A Comprehensive Model for Examining Pre-Service Teachers' Knowledge of Technology Tools for Mathematical Learning: *The T-MATH Framework*

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Abstract
This paper proposes a comprehensive model for understanding the multiple dimensions of knowledge employed by pre-service elementary teachers' when they choose technology for teaching mathematics (Johnston & Moyer-Packenham, in press). The *T-MATH Framework (Teachers' Mathematics and Technology Holistic Framework)* integrates several frameworks, including TPACK (Mishra & Koehler, 2006), MKT (Ball, Thames, & Phelps, 2008), and technology evaluation criteria (Battey, Kafai, & Franke, 2005). This model, which can be used to examine the manner in which pre-service elementary teachers rank and evaluate technology tools for mathematical learning, suggests that there are multiple dimensions to understanding teachers' knowledge of technology for teaching mathematics. The paper reports recommendations for mathematics teacher educators and researchers.

Introduction

This paper posits an integrated model of teachers' *technology knowledge for teaching mathematics*. The model is based on the integration of the relevant literature on technology for teaching mathematics including: TPACK (Technological Pedagogical Content Knowledge) as it applies to the teaching and learning of mathematics (Mishra & Koehler, 2006; Niess, Suharwoto, Lee, & Sadri, 2006); the TPACK framework proposed by Mishra and Koehler (2007); the Domains of Mathematical Knowledge for Teaching (MKT) framework proposed by Ball, Thames, and Phelps (2008); and, the mathematics and technology evaluation criteria proposed by Battey, Kafai, and Franke (2005). By integrating these frameworks and evaluation criteria, this model can be used to investigate pre-service teachers' knowledge as they develop in their evaluation and use of technological tools for mathematics teaching.

Frameworks, Models and Constructs Used to Build the Teachers' Mathematics and Technology Holistic Framework (T-MATH Framework)

The literature includes several theoretical and graphical frameworks which are used to explain the relationships among technology knowledge, pedagogical knowledge, and mathematical content knowledge within the context of mathematics education. Elements in all of these frameworks are valuable in understanding the multi-dimensional nature of teachers' technology knowledge for teaching mathematics. For that reason, we have chosen to integrate these various models in a comprehensive model that is specific to the use of technology for mathematics teaching, which we call the *Teachers' Mathematics and Technology Holistic (T-MATH) Framework*.

The first framework used to develop the T-MATH Framework proposed in this paper is *technology, content, and pedagogical content knowledge,* or simply TPACK. TPACK for mathematics is defined as "the intersection of the knowledge of mathematics with the knowledge of technology and with the knowledge of teaching and learning" (Niess et al., 2006, p. 3750). Mishra and Koehler (2007) represent this intersection of knowledge as a Venn diagram, where each of three circles contains pedagogical knowledge, technological knowledge, and content knowledge, with each circle intersecting the others. At first glance,

one might assume that these three components are distinct entities, but Mishra and Koehler remind researchers that "TP[A]CK is different from knowledge of all three concepts individually" (2007, p. 8). The representation of concepts using the technology, based on pedagogical strategies for teaching the content, and an understanding of how to use the technology to develop those concepts in children demonstrates the complexity of the integrated nature of TPACK.

Early research on TPACK referred to "content" in general; Niess (2008) suggested identifying how TPACK can be expressed within mathematics education. Specifically, she identified various components of TPACK within mathematics education, namely the importance of teachers' knowledge of students, curriculum, and instructional strategies for teaching and learning mathematics with technology. Niess notes that TPACK requires teachers to consider "what the teacher knows and believes about the nature of mathematics, what is important for students to learn, and how technology supports learning mathematics" (p. 2-3). To support this view, Niess (2005) designed a course where pre-service teachers identified technology tools for mathematical learning as well corresponding mathematics and technology standards which could be supported by the technology tools. The results of this study suggest that the pre-service teachers meaningfully engaged in reflection while considering appropriate technology tools for mathematical learning. Although the TPACK model is an integrated model for pedagogical knowledge, technological knowledge, and content knowledge, the T-MATH Framework proposed in this paper goes beyond the TPACK model to make the mathematics in the TPACK model more explicit by aligning types of mathematical knowledge and fidelity with the elements in the TPACK model.

A second framework used to develop the T-MATH Framework proposed in this paper is the Domains of Mathematical Knowledge for Teaching (MKT) graphical framework described by Ball et al. (2008). In describing this framework they note that, based on their empirical results, "content knowledge for teaching is multidimensional" (p. 403). Within the context of their work, three of their six domains are important for the T-MATH Framework that we propose. These three domains are Common Content Knowledge (CCK; "the mathematical knowledge and skill used in settings other than teaching," p. 399), Specialized Content Knowledge (SCK; "the mathematical knowledge and skill unique to teaching," p. 400), and Knowledge of Content and Teaching (KCT; "combines knowing about teaching and knowing about mathematics," p. 401). These three domains of MKT are important for the T-MATH Framework because they provide much greater explication of the type of mathematical knowledge than that which is described in the TPACK framework (where mathematical knowledge is not described at this level of specificity). What the MKT domains bring to the framework is that there are different types of mathematical knowledge in interaction with the different elements of technological, pedagogical, and content knowledge proposed by Niess.

