart is an important phase in six sigma process improvement [1].

The Shewhart control chart uses the information about the process contained in the last point, but it ignores other information given by the entire sequence of points [2-6]. Montgomery [7] pointed out that a Shewhart control chart is applicable only to large shifts in the process. As such, it is insensitive to small process shifts (1.5σ or less). Therefore, Roberts [2] designed the exponentially weighted moving average (EWMA) control chart, and Page [8] developed the cumulative sum（CUSUM）control chart to monitor small process shifts. Since EWMA control charts can be custom tailored to detect any size shift in the process, they are often used to monitor in-control processes for detecting small shifts away from the target. The CUSUM control chart is a sequential analysis technique and is constructed by calculating and plotting a cumulative sum based on the data. In particular, analyzing ARL's (Average run length) for CUSUM control charts shows that they are better than Shewhart control charts at detecting mean shifts (2σ or less). The EWMA and CUSUM control charts have nearly the same detection ability. Jiang et al. [9] used the EWMA control chart to detect small differences in various panel areas and to detect the position and size of MURA defects in a TFT-LCD panel. Several researchers have proposed control chart pattern recognition techniques, namely statistical [10], neural network [12,13], and extracted feature techniques [14]. In recent years, the wavelet-based multi-resolution analysis has been used as a tool to denoise and has been successfully applied to control chart pattern recognition [15]. The advantage of wavelet analysis is that it can handle both stationary and non-stationary data.

Any practical application must consider different manufacturing conditions in the selection of a suitable control chart for monitoring. This paper proposes a control chart based on the wavelet transform for data decomposition and linear combination to detect abnormal process data. This paper discusses and evaluates the different mean shift effects using simulations and compares the proposed method with the ARL. In addition, we use the Wasserman [16] data as an example to explain the proposed method. This comparison and analysis shows that the detection ability of the Wavelet-based variable control chart is excellent in comparison to the EWMA and Q chart.

2. METHODOLOGY

In general, wavelet transform uses the wavelet property to filter noise signals and get smooth signals resembling primitive signal. The difference in this paper is that the wavelet transform is used to get different resolution signal data and to carry out linear combination techniques to emphasize abnormal process data. Using the time-series concept, it is used to linearly combine the different resolution signals and isolate the signal noise (abnormal).

Wavelet analysis is similar to Fourier's analysis in the sense that it decomposes a signal into its basic constituent components. Whereas the Fourier transformed represents the signal as a series of sine waves of different frequencies, the wavelet transform represents signals as linear combinations of wavelets. There are differences between the trigonometric and wavelet base signal representations. This study adopted the Haar wavelet function as its basis. The Haar wavelet transform is a basic wavelet transform function [17,18]. This function was developed using the box function. The Haar function $\varphi(x)$ definition is as follows:

$$\varphi(x) = \begin{cases} 1 & x \in [0,1) \\ 0 & otherwise \end{cases} \qquad \ldots \qquad (1)$$

The Haar wavelet function $\psi(x)$ is:

$$\psi(x) = \begin{cases} 1 & x \in [0,1/2) \\ -1 & x \in [1/2,1) \\ 0 & otherwise \end{cases} \qquad \ldots \qquad (2)$$

Employing the Haar function $\varphi(x)$, the Haar wavelet function can be written as：

$$\psi(x) = \varphi(2x) - \varphi(2x-1) \qquad \ldots \qquad (3)$$

The discrete wavelet transform can describe the regional time frequency characteristic. The Haar transformation operation movements can be regarded as the average difference in the sequence discrete time function, or the average of the difference of calculating two adjoining original data. This research adopted the practice of taking the group as the basis for the wavelet control chart. This procedure can be divided into two stages. The main purpose of stage 1 is to collect the data and to determine the control limits. The Haar function is used as the basis for data decomposition, and the linear combination technique is used to combine different resolution data through wavelet transform decompositions to set up the control chart. Stage 2 applies the data obtained from a process to the future and determines whether the process is being maintained in a state of control. Utilizing the control limits from the first stage, this stage carries on long term control and revision. The following presents the execution steps for the wavelet control chart.

