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FUZZY MULTI-OBJECTIVE LINEAR PROGRAMMING APPROACH FOR SOLVING PROBLEM OF FOOD INDUSTRY

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Abstract. Enterprises and industrial centers need current decision for making products in fast changing market. Uncertainty and yield defined goals make decision making more difficult. In this situation fuzzy logic is used for coping surrounding environment. This paper deals with a fuzzy linear programming model for a problem of food industry. The different types of achievement function such as compensatory and weighted compensatory form associated with equal importance of objectives and non-importance of objectives both have been discussed.

Keywords. Fuzzy Linear Programming, Membership Function, Compensatory Operator.

INTRODUCTION

In present scenario of marketing, decision maker need is to apply fuzzy programming model for optimum profit. In decision making environment such as impression, vague, ill defined, etc. the decision criteria such as goals, constraints, and restrictions may be modeled by FST. Bellman & Zadeh [1-4] applied the concept of FST for solving decision making problems. Zimmerman [5] proposed the first model of Linear programming in fuzzy environment. Then we have a large number of authors used different optimization technique in this area. Various research papers [6-45] are showing the advancement in the field of Fuzzy Mathematical Programming (FMP). Obviously FMP provides compromise solution. In this paper a model of FLP with quadratic form of achievement functions has been presented. Mathematical formulation and the methodology is explained by solving a sample problem.

MATHEMATICAL MODELING

MOLP in fuzzy environment:-
Determine $x = (x_1, x_2, x_3, \ \ldots\ldots\ldots., x_n)$
which optimizes 'k' objectives.

$$F(x) = \{f_1(x), f_2(x), f_3(x), \ldots\ldots., f_k(x)\} \qquad (1.1)$$

subject to 'm' constraints:
$g_i(x) \le b_i, \quad i = 1, 2, 3 \ldots\ldots, m$
$x \ge 01$

Practically we cannot fix $x = (x_1, x_2, x_3, \ldots\ldots., x_n)$ which maximizes all the objectives $f_1(x), f_2(x), \ldots\ldots., f_3(x)$. in such situation DM has to accept compromise solution by setting goals to each of 'k' objectives. Thus the models (1.1) becomes as:

Multi-objective goal linear programming model:
Determine: $x = (x_1, x_2, x_3, \ldots\ldots., x_n)$
which satisfies: (1.2)
$f_i(x) \le p_i\ ; i = 1,2,3, \ldots, k$
$g_j(x) \le b_{j,}\ ; j = 1,2,3, \ldots, m$
$x \ge 0$

Here we have $'k + m'$ goals. The model (1.2) can be solved by goal programming algorithm. In goal programming model there is no control over deviation. Sometimes we find that one goal is more deviated and another goal is very less deviated. To overcome such difficulties, we shall use the concept of FST because rigid goals are always not realistic. Imprecise goals are accepted. DM's have to deal such situations by applying the concept of FST. The coefficients of constraints, objective function or right hand sides are imprecise or both are fuzzy numbers, then DM use fuzzy programming approach.

Fuzzy programming model:
The fuzzy model of (1.2) can be given as:
Determine: $x = (x_1, x_2, x_3, \dots\dots., x_n)$
which satisfies:

$$f_i(x) \underset{\sim}{\leq} p_i,; i = 1,2,3,\dots,k \qquad (1.3)$$

$$g_j(x) \underset{\sim}{\leq} b_{j,}; \ j = 1,2,3,\dots,m$$

$$x \geq 0$$

where '~' sign stands for fuzzification.

The present model (1.3) cannot be solvable. Hence there is need to convert the crisp version of model (1.3) with the help of linear membership function for each fuzzy goal and fuzzy objective. In productions, business, engineering and other fields DMs have to accept imprecise constraints as well as imprecise goals.

