Using A Computer Pen to Investigate Students' Use of Metacognition during Mathematical Problem-Solving

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Abstract

The major aim of this paper is to present the computerized Smartpen as a tool for capturing and exploring students' metacognitive processes as they solve mathematical problems. After sharing their thinking through self-talk or group-talk students worked with others to share their strategies and reflect upon the process. An additional aim for this paper is to share any generalizations that may be helpful for teachers who are helping students strengthen their metacognitive and mathematical problem-solving skills. In this study the Smartpen was used to listen in on undergraduate preservice teachers' problem solving as they explored the Problem of Points, a classic probability problem. Capturing each stroke of the pen as the students wrote while simultaneously engaged in self-/group-talk, each Smartpen session produced a pdf document and accompanying audio for further analysis. Problem solvers who lacked confidence in their problem-solving ability were more reserved in their self-talk and often solved problems unrecorded in advance to appear more successful later during the recorded problem-solving session. It would appear that direct, real-time monitoring by the teacher is needed to capture significant information about students' metacognitive reasoning.

Introduction

The more successful mathematical problem solvers are those who think more deeply *during* the problem-solving process and *after* they have achieved a solution (Pugalee, 2001; Teong, 2003). The term metacognition best characterizes this phenomenon (Legg & Locker Jr., 2009). When teachers attempt to help students become better problem-solvers by employing questioning strategies to tease out critical steps in their thinking verbal or written communication is often a barrier. Students find it difficult to articulate what they were thinking previously when attempting to solve a problem (Hennessey, 2003). The major aims of this paper are to present the Smartpen as a tool for capturing and exploring students' metacognitive processes as they solve mathematical problems and to share generalizations about students' mathematical processing of a classic probability problem known as the Problem of Points.

A computer Smartpen (hereafter simply referred to as a Smartpen) may be useful in helping students to think out-loud or engage in self-talk while they solve problems. The Smartpen stores the written strokes—the image of a student's writing in real-time—and any oral communication that occurs simultaneously during the recording session. Consequently, the communication barrier between the student and the teacher, or between the student and him/herself could become less of a problem. When a student's written work in solving a mathematical problem is completed while using a Smartpen, his/her metacognitive work can receive a boost as the student listens in on any self-talk or group-talk, monitors the solution as it unfolds, and reflects on the process to determine decisions that should be made. Teachers can listen, make observations, note strengths, and determine weaknesses to support intervention as needed.

This paper will share highlights of specific phenomena associated with problem solving for which the Smartpen may assist in monitoring and improving students' metacognitive and mathematical problem solving skills. Specifically we will share a brief introduction to metacognition and problem-solving then move to the context of the classic probability problem—the Problem of Points (Berlinghoff & Gouvêa, 2004). We will also share our findings and generalizations from the investigation of the problem-solving attempts of preservice teachers who worked on the Problem of Points.

Metacognition and Problem Solving: In A Nutshell

The process of thinking about one's own thinking takes place internally—in one's mind—yet the results should also be manifested externally to lead to productive problem solving and to support sharpening of this skill through personal reflection and external intervention by teachers, peers, and learned experiences (Hacker, 2009; Lee, Teo, & Bergin, 2009; Legg & Locker Jr., 2009; Ponnusamy, 2009; Pugalee, 2001; Steif, Lobue, Kara, & Fay, 2010). As early researchers highlighted the importance of metacognition for student learning four key components of the process were identified: (a) verbal reports as data (telling what you know); (b) executive control (directing/managing what you know); (c) self-regulation (monitoring and making decisions about how you will use what you know); and (d) other regulation (internalizing feedback from interactions with others and external experiences) (Brown, Bransford, Ferrara & Campione, 1983 as cited in McKeown & Beck, 2009).

Metacognition has actually been identified as an essential skill in reading and in problem solving (Lee et al., 2009). Problem solving is one of the mathematical process standards of the National Council of Teachers of Mathematics (2000) and an essential component of the Standards of Mathematical Practice of the new Common Core State Standards (2010) in the United States of America. Stronger problem solvers "start by explaining to themselves the meaning of a problem…analyze givens, constraints, relationships, and goals… make conjectures…monitor and evaluate their progress and change course if necessary" (Common Core State Standards Initiative, 2010, p. 6).

Students are often taught specific strategies for problem-solving such as Polya's four-step problem solving model: understand the problem; devise a plan; implement the plan; and looking back (Taylor & McDonald, 2006). However, actual next-steps in the problem-solving process are not easily tracked when students are solving complex problems. Thus the role of metacognition along with the associated skill of communication (NCTM, 2000) can be particularly helpful. Artz and Armour-Thomas (1992) recognized this connection between problem-solving, metacognition, and communication (e.g., thinking out-loud) when they suggested an eight-step problem solving process (with the predominant cognitive levels shown in parentheses): "read (cognitive); understand (metacognitive); analyze (metacognitive); explore (cognitive or metacognitive); plan (metacognitive); implement (cognitive or metacognitive); verify (cognitive or metacognitive); and watch and listen" (p. 142).