Stage 1

Step 1   Collect the training data $\mathbf{x}$, $\mathbf{x} = \left\{ x_1, x_2, ..., x_{2^n} \right\}, n \in \mathbf{Z}$ and group sample size is 2^n. This research suggest selecting n=4 or n=5, because if $n < 4$ the information received may be insufficient and $n > 5$ it may increase the quality cost.

Step 2   Perform the autocorrelation analysis for training data.

Step 3   Use the Haar wavelet transform to decompose the training sample with multiple resolution analysis. S_i, i=1,2,…, is the resolution vector from the multiple resolution analysis. The decomposing procedure is as follows:

$I := 1$

$J := 2$

$M := 2^n$

$For\ L:=1,...,n\ DO$

   $M:=M/2$

   $For\ k:=1,2,...,M\ DO$

      $a_k^{(n-l)} := \left(x_{J \cdot k - I} + x_{J \cdot k} \right) / 2$

      $x_{J \cdot K} := a_k^{(n-l)}$

   END

$STOP$

$RESULT$

$S_1 = \left\{ a_1^{(n-1)}, a_1^{(n-1)}, a_2^{(n-1)}, a_2^{(n-1)}, ..., a_{2^{n-1}}^{(n-1)}, a_{2^{n-1}}^{(n-1)} \right\}$

$$S_2 = \left\{ a_1^{(n-2)}, a_1^{(n-2)}, a_1^{(n-2)}, a_1^{(n-2)}, a_2^{(n-2)}, a_2^{(n-2)}, a_2^{(n-2)}, a_2^{(n-2)}, ..., a_{2^{n-2}}^{(n-2)}, a_{2^{n-2}}^{(n-2)}, a_{2^{n-2}}^{(n-2)}, a_{2^{n-2}}^{(n-2)} \right\} \qquad .$$

.

$$S_n = \left\{ a_1, a_1, ..., a_1 \right\}$$

Step 4   Carry on the linear combination for multiple resolution signals $\sum_{i=1}^{n} c_i S_i$, and get the new

data $\mathbf{y} = \mathbf{x} - \sum_{i=1}^{n} c_i S_i = \left\{ y_1, y_2 ..., y_{2^n} \right\}, c_i \in \mathbf{R}$. In this paper, select to a Kolmogorov-Smirnov method for

normality test for new data $\mathbf{y}$; if the normality assumption is not satisfied, then proceed with the Box-Cox

transformation of $\mathbf{y}$ in order to satisfy the variable control assumption. The Box-Cox transformation is a

parametric power transformation technique in order to reduce non-normality.

Step 5   Draw the control chart, with the control limits *UCL* (Upper Center Line) and *LCL* (Lower Center Line)

defined as follows:

$$UCL = \bar{y} + 3 \frac{\overline{MR}}{1.128} \ ; \ LCL = \bar{y} - 3 \frac{\overline{MR}}{1.128} \qquad ... \qquad (4)$$

in which the moving range is $MR_i = \left| y_i - y_{i-1} \right|, \ \overline{MR} = \frac{1}{n} \sum_{i=1}^{n} MR_i, \ i = 1,2,3...$

