

**Physicists use mathematics to describe physical principles and
mathematicians use physical phenomena to illustrate mathematical formula –
Do they really mean the same?**

Ulrike Böhm¹, Gesche Pospiech², Hermann Körndle³, Susanne Narciss⁴

¹*Center for Teacher Education and Research, Technische Universität Dresden, Germany*

Ulrike.Boehm@mailbox.tu-dresden.de

²*Department of Physics, Technische Universität Dresden, Germany*

Gesche.Pospiech@tu-dresden.de

^{3,4}*Department of Psychology, Technische Universität Dresden, Germany*

Hermann.Koerndle@tu-dresden.de, Susanne.Narciss@tu-dresden.de

Abstract:

Physics and mathematics educators often find themselves in an absurd situation. Students have problems in thinking in an abstract way. In order to help them to overcome these problems mathematical formulae in math classes are often illustrated with physical phenomena which are introduced as authentic applications of mathematics in every day life. However, students in physics classes have also difficulties in gaining a deep understanding of physics. They try to overcome their lack of understanding by memorising facts or using formula (without understanding their meaning).

Given these problems in abstract thinking and understanding of physical phenomena the issue arises what might be the potential confusions regarding a physical phenomenon after having experienced mathematics and physics lessons on this topic? To address this issue we chose an example of geometrical optics – the mirror image.

We generate and evaluate a heuristic framework for describing and exploring the process of understanding a physical phenomenon. This heuristic framework differentiates several scientific models (e.g., physical, mathematical) which are necessary for understanding and explaining the phenomenon. Furthermore, it integrates these models in a multiperspective instructional model (i.e. didacticised model). Using this heuristic framework we analysed the problems in understanding which occur when students have to understand the mirror image.

Introduction

The mirror image is one of the phenomena most studied and most misunderstood in early physics education. Results of numerous studies (e.g. Blumör and Wiesner (1992a), Galili, Bendall, and Goldberg (1993), Jung (1981), La Rosa, Mayer, Patrizi, and Vicentini-Missoni (1984), Gropengießer(1997)) show that the understanding of the mirror image phenomenon is quite unsatisfactory. What is the problem?

At first we have to consider that the misunderstandings are not to be found in physics – but the learners think, that they are not able to understand a physical phenomenon! To explain the mirror image an optical and a non-optical (human) argumentation is needed – and have to be linked to each other. But both sides of this medal cause problems. Already early researchers like AlHazen and Euler said that for the explanation of the mirror image both different kinds of modelling are needed. The first one – the physical argumentation - means the explanation of the mirror image with the help of geometrical optics, and the second one - the non-physical argumentation - is our human interaction with light- the interpretation of the picture on the retina through our brain.

In this area of conflict one answer to the question for problems with the mirror image can be identified. The non-optical argumentation plays a marginal role in school lessons. But there is still another point of view – the mathematical modelling of the mirror image has an important influence on the understanding of the mirror image. This important role is being discussed in this paper.

The starting point is the question: “How do the pupils understand a physical phenomenon?”. We have to look at the process of constructing mental models of a phenomenon. In order to understand how students handle mathematical and physical

knowledge it is necessary to examine their modelling process in physics in a more detailed way.

Understanding physical models

According to Stachowiak's theory of modelling (1973) nobody can describe the real world – only a model of the real world. In particular novices have problems to understand science because they do not understand that teachers are talking about models of the reality and not about reality itself. This is one of the most important problems in modern science teaching – physical theories should be taught in a way acknowledging that these theories are models of the real world. In shaping this processes science teachers have to pay attention on insights in how students acquire scientific concepts (i.e. epistemic processes). The process of constructing mental models during the acquisition of physical knowledge plays an important role in understanding physical phenomena.

To describe and explore the process of understanding physical phenomena through science instruction we developed a heuristic framework which differentiates several scientific models (e.g., physical, mathematical). We assume that in order to help students understand a physical phenomon these different models have to be explained and integrated in science teaching. That is teachers have to develop a didacticised model of the phenomenon which addresses the various scientific models. Hence, the heuristic framework contains five main parts: (1) the phenomenon, (2) the scientific models necessary for explaining and understanding the phenomenon, (3) the teacher, (4) the didacticised model of the phenomenon and (5) the learner which interacts with both, the phenomenon and the didacticised model.