Probability and the Problem of Points

One of the main implications for instruction in probability is that students must be engaged in activity-based, experimental investigations to support their conceptual understanding. The prevalence of misconceptions with regards to probability and the persistence of those misconceptions despite traditional instruction are well-documented (Fischbein & Schnarch, 1997; Khazanov, 2005; Polaki, Lefoka, & Jones, 2000; Shaughnessy, 1992). Metacognitive processes, combined with effective instruction, should help to increase students' awareness and understanding of their misconceptions as well as help to identify ways to overcome them (Fiscbhein & Schnarch, 1997).

Using the probabilistic thinking framework developed by Polaki, Lefoka, and Jones (2000) one might investigate students' understanding of probability with regards to five major constructs (sample space, probability of an event, probability comparisons, conditional probability, and independence) at four different levels, numbered from 1 to 4 (subjective, transitional, informal quantitative, and numeric), respectively. Students operating at the lower level (Level 1) have difficulty appropriately operating in probability situations with anything more than subjective judgments. However, on the high end of the scale (Level 4) where deeper metacognitive thought is applicable, students are able to identify a sample space, predict and assign numerical values for the probability of events, and engage in numerically-supported probability comparisons and generalizations with conditional as well as independent events. The Problem of Points was specifically chosen to engage students in this higher end of the framework.

Subjects
During the spring semesters of 2010 and 2011, convenient samples of preservice teachers preparing to teach children of ages 9-14, consented to participate in this study. The preservice teachers were enrolled in a combined mathematics history-technology course in a medium-sized university (approximately 16,000 students) in the Midwest (USA). The group included 42 students in their second to fourth year of undergraduate study: 23 in spring of 2010 (22 females, 1 male) and 19 in spring of 2011 (14 females, 5 males).

Method
During a four-week period of 12 hours of in-class instruction, with additional instructional assistance provided in an online learning environment, supported by Smartpen pencasts, students explored various probability problems in various multicultural and historical contexts. In 2010 a Pulse™ Smartpen was only distributed to groups of students when engaged in hands-on probability activities in class. However, the need to be better acquainted with the technology dictated distribution of a Smartpen to each student for at least eight weeks (including four weeks prior to the study). Practice engaging in self-talk while writing and doing mathematical problem solving was suggested in specific instructions to students to communicate their inner thoughts as if tutoring a peer or sharing their procedures with the classroom teacher.

The Problem of Points was posed in class (as shared in brief below).

> Our story begins around 1654 as the Chevalier de Méré, a wealthy gambler, requested that Blaise Pascal provide a mathematical validation for a specific dice problem. As Pascal collaborated with Fermat on the solution the study of probability as a field of mathematics began! … Actually there is some confusion about the actual dice problem that the Chevalier first proposed, so the problem we will explore is a simple version of one of his problems that became known as the **Problem of Points**.
>
> > Xavier and Yvon staked $10 each on a coin-tossing game. Each player tosses the coin in turn. If it lands heads up, the player tossing the coin gets a point; if not, the other player gets a point. The first player to get three points wins the $20. Now suppose the game has to be called off when Xavier has 2 points, Yvon has 1 point, and Xavier is about to toss the coin. What is a fair way to divide the $20? (Berlinghoff & Gouvêa, 2004, p. 207)

After giving some initial thought to understanding the problem [while engaging in self-talk] and determining a plan for a possible solution you will join a team of four to provide solutions to Question 1 taken directly from of our course text.

a. Let's call the interrupted-game case described there *(X2, Y1),* meaning that Xavier has two points and Yvon has one. How many other cases are there? What are they?
b. Analyze the rest of the cases from part (a). (Hint: Only two others are "interesting;" you might begin by disposing of the rest quickly.)
c. State your answers for part (b) as fractions or percentages that apply to any amount of money at stake.
d. Suppose the game required 4 points to win. What unfinished case is analogous to the *(X2, Y1)* case of the 3-point game? Generalize your answer to describe cases that result in a $\frac{3}{4}$-to-$\frac{1}{4}$ division of stakes for an unfinished *n*-point game.

... Berlinghoff & Gouvêa (2004, p. 213)

After approximately 10 minutes, students moved into groups to share their individual perspectives about the problem and become informed or inform others about a possible solution. The instructor monitored the groups as they engaged in group-talk, with one member of the group serving as a recorder while writing notes using the Smartpen. Once consensus was reached each group member asked questions of the group as needed to build his/her confidence in sharing the solution to another group or to the entire class.