Step 6   If $y_i \geq UCL$ or $y_i \leq LCL$, this shows that the process is abnormal. We collect samples and go back to Step
2 again after improving the process. If $LCL < y_i < UCL$, then the process is in a steady state, so we use
Step 4 to get the new data $\mathbf{y}$ and Step 5 to calculate control limits to monitor the follow-up data. Figure 1 is
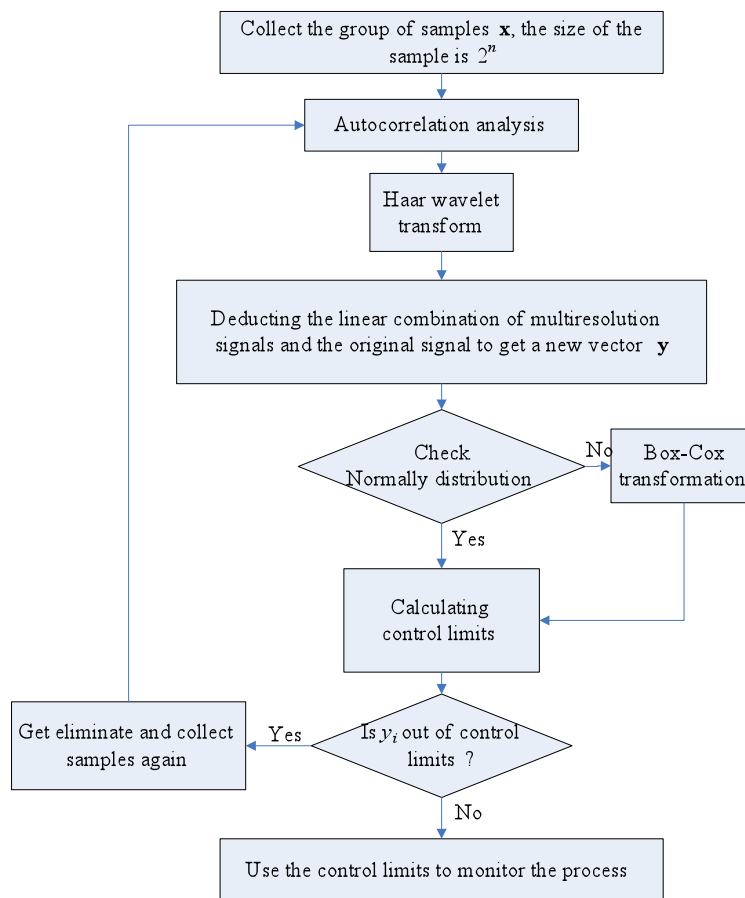a flow chart for stage 1.

Figure 1. The Wavelet control chart analysis procedure for stage 1

Stage 2.

In the second stage, a procedure is used to adopt and reject one old sample, add a new sample, and get a sub sample of all the groups with the same sample size, as in Stage 1. The sub samples are obtained using the Haar wavelet transform and linear combination. The new control receives the observed value and is controlled using the control limits from stage 1. The detailed procedure is as follows:

Step 1   Reject the first point of the Stage 1 original sample, add a new point, and get the sub

sample $\mathbf{x}_{SUB.} = \left\{ x_2, x_3..., x_{2^n+1} \right\}, n \in \mathbf{Z}$.

Step 2   Use the Haar wavelet transform to decompose the multiple resolution analysis of $\mathbf{x}_{SUB.}$. Receive n's

resolution vectors, which are $\mathbf{S}_1, \mathbf{S}_2, ..., \mathbf{S}_n$.

Step 3   Deduct $\mathbf{x}_{SUB.}$ and the linear combination of vectors from Step 2, and receive a new vector $\mathbf{y}$.

$$\mathbf{y} = \mathbf{x}_{SUB.} - \sum_{i=1}^{n} c_i \mathbf{S}_i = \left\{ y_2, y_3,,...., y_{2^n+1} \right\}, \forall c_i \in \mathbf{R}$$

The choice of c_i is based on the Haar-wavelet resolution $\mathbf{S}_i$ from the original data. In practice, the $\mathbf{S}_i$ will

be reduced with the increase of i, until the horizontal line. Therefore, in this paper, we adopted the larger

value of the first $\mathbf{S}_i$ to execute the linear combination.

Step 4   If $y_i \geq UCL$ or $y_i \leq LCL$, it has an abnormal quality and the process must be improved.

If $LCL < y_i < UCL$, the quality is in a stable state, so update the control chart. Between them, LCL and

UCL are the Stage 1 received results.