The third model used to develop the T-MATH Framework proposed in this paper is centered around the criteria proposed by Battey et al. (2005). In their study of pre-service elementary teachers, they identified four main criteria used by the teachers for evaluating mathematics software for use with students: software features, mathematics features, learning features, and motivation features. Further studies which used these four criteria among pre-service elementary teachers noted similar results. These studies found that pre-service teachers emphasized Software Features most often over all other criteria. This finding is important because it suggests that pre-service teachers should consider technology use in mathematics teaching situations as a *mathematical instrument*, not simply as a stand-alone tool. It further demonstrates the challenge that this type of integrative thinking poses for pre-service elementary teachers. In the T-MATH Framework, these criteria highlight a teachers' focus, and that focus reveals the complexity of a teachers' thinking on the use of technology

in mathematics teaching. For example, focusing on a motivation feature indicates less complexity because the teacher is considering pedagogy only, while focusing on a mathematics feature indicates more complexity because the teacher is simultaneously considering pedagogy, technology and the mathematical content.

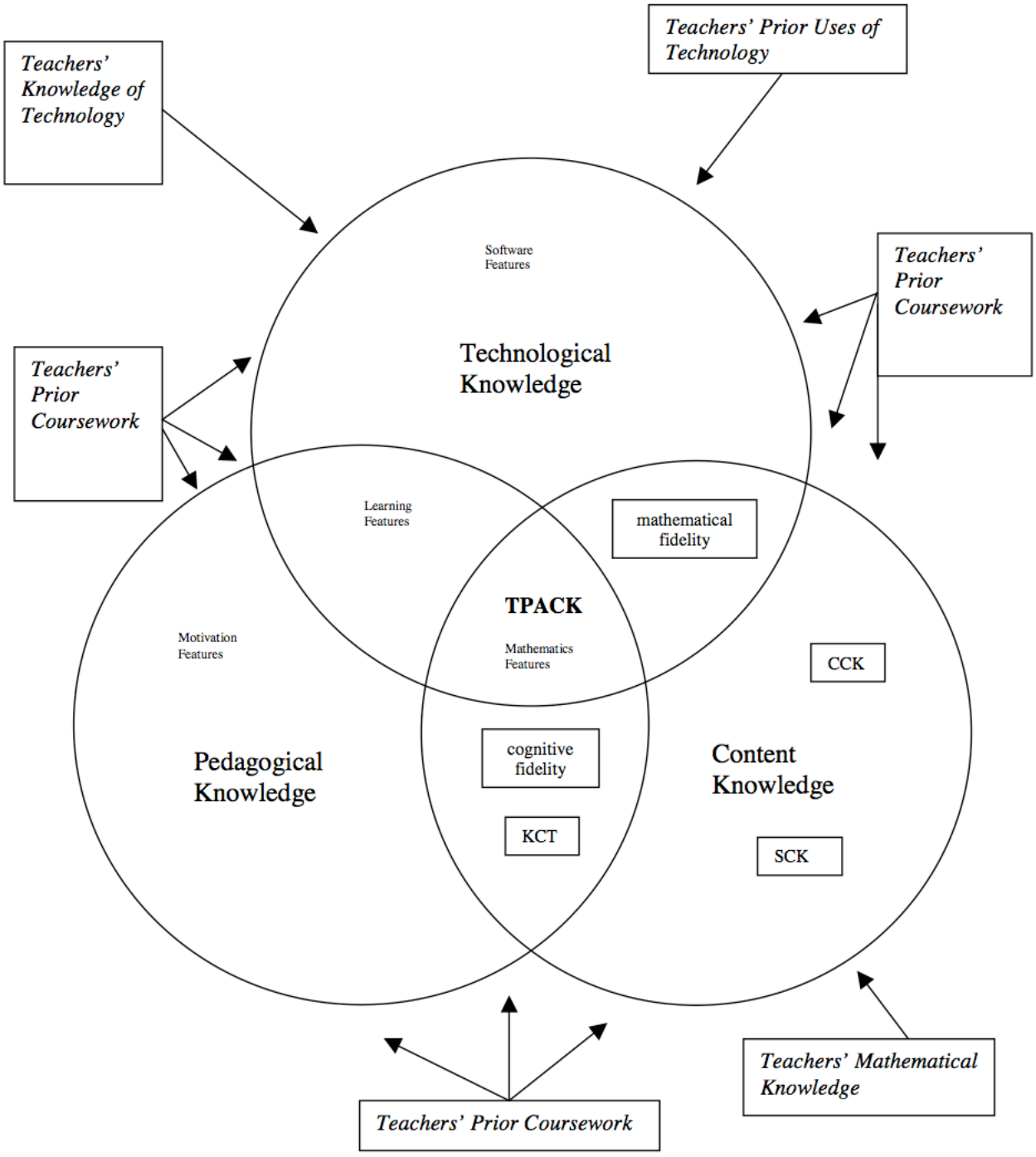
An additional construct used to develop the T-MATH Framework proposed in this paper is *fidelity*. If a tool has high *mathematical fidelity*, "the characteristics of a technology-generated external representation must be faithful to the underlying mathematical properties of that object" (Zbiek, Heid, Blume, & Dick, 2007, p. 1174). Thus, the technology should model procedures and structures of the mathematical system, and be mathematically accurate. A tool is considered to have high *cognitive fidelity* "if the external representations afforded by a cognitive tool are meant to provide a glimpse into the mental representations of the learner, then the cognitive fidelity of the tool reflects the faithfulness of the match between the two" (Zbiek et al., 2007, p. 1176). Thus, a tool which matches the thought processes and procedures of the user has high cognitive fidelity.