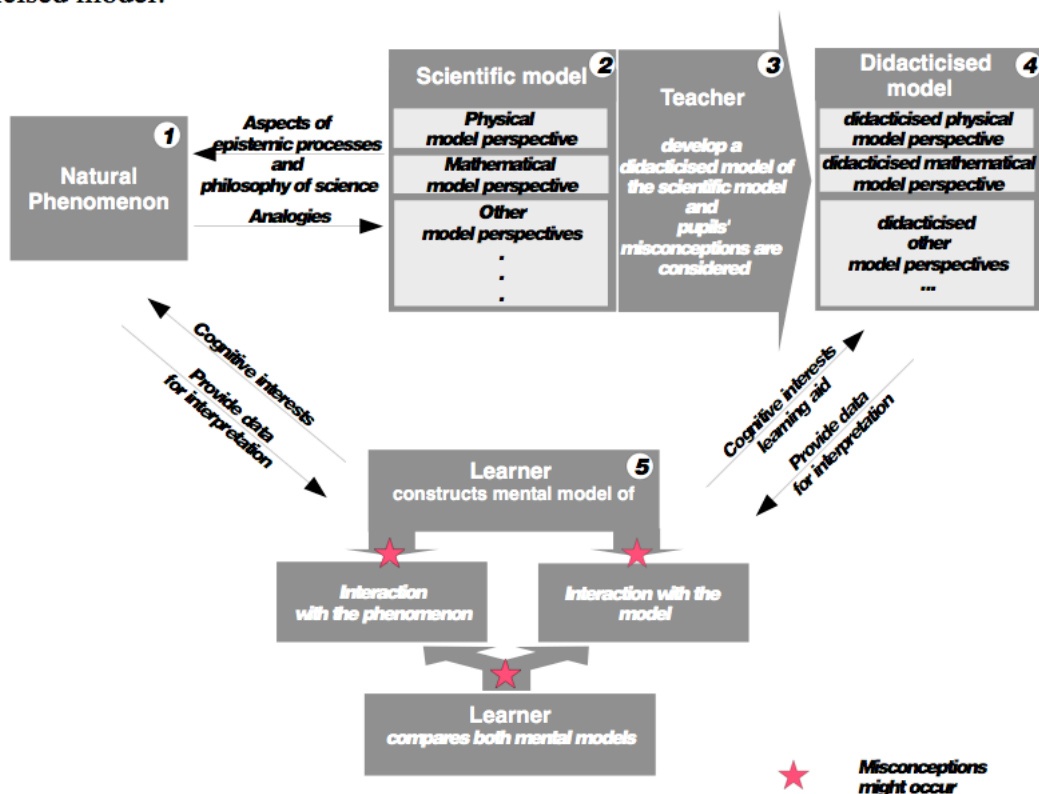


FIGURE 1: Heuristic framework for describing and exploring the process of understanding a physical phenomenon through science instruction (Böhm, Pospiech, Kördle, & Narciss, 2010b, p. 148)

According to this framework, in understanding a physical phenomenon the learner has two models to handle with: (1) the own mental model and (2) the didacticised model of this phenomenon. Hence, in science education a learner is not approaching a physical phenomenon in the way researchers do it. Researchers develop a scientific model, which

describes in the phenomenon as detailed as possible. This scientific model is then examined by experiments and by applying it to the real world.

Learners are mostly taught a didacticised model which a teacher developed especially for the teaching and learning process. The challenging task of the learners is to combine these models with their own models of the phenomenon. This process includes a lot of interactions and is the main cause for misunderstandings in the learning process.

The new idea is to divide the scientific model into different parts of the model according to different science areas. Only all model parts together can explain the phenomenon correctly. If only some (not all) model perspectives are used, the learner is not able to understand the phenomenon in a correct way. With this model it is also possible to discuss the role of the mathematical model perspective in the process of understanding natural phenomena. According to Greca and Moreira (2002) the model of a natural phenomenon is divided into two model perspectives: (1) the physical and (2) the mathematical model perspective. For Greca and Moreira the physical model of a theory is described with linguistic symbols and the mathematical model is described with mathematical symbols; understanding physics in school is achieved if it is possible to predict a physical phenomenon from its physical models. To understand complex physical phenomena (like the mirror image) other perspectives besides the physical and mathematical perspective of modelling are necessary for understanding. The three model perspectives are shown in Figure 2.