Step 5   If new samples come in, repeat Steps 1-4. The first point of the sub sample is rejected, the new point is

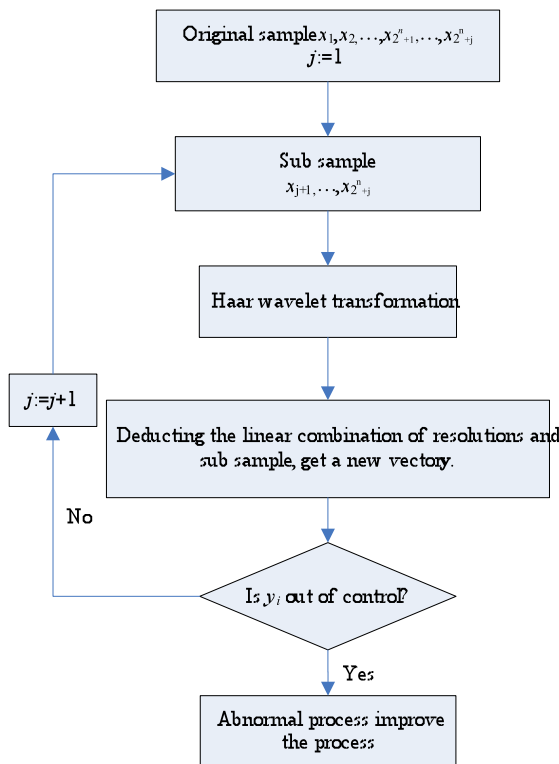added, and a group of new sub samples are acquired. Figure 2 is a flow chart for Stage 2:

Figure 2. The Wavelet control chart analysis procedure for stage 2

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3. SIMULATION AND ANALYSIS

This section discusses and evaluates the different mean shift effects using simulations and compares the proposed method with the ARL. After estimating the wavelet resolutions, the associated linear combination parameters were compared with the EWMA control chart. Suppose that 32 observed values are made independently of each other. From the 23rd point, the mean shift observed value has a standard deviation of 0.1, 0.2, 0.3…, 3. The upward simulation shifts obey the normal distribution. In the normal state, assume the mean $\mu = 0$, and variance $\sigma^2 = 1$. The ARL was compared with the EWMA under different shifts to confirm that the wavelet control chart had the better detection ability.

This length $n = 32 = 2^5$ of samples in the research can achieve 5 levels of resolution, S_1, S_2, S_3, S_4, and S_5, via the wavelet transform. The original data and the linear combination of resolution vectors were transformed, and we

received a new vector $\mathbf{y}$, $\mathbf{y} = \mathbf{x} - \sum_{i=1}^{5} c_i \mathbf{S}_i = \left\{ y_1, y_2, ..., y_{32} \right\}$. In practice, the S_i will be reduced with the increase of i until