Here 'p_i' is replaced by '$p_i + \alpha_i$' and 'b_j' is replaced by '$b_j + \beta_j$'. Where 'α_i' and 'β_j' are tolerance limits for 'i^{th}' fuzzy goal $f_i(x) \underset{\sim}{\leq} p_i$, the linear membership function is defined as:

$$\mu_{f_i}(x) = \frac{(p_i + \alpha_i) - f_i(x)}{\alpha_i}$$

$i = 1,2,3, \dots.,..,..,k$

In similar manner for j^{th} fuzzy constraints $g_j(x) \underset{\sim}{\leq} b_j$, with membership function defined as:

$$\mu_{g_j}(x) = \frac{(b_j + \beta_j) - g_j(x)}{\beta_j}$$

$j = 1,2,3, \dots, \dots, \dots, m$

Using compensatory operator canonical model is given as:

Maximize

$$V(\mu) = (\mu f_1(x))^2 + (\mu f_2(x))^2 + \dots + (\mu f_k(x))^2 + (\mu g_1(x))^2 + (\mu g_2(x))^2 + \dots + (\mu g_m(x))^2$$

s.t. $\mu_{f_i}(x) = \frac{(p_i+\alpha_i)-f_i(x)}{\alpha_i}, i = 1,2,\dots,k$

$$\mu_{g_j}(x) = \frac{(b_j+\beta_j)-g_j(x)}{\beta_j}, j = 1,2,\dots,m \qquad (1.4)$$

$\mu f_i(x) \geq 0,; i = 1,2,3,\dots,k$
$\mu g_j(x) \geq 0,; j = 1,2,3,\dots,m$
$x \geq 0.$

The model (1.4) is solved by Lingo software. The achievement function $V(\mu)$ of (1.4) is replaced by $W(\mu)$ such that $w(\mu) = \sum_{i=1}^{k} \mu_{f_i}(x) + \sum_{j=1}^{m} \mu_{g_j}(x)$, then obtained new model is solved by the software. The solutions are compared for the choice of decision makers need.

Table 1. Data for Modeling

'n' Variables with values								
Variable name	x_1	x_2	x_3	x_4	.	.	.	x_n
Unit profit	p_1	p_2	p_3	p_4	.	.	.	p_n
Expected demand	d_1	d_2	d_3	d_4	.	.	.	d_n
Labour usages	l_1	l_2	l_3	l_4	.	.	.	l_n
Advertisement	a_1	a_2	a_3	a_4	.	.	.	a_n
Discounts	D_1	D_2	D_3	D_4	.	.	.	D_n

Packing charge	o_1	o_2	o_3	o_4	.	.	.	o_n

The model of the problem consists of 5 objectives such as:

$$\begin{aligned}
&\text{Max} \quad Z_1 = p_1x_1 + p_2x_2 + \cdots + \cdots + p_nx_n \\
&\text{Min} \quad Z_2 = l_1x_1 + l_2x_2 + \cdots + \cdots + l_nx_n \\
&\text{Min} \quad Z_3 = a_1x_1 + a_2x_2 + \cdots + \cdots + a_nx_n \\
&\text{Min} \quad Z_4 = D_1x_1 + D_2x_2 + \cdots + \cdots + D_nx_n \qquad (1.5) \\
&\text{Min} \quad Z_5 = o_1x_1 + o_2x_2 + \cdots + \cdots + o_nx_n
\end{aligned}$$

subject to constraints

$x_i \leq d_i, ; i = 1,2,3, \ldots, n$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, \ldots \ldots, x_n \geq 0$

Mathematical model of the problem:

Determine $x = (x_1, x_2, x_3, x_4, x_5)$ which,

$$\begin{aligned}
\text{Maximizes} \quad f_1(x) &= 50x_1 + 40x_2 + 20x_3 + 10x_4 + 5x_5 \\
f_2(x) &= -(10x_1 + 10x_2 + 10x_3 + 10x_4 + 10x_5) \\
f_3(x) &= -(2x_1 + 2x_2 + 2x_3 + 2x_4 + 2x_5) \\
f_4(x) &= -(0.5x_1 + 0.5x_2 + 0.5x_3 + 0.5x_4 + 0.5x_5) \\
f_5(x) &= -(x_1 + x_2 + x_3 + x_4 + x_5) \qquad (1.6)
\end{aligned}$$