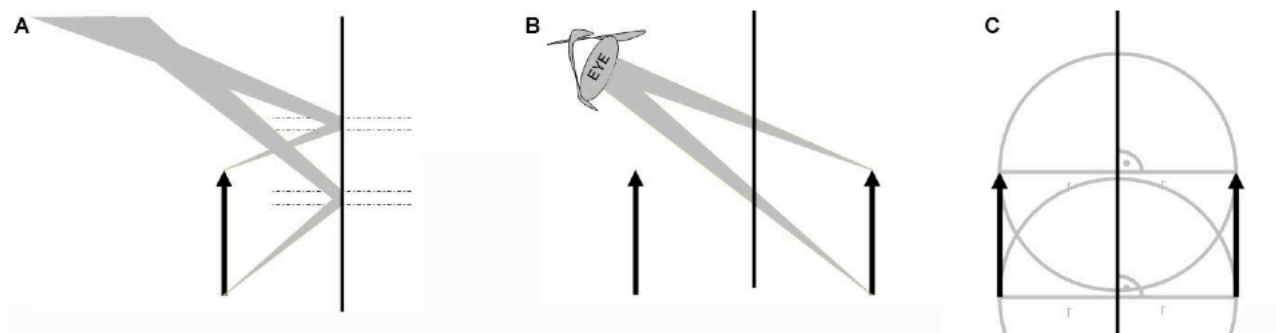


FIGURE 2: (A) physical model (B) “human” model, (C) mathematical model

Understanding mathematical models

The way of thinking in mathematics is totally different to the one in physics. Devlin (1994) describes the core of mathematics in recognizing a pattern. We define abstract objects and look for patterns. At this stage there is no connection to the real world, math is an abstract world. In mathematics abstract definitions and logical consequences of the definition are learned. Mathematicians formulate theorems and find arguments to prove them. Everybody can follow the strict logical rules (if he really wants to) in mathematics. Mathematics is abstract thinking without being linked to the real world.

This, however, does not hold true if mathematics is applied to real situations. This case is described by the modelling circle: (1) mathematical modelling of a physical system, (2) mathematical processing, (3) interpreting the mathematical representation and (4) evaluating the solution by comparing the physical system and the original system.

But we can also adapt the heuristic framework for describing and exploring the process of understanding (see Figure 1). Normally abstract problems have to be illustrated by the teacher, so that the learner wants to solve a given problem - the learner should develop a cognitive interest. Depicting a line reflection e.g. is motivated by folding tasks or using a mirror (see Figure 3). An often used mathematical explanation of the mirror symmetry in beginners' lesson is: “A way to think about it is: if the shape of a figure were to be folded in half over the axis, the two halves would be identical: the two halves are each other's mirror image.” However, if we think physically – the mirror image comes into being only in the mind of the observer. What does it then mean when we talk about ‘each other's mirror

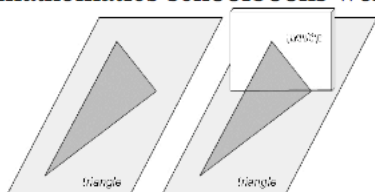
image'? Mathematical knowledge itself, however, is not gained from the illustration of mathematical contents, but from a mathematical discourse.

Problems occurring when linking mathematical with physical models and the impact of prior knowledge

Normally learners have mathematics from the beginning of their school career and only some years later science teaching starts. Thus the learners have a lot of mathematical previous knowledge, which they can use in the physics lessons. How can we succeed in cross linking previous knowledge in Mathematics with new ways of physical thinking? The model of a light beam is used e.g. in beginners' classes to represent optical paths. The model is a single light ray – a half-line in geometry. In contrast to mathematics, models in physics are essentially needed to gain physical knowledge about reality.

The first step to solve this was to understand how students handle their previous knowledge in Mathematics when they start with lessons in Physics. We carried out an investigation to elaborate this process (Böhm, Pospiech, Körndle, & Narciss, 2010). During the process of evaluation we found two very interesting examples of fundamental problems by using the axial symmetry for modelling the mirror image: (1) understanding of the virtual image and (2) left and right conversion of the mirror image.