the horizontal line. Therefore, we adopted the cumulative technique to determine the value of c_i. In this paper, we adopt the larger value of the first $\mathbf{S}_i$ to execute the linear combination. Figure 3(a-e) expresses the different parameters $\left(c_1, c_2, c_3, c_4, c_5 \right)$, and the results of (1,0,0,0,0), (1,1,0,0,0), (1,1,1,0,0), (1,1,1,1,0), and (1,1,1,1,1). Figure 3(a) shows the original data $\mathbf{x}$ after subtracting the $\mathbf{S}_1$ result, and Fig. 3(b) shows the original data $\mathbf{x}$ after subtracting the $(\mathbf{S}_{1+}\mathbf{S}_2)$ result. Figure 3(c) shows the original data $\mathbf{x}$ after subtracting the $(\mathbf{S}_1+\mathbf{S}_2+\mathbf{S}_3)$ result, and Fig. 3(d) shows the original data $\mathbf{x}$ after subtracting the $(\mathbf{S}_1+\mathbf{S}_2+\mathbf{S}_3+\mathbf{S}_4)$ result. Fig. 3(e) shows the original data $\mathbf{x}$ after subtracting the $(\mathbf{S}_1+\mathbf{S}_2+\mathbf{S}_3+\mathbf{S}_4+\mathbf{S}_5)$ result.
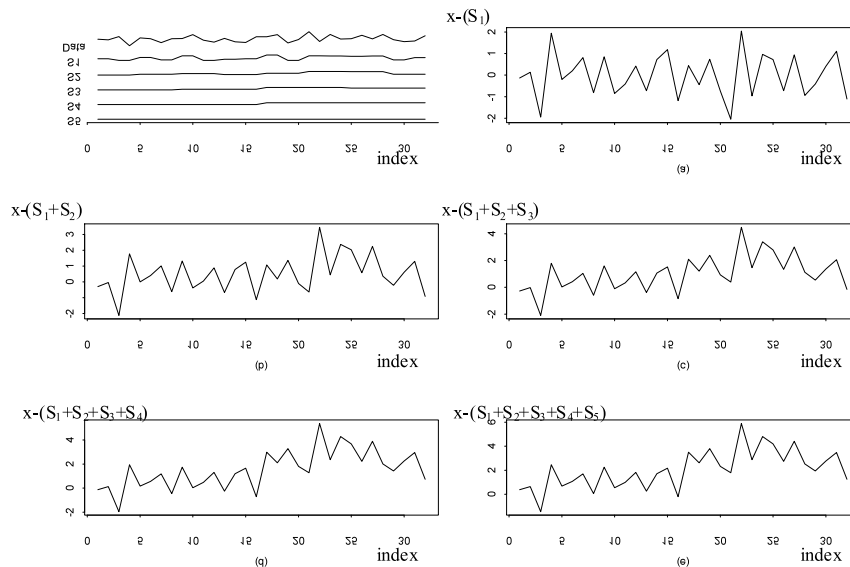
Figure 3. Different resolution situations, difference of linear combination results

Table 1 shows the best values for parameters λ and L of the EWMA control chart, as determined by Lucas and Saccucci [3].

Table 1. The EWMA control chart parameters

Shifts	L	λ
0.1-0.3	2.437	0.03
0.4-0.6	2.615	0.05
0.7-1.1	2.814	0.1
1.4-1.6	2.998	0.25
1.9-2.1	3.054	0.4
2.4-2.6	3.071	0.5
2.9-3	3.087	0.75

Table 2 shows a comparison of different mean shifts. The wavelet control chart and EWMA control chart were used to determine results with ARL under the different mean shifts. The simulation data conditions were designed so that a shift began at the 24th point. The ARL simulation could determine when the process shifted at the 24th point, with a 0.3 standard deviation after this. This research examined whether the EWMA control chart and wavelet control chart could detect this. A horizontal line can be used to approximate the signals S_4 and S_5, as can be seen in Fig. 3. It is already impossible to find the characteristics of the original signal in S_4 and S_5. In Table 2, as the standard deviation of the shifts increased above 0.4, it can be seen that after the approximate signal S_4 and S_5 are joined, the wavelet control chart is already too sensitive. Therefore, we could consider using the parameters (c_1,c_2,c_3,c_4,c_5) =(1,1,1,0,0). When the resolution increases, the approximate signal coarsens.

Table 2. The shift between 0.1 and 3 standard deviation for use in a comparison with the EWMA control chart