Subject to constraints

$$\begin{aligned}
x_1 &\leq 5000 \\
x_2 &\leq 5000 \\
x_3 &\leq 5000 \\
x_4 &\leq 5000 \\
x_5 &\leq 5000 \\
x_1 \geq 0, x_2 &\geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0
\end{aligned}$$

Fuzzy goal programming model:

Determine $x = (x_1, x_2, x_3, x_4, x_5)$

Such that,

$$\begin{aligned}
50x_1 + 40x_2 + 20x_3 + 10x_4 + 5x_5 &\lesssim 26000 \\
10x_1 + 10x_2 + 10x_3 + 10x_4 + 10x_5 &\lesssim 4000 \\
2x_1 + 2x_2 + 2x_3 + 2x_4 + 2x_5 &\lesssim 3000 \\
0.5x_1 + 0.5x_2 + 0.5x_3 + 0.5x_4 + 0.5x_5 &\lesssim 500 \qquad (1.7) \\
x_1 + x_2 + x_3 + x_4 + x_5 &\lesssim 2000 \\
x_1 &\lesssim 5000 \\
x_2 &\lesssim 5000 \\
x_3 &\lesssim 5000 \\
x_4 &\lesssim 5000 \\
x_5 &\lesssim 5000
\end{aligned}$$

CRISP MODEL:

Compensatory model for this problem:

$$\text{Max} \quad (\mu_1)^2 + (\mu_2)^2 + (\mu_3)^2 + (\mu_4)^2 + (\mu_5)^2 + (\mu_6)^2 + (\mu_7)^2 + (\mu_8)^2 + (\mu_9)^2 + (\mu_{10})^2$$

Subject to constraints

$$\mu_1 = \frac{26000 - (50x_1 + 40x_2 + 20x_3 + 10x_4 + 5x_5)}{1000}$$

$$\mu_2 = \frac{4500 - (10x_1 + 10x_2 + 10x_3 + 10x_4 + 10x_5)}{500}$$

$$\mu_3 = \frac{3200 - (2x_1 + 2x_2 + 2x_3 + 2x_4 + 2x_5)}{200}$$
$$\mu_4 = \frac{600 - (0.5x_1 + 0.5x_2 + 0.5x_3 + 0.5x_4 + 0.5x_5)}{100}$$
$$\mu_5 = \frac{2000 - (x_1 + x_2 + x_3 + x_4 + x_5)}{200}$$
$$\mu_6 = \frac{5500 - x_1}{500}$$
$$\mu_7 = \frac{5500 - x_2}{500}$$
$$\mu_8 = \frac{5500 - x_3}{500}$$
$$\mu_9 = \frac{5500 - x_4}{500}$$
$$\mu_{10} = \frac{5500 - x_5}{500}$$
$$\mu_1 \text{ and } \mu_{10} > 0$$

RESULTS AND DISCUSSIONS

In this paper, five objectives are optimizing with the help of fuzzy logic. Each objective is related with food industries that help to optimize linearly as maximization or minimization of the system. The whole model is validated into two phases such as linear and quadratic model where quadratic model is efficient that linear model. The quadratic model optimizes the decision variables by applying some degree where linear model optimizes without any degree. The values of decision variables are changes during performance evaluation. Each decision variable is depends on the combine model of objectives functions with constraints. During iteration of the simulation model, constraints are controlled by membership function of fuzzy logic. Fuzzy logic helps to tune the current value as the optimum value by the help of optimset. The optimset is an artificial variable that helps to adjust the constraints of the objective functions. Finally, fuzzy multi-objective linear programming produce better results that linear multi-objective programming.

CONCLUSION

In this paper, fuzzy multi-objective linear programming approach has been proposed for solving problem of food industry. The basic aim of this approach is to solve uncertainty related information in the food industry by using different achievement functions. The proposed approach is the fusion of fuzzy logic and multi-objective optimization techniques. The multi-objective optimization technique is used to optimize multiple conflicting objectives that arise during food processing. Fuzzy logic is used to makes tolerant of each objective with the help membership function. The compensatory operator has been used in linear and quadratic form for achievement function of each objective. The proposed model has been fuzziness in term of objectives as well as constraints and the obtained solution provides the amount of each product in order to gain optimal value of the profit.

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