The Teachers' Mathematics & Technology Holistic Framework (T-MATH Framework)

The frameworks and constructs discussed in the previous section demonstrate the *integrated* nature of knowledge and the *complexity* of knowledge specifically as it is needed by teachers who want to use technology to teach mathematics effectively. Because each framework and construct informs different aspects of teachers learning to use technology for teaching mathematics, we propose a comprehensive model that takes into account elements of each framework in an integrated way which we call the *Technology Knowledge for Teaching Mathematics (T-MATH) Framework*.

This model in Figure 1 is a specific extension of TPACK, forming a model of teachers' knowledge of mathematical TPACK. The proposed model begins with the Mishra and Koehler TPACK framework (2007), which includes Technological Knowledge, Pedagogical Knowledge, and Content Knowledge in three circles in a Venn diagram. Next we map onto this framework Ball et al.'s (2008) three domains of knowledge including: Common Content Knowledge (Content Knowledge circle), Specialized Content Knowledge (Content Knowledge circle), and Knowledge of Content and Teaching (intersection of the Pedagogical and Content Knowledge circles). Finally we consider the constructs of mathematical fidelity (intersection of the Technological and Content Knowledge circles) and cognitive fidelity (intersection of the Pedagogical and Content Knowledge circles).

Figure 1. Teachers' Mathematics and Technology Holistic (T-MATH) Framework



Interpreting a Teacher's Location in the T-MATH Framework

In the T-MATH Framework, we propose that when prior researchers (Battey et al., 2005; Johnston, 2008; 2009) reported that pre-service elementary teachers focused on various features of teaching mathematics with technology (e.g., software, mathematics, learning, motivation), their focus on these features reflects important information about the dimensions of their technology knowledge for teaching mathematics. For example, when pre-service teachers focus their selection of technology tools for mathematics teaching on Software Features (which shows their Technological Knowledge) or Motivation Features (which shows their Pedagogical Knowledge), this is a "one dimensional" focus. That is, they are interested solely in an aspect of the technology tool that is not explicitly linked to students' learning or

the learning of mathematics. For example, identifying features such as "has clear directions" (Software Feature) or "is fun for students to use" (Motivation Feature) could apply to many different technologies or learning situations and does not consider how the features are related to teaching and learning mathematics concepts. On Figure 1, we have positioned Software Features and Motivation features in the Technological Knowledge and Pedagogical Knowledge circles, respectively. Pre-service teachers who focus on these features are exhibiting a singular focus and no intersection with other knowledge areas in the model, thus reflecting a less integrated knowledge.

Within the T-MATH Framework, we propose that when pre-service teachers focus their selection of technology tools for mathematics teaching on Learning Features (which shows their knowledge of how the technology is related to student learning), this is a "two dimensional" or integrated focus. That is, they are connecting features of the technology to students' learning with the technology. For example, identifying a feature such as "applicable to what we are learning in the classroom" connects the technology with the pedagogy of the classroom, considering the implications of both the technology and the pedagogy. On Figure 1, we have positioned Learning Features in the intersection of the two circles for Technological Knowledge and Pedagogical Knowledge. We propose that identifying a Learning Feature requires teachers to make a connection between technology and pedagogy, and thus reflects a more integrated type of knowledge with respect to learning and the use of technology. For example, when pre-service teachers consider the use of embedded buttons on an applet which allow the selection of "easy" or "difficult" mathematics problem items, this allows for learning differentiation afforded by the technology.