Shifts	EWMA	Wavelet control chart (c_1,c_2,c_3,c_4,c_5)				
		(1,0,0,0,0)	(1,1,0,0,0)	(1,1,1,0,0)	(1,1,1,1,0)	(1,1,1,1,1)
		ARL				
0.1	NA	NA	NA	NA	15.45	16.07
0.2	NA	NA	NA	NA	16.47	15.14
0.3	NA	NA	NA	NA	17.85	16.23
0.4	NA	NA	NA	23.4	18.79	17.42
0.5	27.65	NA	NA	23.53	19.98	18.54
0.6	27.73	NA	NA	23.72	20.8	19.57
0.7	27.85	NA	NA	23.94	21.86	20.06
0.8	28.36	NA	NA	24.3	21.97	20.54
0.9	28.44	NA	NA	24.55	22.15	20.19
1	28.46	NA	NA	24.73	22.18	20.11
1.1	28.62	NA	NA	24.89	22.23	20.06
1.2	NA	NA	NA	24.87	22.28	19.87
1.3	NA	NA	NA	24.79	22.1	19.87
1.4	27.36	NA	NA	24.48	21.84	19.71
1.5	27.18	NA	NA	24.41	21.51	19.13
1.6	26.96	NA	NA	24.26	21.34	19.3
1.7	NA	NA	NA	24.18	21.23	19.08
1.8	NA	NA	27.29	23.95	20.97	18.42
1.9	26.06	NA	27.2	23.87	20.84	18.07
2	25.84	NA	26.99	23.69	20.6	18
2.1	25.6	NA	27.02	23.58	20.34	17.43
2.2	NA	NA	26.79	23.52	20.24	17.46
2.3	NA	NA	26.6	23.46	20	17.22
2.4	25.04	NA	26.47	23.38	19.88	16.52
2.5	24.95	NA	26.26	23.34	19.73	16.47
2.6	24.81	NA	26.09	23.28	19.58	16.5
2.7	NA	NA	25.83	23.28	19.46	16.25
2.8	NA	NA	25.61	23.09	19.18	15.41
2.9	24.4	NA	25.39	23.07	18.98	15.33
3	24.31	NA	25.23	23.08	18.84	14.87

4. EXAMPLES

In this paper, we use the Wasserman [16] data as an example to show the execution procedure of the wavelet-based variable control chart and for comparison with the EWMA and Q-charts. This process uses a sample size of n=38; the first 15 points are normal process states $\mathbf{X}_t \sim N(0,1)$, t=1, 2… 15. The 16th shifts up 1 standard deviation $\mathbf{X}_t \sim N(1,1)$, t=16, 18…, 38.

## 4.1 The first stage procedure
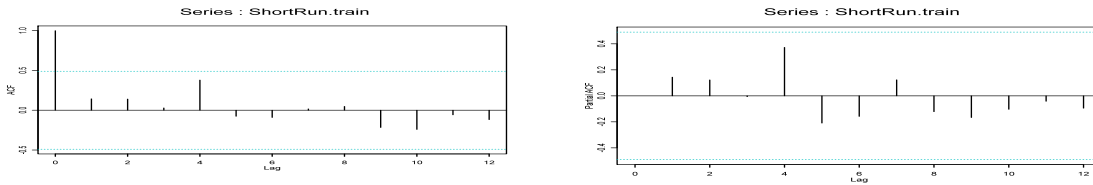
(a) Autocorrelation function   (b) Partial autocorrelation function

Figure 4. Autocorrelation analysis result

Step 1.   Use n =16 samples as training samples, draw ACF and PACF, and perform an autocorrelation analysis.

Figure 4 shows that the autocorrelation function has a time-series trend, and the partial autocorrelation function is not outside the confidence interval. These points are independent of each other among the observed sample values.

Step 2. Multiple resolution analysis of the sample via the Haar wavelet transforms.
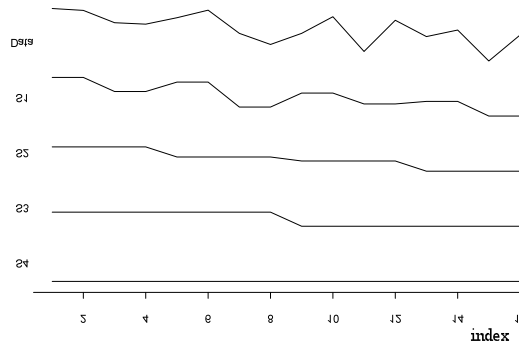
Figure 5. The multiple resolution analysis results

We can get 4 levels of resolution vector, $\mathbf{S}_1$, $\mathbf{S}_2$, $\mathbf{S}_3$, and $\mathbf{S}_4$ from Fig. 5. Here, this data utilizes the two vectors, S1 and S2, to transform the original data.

Step 3.   It uses the resolution and gets a new vector $\mathbf{y}$ after Step 2 deducts the original data; that is to say, $\mathbf{y}$ =$\mathbf{x}$-($\mathbf{S}_1$+ $\mathbf{S}_2$). Perform a normality test for the sample after the change. From the K-S test we get a *p-value* of 0.5 more than the significance level $\alpha = 0.05$. Therefore, we can show that the observed value after changing obeys the normality assumption.