At first mathematics schoolbooks were analysed with respect to their definition of symmetry.



A figure is symmetrical if one half is the mirror image of the other half. That is why the axis of symmetry of a figure is called mirror axis.

FIGURE 3: The definition of axial symmetry by using the mirror image

This example does not only contain abstract mathematical content to define axial symmetry. It is not an exact mathematical definition like: “A plane figure is symmetrical if it has at least one identical image which is mapped on itself by e.g. line reflection and rotation.

In using the mirror image to define axial symmetry we forget that the mirror image is not existing – it is ‘only’ a stimulus on our retina and its interpretation through the brain. The mirror image is not really existing like the other image in line-reflection. But this is not mentioned in math lessons. Thus physics teachers should not be surprised that students have problems to understand the virtual image when axial symmetry for modelling the mirror image is used – without mentioning the eye as a mapping system.

The second problem is, that not in every case we are able to see the mirror image. In Figure 4 two cases are demonstrated: Only cases comparable to Figure 4, picture A, looking diagonally onto the mirror we can see an image. In case Figure 4, picture B, if we looking perpendicular at the axis of the mirror we can not see an image – but mostly exactly this case is used in physics lesson to model the mirror image by using the former knowledge of the learners from the math classes – the axial symmetry.

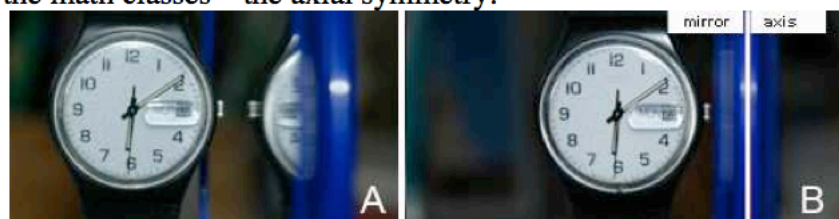


FIGURE 4: The definition of axial symmetry by using the mirror image

It is absolutely strange that for modelling the mirror image the only case of looking on the mirror is taken in which we can not see an image – and this is not told to the learners! When case B in Figure 4 is used pictures like the one in Figure 5 (in physics lessons) can be drawn.

In both drawings the eye does not play any role at all. But in this case, the mirror image does not exist – it only exists in the mathematical case – in line-reflection.

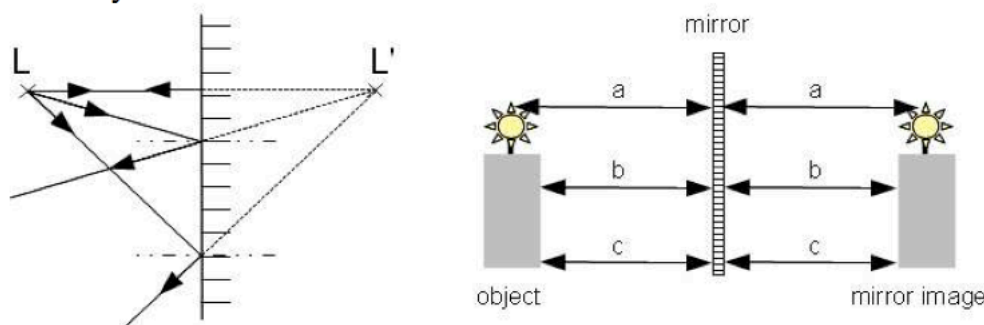


FIGURE 5: Drawings taken from physics text books to explain the mirror image.

If we look perpendicular at the drawing, we see that the left and right sides of the object and the mirror image are changed. But the mirror does not change left and right, the mirror changes front and back! This is easy to accept, if we know that we model the only case when no mirror image can be seen. We have to turn our head and look into the mirror – so the change of left and right becomes a change of front and back.

Conclusions

Every time models are used in physics education. The fact has to be taken in account that not reality itself, only a model of reality is being described. The model should fit the reality very well. We must pay attention to the role the used model has in its original meaning. On the other hand, if we are using examples to illustrate abstract structures in mathematics education we think about the physical understanding of the real phenomenon. In the case of the mirror image mathematical and physical explanations do not go hand in hand. The learner has no chance to construct adequate mental models in each subject.

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