Finally, in the T-MATH Framework, we propose that when pre-service teachers focus their selection of technology tools for mathematics teaching on Mathematics Features, this is highly complex, representing a "multi-dimensional" focus and requiring highly specialized and integrated knowledge. On Figure 1, we have positioned Mathematics Features at the intersection of the three circles. We propose that identifying a Mathematics Feature requires teachers to make multiple connections among technology, pedagogy, mathematics (including CCK, KCT, and SCK), and mathematical and cognitive fidelity. Let us further examine the complexity of this placement. The circle of Content Knowledge itself, specific to our model, includes Common Content Knowledge and Specialized Content Knowledge (i.e., mathematical knowledge and skill; Ball et al., 2008). At the intersection of Technological and Content Knowledge is mathematical fidelity. The intersection of the Content Knowledge circle and the Pedagogical Knowledge circle integrates knowledge, and is reflective of Knowledge of Content and Teaching (i.e., combines knowing about teaching and knowing about mathematics; Ball et al., 2008) and cognitive fidelity. The intersection of the Content, Pedagogy, and Technology circles are at the highest levels of complexity because they integrate each of the types of knowledge discussed here, thereby forming the total package.

The positioning of Mathematics Features in the three-circle intersection indicates the complexity of this focus for teachers. We propose that when pre-service teachers focus on Mathematics Features, they must have a deep understanding of technology, pedagogy, *and* mathematics. Specifically, Mathematics Features can focus on three primary areas, namely: *Provides multiple representations of mathematical concepts; Links conceptual understanding with procedural knowledge*; and *Connects multiple mathematical concepts*. In addition, their mathematical knowledge can be the type of knowledge that is used in settings other than teaching (CCK), the mathematical knowledge unique to teaching (SCK), or the knowledge that combines teaching and knowing about mathematics (KCT) (Ball et al., 2008). The identification of a teacher located in the proposed framework and focused on the mathematics features would indicate much more complexity in that teacher's technology knowledge for teaching mathematics.

Conclusion

In this paper we have proposed the *Technology Knowledge for Teaching Mathematics (T-MATH) Framework*. This framework is a specific extension of TPACK that integrates MKT, fidelity (i.e., cognitive and mathematical), and criteria for evaluating technology (i.e., motivation, learning, software, and mathematics features). This framework demonstrates the complex, multi-dimensional nature of a teacher using technology to teach mathematics. It can be used to examine teachers' mathematics lessons or to observe mathematics instruction that integrates technology to determine the complexity of a teacher's knowledge in planning and teaching mathematics with technology. It can also be a framework for teaching pre-service teachers to design technology experiences for their K-12 students that integrate technology in mathematics teaching. Our hope is that this framework illuminates that complexity, and by making these complex elements explicit, helps to focus on the critical elements necessary for consideration and integration when teaching mathematics with technology.

References

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*, 389-407.

Battey, D., Kafai, Y., & Franke, M. (2005). Evaluation of mathematical inquiry in commercial rational number software. In C. Vrasidas & G. Glass (Eds.), *Preparing teachers to teach with technology* (pp. 241-256). Greenwich, CT: Information Age Publishing.

Johnston, C. J. (2009). Pre-service elementary teachers planning for mathematics instruction: The role and evaluation of technology tools and their influence on lesson design. Unpublished doctoral dissertation, George Mason University.

Johnston, C. J. (2008). Pre-service teachers' criteria for evaluating technology tools for mathematics learning. Unpublished doctoral pilot study, George Mason University.

Johnston, C. J. and Moyer-Packenham, P. S. (In press). A model for examining the criteria used by pre-service elementary teachers in their evaluation of technology for mathematics teaching. In R. Ronau, C. Rakes, and M. Niess (Eds.), *Educational technology, teacher knowledge, and classroom impact: A research handbook on frameworks and approaches*. Hershey, PA: IGI Global.

Mishra, P., & Koehler, M. (2007). Technological pedagogical content knowledge (TPCK): Confronting the wicked problems of teaching with technology. In C. Crawford et al. (Eds.), *Proceedings of Society for Information Technology and Teacher Education International Conference 2007* (pp. 2214-2226). Chesapeake, VA: Association for the Advancement of Computing in Education.

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record, 108*, 1017-1054.

Niess, M. L. (2008, June). Developing Teachers' Technological Pedagogical Content Knowledge (TPCK) with Spreadsheets. Paper presented at the annual meeting of the International Society for Technology in Education, National Educational Computing Conference, San Antonio, TX.

Niess, M. L., Suharwoto, G., Lee, K., & Sadri, P. (2006, April). Guiding inservice mathematics teachers in developing TPCK. Paper presented at the annual meeting of the American Education Research Association, San Francisco, CA.

Zbiek, R., Heid, M. K., Blume, G. W., & Dick, T. P. (2007). Research on technology in mathematics education: A perspective of constructs. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 1169-1206). Reston, VA: National Council of Teachers of Mathematics.