Step 4.   Calculate the control limits.

$$UCL = \overline{\mathbf{y}} + 3 \times \frac{\overline{MR}}{1.128} = 2.835 \; ; \quad LCL = \overline{\mathbf{y}} - 3 \times \frac{\overline{MR}}{1.128} = -2.787$$

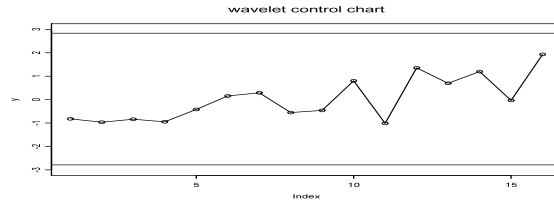Step 5.   Draw the wavelet control chart (Fig. 6).

Figure 6. State of the training sample

The result shown in Fig. 6, the normal state, utilizes this training sample to produce control limits to carry out the follow-up control.

4.2   The second stage procedure

Figure 7 shows the control results for all the sub samples. We can observe from Fig. 7 that the 23rd sample point, when it is controlled to the 26th sample point, is out of control on the control chart. That is, the 10th point after the abnormal event can be detected on the wavelet control chart. However, while controlling to the 24th sample point, we also find that the 23rd is close to the control limit.

According to Figs. 8(a) and (b), the wavelet control charts are at the same time because of entering the sample differences. After the multiple resolution analysis via the wavelet transform, this leads to the fact that every sub observed value within the same time of the sample exhibits a change. Figure 9 shows that the observed value is at $t=26$ and that the 23rd is out of control. With traditional EWMA or CUSUM control charts, comprehensive information is produced from the observed values that arrive before the present observed value. The wavelet control chart used in this research was developed to synthesize the value information that precedes the observed value information with the information that follows it. As shown by Wasserman [16], the dynamic EWMA control chart method just detects and measures the 33rd point. The Q-chart cannot detect or measure an abnormal situation. Therefore, the wavelet control chart has good detecting ability.
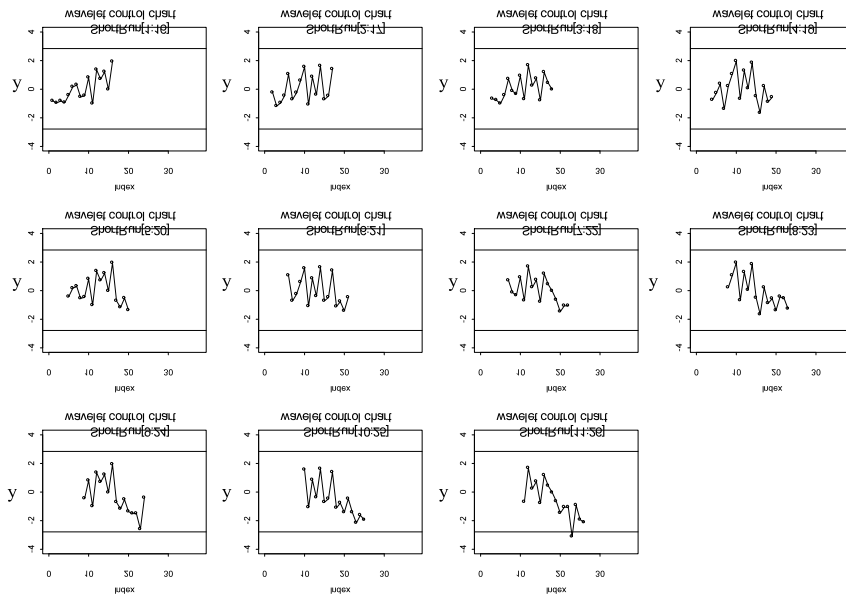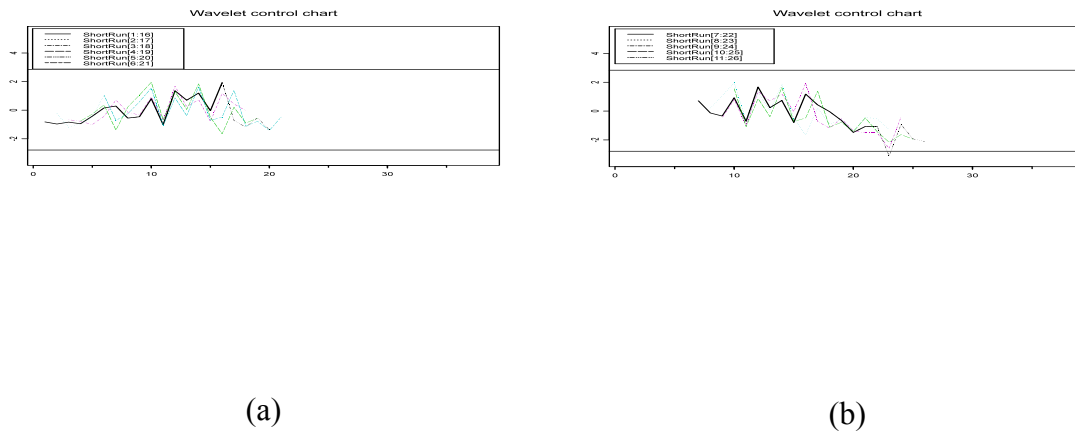