All pencasts (i.e., the oral recordings and the accompanying pdf documents) were uploaded to a special class email address for analysis by the instructor. A mixed qualitative-quantitative design was used to analyze the data gathered. A comparison of the means of the pre-/post-assessment on probability was conducted using a *t*-test (in 2010, but a post-assessment was not given in 2011). Constant comparative and content analysis qualitative methods were used to investigate the categories of responses regarding metacognitive thought and the alignment of comments with the probability framework. The findings are shared in brief here.

Results

Self-talk sessions indicated that additional targeted training would be needed to help students truly share the details of their thoughts as they solved a problem. Most students appeared to lack confidence in sharing a "work in progress," and preferred to share their more polished results in a recording *after* attempting to solve the problem one or more times without the pen. Students with better than average scores on the pre-assessment of probability content knowledge were more likely to share more details of their thinking process or engage in the verbal reports of telling with such statements as "What I did was...I assumed that… The approach I'm taking is… I came up with …I thought there were 3 coin tosses at first rather than 3 [points] to win. That's the way I understood it. … I always looked at what the next toss would be." However, students who shared such comments often engaged in self-talk much more than they wrote. (See Figure 1.) For example, one student shared her metacognitive thoughts quite eloquently for almost 6 minutes before writing anything with the pen!

Group conversations featured much better sharing of metacognitive thoughts as students explained their positions, questioned the thoughts of others, and contributed to the solution. Since each student had expectations for later sharing with the class, each student clearly wanted to be prepared and often sought to confirm their understanding of the problem and the solution more than once. For example, one student who shared her thinking quite well with the group often asked at interim steps: "Do you understand that?" Another student, who often responded affirmatively to those questions as she indicated that she understood, waited until the last step of the solution and asked again, "Is it [i.e., the goal] the first to get to 3, or is it three (3) rolls?"

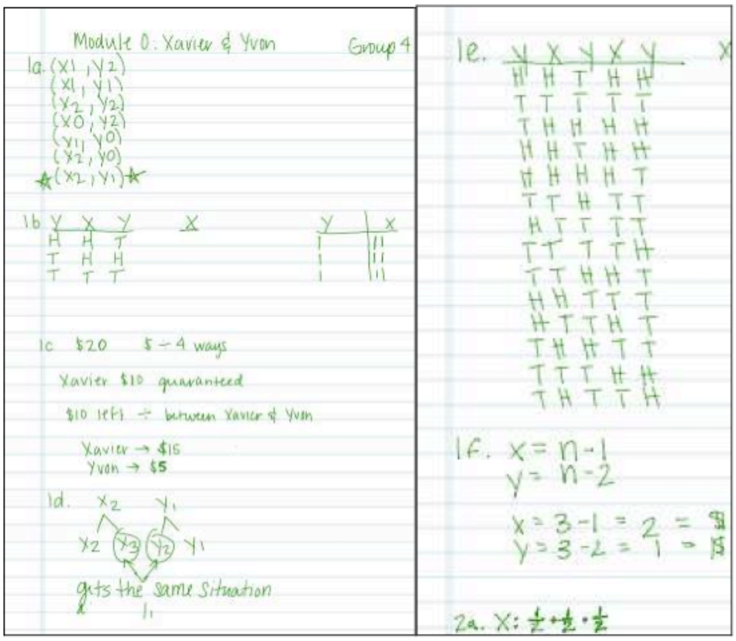

Figure 1. Two-page copy of pdf document resulting from Group 4's pencast.

Conclusion

It appears that using a Smartpen to support students' metacognitive thought while engaging in mathematical problem-solving may increase their awareness of metacognitive processes, but will do less to share it with others. Many of the students we encountered were often too self-conscious to share their unpolished thoughts. Perhaps they believe that the classroom teacher or the high achievers in class do not have moments of uncertainty during their attempts to solve mathematical problems. Perhaps we should be sure to have students share portions of their decision making process as they report their solutions to class. Of the four historical roots of metacognition, we believe we approached three: the verbal reports of telling, a hint of self-regulation, and metacognition that comes from interacting with other sources and experiences. However, most difficult to capture—and perhaps a source of greatest insight for helping novice problem solvers—was the executive control (directing/managing thoughts). Our informal experiences with classroom teachers, undergraduates, and high school students seems to indicate support our findings in this study: a pattern of solving problems "off the record" and sharing only best attempts with others. Most of the decisions about what to do and think next is still happening in the minds of students—virtually uncaptured, for the most part—because of fear of imperfection or appearing cognitively disheveled. We would like to suggest that students need many more experiences working in groups to share and defend solutions (through each step of the problem-solving process) to increase the awareness of the many moments of uncertainty during a problem-solving process that may eventually lead to a beautiful solution to a problem. To tap into the metacognitive thought that occurs throughout the problem-solving process we would suggest teachers or researchers sit and observe/question students while they are solving a problem and students should be encouraged to prepare to share a solution—along with accompanying decisions (executive control).

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