Figure 7. The sub sample of every stage

(a)   (b)

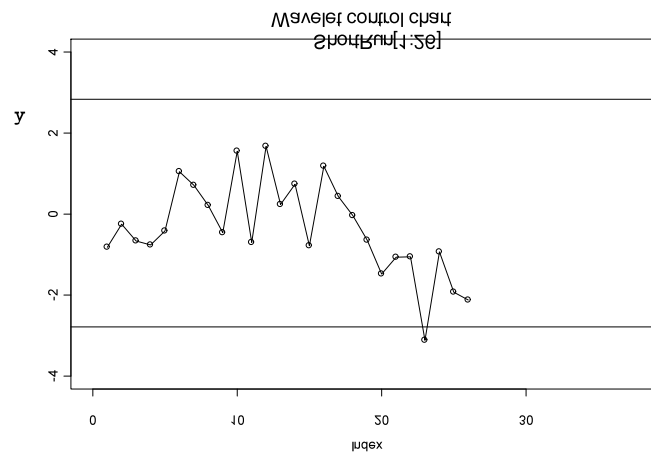**Figure 8. The changes of 12 groups of sub samples by time unit**

Figure 9. Wavelet control chart

5. CONCLUSIONS

This study used the Shewhart control chart to structure wavelet and linear combination resolutions to identify quality characteristic data. Its purpose was to emphasize abnormal information points, making the traditional Shewhart control chart detect and examine the weak variation changes in a process. Utilizing simulations, the wavelet and EWMA control charts were compared in identifying 0.3 standard deviation process shifts, with the following results: (1) The EWMA and wavelet control charts were not good at detection, and (2) for 0.4 standard deviation shifts, the wavelet control chart was superior to the EWMA control chart.

In addition, to verify the method in practice, the data from the study by Wasserman [16] was adopted and the dynamic EWMA and Q-chart were used. The wavelet control chart was relatively good at detecting process shifts. The wavelet linear combination of original data to produce the best linear combination is the main direction for future research.

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BIOGRAPHICAL SKETCH

Chien-Chih Wang is an Associate Professor in the Industrial Engineering and Management at Ming Chi University of Technology in Taiwan. He received the B.S. degree in applied mathematic from National Chung Hsing University (Taiwan) in 1992, and the M.S. degree in statistics from the University of National Cheng Kung University (Taiwan) in 1994, and the Ph.D. degree in industrial engineering and management from Yuan Ze University (Taiwan) in 2001. His current research activities include machine vision in manufacturing, quality engineering and statistics in industrial application.

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